

# Teaching Math in Middle School



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Say hello.

Tell us about yourself and the math you support.



# Teaching Math in Middle School

Using MTSS to Meet All Students' Needs



Leanne R. Ketterlin-Geller, Sarah R. Powell,  
David J. Chard, & Lindsey Perry

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## Teaching Math in Middle School

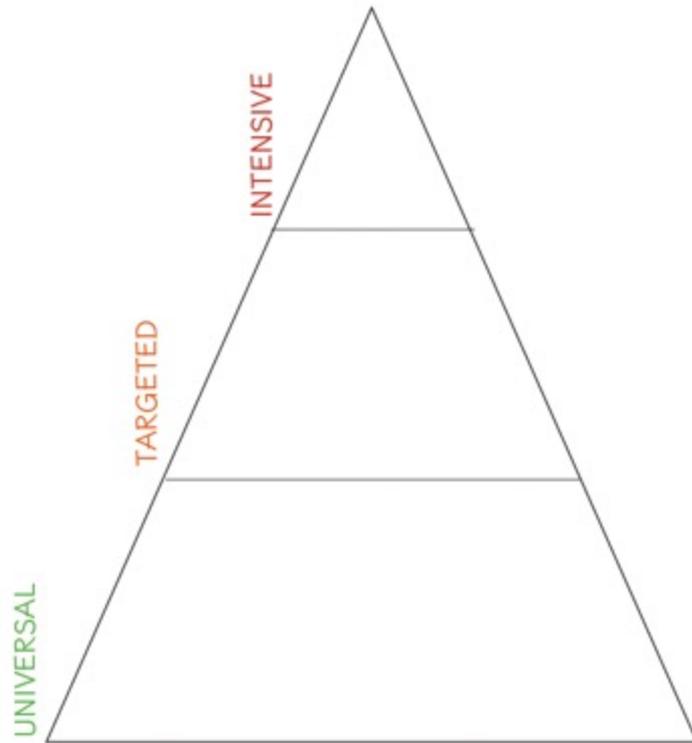
Ketterlin-Geller, Powell, Chard, & Perry (2019)

Sarah R. Powell, Ph.D.

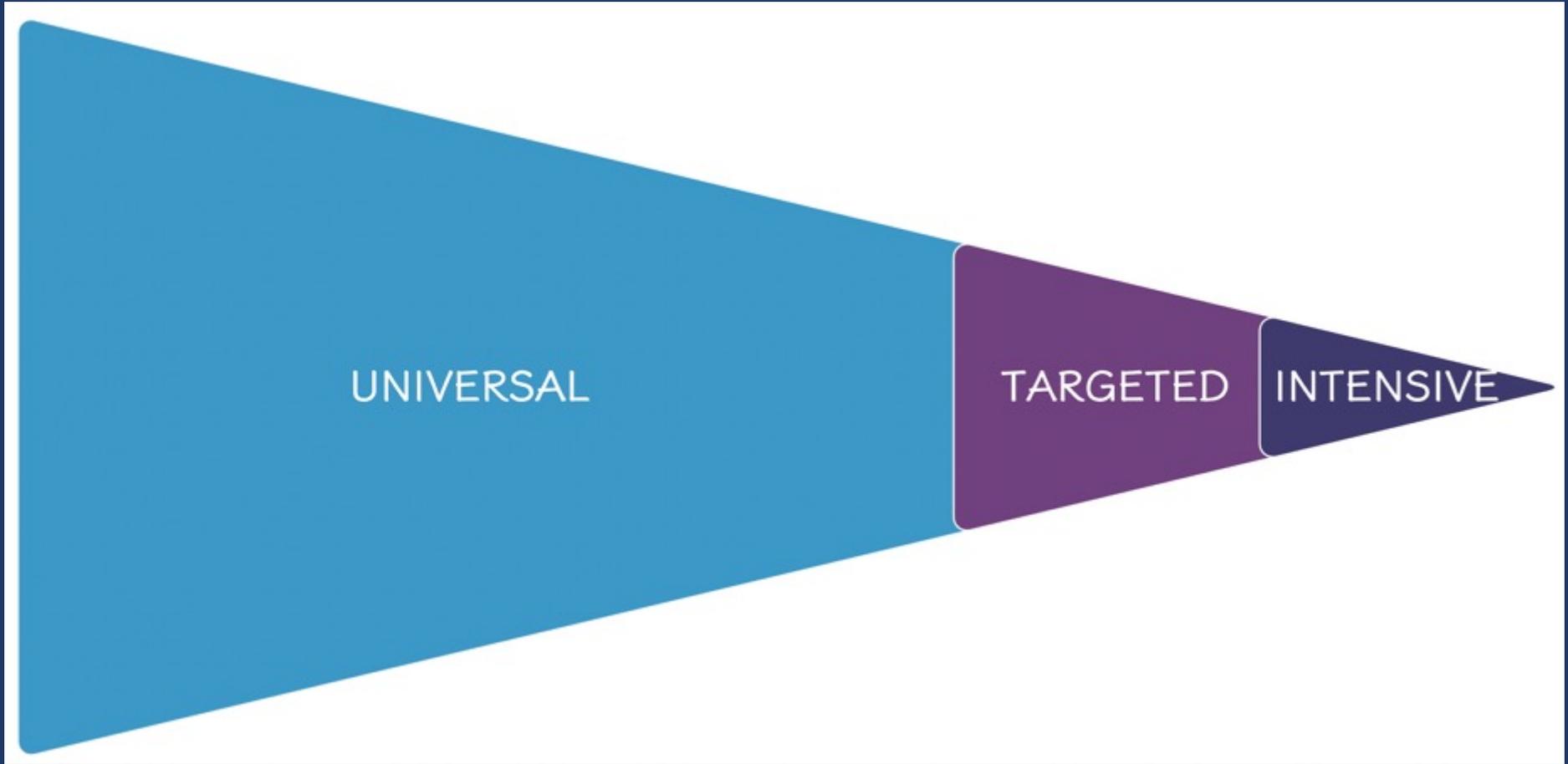
[srpowell@utexas.edu](mailto:srpowell@utexas.edu)

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# Multi-Tiered Systems of Support (MTSS)



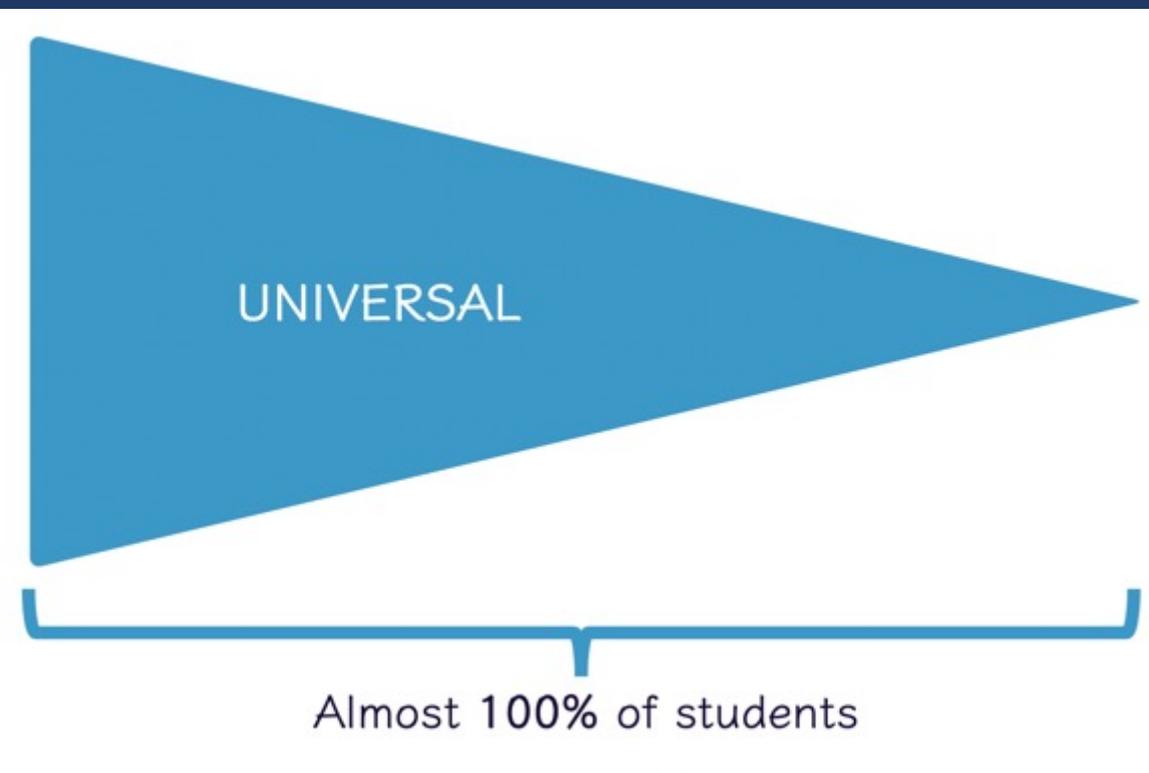
Also known as **Tier 1** or **primary prevention**

Designed for all students

Occurs in general education classroom

Almost all students participate

~80% of students need **only** universal intervention



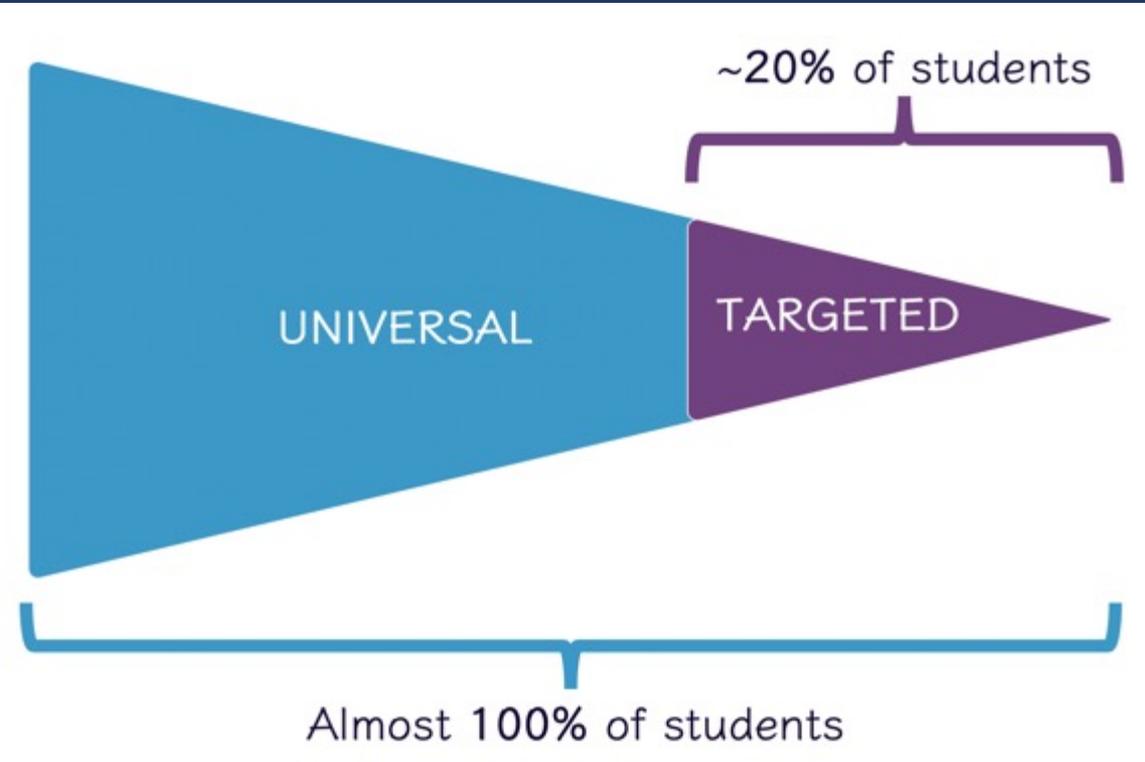
Also known as Tier 2 or secondary prevention

