Ball Lightning as a Hot, Highly Charged, Sphere of Air

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Abstract.

An exceptionally simple model of ball lightning is proposed that describes it as a highly charged sphere of hot, conductive, air surrounded by colder air. This conductive sphere possesses a net excess of charge. This charge will create a corona discharge that will heat the surrounding air thus maintaining the temperature of the ball itself. Numerical simulations are presented which give results that would appear to agree with what many witnesses have reported. It is argued that such spheres are likely positively charged.

1. Introduction.

Ball lightning is a peculiar atmospheric phenomenon usually, but not always, associated with lightning strikes (Uman 1969; Rakov et. al. 2003). Balls of light are observed to move about, being smaller than a golf ball or even several meters in diameter. Their color is often red, orange, or white. They may last anywhere from several seconds to several minutes but 5 sec seems typical (Rakov et. al. 2003). Their demise is often violent as they frequently explode (either spontaneously or as a result of contact with objects). Many theories have been put forward to explain these strange observations. Some are chemical in nature; burning silicon has been suggested (Abrahamson et. al. 2000). Cen et. al. (2014) recently captured what may well be the spectrum of a ball lightning produced by a cloud-to-ground strike. Lines for silicon, iron, and calcium were observed in addition to lines for neutral atomic nitrogen and oxygen. These latter are not surprising and suggest that the phenomenon is significantly cooler than the parent lightning channel, in the spectrum of which were observed lines for ionized nitrogen and oxygen. The other elements must surely have come from the soil. But it is hard to say whether they played any active role in the ball lightning process. Kapitza (1955) proposed a a model in which microwave radiation is trapped within a spherical cavity. Other authors have extended this idea (Wu 2016). Ranada (2000) has pictured it as a force-free, topologically entangled, magnetic knot (also see (Tsui 2003)). Different suggestions include black holes (Rabinowitz 2002) and Rydberg matter (Manykin et. al. 2006). Morrow (2018) envisions a sphere of positive ions that attracts burning, negatively charged, particles into it. These account for the ball's luminosity.

The present model concentrates on the thermodynamics of a sphere of very hot air at 1 atm. This problem was examined by Lowke et. al. (1969) who found that such spheres would probably not support their temperature or luminosity long enough to be viable candidates for ball lightning. Tesla (1899) had speculated that a small sphere of hot, diaphanous, air might conduct charge from sky to ground, thus maintaining its temperature. Uman et. al. examined a similar idea (1966).

There is considerable evidence to suggest that these lightning balls may be charged (Charman 1972). They are sometimes observed to move towards or along conducting surfaces. Very significantly, they have been described as giving off filamentous or corona discharges as well as hissing sounds. Ozone and acrid smells associated with such discharges are also reported. Several people, for instance Georg Richmann (Clarke 1983), seem to have been electrocuted by the phenomenon. We will take this evidence seriously and propose a model of ball lightning that pictures it as a sphere of very hot, conductive, air bearing an intense charge. The energy generated by the dissipation of this charge will serve to keep the inside of the ball hot.

2. A Model of Ball Lightning.

When a lightning strike comes down it will certainly create quite a bit of highly heated and partially ionized air owing to the tremendous energies involved. Suppose that the hot and conductive mass of air produced by this process manifests an imbalance in its contained charge. (We will discuss further how such a situation might come about below). It is not clear, right now, whether the excess charge should be regarded as positive or negative. We imagine this plasma would start off with a temperature on the order of at least $\sim 4,000\ K$. (Note that we are not suggesting that the ball is, literally, formed out of the lightning channel itself. It is, rather, a consequence of the channel's striking down.) Ordinarily, such a mass of heated air would be expected to cool in a very short time (Lowke et. al. 1969). We propose that, because it is highly charged, it remains heated under the influence of its own corona discharge. We will examine and numerically simulate this situation below. While no very detailed mechanism can be provided, at this time, to explain the formation of such a region of hot, highly charged, air it is worth pointing out that a lightning strike generates both charge and highly heated air in great abundance; these are the only raw materials necessary for our model to work. It requires no external source of power. The excess charges are, initially, trapped by the interface between the very hot air and the cold, non-conductive, air outside the ball. We speculate that some of these charges will escape, or be neutralized, in a process somewhat reminiscent of thermionic emission. We are not sure yet if these excess charges are positive ions or electrons. But we assume them to be distributed uniformly throughout the ball. It might be objected that these excess charge carriers would generate a significant electric field and immediately be expelled from the ball by their own electrostatic force. But the air inside the ball is considerably ionized owing to its high temperature. The Debye length is very small. The excess charge carriers will feel little force from their companions causing them to move outward. This matter will be examined in greater detail presently.

Now an obvious question comes to mind – why is ball lightning round? Suppose that our mass of highly charged air starts out in some irregular shape. Suppose it has a sharp spike or protrusion in some area. Around such a protrusion the electric field will be extraordinarily high and we expect the rate of charge emission there will surely be greater. The surface charge density will drop in this region quite rapidly. Also, it will lose heat more quickly in this area. Likewise, if there were to be a more concave or flat area on the mass, the electric field there would be weaker. The surface charge density would diminish more slowly in that region. In this way the mass would tend to work itself into a situation where its surface charge density was everywhere the same. This, in turn, requires that it be *spherical* in shape. But it would not be very stable. Any serious turbulence in the atmosphere would tear it apart. And, if it came into contact with an object, it would rapidly dissipate. Ball lightning would be favored in areas of calm wind. It would stand a better chance of lasting if it formed rather far away from any objects it might encounter. Such a ball – having almost no density – would be quite buoyant. But it would also, as a charged object, be influenced by local electric fields which might draw it downwards (*vide infra*). Only under those happy circumstances where everything works out right do we see ball lightning. In most cases it probably floats away or crashes into the ground unnoticed.

We assume that the pressure here is everywhere $P_0 = 1$ atm (101, 000 N/m^2). η (the number density) = P_0/k T always. We assume that β k T is the average energy of any particular particle. The initial conditions inside the ball, at t = 0, are taken to be those of air at 1 atm pressure and 4, 000 K. It may well have been hotter to begin with. But above about 8, 000° the ball will cool very quickly due to its emission of light. And even much above 6, 000° it would glow too brightly to be a realistic ball lightning. At 4, 000° it would consist, primarily, of partially ionized nitrogen and oxygen molecules. Outside will be an area of corona discharge.

Let us suppose that a lightning strike has come down and deposited such a hot sphere with a radius R_0

and an initial charge Q_0. Outside this charged sphere will be an electric field – perhaps very strong. This will produce some ionization in the air around it. If the sphere is negatively charged electrons will leak away and escape from it. If it is positively charged electrons will flow to it from infinity. This will produce further electrons and positively charged ions outside the ball. Some of these will eventually fall back upon the sphere, thus reducing its charge. Some electrons will neutralize positive ions. There will also be chemical reactions taking place between the various gas molecules and ions in this very energetic region. We have no reliable way of estimating just how fast these processes will occur. But they all seem more-or-less related to the electric field strength at the surface of the ball. So let us guess that $\partial_t Q(t) = -\alpha Q(t)$ where α is some unknown constant and $Q(t)$ is the charge of the ball. α measures the rate of corona discharge as a function of the ball's net charge. Thus we will assume $Q(t) = Q_0 e^{-\alpha t}$.

Now we imagine that each unit of charge generated as a result of this complicated process makes its way from the ball's surface to r = ∞ , or vice-versa, at a sort-of constant drift velocity and that it does so very quickly in relation to the ball's lifetime; the energy it gains from the ball's electrical potential mostly goes into heating the gas it passes through. One can simply picture surrounding the ball with shells of radii (r, r+dr) and asking how much energy the charge gives out in moving through each. One must also consider how many particles there are in each shell that will have to share this energy. Considering β, one can easily find the local rate of temperature increase. This will be greatest near the ball's surface and much less significant far away. But the local temperature will also change at a rate $D(T(r, t)) \nabla^2 T(r, t)$ in consequence of thermal diffusion where we write the thermal diffusivity as a function of temperature. We find:

1) $\partial_t T(r, t) = \frac{-(\partial_t Q(t)) Q(t)}{16 \pi^2 \epsilon_0 r^4 \beta k \eta} + D(T(r, t)) \nabla^2 T(r, t)$.

We suppose that $\partial_t Q(t) \approx -\alpha Q(t)$. The local temperature will also change owing to the motion of the air in our model as it expands and contracts. This term, $- v \cdot \nabla T(r, t)$, is reduced in Lowke et. al. ([1969](#)) to $- r^{-2} \partial_r T[r, t] \int_0^r \frac{1}{C_p} \partial_T (1/\rho) \partial_{r'}(r'^2 \kappa \partial_{r'} T[r', t]) dr'$. This, in turn, is based on ([Christmann 1967](#)). If we make the simplifying approximation that C_P is roughly constant we find:

2) $\partial_t T(r, t) = \frac{\alpha Q_0^2 e^{-2\alpha t}}{16 \pi^2 \epsilon_0 r^4 \beta P_0} T(r, t) + D(T(r, t)) [\nabla^2 T(r, t) - (\partial_r T(r, t))^2/T(r, t)]$.

We cannot neglect cooling due to radiative losses. We designate this term $W(T(r, t))$ (W/m^3). Therefore:

3) $\partial_t T(r, t) = \frac{\alpha Q_0^2 e^{-2\alpha t}}{16 \pi^2 \epsilon_0 r^4 \beta P_0} T(r, t) + D(T(r, t)) [\nabla^2 T(r, t) - (\partial_r T(r, t))^2/T(r, t)] - \frac{W(T(r, t))}{P_0 \beta} T(r, t)$.

We do not care, at this stage, whether the excess charge is positive or negative since the net energetics are the same in both cases.

3. Numerical Simulations.

In order to see whether Equation 3) describes any interesting physics it must be solved numerically. In order to do this we need expressions for $D(T(r, t))$ and $W(T(r, t))$ that are both reasonably accurate and simple for a

computer to work with. $D(T(r, t))$ can be estimated by fitting the thermal conductivity (κ) of very hot air from (Yos 1963 and Engineeringtoolbox.com) to a curve and recalling that $D(T(r, t)) = \kappa / (\rho \, C_P)$.

4) $D(T) =$ Exp[.0010724 T - 9.7457] provides a fairly good approximation. (Its units are m^2/sec.)

For $W(T(r, t))$ data can be extracted from figures in (Lowke et. al. 1969) and fit rather well to a simple curve (see Supplementary Material *Mathematica*). We will employ

5) $W(T) =$ Exp[.00115 T + 1.5567] for the energy lost to light by our model. (Its units are W/m^3.)