Designed for students experiencing difficulty in academics or behavior

Can occur inside or outside of the classroom

Provided in conjunction with universal intervention

~20% of students require targeted intervention

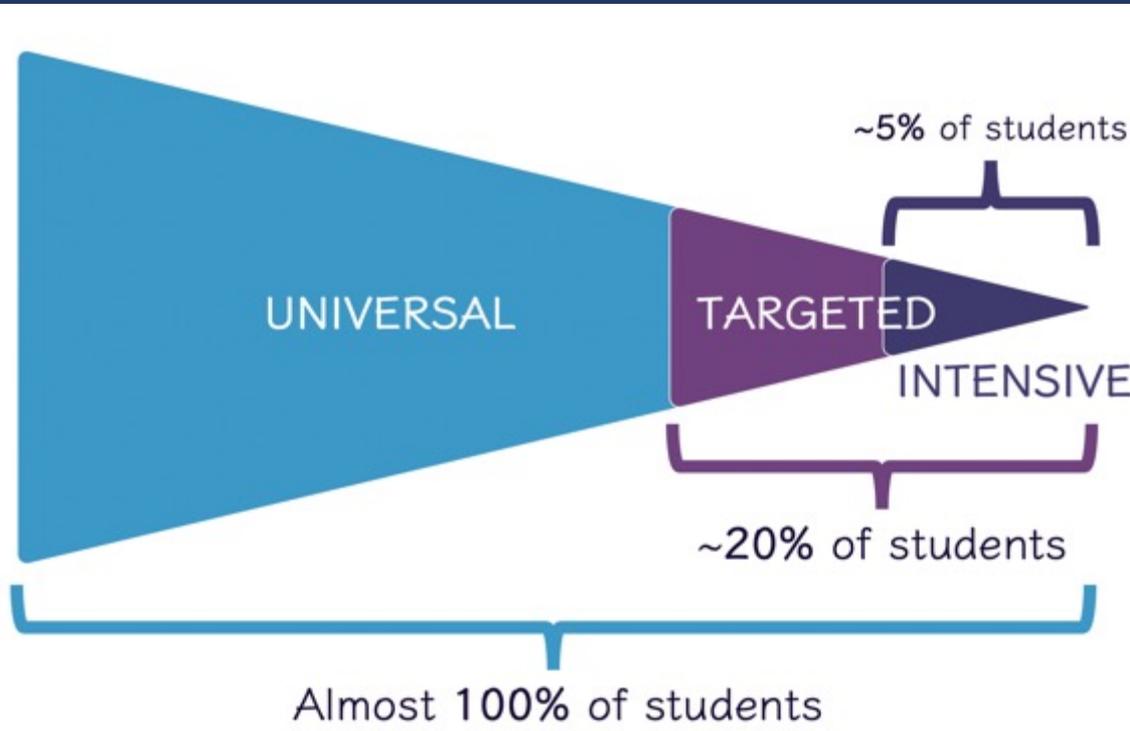


Also known as Tier 3 or tertiary prevention

Designed for students who demonstrate inadequate response to universal and targeted intervention

Occurs within or outside of special education

~5% or less of students require intensive intervention





Describe the MTSS frameworks you have used or are familiar with.

What more would you like to know about MTSS in math?



# Components Across Tiers

## UNIVERSAL

- Universal evidence-based practices
- Screening of all students
- Progress monitoring of at-risk students
- Data-based decision making

## TARGETED

- Targeted evidence-based practices
- Progress monitoring
- Data-based decision making

## INTENSIVE

- Diagnostics
- Intensive evidence-based practices
- Progress monitoring
- Data-based decision making



# Evidence-Based Practices

Instructional techniques supported by high-quality research demonstrating meaningful effects on student outcomes

## UNIVERSAL

- Universal evidence-based practices
- Screening of all students
- Progress monitoring of at-risk students
- Data-based decision making

## TARGETED

- Targeted evidence-based practices
- Progress monitoring
- Data-based decision making

## INTENSIVE

- Diagnostics
- Intensive evidence-based practices
- Progress monitoring
- Data-based decision making



# Progress Monitoring

Using reliable and valid assessments to determine adequacy of response to intervention

## UNIVERSAL

- Universal evidence-based practices
- Screening of all students
- Progress monitoring of at-risk students
- Data-based decision making

## TARGETED

- Targeted evidence-based practices
- Progress monitoring
- Data-based decision making

## INTENSIVE

- Diagnostics
- Intensive evidence-based practices
- Progress monitoring
- Data-based decision making



# Data-Based Decision Making

Making decisions about adequacy of student response using data from progress-monitoring measures

## UNIVERSAL

- Universal evidence-based practices
- Screening of all students
- Progress monitoring of at-risk students
- Data-based decision making

## TARGETED

- Targeted evidence-based practices
- Progress monitoring
- Data-based decision making

## INTENSIVE

- Diagnostics
- Intensive evidence-based practices
- Progress monitoring
- Data-based decision making



# Assessments

Using reliable and valid assessments to determine which students require support and how to adapt intervention.

## UNIVERSAL

- Universal evidence-based practices
- Screening of all students
- Progress monitoring of at-risk students
- Data-based decision making

## TARGETED

- Targeted evidence-based practices
- Progress monitoring
- Data-based decision making

## INTENSIVE

- Diagnostics
- Intensive evidence-based practices
- Progress monitoring
- Data-based decision making

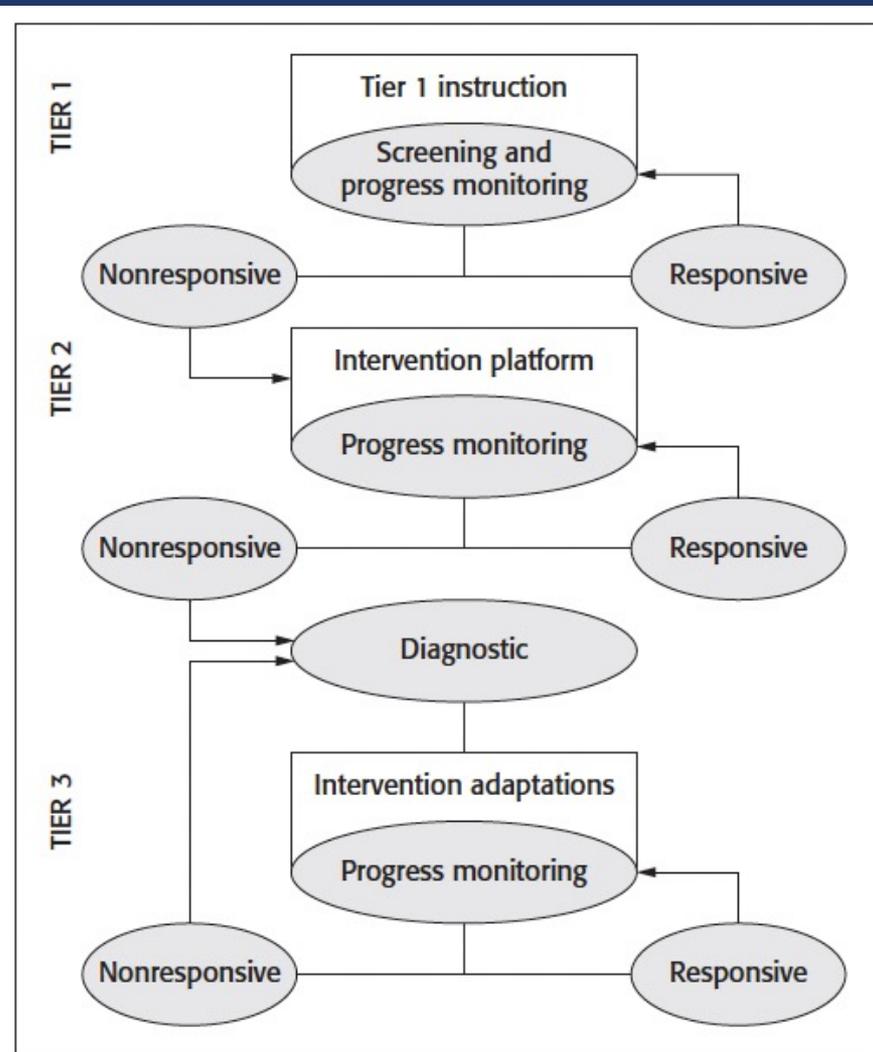




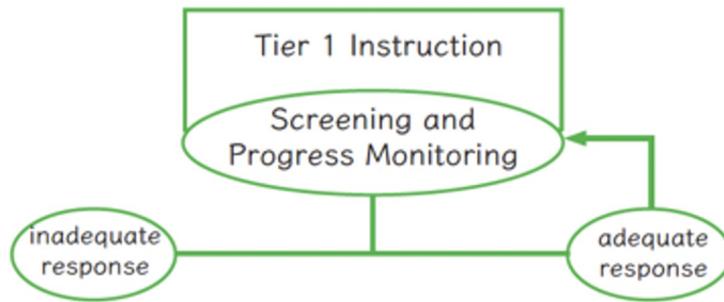
At your school, which component is a strength?

Which component is an opportunity for growth?





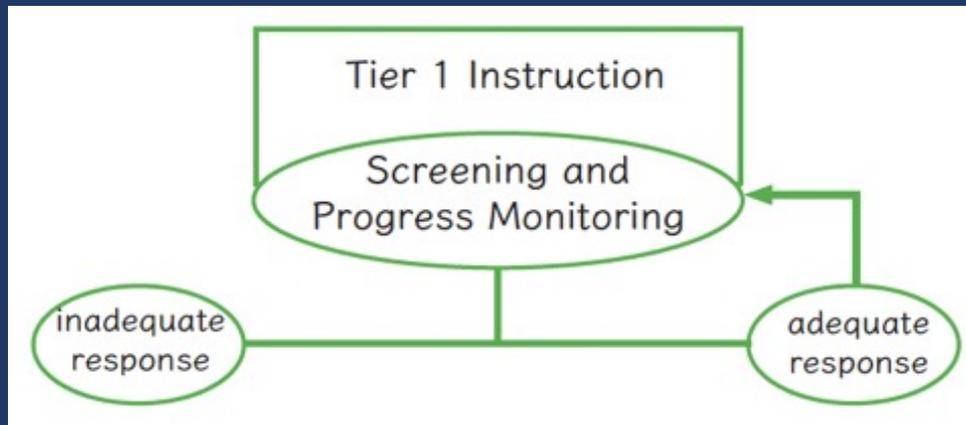
**Figure 2.2.** The three tiers of intervention typically included in the multi-tiered systems of support (MTSS) framework.