These expressions work only in the temperature ranges we are dealing with. (The thermal conductivity of air behaves very differently, for instance, above about 9, 000 K.) This is not a problem, however, since we are only simulating temperatures below about 4000 K. We will set $\beta = 3.3$ since this a normal value for air under our circumstances. We obtain:

6) $\partial_t \, T(r, t) = \frac{\alpha \, Q_0^2 \, e^{-2\alpha t}}{16 \, \pi^2 \, \epsilon_0 \, r^4 \, 3.3 \, P_0} T(r, t) \, U(r - R_0) + D \, (T(r, t)) \left[\nabla^2 \, T(r, t) - (\partial_r \, T \, (r, t))^2 / T(r, \, t) \right] - \frac{W(T(r, t))}{P_0 \, 3.3} \, T(r, t)$

where U represents the unit step function. In this way we take account of the fact that energy production is only taking place outside the ball. R_0 denotes the radius within which the charge was originally deposited. It is true that the ball will contract a bit as it cools. This effect is relatively insignificant, however, so we ignore it in the interest of convenience. Equation 6) leaves us only three parameters to adjust – α, Q_0, and R_0; everything else is set rigidly by Nature.

The numerical solution of this non-linear PDE is not very difficult. The results shown here were obtained using *Mathematica* running on an ordinary computer. The solutions seldom take more than a few minutes to obtain. They have been checked and found to be reasonably robust in the sense that small, physically inconsequential, changes to the initial conditions do not materially change the results; we are likely looking at real solutions, not artifacts. The details are described in Supplementary Material *Mathematica*. This PDE can also be solved using FlexPDE. FlexPDE employs the finite element method as opposed to the method of lines which is used by *Mathematica*. This results in a much more accurate simulation of the step function-like behavior of our initial conditions. The solutions provided by each agree to within fairly minimal errors. These details are described in Supplementary Material Flex PDE.

We have no idea what value to select for α but we do know that it is related to the ball's lifetime. Suppose our ball starts with a radius of .1 m (which is not atypical of many reported cases). Such a ball might be expected to last on the order of 10 sec. So let us choose $\alpha = .1 \ \text{sec}^{-1}$ just to see if anything reasonable happens. We assume the initial temperature is 4000° inside the ball and 300° outside. Guessing at values for Q_0, we find the most interesting results occurring when Q_0 is about .0015 C. Much below this there is simply not enough charge to heat the ball sufficiently to prolong its life – the ball quickly cools and dies (fig. 1). It produces little visible light. For $Q_0 = .0015$ the ball quickly cools but then remains close to 3000°, and easily visible, for at least 10 sec which brings it well within the range reported by witnesses. Below is plotted its temperature profile as a function of time (fig. 2). We plot the decay of the ball's core temperature with and without charge (fig. 3). (See Supplementary Material *Mathematica*.) Its maximum luminosity is about 1.6 W. This would correspond

to the visible light output from a 80 W incandescent light bulb, assuming these are about 2% efficient ([Armaroli et. al. 2011]). This is not unlike what many witnesses report. Since a lightning strike is believed to usually bring down about 10 C of charge (Uman 1969), a figure of $Q_0 = .0015$ C is hardly unreasonable. If the initial charge had to be 1000 C we would wonder where that much charge came from.
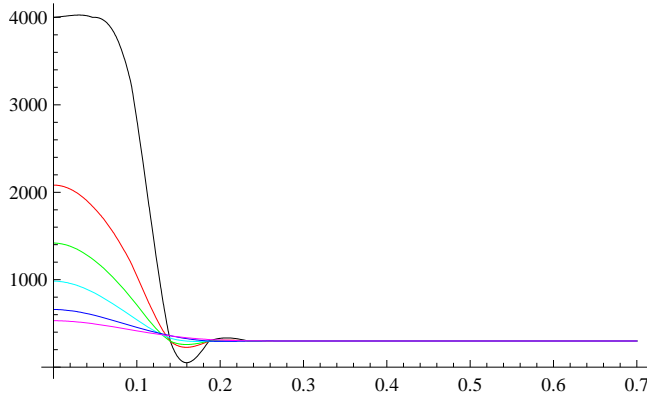

fig. 1

The calculated temperature profile (K) of an $R_0 = .1$ m, $Q_0 = 0$ C ball as a function of radius (m) plotted for $t = 0$ (black), 2 (red), 5 (green), 10 (blue-green), 20 (blue), 30 (purple) sec.
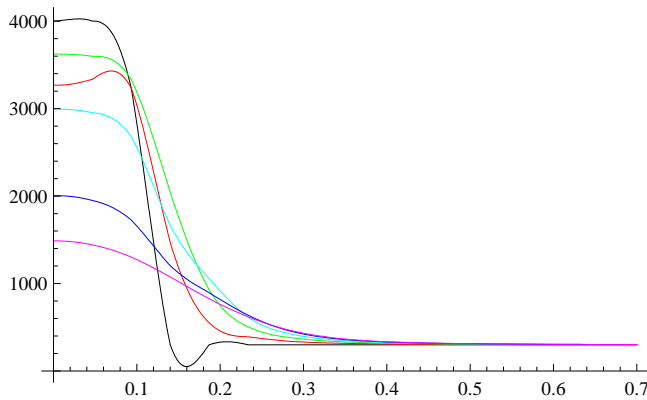
fig. 2

The calculated temperature profile (K) of an $R_0 = .1$ m, $Q_0 = .0015$ C, $\alpha = .1$, ball as a function of radius (m) plotted for $t = 0$ (black), 2 (red), 5 (green), 10 (blue-green), 20 (blue), 30 (purple) sec.
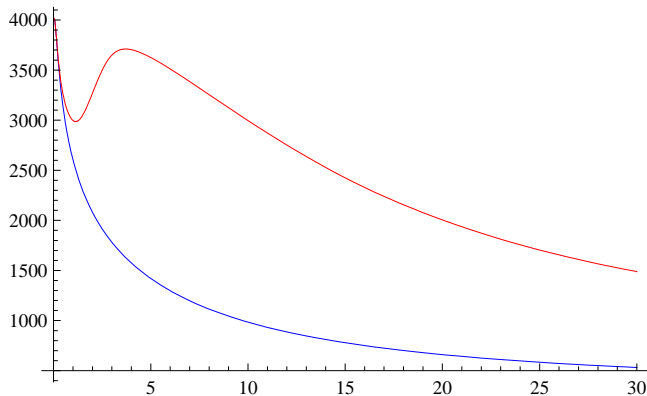
fig. 3

The calculated core temperature (K) of an $R_0 = .1$ m, $Q_0 = .0015$ C (red) and $Q_0 = 0$ (blue), ball as a function of time (sec).

Below (fig. 4) we plot the luminosity profile of this ball as a function of time. As a witness might observe it, it would have no well-defined or hard-edged boundary. Most of the light would come from inside the ball, however. This is generally consistent with reports (Charman 1972).
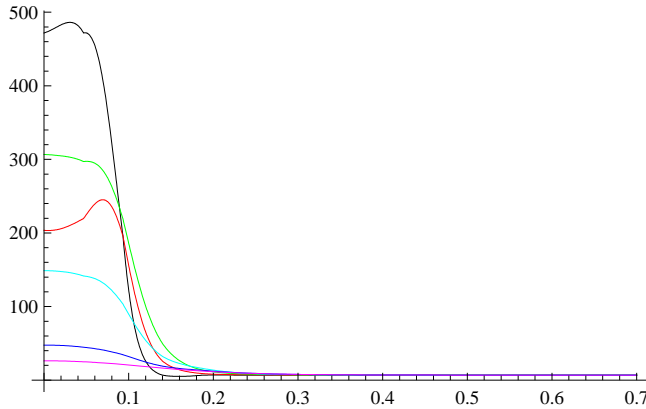

fig. 4

The calculated luminosity profile (W/m^3) of an $R_0 = .1$ m, $Q_0 = .0015$ C ball plotted for $t = 0$ (black), 2 (red), 5 (green), 10 (blue-green), 20 (blue), 30 (purple) sec.

For $Q_0 = .0015$ and $\alpha = .01$ the ball is far less effective at heating its outside environment. It never achieves any noticeable brightness. If α is increased to 1 the ball rapidly dissipates it charge. It glows brightly but only over about 3 sec (see Supplementary Material *Mathematica*).

Consider the $Q_0 = .0015$ C case. Examining the electric field immediately beyond this ball's surface (at t = 0) we find that it is about 1.4×10^9 V/m – an enormous value ~3 orders of magnitude greater than the breakdown field strength for normal air. This condition would exist out to several meters beyond the ball. In some ways this is a good result – we are, at least, guaranteed a vigorous corona discharge to keep the ball hot. Of course, we do not think that the actual field inside the corona discharge is likely to be quite so high. It is, probably, screened somewhat by ions within the discharge itself. Owing to the strong field produced by the ball, there will always tend to be a net excess of oppositely charged ions close to the ball and more like-charged ions farther away. We have no reliable means of calculating just how much the effective field would be reduced. But it would have to still be quite large. (If it became too small there would no longer be a sufficient electric field to promote the creation of ions in the corona and this region would then fully re-experience the ball's unscreened field and ionize again.) This is problematic in that, were the ball to get too close to any object (e.g. a grounded conductor) onto which it could release its excess charge, the result would likely be a mini-lightning strike and the quick and violent destruction of our ball. This may explain the explosive deaths of some ball lightnings. No wonder ball lightning is so rare and fleeting – very favorable circumstances would have to exist for it to live out its 10 sec lifetime.

In light of the diminution of the effective field within the region of corona discharge our simulations can be looked upon as giving unrealistically long-lived solutions – being weaker, the real field would not allow the current flow to give up so much heat-energy. Not knowing how to model the corona we can only look upon

our numerical simulation as a kind of best-case scenario. We can also consider a worst-case scenario in which the effective E field outside the ball is so well-screened that it is at only about the breakdown field for air (which we will take to be $3X10^6$ V/m) out to some rather large distance from the ball's surface. This field is only ca. 10^{-3} that assumed above. The energy produced by the escaping charges is, therefore, reduced by this factor. Hence αQ_0 might have to be roughly 1000 times that employed before or the ball would cool much too quickly. As discussed above, there is only so large α can become. Assuming $\alpha = .1$ we find $Q_0 \approx 1$ C in this case; this is an unrealistic number, even given the total charge a lightning strike brings down. But we can calculate the results for a ball of this kind. We get a solution that is physically reasonable for as little as $Q_0 = .03$ C (fig. 5). This is because heat generation now drops off only as $\frac{1}{r^2}$. The ball expands to about twice its original size and lasts over 20 sec. Obviously, both scenarios are unrealistically extreme, and the latter one not strictly possible mathematically. The truth must lie somewhere in between.
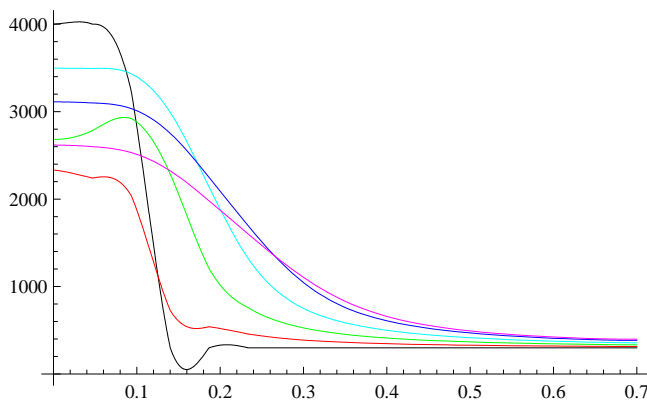

fig. 5

The calculated temperature profile (K) of an $R_0 = .1$ m, $Q_0 = .03$ C, $\alpha = .1$, ball with a constant external field as a function of radius (m) plotted for $t = 0$ (black), 2 (red), 5 (green), 10 (blue-green), 20 (blue), 30 (purple) sec.

This ball, having little mass, would feel an upward force of about $5X10^{-2}$ N. But there could very well be ambient electric fields in its environment that might counteract this. The ball is a (highly) charged object and will be subject to these fields. Such balls as are observed are, necessarily, somewhat close to the ground. If a .0015 C ball attracted about $1.3X10^{-8}$ C of charge in the ground below it it would remain stable at about a height of 2 m. There might also be other E fields around that might help stabilize it. In any case, the ball's charge must be distributed uniformly throughout it; otherwise it would be torn apart. It may be objected that, as the ball's charge decays away, there will be less downward force holding it in place. True enough. But, while this process is occurring, the ball will generally be cooling and becoming less buoyant. Also, it will have had additional time to attract more charges into the ground beneath it. This may further help it not to convect away and dissipate. (Interestingly, the electric field at the Earth's surface, under normal circumstances, is ca. 100 V/m which is just about enough to keep the .0015 C ball in place assuming that it is positively charged.)

We must say that ball lightning occurs only under rather fortunate circumstances where the electric field in the surrounding environment balances off its natural buoyancy. But it is significant that we are, at least, able to say this. Not every model of ball lightning that proposes it to be hot can explain its not floating away. And the Cen spectra indicate clearly that these balls are hot indeed. We have to consider the possibility that the formation of hot, charged, masses of air is a rather common phenomenon occurring when lightning strikes

down. Ball lightning would be uncommon only because unusual conditions are required for its survival. This possibility is not, necessarily, untestable. Perhaps what we see as ball lightning represents only the most energetically extreme manifestation of a more general phenomenon. Cooler, less highly charged, balls might well form even more easily and frequently. But these would be invisible to the naked eye. It is possible to trigger lightning strikes using model rockets attached to wires and attempts to create ball lightning in this way have been made (Hill et al. 2010). If the aftermath of such strikes could be filmed using highly sensitive night-vision or thermal-imaging equipment we might detect the frequent formation of such "sub-critical" balls.

      Larger and smaller balls can be modeled in the same way. Equation 6) is non-linear and no simple scaling arguments can be made regarding its behavior – each case must be considered individually. We examine a ball with $R_0 = 1$ m. This is close to the largest size reported. If $Q_0 = 0$ the ball cools over about 90 sec. Interesting results are observed for $Q_0 = .1$ C (fig. 6). For this value it initially swells. Its luminosity is very great.
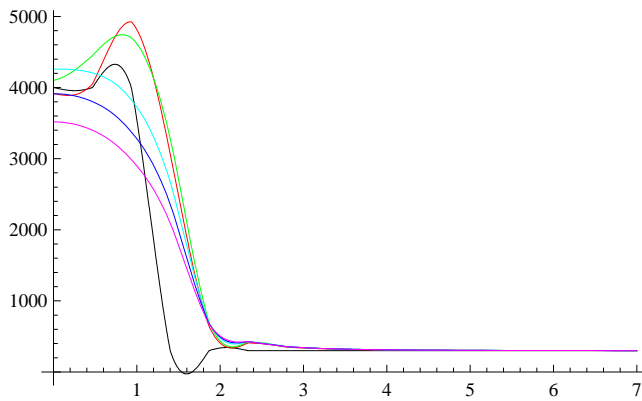

fig. 6

The calculated temperature profile (K) of an $R_0 = 1$ m, $Q_0 = .1$ C, $\alpha = .1$, ball as a function of radius (m) plotted for $t = 0$ (black), 10 (red), 20 (green), 40 (blue-green), 60 (blue), 90 (purple) sec.

      An uncharged small ball with an initial radius of .01 m stays above 2000° only about .02 sec. For $Q_0 = .00006$ C it lives about 5 sec and achieves a luminosity of $\sim .0025$ W (fig. 7). (See Supplementary Material *Mathematica*.) Its radius also swells quickly.
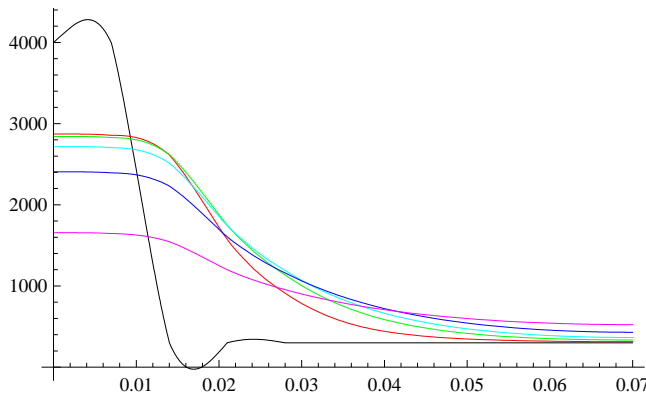

fig. 7

The calculated temperature profile (K) of an $R_0 = .01$ m, $\alpha = .1$, $Q_0 = .00006$ C ball as a function of radius (m) plotted for $t = 0$

(black), 1 (red), 2 (green), 3 (blue-green), 5 (blue), 10 (purple) sec.

4. Discussion.

If the charge of the ball is .0015 C there will be about 10^{16} charge carriers to begin with. It is, of course, quite arbitrary which particles we choose to call "charge carriers." They are just any ions or electrons that happen to be in excess. If the initial radius of the ball is .1 m, and if these are uniformly distributed and confined within this radius, the average distance between them and their nearest neighbors will be about 10^{-6} m. If the temperature in this region is about $4,000\,K$ the oxygen molecules (which we concentrate on since O_2 has the lowest ionization energy) are about 10^{-4} % ionized (as follows from the Saha equation). The predominant charged species will be O_2^+ and electrons (Bauer 1990). The Debye length is therefore about $5X10^{-6}$ m. It is reasonable to invoke this mechanism to the extent that the number density of screening ions in the environment exceeds that of the 'charge carriers.' Here they are about the same ($6X10^{17}$ and $2.5X10^{18}$ m^{-3}, respectively). Also, the presence of so many positive ions or electrons will tend to suppress somewhat the ionization of O_2. This further compromises our screening mechanism. If, however, we imagine that the air inside the ball contains even a very small number of atoms with low ionization energies (e.g. sodium) the number density of 'screening particles' will be increased greatly. The effect of such contaminants on uncharged balls has been studied by Lowke et. al. (1969). A sodium atom concentration of about .01% would provide more than enough 'screening particles' to ensure the stability of the ball. They would contribute a number density of around $7X10^{19}$ m^{-3} to the 'screening particles.' This concentration is considerably less than those examined by Lowke. (The presence of such impurities may also serve to affect the ball's color.) We must also think about this in a somewhat different way (Chen, 1974) and estimate the number of 'screening particles' – those present owing to the Saha equation – inside a sphere of radius the Debye length. This is $(4\,\pi/3) \left(\frac{\epsilon_0\,k}{e^2}\right)^{3/2} T^{3/2}\,\eta^{-1/2}$ (where η is the number density of the 'screening particles') and it must be quite a bit greater than one for effective Debye screening to be possible. We find it is $> \sim40$ for our .0015 C example during the time when heat production is mostly taking place (with or without the highly ionizable atoms). It is also necessary that some degree of local thermodynamic equilibrium prevail. It is true that the screening becomes less effective as the ball cools. But, as it cools, the 'charge carriers' themselves are escaping and becoming fewer in number. Well-screened, the charge carriers will only feel repulsive forces from other charges very nearby them. And these forces might just as easily push them towards the center of the ball as outwards. As long as the charge carriers, however defined, remain well-screened the charge will not all immediately fly away to infinity. It enjoys a natural mechanism of containment.

Positively charged ions have small mean free paths ($\sim7X10^{-8}$ m) and are hugely massive – they just cannot go very far by diffusion. A nitrogen molecule in air at STP travels about 7 mm in 1 sec and 21 mm in 10 sec which is insignificant in the context of our problem (though maybe less so for the .01 m ball considered above). They will go a bit farther in hotter air but this will not alter our conclusions very drastically; by the time these charge carriers move very far in relation to the ball's size most of the excess charge will have been expended anyway and heat production will have largely ceased. We do not much care what happens to them afterwards. This justifies our treatment of the unit step function in Equation 6) if we conclude that the ball is *positively charged*. It is harder to estimate the mean free path for electrons but it is generally at least 100 times that of ions and molecules. If the excess charge carriers are electrons they will quickly diffuse into the region where the electric field is largely unscreened and be propelled away to infinity. This would necessarily result in a situation where α would be quite large. This is not compatible with our mechanism, as has been noted above.

If the ball is positively charged the electrons will be attracted back towards it.

While the screening mechanism outlined above seems adequate to confine the number of 'charge carriers' we require, it does appear fairly stretched to its limit. There is only so hot the ball can be and only so many highly ionizable atoms it can plausibly contain. It is difficult to imagine how our ball could confine a charge that was an order of magnitude or more greater than the .0015 C assumed above. To contain similarly an initial charge that was tenfold greater would require a concentration of highly ionizable atoms that was roughly 1% which is rather enormous. The additional light produced by this many atoms, were they sodium, would also cause the ball to cool more rapidly (Lowke et. al. 1969). Furthermore, the Chen factor mentioned above becomes smaller as we add more ionizable atoms which reduces the viability of our screening mechanism. To contain much more charge would require concentrations that are plainly impossible. Thus our theory precludes the formation of incredibly bright and intense balls. If Q_0 were even .015 C the temperature would increase to over 9, 000 K out to .2 m in 0.3 sec! Clearly, such phenomena are never reported.