Tier 1 Instruction:

Screening and Progress Monitoring:

Decision Making:



- Core instruction utilizes **evidence-based practices**
- All students **screened** (universal screener)
- Students scoring below a cut-score are suspected **at risk** for math difficulties
- Suspected **at-risk students** monitored for 6 to 10 weeks during primary prevention using **progress monitoring**

evidence-based practice

A practice that  
has shown  
consistent and  
positive results



evidence-based practice



evidence-based intervention

An intervention (i.e., packaged program) that has shown **consistent and positive** results



evidence-based practice



evidence-based intervention

evidence-based strategy

A method or strategy that has shown consistent and positive results



evidence-based practice



evidence-based intervention

evidence-based strategy

promising practice

A method or strategy that has shown a positive result



evidence-based practice



evidence-based intervention

evidence-based strategy

promising practice

~~no or negative  
evidence~~





# Websites

## ies.ed.gov/ncee/wwc/

The screenshot shows the homepage of the IES WWC What Works Clearinghouse. At the top, there is a navigation bar with the IES WWC logo, the text 'What Works Clearinghouse', a 'MENU' button, and a search bar with a 'Go' button. Below the navigation bar is a green banner with the text 'Select topics to Find What Works based on the evidence'. Underneath the banner is a grid of 12 topic icons: Literacy, Mathematics, Science, Behavior, Children and Youth with Disabilities, English Learners, Teacher Excellence, Dropout Prevention, Early Childhood (Pre-K), K-12 Kindergarten to 12th Grade, Path to Graduation, and Postsecondary. Below the grid are two main sections: 'WELCOME TO THE WHAT WORKS CLEARINGHOUSE' and 'HIGHLIGHTS'. The 'WELCOME' section contains a paragraph about the clearinghouse's mission. The 'HIGHLIGHTS' section features a 'WEBINAR' card titled 'Finding Evidence: New Resources for Education Researchers from the What Works Clearinghouse'. At the bottom, there is a 'QUICK LINKS' section with three buttons: 'INTERVENTION REPORTS', 'PRACTICE GUIDES', and 'REVIEWS OF INDIVIDUAL STUDIES'.

IES WWC What Works Clearinghouse

Search Go

Select topics to **Find What Works** based on the evidence

- Literacy
- Mathematics
- Science
- Behavior
- Children and Youth with Disabilities
- English Learners
- Teacher Excellence
- Dropout Prevention
- Early Childhood (Pre-K)
- K-12 Kindergarten to 12th Grade
- Path to Graduation
- Postsecondary

**WELCOME TO THE WHAT WORKS CLEARINGHOUSE**

The What Works Clearinghouse (WWC) reviews the existing research on different *programs, products, practices, and policies* in education. Our goal is to provide educators with the information they need to make evidence-based decisions. We focus on the results from *high-quality research* to answer the question "What works in education?" Find more information **about the WWC.**

**HIGHLIGHTS**

**WEBINAR**

**Finding Evidence: New Resources for Education Researchers from the What Works Clearinghouse**

**QUICK LINKS**

- INTERVENTION REPORTS
- PRACTICE GUIDES
- REVIEWS OF INDIVIDUAL STUDIES





# Websites

## www.evidenceforessa.org

**EVIDENCE for ESSA**

ABOUT WHO WE ARE FAQs CONTACT US

MATH PROGRAMS READING PROGRAMS

Search by Program Name

### Proven Programs Successful Students

Your new standard for the most up-to-date and reliable information on programs that meet ESSA evidence standards.

LEARN MORE ABOUT ESSA >

#### Find Evidence Based Programs

Selecting a program is part of a larger [integrated process](#) as recommended by the U.S. Department of Education.

#### Reading Programs

- > Elementary
- > Middle/High School

#### Math Programs

- > Elementary
- > Middle/High School



# Websites

## www.edreports.org

A screenshot of the edreports.org website. The header is dark teal and contains the edreports.org logo on the left, social media icons for Facebook, Twitter, and LinkedIn in the center, and two buttons: 'EMAIL SIGN-UP | GO' and 'SEARCH' on the right. Below the header, the main content area features a blurred background image of hands. The text reads: 'We provide reports that help you evaluate educational materials because high quality content matters to teachers, to kids, and to our collective future.' Below this text are two yellow buttons: 'EXPLORE REPORTS' and 'UP NEXT'.

 [Compare Materials](#) • [Reports Center](#) • [Resources](#) • [Impact](#) • [About Us](#)

[EMAIL SIGN-UP | GO](#) [SEARCH](#)

**We provide reports that help you evaluate educational materials because high quality content matters to teachers, to kids, and to our collective future.**

[EXPLORE REPORTS](#)

[UP NEXT](#)





# Websites

www.intensiveintervention.org

## National Center on INTENSIVE INTERVENTION

at American Institutes for Research

Coaches' Corner

Search

Advanced Search

Resources

Tools Charts

Implementation Support

Instructional Support

About Us

### Interactive DBI Process

**Intensive intervention** helps students with severe and persistent learning or behavioral needs. The Center's approach to intensive intervention is **data-based individualization (DBI)**.

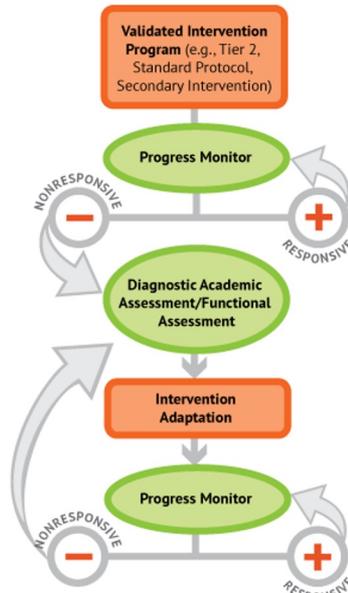
#### What is DBI?

DBI is a research-based process for individualizing and intensifying interventions through the systematic use of assessment data, validated interventions, and research-based adaptation strategies.

Click on the components in the graphic to learn more about the steps in the DBI process and find relevant resources.

**Read more about DBI and view this video to learn why intensive intervention is critical.**

**View the NCII web tour to learn about navigating the site.**



### The NCII Newsletter

Signup for our newsletter and updates!

Subscribe

See us on:

### 15 New Math Videos on Place Value Computation



Do you have students struggling with place value computation? This series of videos focus on multiple strategies to help students struggling with addition, subtraction, multiplication and division.

**Watch the videos.**

### New Reading Lessons & Activities

Looking for lessons to support decoding, vocabulary instruction, comprehension and more? Check out our new sample lessons that include modeling, error correction, practice, and fluency building.

**View the reading lessons.**



evidence-based practice



evidence-based intervention

evidence-based strategy

promising practice

Assessment data to show results

Improvement from before intervention

Improvement compared to no treatment students

Replication

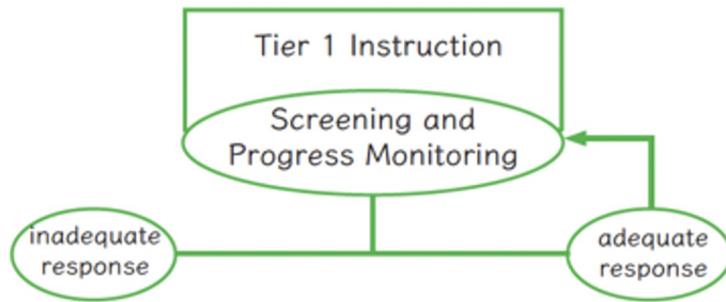
Multiple researchers

Multiple students

Multiple times

Setting and students similar to your own





Tier 1 Instruction:

- Evidence-based practices

Screening and  
Progress Monitoring:

Decision Making:

# Number Identification

6 16 23 10 17

38 97 20 15 24

14 33 11 79 8

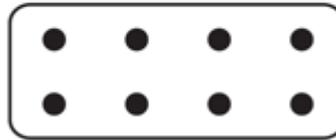
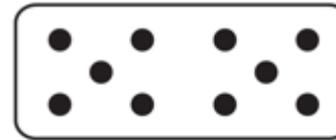
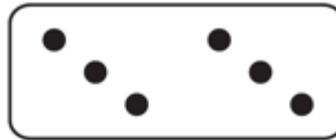
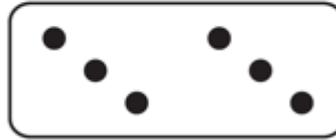
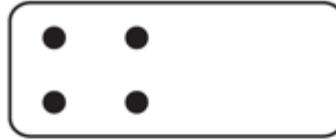
21 19 93 3 49

4 30 12 9 1

28 7 27 2 13



# Quantity Discrimination



# Missing Number

13 \_\_\_ 15 16

50 60 \_\_\_ 80

40 45 \_\_\_ 55

50 51 \_\_\_ 53

23 \_\_\_ 25 26

15 20 \_\_\_ 30

27 28 \_\_\_ 30

38 48 \_\_\_ 68

75 \_\_\_ 85 90

83 \_\_\_ 85 86

Acadience® Math / Computation Grade 4  
Benchmark 1 / Form A

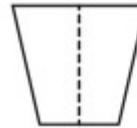
Total: \_\_\_\_\_

1. $\begin{array}{r} 527 \\ +320 \\ \hline \end{array}$	2. $\begin{array}{r} 4778 \\ +2242 \\ \hline \end{array}$	3. $8\frac{4}{5} - 6\frac{2}{5} =$	4. $\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$	5. $4\overline{)573}$
6. $\begin{array}{r} 197 \\ - 74 \\ \hline \end{array}$	7. $\frac{5}{8} + \frac{2}{8} =$	8. $\begin{array}{r} 7273 \\ - 387 \\ \hline \end{array}$	9. $\begin{array}{r} 19 \\ \times 11 \\ \hline \end{array}$	10. $9\frac{7}{12} - 1\frac{4}{12} =$
11. $8\overline{)642}$	12. $7\overline{)49}$	13. $\begin{array}{r} 99 \\ \times 72 \\ \hline \end{array}$	14. $\frac{1}{4} + \frac{2}{4} =$	15. $\begin{array}{r} 526 \\ \times 6 \\ \hline \end{array}$
16. $8\frac{9}{10} - 1\frac{5}{10} =$	17. $\frac{1}{3} + \frac{1}{3} =$	18. $\frac{9}{12} - \frac{2}{12} =$	19. $\begin{array}{r} 829 \\ \times 7 \\ \hline \end{array}$	20. $6\overline{)939}$
21. $3\overline{)397}$	22. $\begin{array}{r} 65 \\ \times 23 \\ \hline \end{array}$	23. $\begin{array}{r} 2414 \\ - 668 \\ \hline \end{array}$	24. $\begin{array}{r} 7568 \\ +1638 \\ \hline \end{array}$	25. $\begin{array}{r} 34 \\ \times 12 \\ \hline \end{array}$

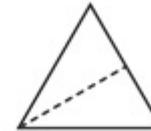
Computation

Total: \_\_\_\_\_

1. Is the dotted line a line of symmetry for each shape? Write "yes" or "no" in the space provided below each shape.



\_\_\_\_\_



\_\_\_\_\_



\_\_\_\_\_

2. Compare the number in Box 1 with the number in Box 2. Fill in the blank with > (greater than), = (equal to), or < (less than):

Box 1	>, =, <	Box 2
835		751
333		613
131		168

3. List three numbers that are multiples of 4:

\_\_\_\_\_

4. Jake read 17 books over the summer that were nonfiction and 43 books that were fiction. His friend Ross read 38 books total. How many more books did Jake read than Ross? \_\_\_\_\_ books.