The high field strength around the ball is difficult to understand. We have already observed that it may be screened somewhat. But it would have to be quite large for the ball to sustain itself according to our mechanism. Very large electric fields, some probably exceeding $3X10^6$ V/m, undoubtedly exist between thunderclouds and the ground. But the entire sky does not break down instantaneously. The circumstances around our ball may be similar – its quasi-stable state only exists as long as it cannot find anything onto, or any way in which, which to release its charge. When it does, it dies explosively. In the meantime it releases what charge it can through a positive corona discharge. And for a positively charged ball to explode it has to be hit by a mini-lightning strike coming from a source of electrons that may be far away where the field is small. This process may have a more difficult time occurring than were the ball negatively charged; then the electrons would already be in an area where the field was intense.

It is, also, rather easier to understand the formation of a positively charged ball. If the initial mass of hot, ionized, air were to find itself around some positively charged object some of its electrons, which have great mobility, might be drawn away, thus leaving it with a net positive charge. We could even imagine something like a very brief mini-lightning strike carrying away the electrons by providing a conducting path between the nascent ball and the object.

5. Conclusion.

The strongest argument in favor of the above-described model is its ability to produce results that agree fairly well with what has been reported in spite of being rather inflexible – it has only three adjustable parameters, R_0, Q_0 and α. And its agreeable results only come about when these parameters are in what would seem to be a physically reasonable range. Everything else is determined by the relatively straightforward physics of the model and by numbers that can be looked up in, or extrapolated from, the literature. Also, it does not require any external source of power; its energy comes only from the heat and charge that were in the lightning channel that created the ball. Moreover, it provides an explanation for the spherical shape of the phenomenon. The ball's charge provides a mechanism whereby its natural buoyancy may be counteracted. The presence of NI and OI lines in the Cen ball lightning spectra is consistent with this model (although it is consistent with some other models as well). This model also explains the lack of a sharply-defined boundary to the ball – something eyewitnesses have mentioned. It explains the reported hissing and corona discharges associated with the phenomenon – something very few other models do. It explains why some people have been shocked, and even killed, by the phenomenon and why it is frequently observed to explode. It is also important to point out that we

are able to provide a rough mechanism whereby the ball's excess charge is, mostly, contained within its initial radius. This is only possible owing to some fortuitous results from the Saha and Debye equations as they apply to our situation. There is no obvious reason why such favorable coincidences should have occurred accidentally. This model also places a limit on just how large Q_0 can be, thus ruling out bizarre, super-bright, balls.

A shortcoming of this model is its inability to predict α – there is, admittedly, something circular about assuming that $\alpha = .1 \; \text{sec}^{-1}$ because ball lightning lasts about 10 sec. But, still, it is significant that we get physically plausible solutions for α in this range; if the model only worked for $\alpha = 10^7$ or $10^{-15} \; \text{sec}^{-1}$ we would have to say it was nonsense. The surprisingly high electric field strength immediately outside the ball is somewhat perplexing. Knowing of no reliable way to simulate such a situation, we have contented ourselves with a simple model that deals only with the net energetics of each unit of charge that leaves the ball and departs to infinity. We pay no attention to the details of its departure. And the assumption that the ball's charge decays exponentially with time is, itself, arbitrary. But this is not really a bad assumption. Regardless of the details of the corona discharge process, we know two things with certainty: The ball's charge will decay away and it will do so over some fairly definite period of time (which we may as well call $1/\alpha$). Modeling this process crudely as $Q(t) = Q_0 \; e^{-\alpha t}$, or trying to simulate $Q(t)$ more realistically, might not produce very different or more reliable answers given the approximate way in which we approach the problem and in which computers solve PDEs.

Acknowledgement.

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(Since ArXiv does not support the posting of multiple pdf. files I have simply attached the
  Supplementary Material. Both files are also available as Mathematica notebooks and pde. files,
 respectively. These could easily be run by any user having these programs on his
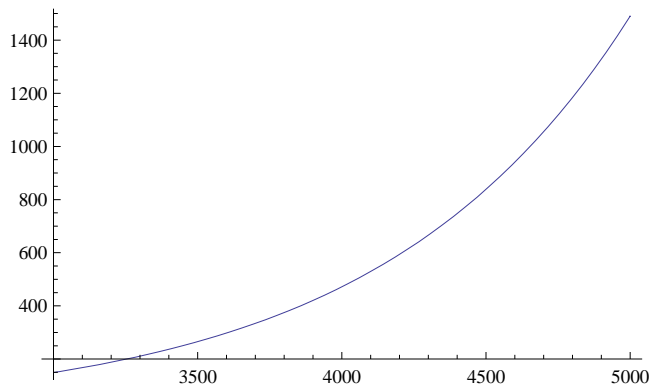 computer. They are readily obtainable by contacting the author at the above email address.)

Supplementary Material Mathematica.

Below are the programs used :

W[t_] := Exp[.00115 t + 1.5567]

This is the fomula that gives the approximate luminosity of very hot air in W/m^3. It is derived
  from the data graphed in (Lowke 1969). We demand that W[3000] = 150 and W[5000] = 1500.

Plot[W[T], {T, 3000, 5000}]

Thermal diffusivity was calculated from data
 in (Yos 1963) as described in the text and approximated as :

D_0[T_] := Exp[.0010724 T - 9.7457]

```
Plot[D₀[T], {T, 1000, 5000}]
```



```
Ball[α_, Qo_] := NDSolve[

  {∂ₜ T[x, y, z, t] == (.5 - .5 Tanh[ 300 - T[x, y, z, t]]) (D₀ [T[x, y, z, t]] (∂ₓ,ₓT[x, y, z, t] +

          ∂_y,y T[x, y, z, t] + ∂_z,z T[x, y, z, t] - (1 / (x^2 + y^2 + z^2 + .00001)

            (x ∂ₓ T[x, y, z, t] + y ∂_y T[x, y, z, t] + z ∂_z T[x, y, z, t])^2 ) / T[x, y, z, t]) +

      ((.5 + .5 Tanh[1000 (√(x^2 + y^2 + z^2) - .1)]) 2146 α Qo^2 Exp[-2 α t] T[x, y, z, t] /

        (x^2 + y^2 + z^2 + .00001) ^2) - T[x, y, z, t] W[T[x, y, z, t]] / (101 000 × 3.3)),

   T[x, y, z, 0] == 300 + 3700 (.5 - .5 Tanh[100 (√(x^2 + y^2 + z^2) - .1)]) ,

   T[-.7, y, z, t] == T[.7, y, z, t],
   T[x, -.7, z, t] == T[x, .7, z, t],
   T[x, y, -.7, t] == T[x, y, .7, t]},
  T, {x, -.7, .7}, {y, -.7, .7},
  {z, -.7, .7},
  {t, 0, 30},
  Method → {"MethodOfLines",
    "SpatialDiscretization" → {"TensorProductGrid", MaxPoints → 30}}]

P[X_] := Show[Plot[T[x, 0, 0, 0] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → Black],
  Plot[T[x, 0, 0, 2] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 0]],
  Plot[T[x, 0, 0, 5] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 0]],
  Plot[T[x, 0, 0, 10] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 1]],
  Plot[T[x, 0, 0, 20] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 0, 1]],
  Plot[T[x, 0, 0, 30] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 1]]]

PW[X_] := Show[Plot[W[T[x, 0, 0, 0] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → Black],
  Plot[W[T[x, 0, 0, 2] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 0]],
  Plot[W[T[x, 0, 0, 5] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 0]],
  Plot[W[T[x, 0, 0, 10] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 1]],
  Plot[W[T[x, 0, 0, 20] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 0, 1]],
  Plot[W[T[x, 0, 0, 30] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 1]]]
```

It will be noted that a few adjustments have been employed to "help"
  Mathematica do its work. Since the equation will become singular at r =
 0 we add a very small constant to the r ^ 2 term. We have checked that its value does not
   influence the solutions in any significant way. We also replace the Step Functions with steep,
but continuous Tanh functions. This prevents the program from introducing
   any artifactual discontinuities. We do not want the boundaries to be
   any farther away than where the interesting physics is taking place –
  there the computer is just wasting its time. So we use – .7 < x <
 .7 (for the R_0 = .1 m ball) and employ periodic boundary conditions. We cut ∂_t T off
  if T goes below 300. Its doing this introduces instabilities and is, obviously,
unphysical anyway. MaxPoints specifies the level of discretization the computer
 uses in representing things over the spatial dimensions. If it is too small the
 solution will be inaccurate and unreliable. If MaxPoints is set too high the
 computer may not have enough memory to calculate the solution. A robust solution
 should not depend much on MaxPoints in the regime where we can calculate it.

Ball[0, 0]

NDSolve::mxsst : Using maximum number of grid points

      30 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

      30 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫
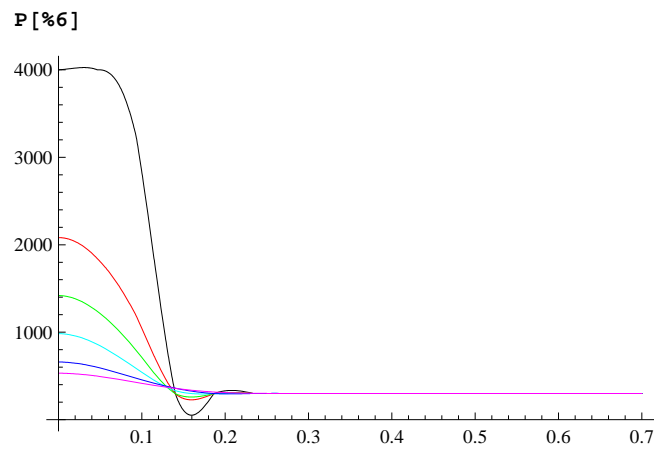
NDSolve::mxsst : Using maximum number of grid points

      30 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

{{T → InterpolatingFunction[{{-0.7, 0.7}, {-0.7, 0.7}, {-0.7, 0.7}, {0., 30.}}, <>]}}

Plot[T[0, 0, 0, t] /. %6, {t, 0, 30}, PlotRange → All, PlotStyle → RGBColor[0, 0, 1]]
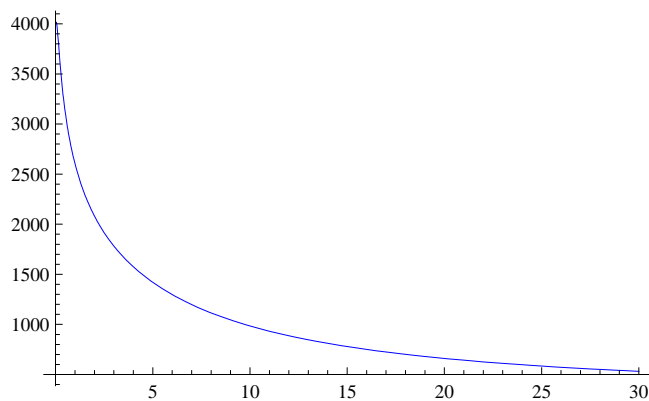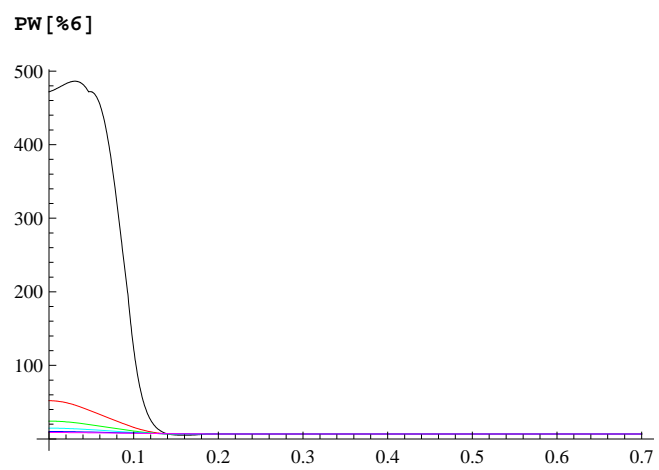
P[%6]

PW[%6]

```
Ball[.1, .0015]
```

NDSolve::mxsst : Using maximum number of grid points

     30 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

     30 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

NDSolve::mxsst : Using maximum number of grid points

     30 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

NDSolve::eerr :

  Warning: Scaled local spatial error estimate of 34.586393588334` at t = 30.` in the direction of independent

     variable x is much greater than prescribed error tolerance. Grid spacing with 31 points may be

     too large to achieve the desired accuracy or precision. A singularity may have formed or you may

     want to specify a smaller grid spacing using the MaxStepSize or MinPoints method options. ≫

```
{{T → InterpolatingFunction[{{-0.7, 0.7}, {-0.7, 0.7}, {-0.7, 0.7}, {0., 30.}}, <>]}}
```

**Because we are using this .0015 C ball as our primary example we will make the figures a little more
precise by correcting a small Mathematica artifact. Owing to its inability to model the initial
conditions perfectly Mathematica generates slightly asymmetiric solutions - T[x, 0, 0, t] ≠
T[-x, 0, 0, t] exactly, for instance. We will average them to try to smooth away this
small error. We will only make this extra effort for this, our exemplary, ball.**

```
Tm[x_, y_, z_, t_] := (T[x, y, z, t] + T[-x, -y, -z, t]) / 2
```

```
Pm[X_] := Show[Plot[Tm[x, 0, 0, 0] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → Black],
  Plot[Tm[x, 0, 0, 2] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 0]],
  Plot[Tm[x, 0, 0, 5] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 0]],
  Plot[Tm[x, 0, 0, 10] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 1]],
  Plot[Tm[x, 0, 0, 20] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 0, 1]],
  Plot[Tm[x, 0, 0, 30] /. X, {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 1]]]
```

```
PWm[X_] := Show[Plot[W[Tm[x, 0, 0, 0] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → Black],
  Plot[W[Tm[x, 0, 0, 2] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 0]],
  Plot[W[Tm[x, 0, 0, 5] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 0]],
  Plot[W[Tm[x, 0, 0, 10] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 1]],
  Plot[W[Tm[x, 0, 0, 20] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[0, 0, 1]],
  Plot[W[Tm[x, 0, 0, 30] /. X], {x, 0, .7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 1]]]
```

Pm[%5]

PWm[%5]
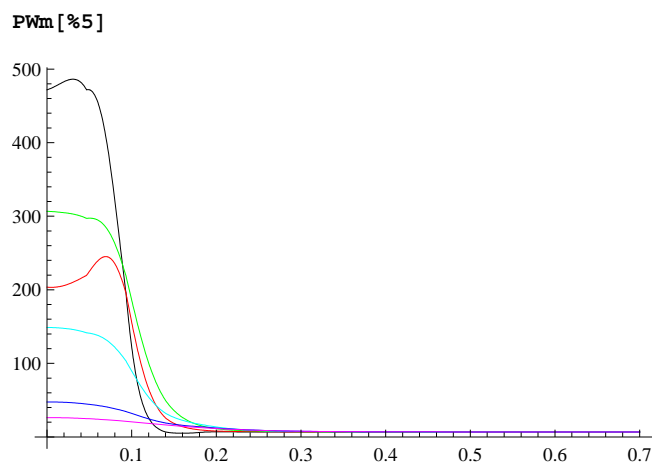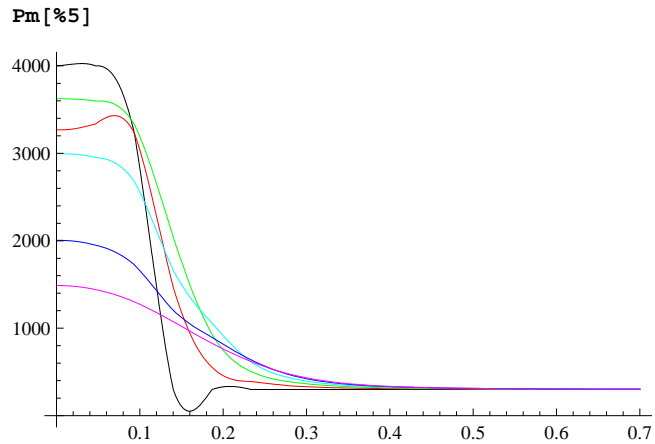
**Its luminosity is calculated. Note that we only integrate out to
.12. This is because our W[T] does not cut off quite realistically at low
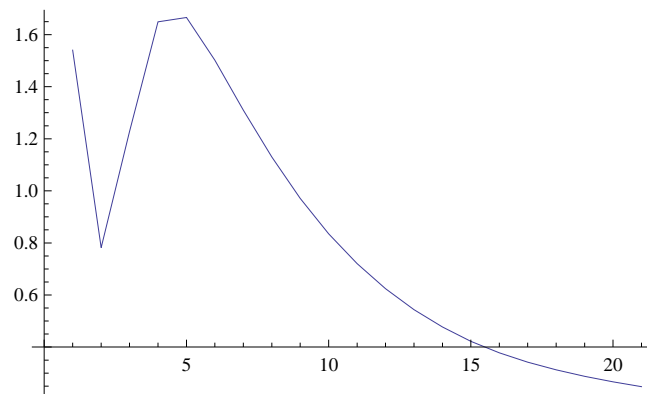temperatures and artifacts would be introduced if we integrated out to .7 :**

`F[x_, t_] := x^2 W[Tm[x, 0, 0, t] /. %4][[1]]`
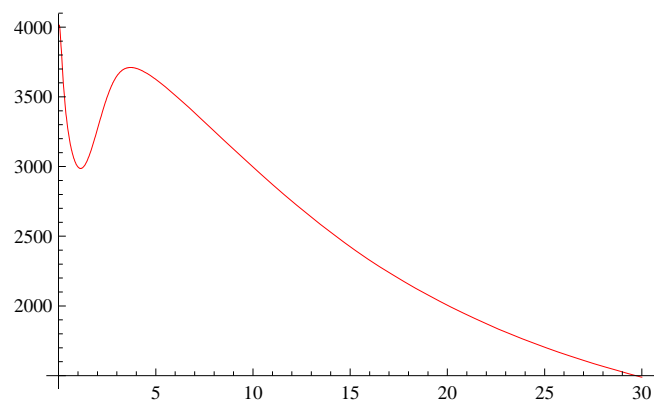
`L[t_] := 4 π NIntegrate[F[x, t], {x, 0, .12}]`

`M = Table[L[t], {t, 0, 20, 1}]`

```
{1.54044, 0.782025, 1.22707, 1.64906, 1.6655, 1.50196, 1.31019,
 1.12989, 0.971007, 0.834996, 0.720231, 0.624126, 0.543929, 0.477056,
 0.421918, 0.377673, 0.34188, 0.312385, 0.287563, 0.26625, 0.247795}
```

ListPlot[%, Joined → True]

Plot[T[0, 0, 0, t] /. %11, {t, 0, 30}, PlotRange → All, PlotStyle → RGBColor[1, 0, 0]]

Show[%7, %19]
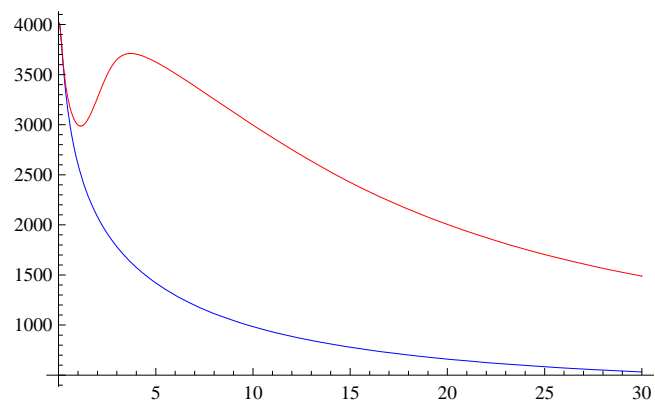
Ball[.1, .001]

NDSolve::mxsst : Using maximum number of grid points

     30 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

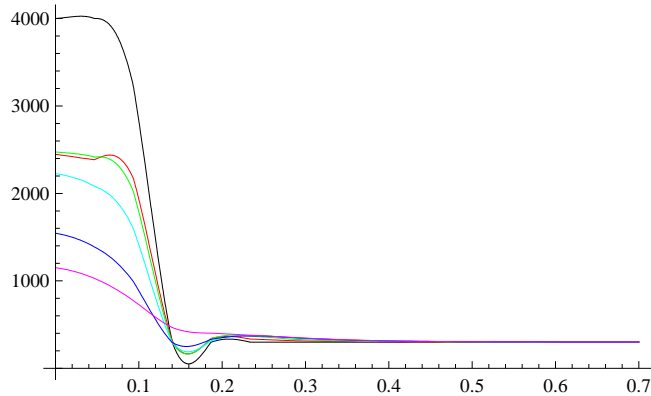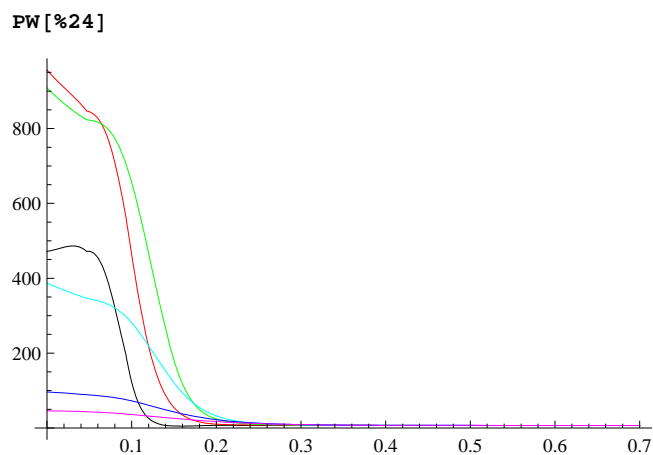   30 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