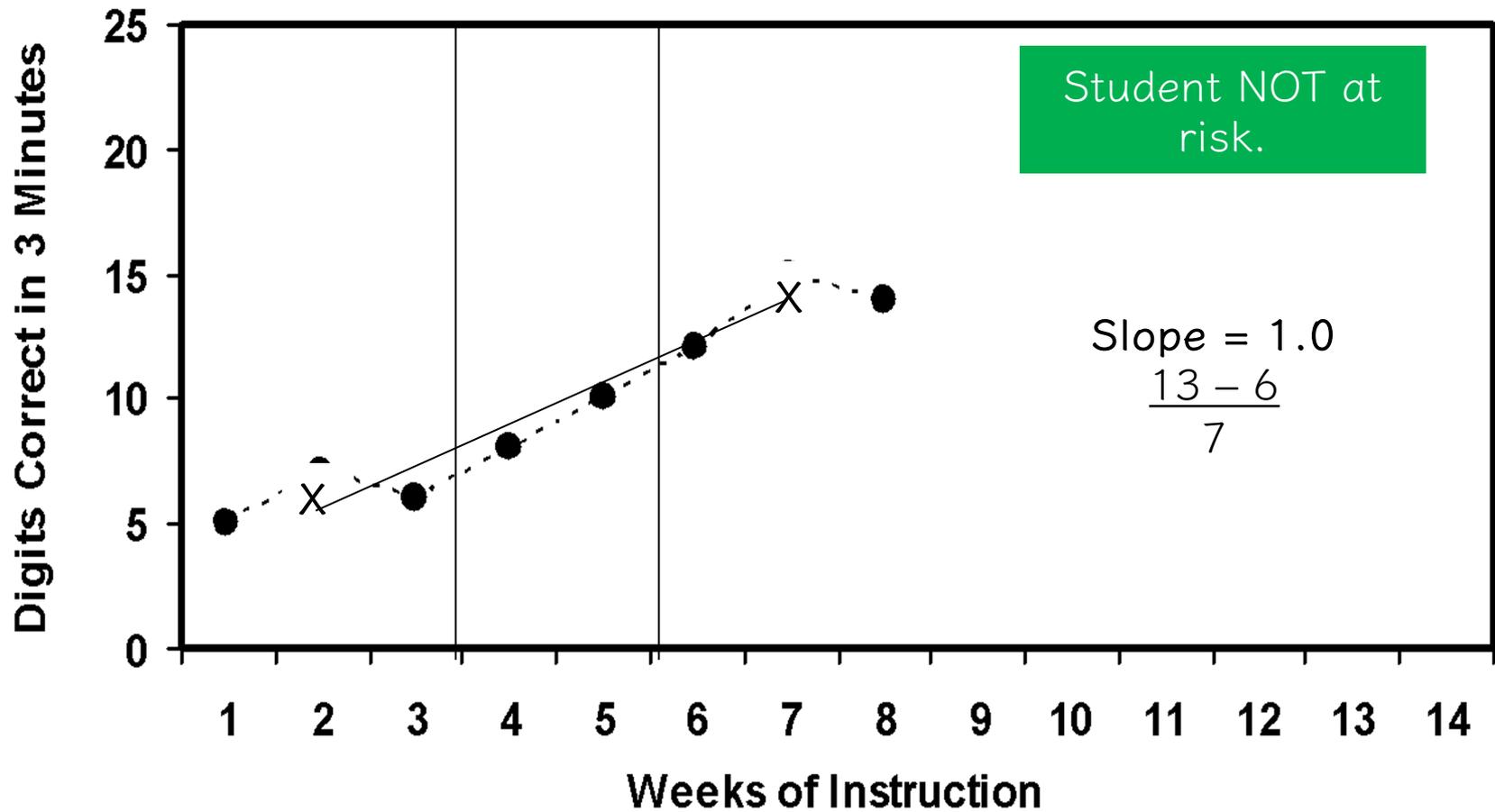
5. Compare the decimal in Box 1 with the decimal in Box 2. Fill in the blank with > (greater than), = (equal to), or < (less than):

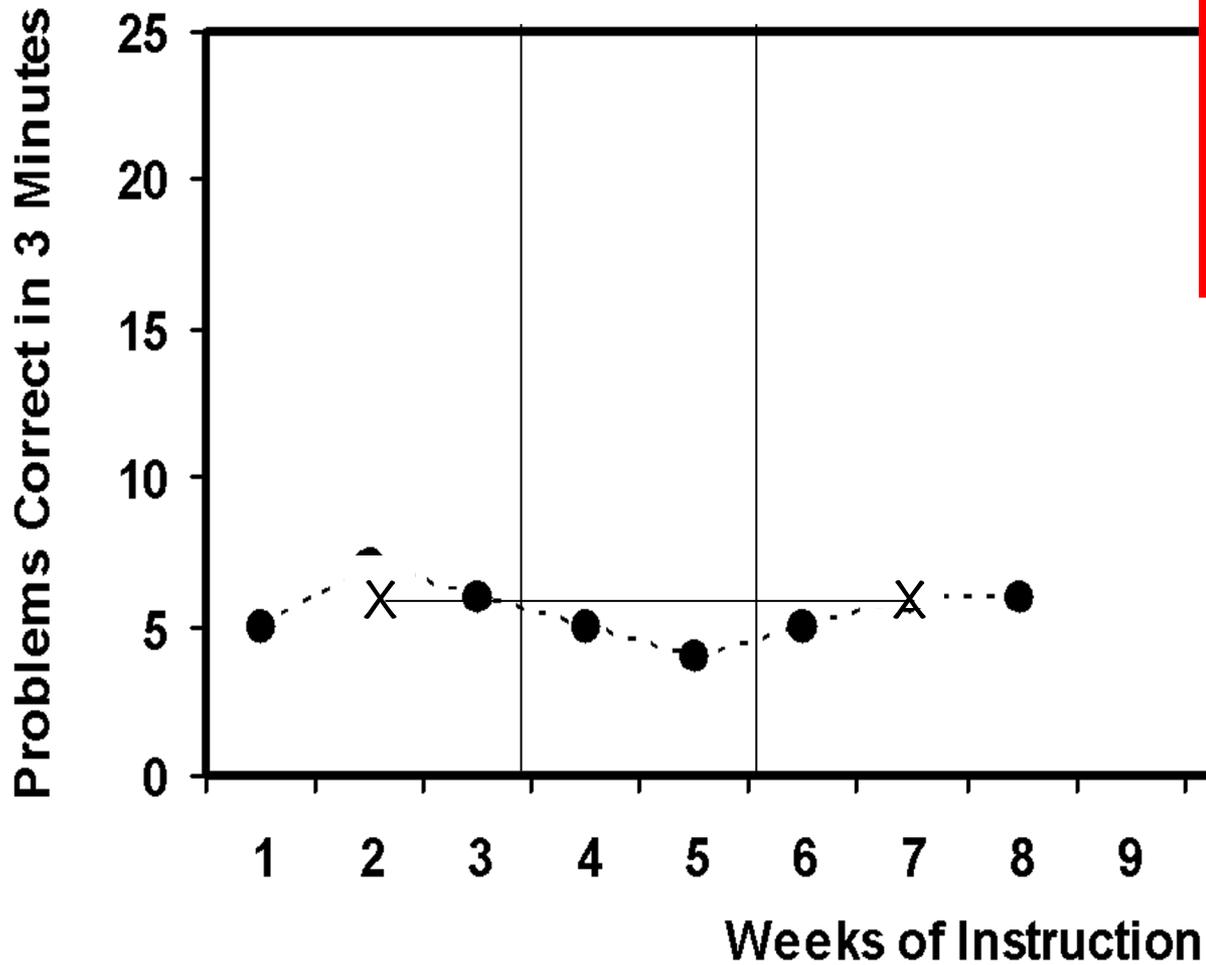
Box 1	>, =, <	Box 2
0.47		0.25
0.39		0.68
0.89		0.91

6. We rented a movie that was 2 hours and 15 minutes long. How many minutes total was the movie? \_\_\_\_\_ minutes.

# Concepts and Applications







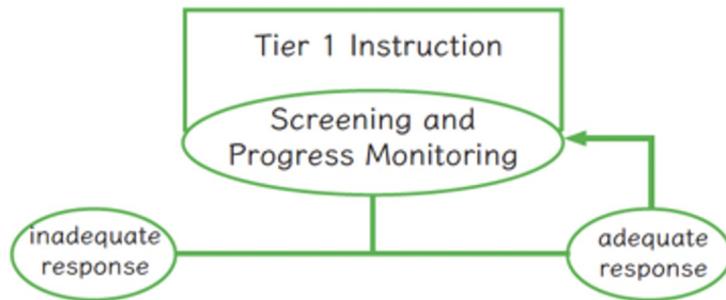
Student AT RISK.

Requires  
additional  
support!

Slope = 0.0

$$\frac{6 - 6}{7}$$





Tier 1 Instruction:

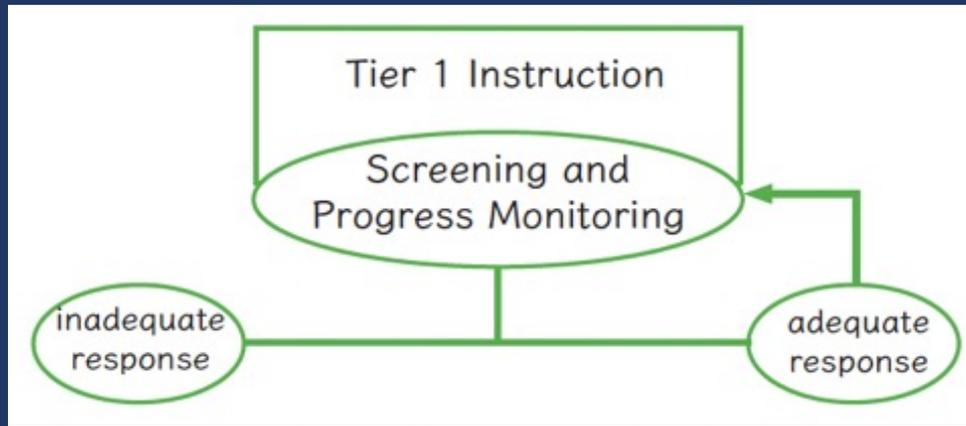
- Evidence-based practices

Screening and Progress Monitoring:

- Reliable measures with normative data
- Usually administered fall, winter, spring
- Reliable measures, administered regularly

Decision Making:

- After 6-10 weeks, student risk status is **confirmed** or **disconfirmed**



Describe your school's Tier 1 strengths.

Describe your school's Tier 1 opportunities for growth.