NDSolve::mxsst : Using maximum number of grid points

     30 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

NDSolve::eerr :

  Warning: Scaled local spatial error estimate of 22.66450811222414` at t = 30.` in the direction of independent

     variable x is much greater than prescribed error tolerance. Grid spacing with 31 points may be

     too large to achieve the desired accuracy or precision. A singularity may have formed or you may

     want to specify a smaller grid spacing using the MaxStepSize or MinPoints method options. ≫

{{T → InterpolatingFunction[{{-0.7, 0.7}, {-0.7, 0.7}, {-0.7, 0.7}, {0., 30.}}, <>]}}

P[%]

Ball[.1, .002]

NDSolve::mxsst : Using maximum number of grid points

     30 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫
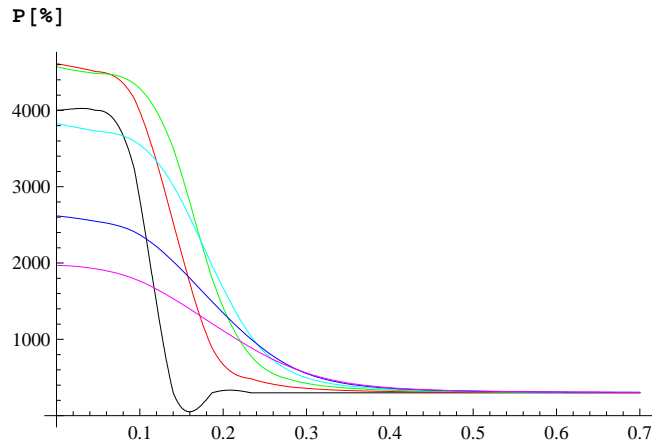
NDSolve::mxsst : Using maximum number of grid points

     30 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

NDSolve::mxsst : Using maximum number of grid points
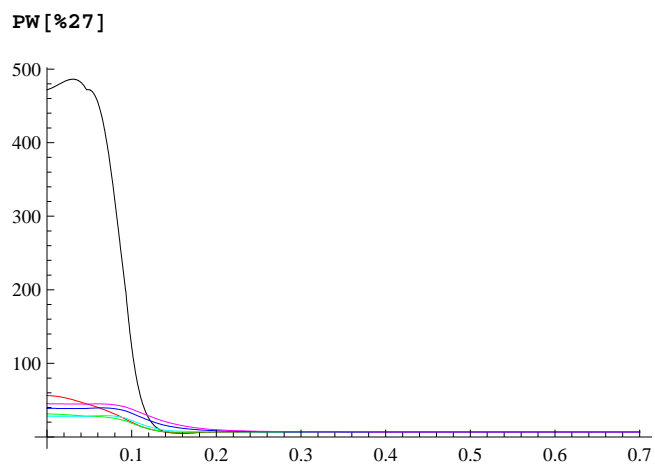
     30 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

{{T → InterpolatingFunction[{{-0.7, 0.7}, {-0.7, 0.7}, {-0.7, 0.7}, {0., 30.}}, <>]}}

P[%]

PW[%24]

Here is if alpha gets smaller.

Ball[.01, .0015]

NDSolve::mxsst : Using maximum number of grid points

    30 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

    30 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

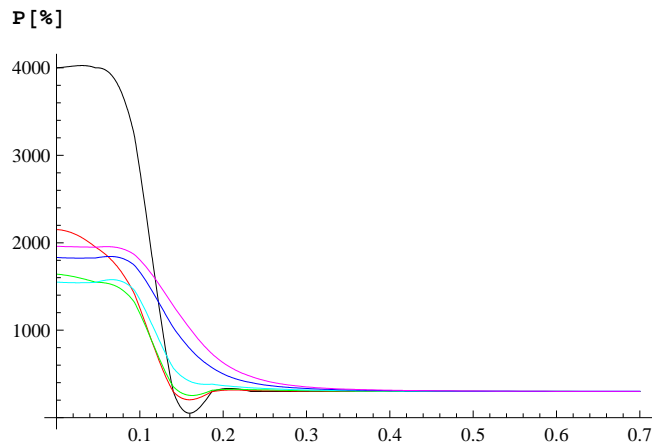NDSolve::mxsst : Using maximum number of grid points

    30 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

NDSolve::eerr :

  Warning: Scaled local spatial error estimate of 45.44476606741424` at t = 30.` in the direction of independent

    variable x is much greater than prescribed error tolerance. Grid spacing with 31 points may be

    too large to achieve the desired accuracy or precision. A singularity may have formed or you may

    want to specify a smaller grid spacing using the MaxStepSize or MinPoints method options. ≫

{{T → InterpolatingFunction[{{-0.7, 0.7}, {-0.7, 0.7}, {-0.7, 0.7}, {0., 30.}}, <>]}}

P[%]

PW[%27]

Here is if α gets larger.

Ball[1, .0015]

NDSolve::mxsst : Using maximum number of grid points

    30 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

    30 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

NDSolve::mxsst : Using maximum number of grid points

    30 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

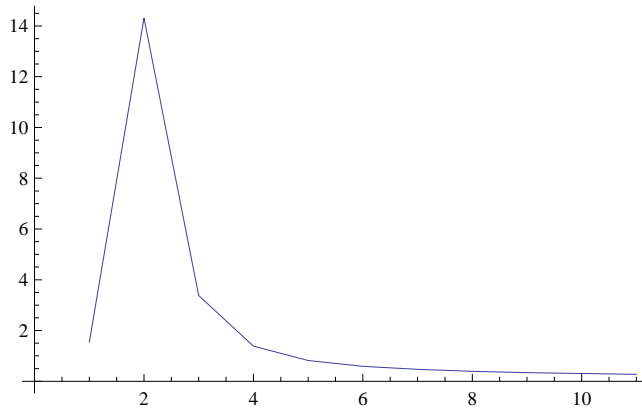{{T → InterpolatingFunction[{{-0.7, 0.7}, {-0.7, 0.7}, {-0.7, 0.7}, {0., 30.}}, <>]}}

P[%]

PW[%30]
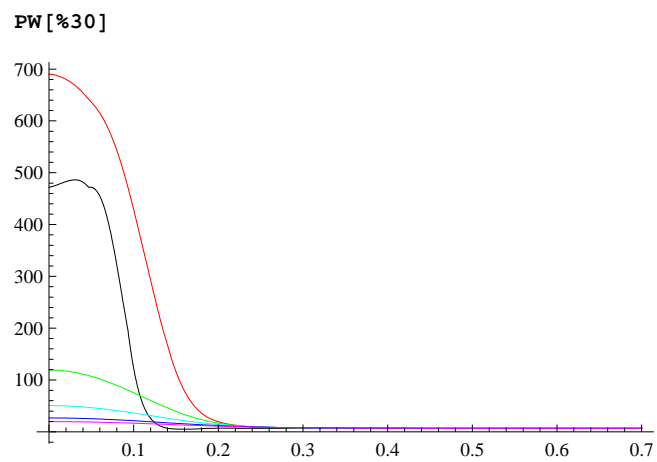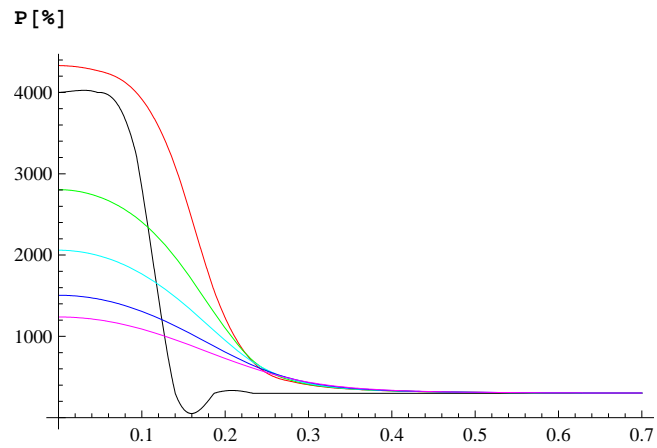
We calculate its luminosity :

L[t_] := 4 π NIntegrate[x^2 W[T[x, 0, 0, t]] /. %4[[1]], {x, 0, .12}]

M = Table[L[t], {t, 0, 10, 1}]

{1.54044, 14.3156, 3.37901, 1.38225, 0.821507,
 0.590751, 0.468789, 0.393651, 0.342448, 0.305203, 0.276776}

```
ListPlot[%6, Joined → True, PlotRange → All]
```


Below we describe a ball surrounded by a constant field of 3 X10 ^ 6 V / m.

```
SBall[α_, Qo_] := NDSolve[

  {∂_t T[x, y, z, t] == (.5 - .5 Tanh[ 300 - T[x, y, z, t]]) (D_0 [T[x, y, z, t]] (∂_{x,x}T[x, y, z, t] +

      ∂_{y,y}T[x, y, z, t] + ∂_{z,z}T[x, y, z, t] - (1 / (x^2 + y^2 + z^2 + .00001)

        (x ∂_x T[x, y, z, t] + y ∂_y T[x, y, z, t] + z ∂_z T[x, y, z, t])^2 ) / T[x, y, z, t]) +

      ((.5 + .5 Tanh[1000 (√(x^2 + y^2 + z^2) - .1)]) 2146 (2 × 10 ^ -3) α Qo

        Exp[-α t] T[x, y, z, t] / (x^2 + y^2 + z^2 + .00001)) -

      T[x, y, z, t] W[T[x, y, z, t]] / (101 000 × 3.3)),

   T[x, y, z, 0] == 300 + 3700 (.5 - .5 Tanh[100 (√(x^2 + y^2 + z^2) - .1)]) ,

   T[-.7, y, z, t] == T[.7, y, z, t],
   T[x, -.7, z, t] == T[x, .7, z, t],
   T[x, y, -.7, t] == T[x, y, .7, t]},

  T, {x, -.7, .7}, {y, -.7, .7},
  {z, -.7, .7},
  {t, 0, 30},
  Method → {"MethodOfLines",
    "SpatialDiscretization" → {"TensorProductGrid", MaxPoints → 30}}]

SBall[.1, .03]
```

NDSolve::mxsst : Using maximum number of grid points

    30 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

    30 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

NDSolve::mxsst : Using maximum number of grid points

    30 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

```
{{T → InterpolatingFunction[{{-0.7, 0.7}, {-0.7, 0.7}, {-0.7, 0.7}, {0., 30.}}, <>]}}
```