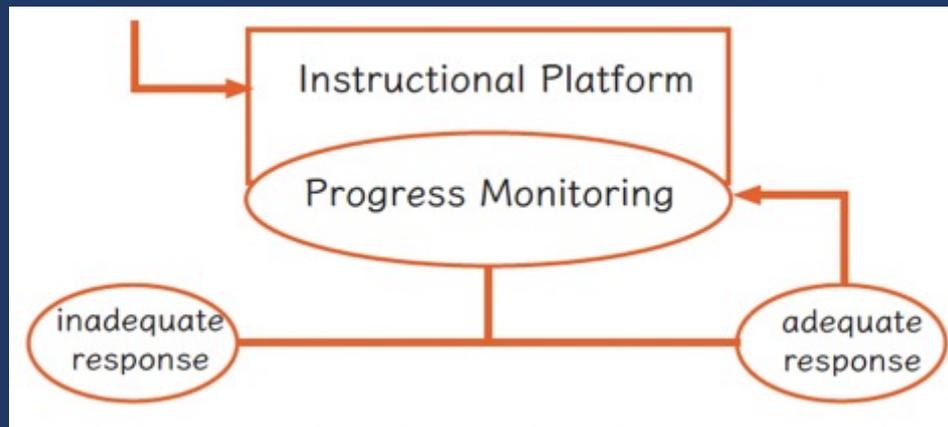




Instructional Platform:

Progress Monitoring:

Decision Making:



- Students are tutored in small groups using **evidence-based practices**
- Tutoring takes place three or four times a week
- Each tutoring session lasts 30 to 60 minutes
- Tutoring lasts 10 to 20 weeks
- Progress monitoring continues weekly

# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving  
instruction



Explicit  
instruction

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



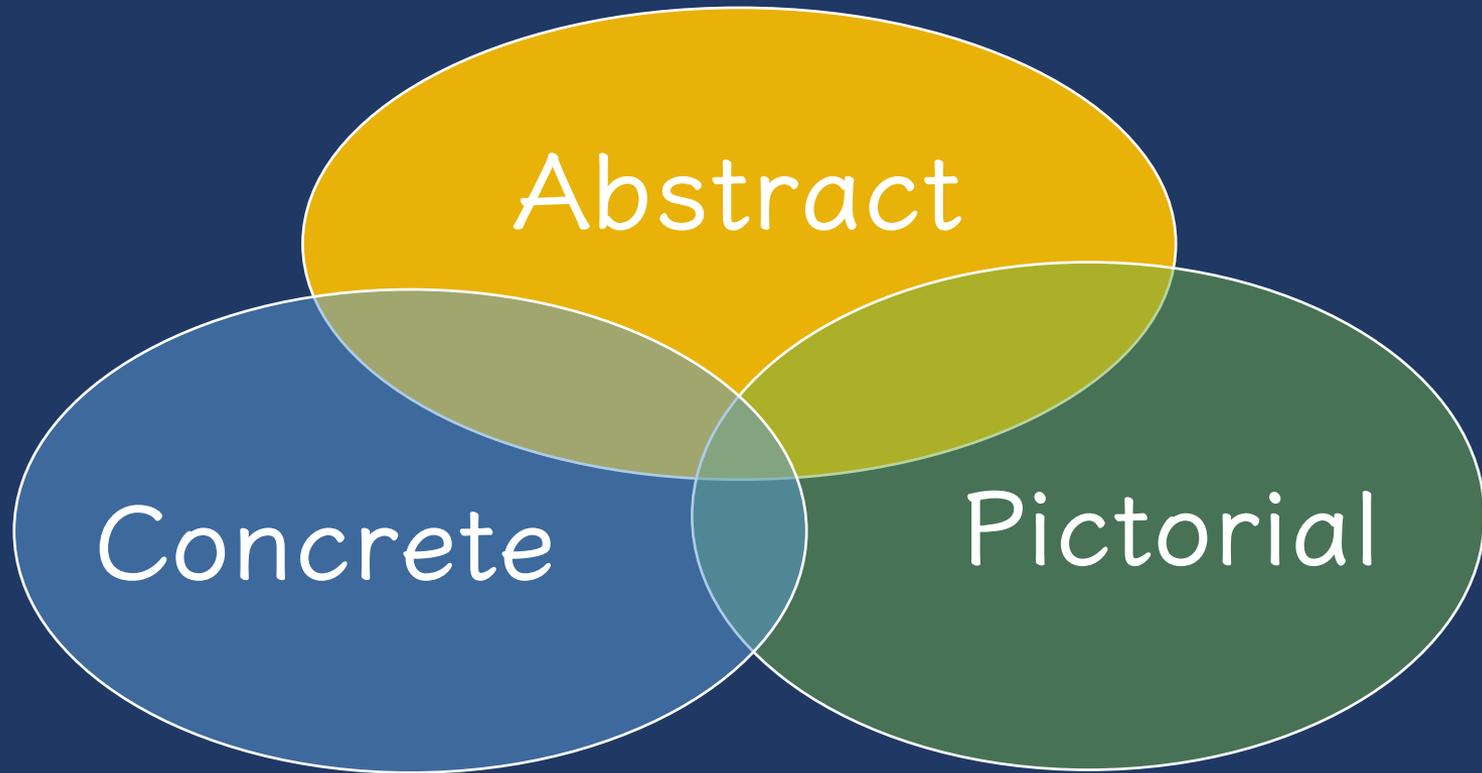
Precise  
language

Use formal math language

Use terms precisely



Multiple  
representations



# Fluency building

Addition

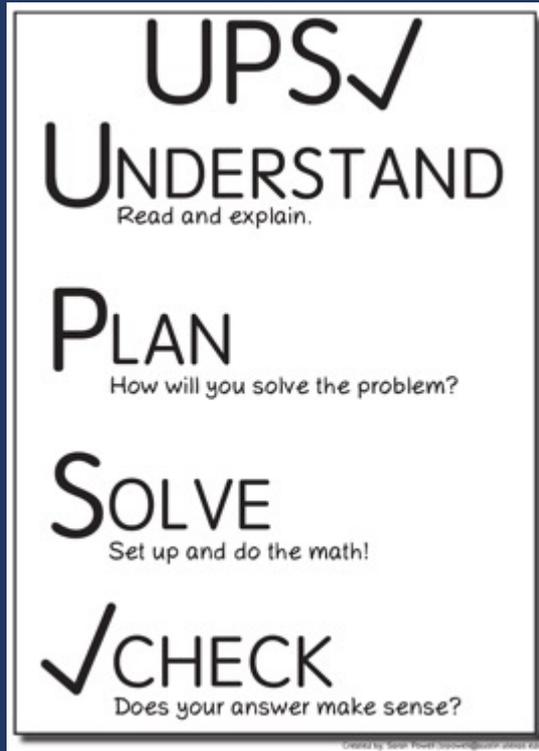
Subtraction

Multiplication

Division



Problem solving  
instruction



Total

Difference

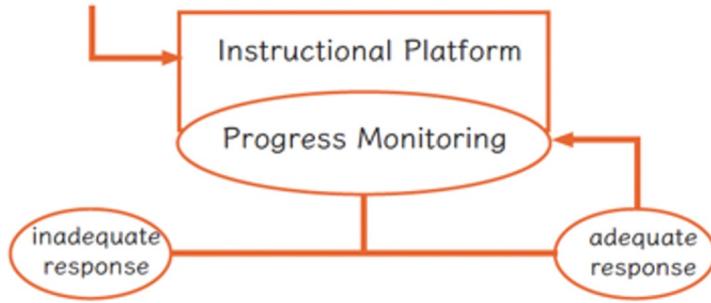
Change

Equal Groups

Comparison

Ratios/Proportions





Instructional Platform:

Explicit instruction

Precise language

Multiple representations

Fluency building

Problem solving instruction

Progress Monitoring:

Decision Making:

# Progress Monitoring Considerations

- Skills to be measured—age and grade appropriate
- Cost and training requirements
- Administration and scoring time
- Data management
- Technical rigor (consider population)
  - Reliability
  - Validity
  - Evidence of being sensitive to change
  - Alternate/parallel forms





Instructional Platform:

Explicit instruction

Precise language

Multiple representations

Fluency building

Problem solving instruction

Progress Monitoring:

- Reliable measures, administered regularly
- Efficient and easy to administer
- Skills assessed serve as indicators of general knowledge

Decision Making:

# Setting Goals

Benchmark

Slope (ROI)