P[%]

PW[%34]
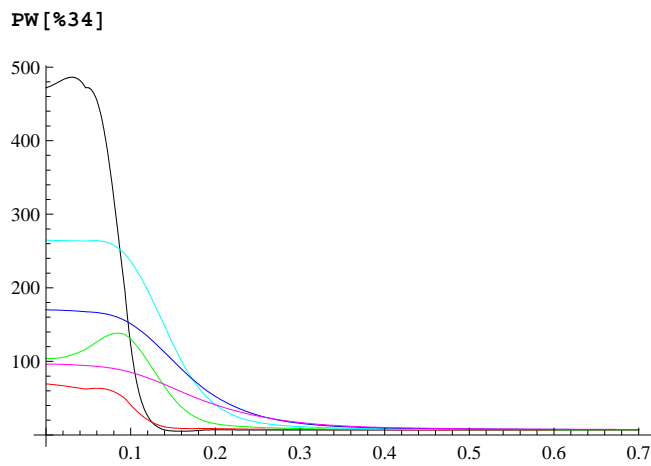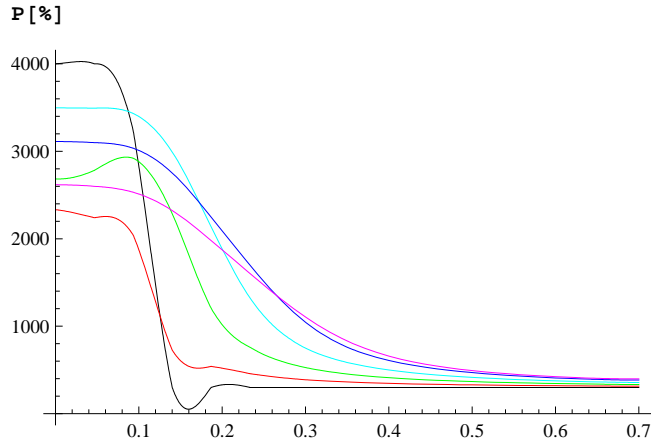
Here we describe a ball with a radius of 1 m :

```
BigBall[α_, Qo_] := NDSolve[

  {∂_t T[x, y, z, t] == (.5 - .5 Tanh[ 300 - T[x, y, z, t]]) (D_0 [T[x, y, z, t]] (∂_{x,x} T[x, y, z, t] +

        ∂_{y,y} T[x, y, z, t] + ∂_{z,z} T[x, y, z, t] - (1 / (x^2 + y^2 + z^2 + .00001)

          (x ∂_x T[x, y, z, t] + y ∂_y T[x, y, z, t] + z ∂_z T[x, y, z, t])^2 ) / T[x, y, z, t]) +

      ((.5 + .5 Tanh[100 (√(x^2 + y^2 + z^2) - 1)])) 2146 α Qo^2 Exp[-2 α t] T[x, y, z, t] /

        (x^2 + y^2 + z^2 + .00001)^2) - T[x, y, z, t] W[T[x, y, z, t]] / (101 000 × 3.3)),

   T[x, y, z, 0] == 300 + 3700 (.5 - .5 Tanh[100 (√(x^2 + y^2 + z^2) - 1)]) ,

   T[-7, y, z, t] == T[7, y, z, t],

   T[x, -7, z, t] == T[x, 7, z, t],

   T[x, y, -7, t] == T[x, y, 7, t]},

  T, {x, -7, 7}, {y, -7, 7}, {z, -7, 7},

  {t, 0, 90},

  Method → {"MethodOfLines",

    "SpatialDiscretization" → {"TensorProductGrid", MaxPoints → 30}}]
```

BigBall[.1, 0]

NDSolve::mxsst : Using maximum number of grid points

  30 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

  30 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

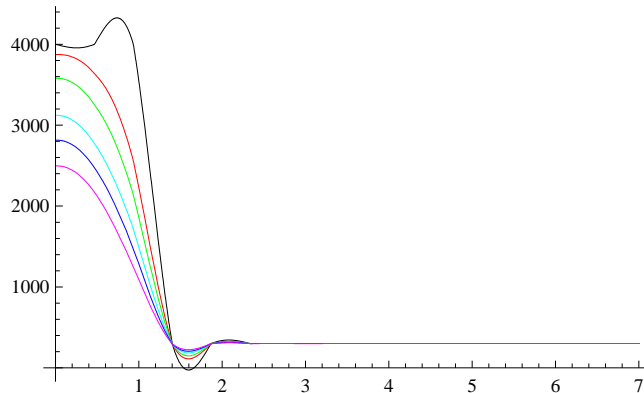NDSolve::mxsst : Using maximum number of grid points

  30 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

{{T → InterpolatingFunction[{{-7., 7.}, {-7., 7.}, {-7., 7.}, {0., 90.}}, <>]}}

```
PBig[X_] := Show[Plot[T[x, 0, 0, 0] /. X, {x, 0, 7}, PlotRange → All, PlotStyle → Black],
  Plot[T[x, 0, 0, 10] /. X, {x, 0, 7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 0]],
  Plot[T[x, 0, 0, 20] /. X, {x, 0, 7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 0]],
  Plot[T[x, 0, 0, 40] /. X, {x, 0, 7}, PlotRange → All, PlotStyle → RGBColor[0, 1, 1]],
  Plot[T[x, 0, 0, 60] /. X, {x, 0, 7}, PlotRange → All, PlotStyle → RGBColor[0, 0, 1]],
  Plot[T[x, 0, 0, 90] /. X, {x, 0, 7}, PlotRange → All, PlotStyle → RGBColor[1, 0, 1]]]
```

PBig[%38]

BigBall[.1, .1]

NDSolve::mxsst : Using maximum number of grid points

  30 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

  30 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

NDSolve::mxsst : Using maximum number of grid points

  30 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

{{T → InterpolatingFunction[{{-7., 7.}, {-7., 7.}, {-7., 7.}, {0., 90.}}, <>]}}

PBig[%]
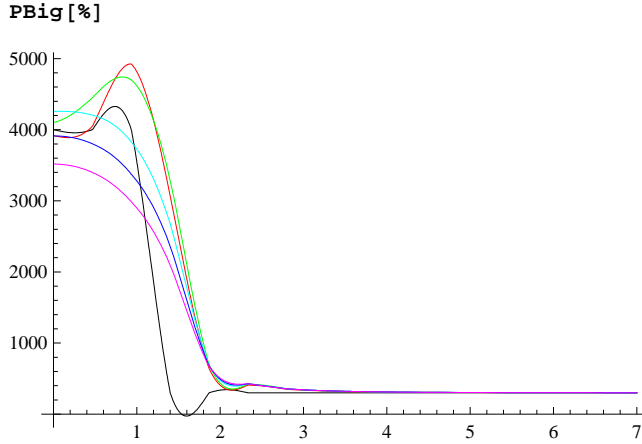
Below we describe a ball with a radius of only .01 m.

```
SmallBall[α_, Qo_] := NDSolve[
```

$$\{\partial_t\, T[x, y, z, t] == (.5 - .5\,\text{Tanh}[\,300 - T[x, y, z, t]\,])\,\Big(D_0\,[T[x, y, z, t]]\,\big(\partial_{x,x}T[x, y, z, t] +$$

$$\partial_{y,y}T[x, y, z, t] + \partial_{z,z}T[x, y, z, t] - \big(1\,/\,(x{\wedge}2 + y{\wedge}2 + z{\wedge}2 + .000001)$$

$$\big(x\,\partial_x\,T[x, y, z, t] + y\,\partial_y\,T[x, y, z, t] + z\,\partial_z\,T[x, y, z, t]\big){\wedge}2\,\big)\,/\,T[x, y, z, t]\big) +$$

$$\Big(\big(.5 + .5\,\text{Tanh}\big[10\,000\,\big(\sqrt{x{\wedge}2 + y{\wedge}2 + z{\wedge}2} - .01\big)\big]\big)\Big)\,2146\,\alpha\,\text{Qo}{\wedge}2\,\text{Exp}[-2\,\alpha\,t]\,T[x, y, z, t]\,/$$

$$(x{\wedge}2 + y{\wedge}2 + z{\wedge}2 + .000001){\wedge}2\Big) - T[x, y, z, t]\,W[T[x, y, z, t]]\,/\,(101\,000 \times 3.3)\Big),$$

$$T[x, y, z, 0] == 300 + 3700\,\Big(.5 - .5\,\text{Tanh}\big[1000\,\big(\sqrt{x{\wedge}2 + y{\wedge}2 + z{\wedge}2} - .01\big)\big]\Big)\,,$$

$$T[-.07, y, z, t] == T[.07, y, z, t],$$

$$T[x, -.07, z, t] == T[x, .07, z, t],$$

$$T[x, y, -.07, t] == T[x, y, .07, t]\},$$

```
 T, {x, -.07, .07}, {y, -.07, .07},
 {z, -.07, .07},
 {t, 0, 10},
 Method → {"MethodOfLines",
    "SpatialDiscretization" → {"TensorProductGrid", MaxPoints → 20}}]
```

```
PS[X_] := Show[Plot[T[x, 0, 0, 0] /. X, {x, 0, .07}, PlotRange → All, PlotStyle → Black],
  Plot[T[x, 0, 0, 1] /. X, {x, 0, .07}, PlotRange → All, PlotStyle → RGBColor[1, 0, 0]],
  Plot[T[x, 0, 0, 2] /. X, {x, 0, .07}, PlotRange → All, PlotStyle → RGBColor[0, 1, 0]],
  Plot[T[x, 0, 0, 3] /. X, {x, 0, .07}, PlotRange → All, PlotStyle → RGBColor[0, 1, 1]],
  Plot[T[x, 0, 0, 5] /. X, {x, 0, .07}, PlotRange → All, PlotStyle → RGBColor[0, 0, 1]],
  Plot[T[x, 0, 0, 10] /. X, {x, 0, .07}, PlotRange → All, PlotStyle → RGBColor[1, 0, 1]]]
```

Here is the uncharged case.