Intra-  
individual



# Determining Response

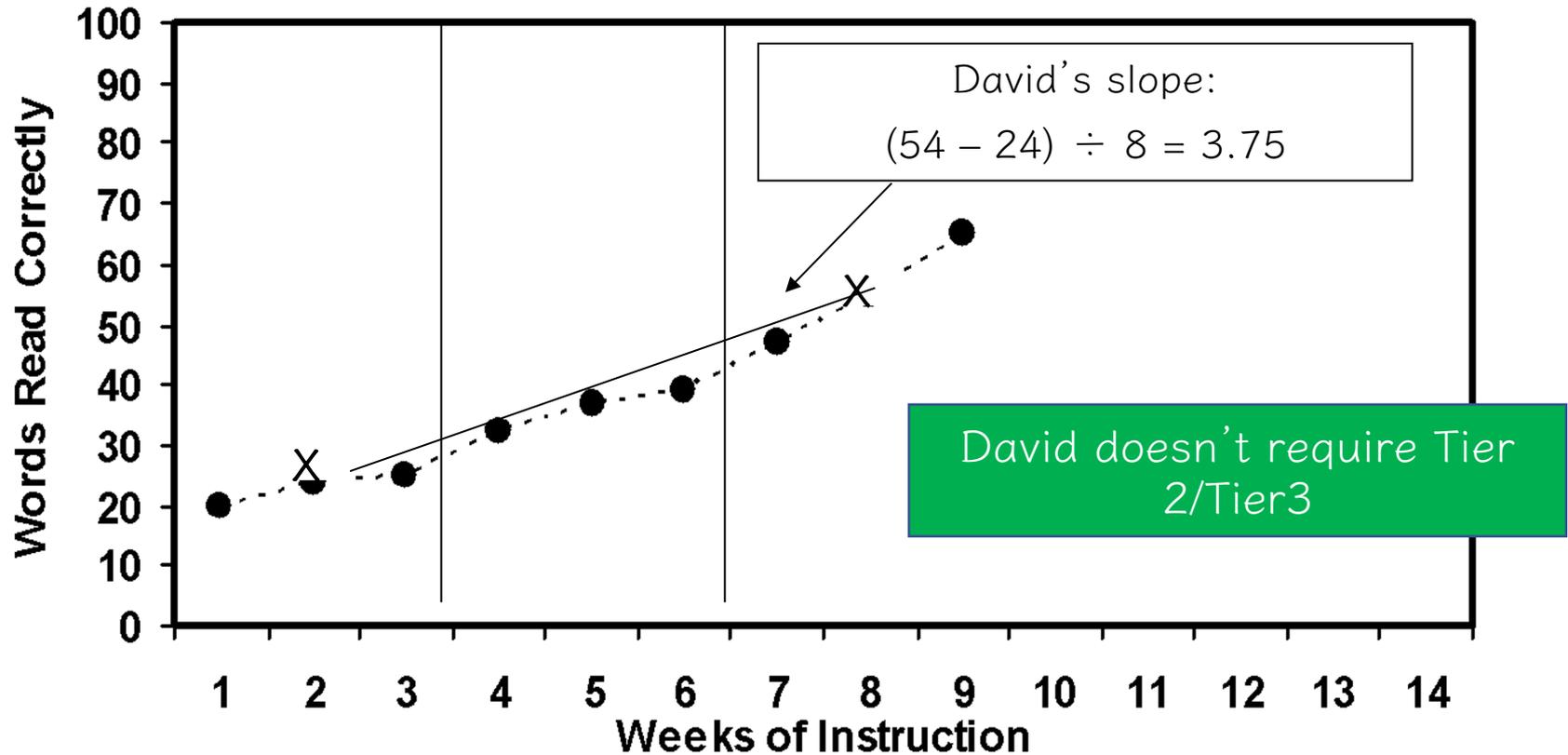
Four most recent, consecutive scores



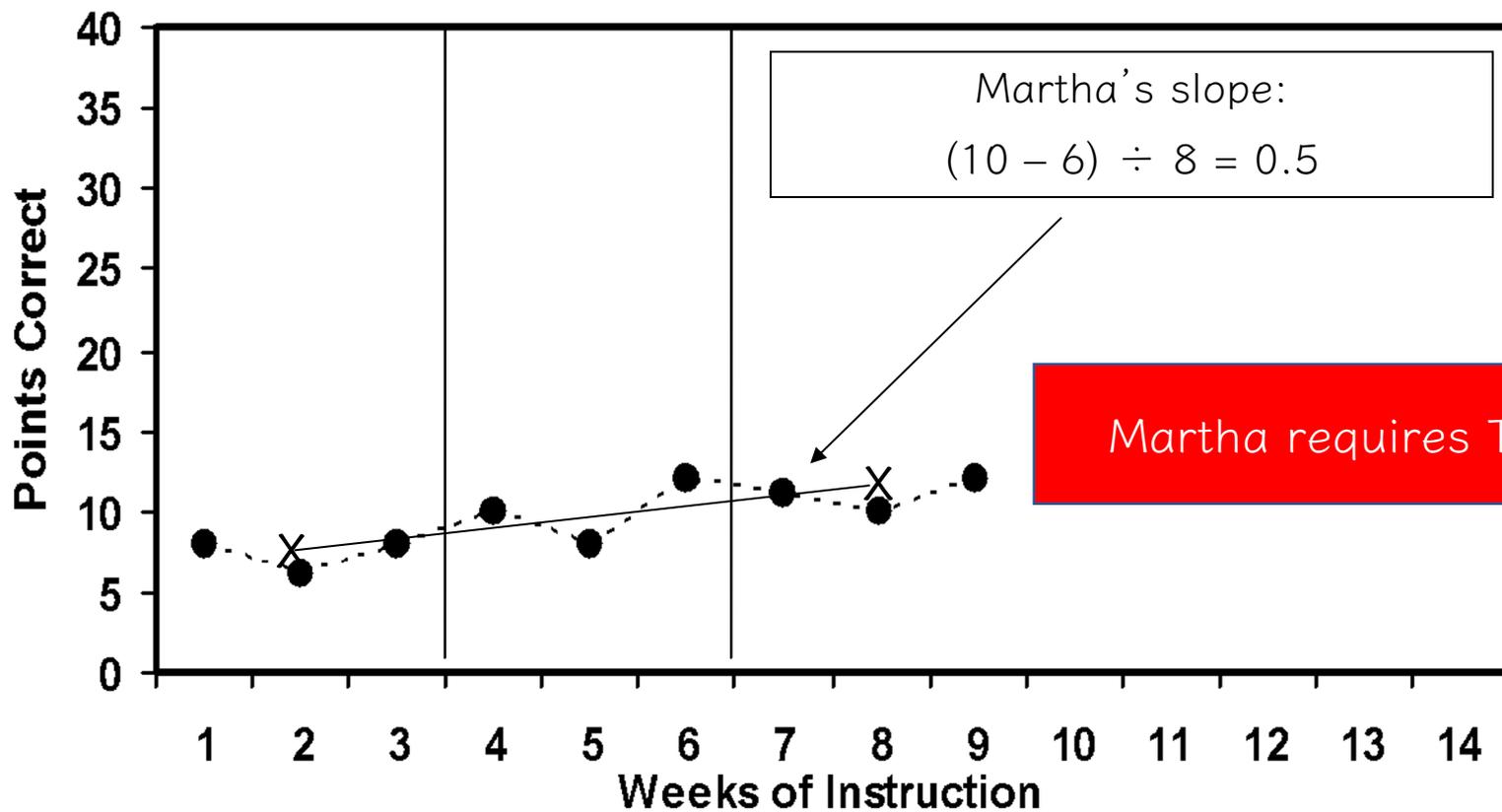
Trendline

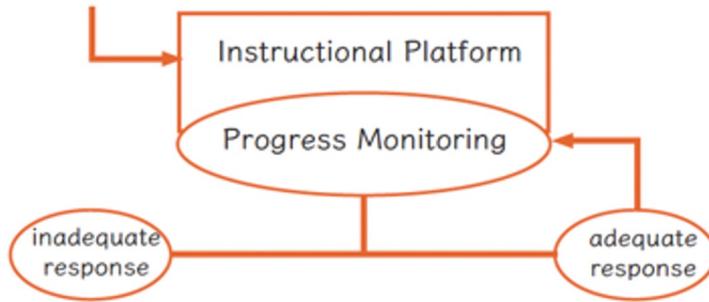


# David



# Martha





Instructional Platform:

Explicit instruction

Precise language

Multiple representations

Fluency building

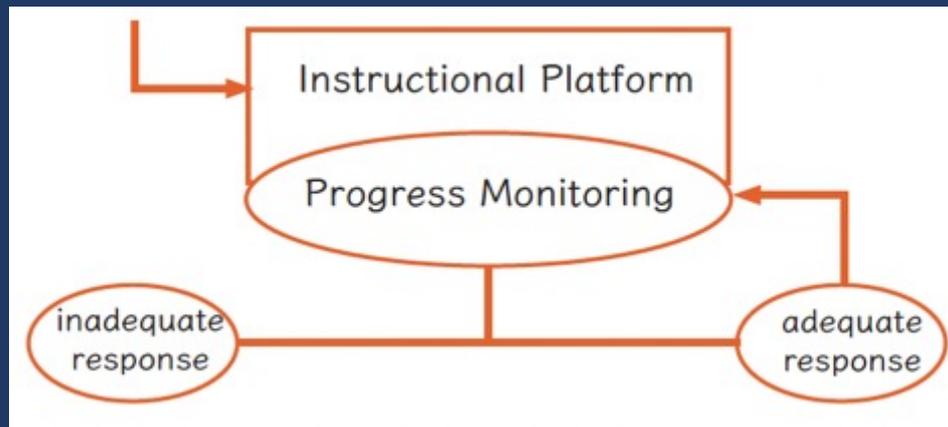
Problem solving instruction

Progress Monitoring:

- Reliable measures, administered regularly
- Efficient and easy to administer
- Skills assessed serve as indicators of general knowledge

Decision Making:

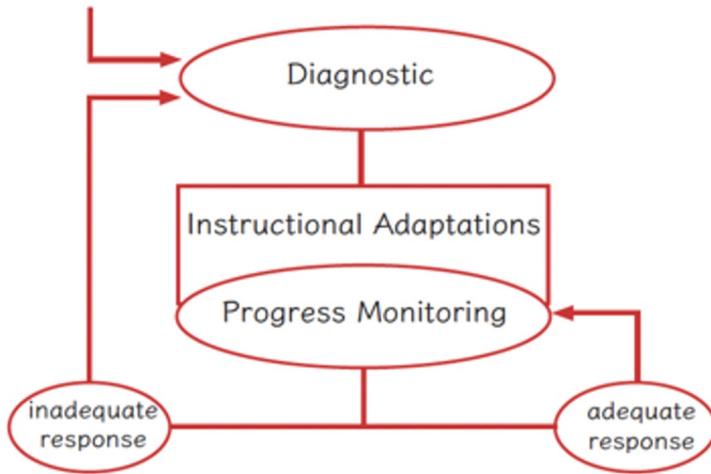
- Teachers set goals
- After 10-20, student progress is determined
  - Adaptations to instructional platform
  - Intensify support (Tier 3)



Describe your school's Tier 2 strengths.

Describe your school's Tier 2 opportunities for growth.

Chapter 2:  
INTENSIVE



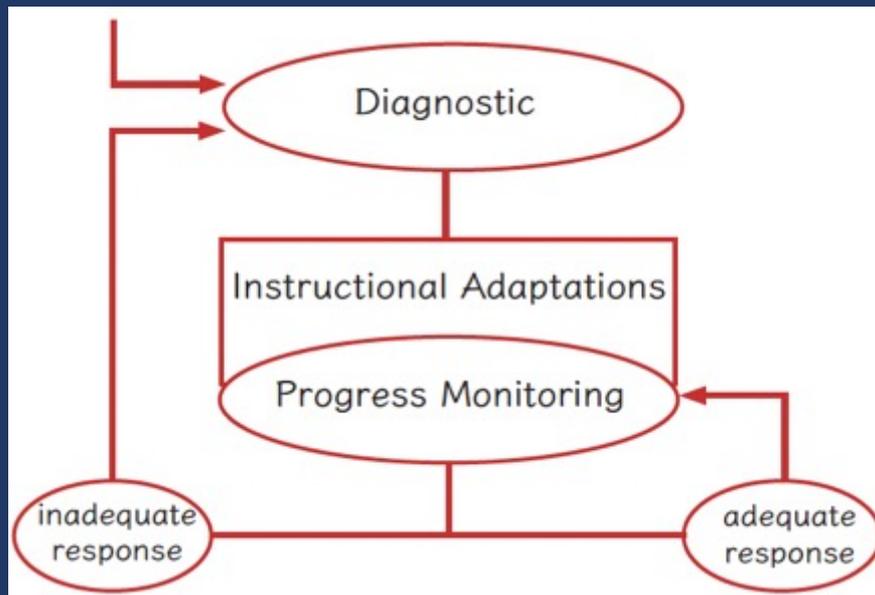
Diagnostic:

Instructional Adaptations:

Progress Monitoring:

Decision Making:





- Diagnostics are conducted
- Adaptations are made to the student's intervention
- Student progress is monitored weekly
  - With adequate slopes or end levels, students return to Tier 1 or 2

## Implement with greater fidelity

Ensure that you are implementing the intervention or strategy with fidelity

### Cover, Copy, and Compare:

1. Create a sheet for the student. This sheet should contain 10 problems and cover material the student needs to practice. All problems should be answered.
2. Ask the student to look at each problem and read it aloud.
3. Ask the student to cover the problem with an index card.
4. Ask student to copy the entire problem to the right of the covered problem.
5. Ask student to lift up index card and compare his or her copy to the original.
6. Repeat for all problems.
7. Conduct three times per week.

### Math Fact Flash Cards

- \_\_ Tutor greets student.
- \_\_ Tutor starts timer.
- \_\_ Tutor begins flash card activity immediately.
- \_\_ Tutor reminds student of flash card procedures; answers questions if necessary.
- \_\_ Tutor sets timer for 1 minute.
- \_\_ Tutor allows student to respond to cards.
- \_\_ Tutor prompts student to Count Up if incorrect.
- \_\_ Tutor stops presenting cards when timer goes off.
- \_\_ Tutor prompts student to count correct cards.
- \_\_ Tutor encourages student to "beat the score" on next round.
- \_\_ Tutor sets timer for 1 minute.
- \_\_ Tutor allows student to respond to cards.
- \_\_ Tutor prompts student to Count Up if incorrect.
- \_\_ Tutor stops presenting cards when timer goes off.
- \_\_ Tutor prompts student to count correct cards.
- \_\_ Tutor prompts student to graph the high score.
- \_\_ Tutor records flash card score in attendance log.
- \_\_ Tutor rewards student with gold coin.

### Word Problem Warm-Up

- \_\_ Tutor presents word problem from previous session.
- \_\_ Tutor encourages student to talk through problem.
- \_\_ Tutor assists with explanation, as needed.
- \_\_ Tutor rewards student with gold coin.

### Tutoring Lesson

- \_\_ Tutor begins tutoring lesson immediately.
- \_\_ Tutor prompts student to describe Counting Up strategy.
- \_\_ Tutor quizzes student on 4 math facts, re-quizing if incorrect.
- \_\_ Tutor presents story problem #1.
- \_\_ Tutor allows time for student to respond.
- \_\_ Tutor praises/corrects student's response.
- \_\_ Tutor rewards student with gold coin.

- \_\_ Tutor presents story problem #2.
- \_\_ Tutor allows time for student to respond.
- \_\_ Tutor praises/corrects student's responses.
- \_\_ Tutor rewards student with gold coin.
- \_\_ Tutor presents story problem #3.
- \_\_ Tutor allows time for student to respond.
- \_\_ Tutor praises/corrects student's responses.
- \_\_ Tutor rewards student with gold coin.

### Sorting Activity

- \_\_ Tutor begins sorting activity immediately.
- \_\_ Tutor reminds student of sorting procedures and answers questions as necessary.
- \_\_ Tutor sets timer for 2 minutes.
- \_\_ Tutor reads cards out loud for student.
- \_\_ Tutor allows student to place cards on sorting mat without interrupting.
- \_\_ Tutor prompts student to stop when timer goes off.
- \_\_ Tutor goes through correction procedure with up to 3 cards from "incorrect" pile.
- \_\_ Tutor goes through cards with student, counting the number of correct cards.
- \_\_ Tutor rewards student with gold coin.
- \_\_ Tutor records sorting cards score on Attendance Log.

### Pirate Problems Daily Review

- \_\_ Tutor begins Pirate Problems Daily Review immediately.
- \_\_ Tutor reminds student of Pirate Problems procedures; answers questions as necessary.
- \_\_ Tutor sets timer for 2 minutes.
- \_\_ Tutor allows student to work independently for 2 minutes.
- \_\_ Tutor prompts student to stop when timer goes off.
- \_\_ Tutor sets timer for 2 more minutes (for word problem on back).
- \_\_ Tutor allows student to work independently for 2 more minutes.
- \_\_ Tutor prompts student to stop when timer goes off.
- \_\_ Tutor corrects the problems while student watches.
- \_\_ Tutor models Counting Up strategy for incorrectly answered items.
- \_\_ Tutor writes score on corner of sheet.
- \_\_ Tutor records Pirate Problems score in attendance log.
- \_\_ Tutor rewards student with gold coin.
- \_\_ Tutor prompts student to count coins and mark on map.
- \_\_ Tutor dismisses student to return to class.
- \_\_ Tutor stops timer.
- \_\_ Tutor records time of session in attendance log.
- \_\_ Tutor records date in attendance log.



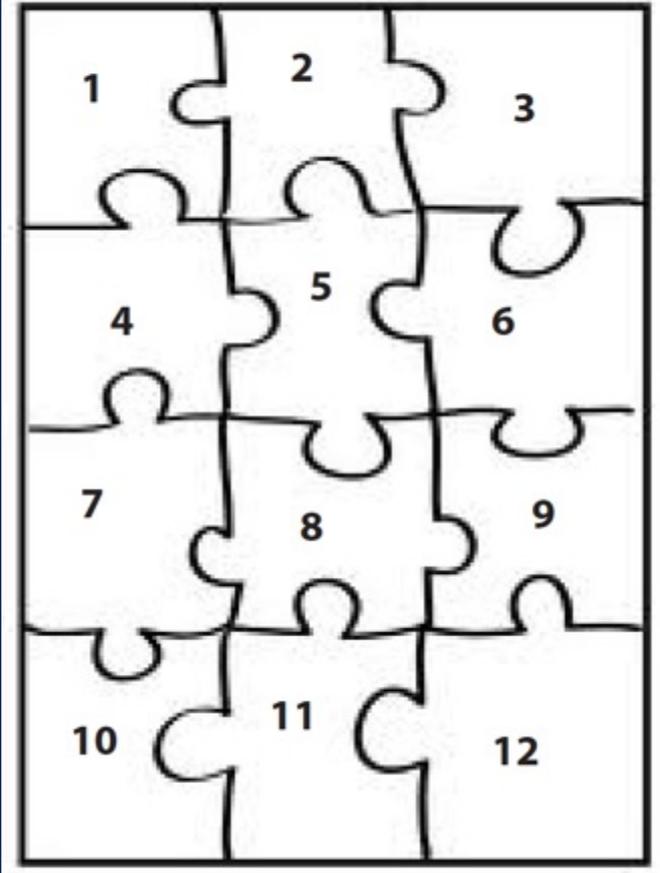
Implement with greater fidelity

Embed behavioral supports

May want to incorporate strategies to improve self-regulation and minimize nonproductive behavior

UPSCheck  
Understand  
Plan  
Solve  
Check

## PUZZLE



Implement with greater fidelity

Embed behavioral supports

Increase dosage

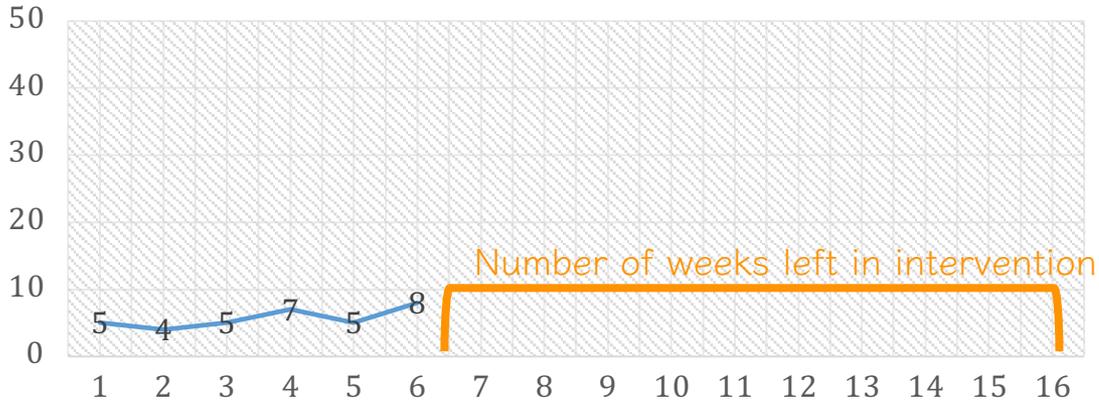
Conduct longer sessions, more sessions per week, or more weeks within DBI



## September

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	1	2	3

## Maria's Progress



Implement with greater fidelity

Embed behavioral supports

Increase dosage

Adapt mathematics content



Implement with greater fidelity

Embed behavioral supports

Increase dosage

Adapt mathematics content

Utilize explicit instruction

Make sure you're doing it! And do it well!

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



Implement with greater fidelity

Embed behavioral supports

Increase dosage

Adapt mathematics content

Utilize explicit instruction

Explicitly teach transfer

Explicitly teach how current learning relates to other learning

$$\begin{array}{r} 405 \\ + 16 \\ \hline \end{array} \quad \begin{array}{r} 4305 \\ + 216 \\ \hline \end{array}$$

Marney baked 89 cookies and sold 40 cookies at the bake sale. How many cookies does Marney have left?

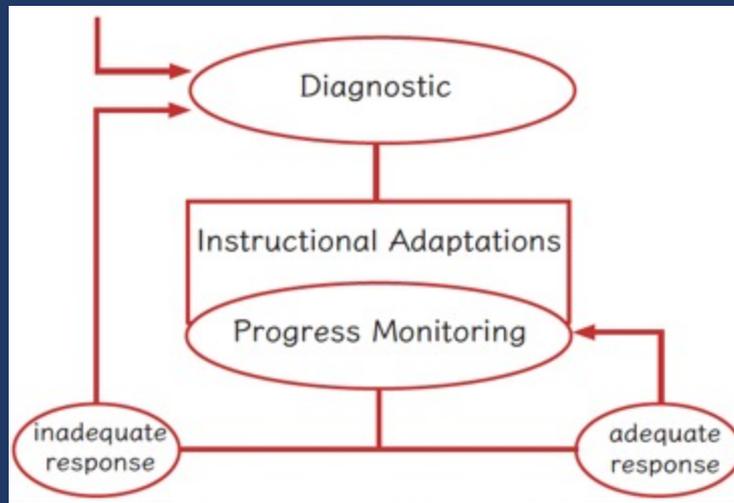
Marney had \$89 and spent \$40 on shoes. How much does Marney have left?

Marney had \$89 and spent \$40 on shoes. How much money will Marney have after buying the shoes?









Describe your school's Tier 3 strengths.

Describe your school's Tier 3 opportunities for growth.



# Teaching Math in Middle School

Using MTSS to Meet All Students' Needs



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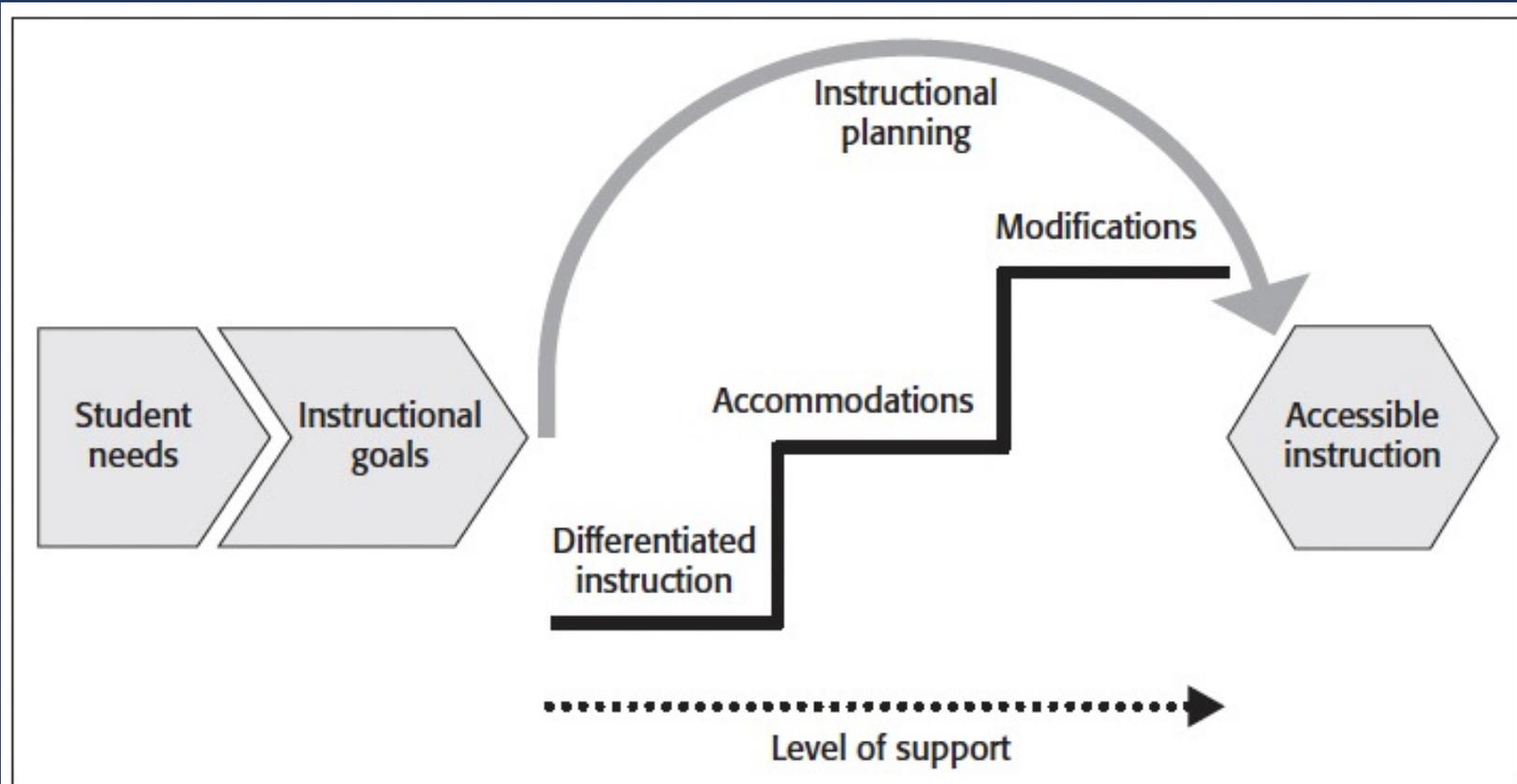
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Chapter 3: Differentiation, Accommodation, and Modification

Chapter 5: Evidence-Based Practices





**Figure 3.1.** Relationship between a student’s needs and the level of support required to enhance accessibility. (From Ketterlin-Geller L.R., Jamgochian E.M. [2011] *Instructional Adaptations: Accommodations and Modifications That Support Accessible Instruction*. In: Elliott S., Kettler R., Beddow P., Kurz A. [eds.] *Handbook of Accessible Achievement Tests for All Students*, 131–146. Springer, New York, NY.)