SmallBall[.1, 0]

NDSolve::mxsst : Using maximum number of grid points

      20 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

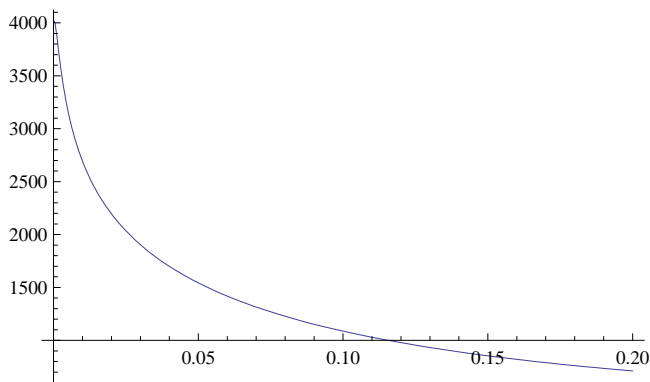   20 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

NDSolve::mxsst : Using maximum number of grid points

      20 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

`{{T → InterpolatingFunction[{{-0.07, 0.07}, {-0.07, 0.07}, {-0.07, 0.07}, {0., 10.}}, <>]}}`

Plot[T[0, 0, 0, t] /. %, {t, 0, .2}, PlotRange → All]

SmallBall[.1, .00006]

NDSolve::mxsst : Using maximum number of grid points

      20 allowed by the MaxPoints or MinStepSize options for independent variable x. ≫

NDSolve::mxsst : Using maximum number of grid points

      20 allowed by the MaxPoints or MinStepSize options for independent variable y. ≫

NDSolve::mxsst : Using maximum number of grid points

      20 allowed by the MaxPoints or MinStepSize options for independent variable z. ≫

General::stop : Further output of NDSolve::mxsst will be suppressed during this calculation. ≫

NDSolve::eerr :

  Warning: Scaled local spatial error estimate of 707.9613016136069` at t = 10.` in the direction of independent

      variable x is much greater than prescribed error tolerance. Grid spacing with 21 points may be

      too large to achieve the desired accuracy or precision. A singularity may have formed or you may

      want to specify a smaller grid spacing using the MaxStepSize or MinPoints method options. ≫

`{{T → InterpolatingFunction[{{-0.07, 0.07}, {-0.07, 0.07}, {-0.07, 0.07}, {0., 10.}}, <>]}}`

PS[%]
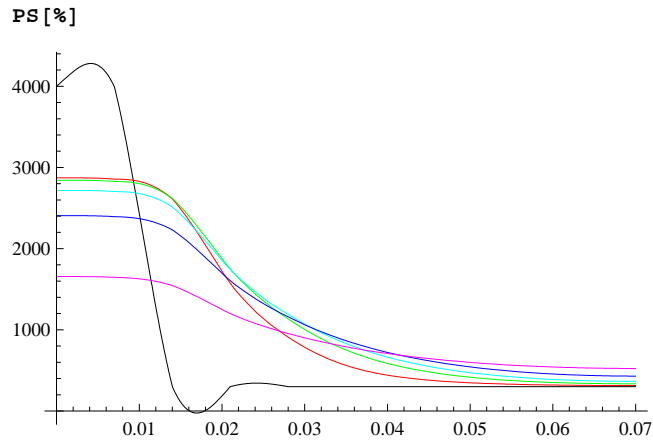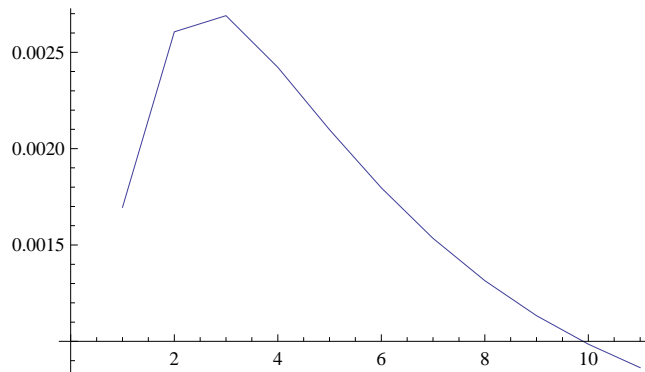

We calculate its luminosity :

`L[t_] := 4 π NIntegrate[ x ^ 2 W[T[x, 0, 0, t]] /. %47[[1]], {x, 0, .02}]`

`M = Table[L[t], {t, 0, 10, 1}]`

{0.00169541, 0.00260569, 0.00268962, 0.00242142, 0.00209877,
 0.00179656, 0.00153463, 0.00131503, 0.00113384, 0.000985329, 0.000864023}

`ListPlot[%, Joined → True]`

Below we calculate, using the Saha equation,
a rough estimate of the number density of O_2^+ ions (these will mostly prdominate at lower
**   temperatures owing to the rather low ionization energy of molecular oxygen) :**

`FO2[T_] := ne /. NSolve[ne ^ 2 / ((.2 × 7 × 10 ^ 27 / T) - ne) ==`
`    (2 (3 / 5) ((7.47 × 10 ^ -8 ) T ^ -.5) ^ -3 ) Exp[- 1.96 10 ^ -18 / (1.38 × 10 ^ -23 T)], ne][[2]]`

`FO2[5500]`

4.26812×10^{19}

`FO2[5000]`

1.14595×10^{19}

`FO2[4500]`

2.30343×10^{18}

FO2[4000]

3.11091×10^{17}

FO2[3500]

2.38187×10^{16}

FO2[3000]

7.79028×10^{14}

FO2[2500]

6.54151×10^{12}

FO2[2000]

5.09718×10^{9}

From this we can determine the Debye length
$\big($**Note that we must double the numbers above to get the number density of ' screening**
 particles ' since, for each O_2^+ produced, there will also be an electron.$\big)$ **:**

DebO2[T_] := 1 / Sqrt[2 FO2[T] 1.13 × 10 ^ 11 (1.6 × 10 ^ -19) ^ 2 / (1.38 × 10 ^ -23 T)]

DebO2[5500]

5.54407×10^{-7}

DebO2[5000]

1.02016×10^{-6}

DebO2[4500]

2.15866×10^{-6}

DebO2[4000]

5.53798×10^{-6}

DebO2[3500]

0.0000187215

DebO2[3000]

0.0000958406

DebO2[2500]

0.000954765

DebO2[2000]

0.0305925

Here we calculate the number of ' screening particles ' inside a sphere of the Debye radius :

ChenO2[T_] := (4 Pi / 3) DebO2[T] ^ 3 2 FO2[T]

ChenO2[5500]

60.9313

ChenO2[5000]

101.926

ChenO2[4500]

194.109

ChenO2[4000]

442.65

ChenO2[3500]

1309.35

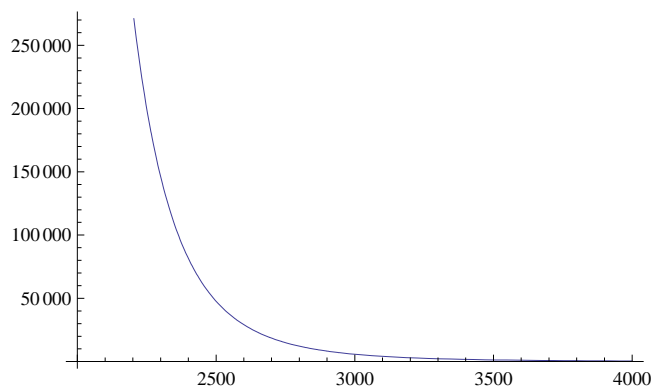**ChenO2[3000]**

5745.39

ChenO2[2500]

47 696.4

ChenO2[2000]

1.22263×10^6

Plot[ChenO2[T], {T, 4000, 2000}]

Below are the results for sodium if it were present at .01 % in the ball :

FNa[T_] := ne /. NSolve[ne^2 / ((7 × 10^23 / T) − ne) ==
(((7.47 × 10^-8) T^−.5)^−3) Exp[− 10^−18 / (1.38 × 10^−23 T)], ne][[2]]

FNa[5500]

1.19571×10^{20}

FNa[5000]

1.11268×10^{20}

FNa[4500]

7.63218×10^{19}

FNa[4000]

3.40607×10^{19}

FNa[3500]

9.81425×10^{18}

FNa[3000]

1.71847×10^{18}

FNa[2500]

1.47169×10^{17}

FNa[2000]

3.7168×10^{15}

DebNa[T_] := 1 / Sqrt[2 FNa[T] 1.13 × 10 ^ 11 (1.6 × 10 ^ -19) ^ 2 / (1.38 × 10 ^ -23 T)]

DebNa[5500]

3.31233×10^{-7}

DebNa[5000]

3.2739×10^{-7}

DebNa[4500]

3.75014×10^{-7}

DebNa[4000]

5.2926×10^{-7}

DebNa[3500]

9.22297×10^{-7}

DebNa[3000]

2.04059×10^{-6}

DebNa[2500]

6.36542×10^{-6}

DebNa[2000]

0.0000358258

ChenNa[T_] := (4 Pi / 3) DebNa[T] ^ 3 2 FNa[T]

ChenNa[5500]

36.4037

ChenNa[5000]

32.7103

ChenNa[4500]

33.7217

ChenNa[4000]

42.3037

ChenNa[3500]

64.5042

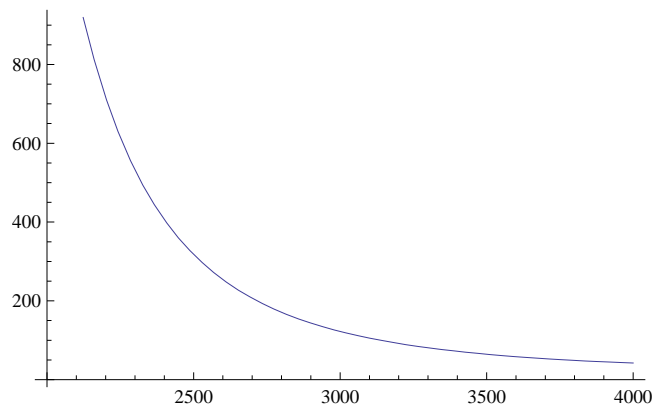**ChenNa[3000]**

122.328

ChenNa[2500]

317.992

ChenNa[2000]

1431.78

Plot[ChenNa[T], {T, 4000, 2000}]

Supplementary Material FlexPDE

```
title ' 1 D Nested Sphere '

coordinates
sphere1 {default coordinate name is "R"}

variables
temp

definitions
R1 = .1
R2 = .7
Tout = 300
Tin = 4000
T0 = Tout
alpha = .1
Q = .0015
S = UStep (r - .1) * 2146 * alpha * Q ^ 2 * Exp (-2 * alpha * t) * temp / (r ^ 4) -
   temp * Exp (.00115 * temp + 1.5567) / (101 000 * 3.3) {source}

initial values
temp = T0
! temp = swage (R - R1, Tin, Tout, R1 / 10) ! smoothly
   transition the temperature discontinuity to avoid FlexPDE trying

equations
temp : (Exp (.0010724 * temp - 9.7457)) * (div (grad (temp)) - (DR (temp) ^ 2 / temp)) + S =
 dt (temp) {the heat equation}


boundaries
region 1
T0 = Tin
start (0) line to (R1)

region 2
T0 = Tout
start (R1) line to (R2)


time 0 by 1 e - 4 to 30
```