## Differentiated Instruction

Intentionally changing the design and/or delivery of instruction to support students' access to learning

Content expectations do not change

## Accommodations

Changes to the presentation, setting, timing or schedule, and response mode of instruction

Does not change the instructional objective

## Modifications

Changes the instructional objective



Differentiated  
Instruction

Accommodations

Modifications

Changes to the presentation, setting, timing or schedule, and response mode of instruction

**Table 3.3.** Some examples of presentation changes

Accommodations	Modifications
<ul style="list-style-type: none"><li>• Audio- or video-record a lesson instead of taking notes</li><li>• Read the directions and/or problems aloud to the student</li><li>• Increase the font size (e.g., enlarge text, use a magnification device)</li><li>• Increase the contrast or differentiation of information included in visual representations (e.g., use color to help students identify corresponding sides on similar figures)</li><li>• Increase white space on assignments</li><li>• Reduce the number of items on a page</li><li>• Allow the student to use a screen reader</li><li>• Provide tactile prompts such as physical guidance or raised-line paper</li><li>• Allow the student to use highlighters</li><li>• Provide the student with a copy of notes or class presentations before the lesson begins</li><li>• Allow the student to use a dictionary that does not include mathematical terms</li></ul>	<ul style="list-style-type: none"><li>• Allow the student to read shorter versions of the mathematics textbook that may not contain grade-level vocabulary</li><li>• Shorten story problems by reducing the number of relevant mathematical steps needed to respond</li><li>• Reduce the reading expectation for word problems (e.g., removing irrelevant information)</li><li>• Allow the student to use a dictionary on tests that includes mathematical terms</li></ul>



Differentiated  
Instruction

Accommodations

Modifications

Changes to the presentation, setting, timing or schedule, and response mode of instruction

**Table 3.4.** Some examples of setting changes

Accommodations	Modifications
<ul style="list-style-type: none"><li>• Change the location where the student is completing the assignment</li><li>• Provide a separate location for the student to complete the assignment</li><li>• Allow the student to use a physical device to reduce distractions (i.e., headphones or study carrel)</li><li>• Allow the student to complete an assignment in a small group or in a room with fewer students</li><li>• Use specialized lighting or acoustic devices</li></ul>	<ul style="list-style-type: none"><li>• Allow the student to work with a partner on a task that is intended to be completed alone</li></ul>



Differentiated  
Instruction

Accommodations

Modifications

Changes to the presentation, setting, timing or schedule, and response mode of instruction

**Table 3.5.** Some examples of timing or scheduling changes

Accommodations	Modifications
<ul style="list-style-type: none"><li>• Provide longer time for the student to complete an assignment, as needed</li><li>• Allow the student to take multiple breaks while completing an assignment</li><li>• Allow the student to take a test at a certain time of the day (e.g., first thing in the morning)</li></ul>	<ul style="list-style-type: none"><li>• Provide more time for the student to respond to an assignment or test that is intended to be timed (e.g., allow twice as much time as intended)</li><li>• Extend the number of sessions a student has to complete an assignment or test that is intended to be timed (e.g., allow the student to take a test over 2 days)</li></ul>



Differentiated  
Instruction

Accommodations

Modifications

Changes to the presentation, setting, timing or schedule, and response mode of instruction

**Table 3.6.** Some examples of response mode changes

Accommodations	Modifications
<ul style="list-style-type: none"><li>• Allow the student to write responses to assignments instead of speaking them aloud</li><li>• Allow the student to speak responses to assignments instead of writing them</li><li>• Allow the student to use a communication device</li><li>• Allow audio recording of teachers' presentations</li><li>• Use a scribe</li><li>• Use concrete objects and manipulatives</li><li>• Use a graphic organizer to organize one's thoughts</li><li>• Use a calculator or multiplication chart on an assignment that <i>does not</i> assess computation</li></ul>	<ul style="list-style-type: none"><li>• Require fewer items on an assignment or fewer assignments</li><li>• Reduce the number of answer options on a multiple-choice assignment</li><li>• Describe one's thinking (i.e., explanation, justification) at a reduced depth</li><li>• Use a calculator or multiplication chart on an assignment that <i>does</i> assess computation</li><li>• Have material scored using a different rubric or level of expectations than other students</li></ul>



Differentiated  
Instruction

Accommodations

Modifications



Describe your strengths with accommodations and modifications.

Describe your opportunities for growth with accommodations and modifications.



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# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving  
instruction



evidence-based practice



evidence-based intervention

evidence-based strategy

promising practice

Assessment data to show results

Improvement from before intervention

Improvement compared to no treatment students

Replication

Multiple researchers

Multiple students

Multiple times

Setting and students similar to your own

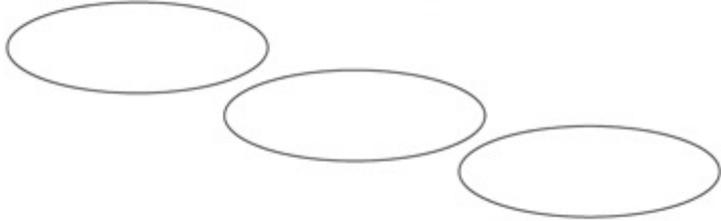




How do you locate and identify evidence-based practices?  
Which evidence-based practices do you plan to use?

Instructional Platform

Instructional Delivery



Instructional Strategies



# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving  
instruction



# Explicit Instruction



Chapter 5: Explicit Instruction

MODELING

PRACTICE

SUPPORTS

Page 69



# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

## INSTRUCTIONAL STRATEGIES



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

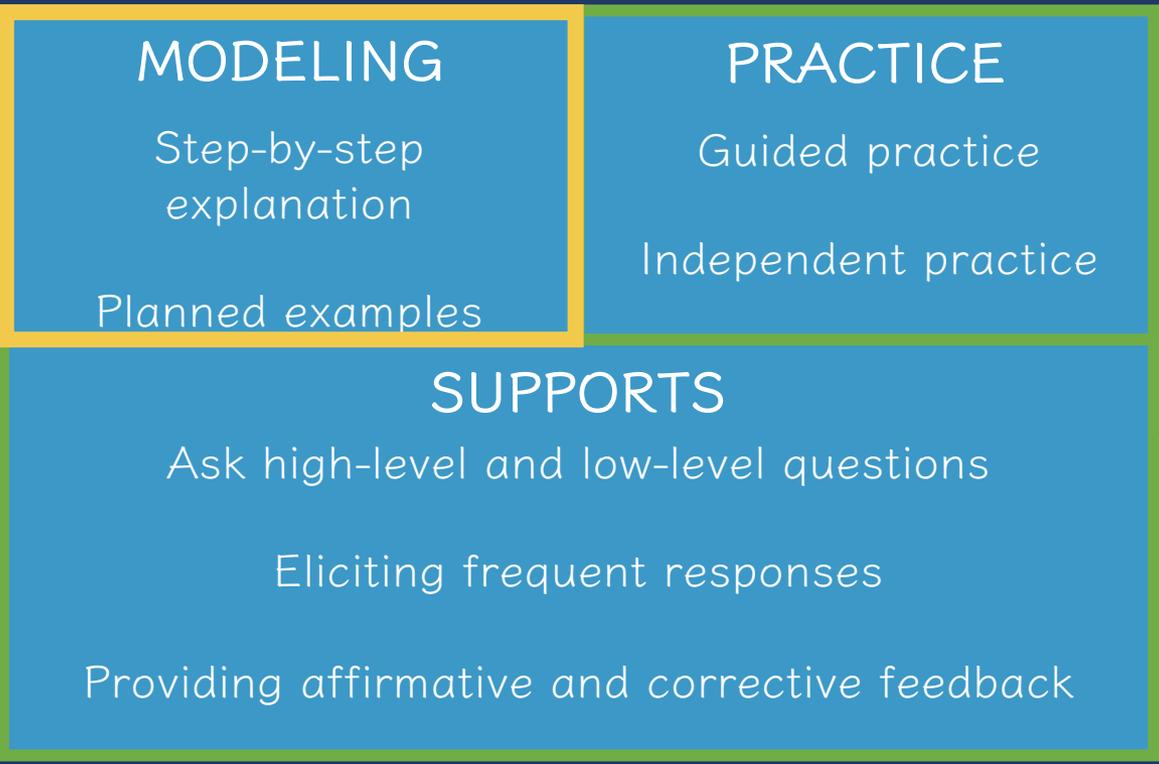
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



Modeling is a dialogue between the teacher and students.



Modeling includes a step-by-step explanation of how to do a math problem.

A teacher may do 1 modeled problem or several.

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback





“Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether.”





“Let’s solve this problem. What’s the problem?”

“26 plus 79.”



“To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?”

“Add.”



“How did you know we want to add?”

“There’s a plus sign.”





“The plus sign tells us we want to add. To add, let’s use the partial sums strategy. What strategy?”

“Partial sums.”



“With the partial sums strategy, we start adding in the greatest place value. What’s the greatest place value in this problem?”

“The tens.”



“So, let’s add the tens. What’s 20 plus 70?”

“90.”





“20 plus 70 equals 90. Let’s write 90 right here below the equal line. What will we write?”

“90.” 

“90 is the partial sum when you add the tens. What does 90 represent?”

“It’s the partial sum of adding 20 plus 70.”



“Now, let’s add the ones. What should we add?”

“6 plus 9.” 





“6 plus 9 equals what?”

“15.” 

“Let’s write 15 below the 90.  
Where do we write the 15?”

“Below the 90.” 

“15 is the partial sum when you add the ones.  
Now, let’s add the partial sums together. What will we add?”

“90 plus 15.” 





“What’s 90 plus 15?”

“How did you add those numbers?”

“So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?”

“105.”



“I added 90 plus 10 then added 5 more.”



“We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums.”



Modeling needs to include planned examples.

## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

These examples should be sequenced so easier skills lead to more difficult skills.



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

Practice continues as a dialogue between the teacher and students.

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Guided practice is practice in which the teacher and students practice problems together.



“Let’s work on a problem together.”



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Independent practice is practice in which the students practice independently with teacher support.



“Now, you’ll practice a problem on your own. Use your attack strategy!”



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

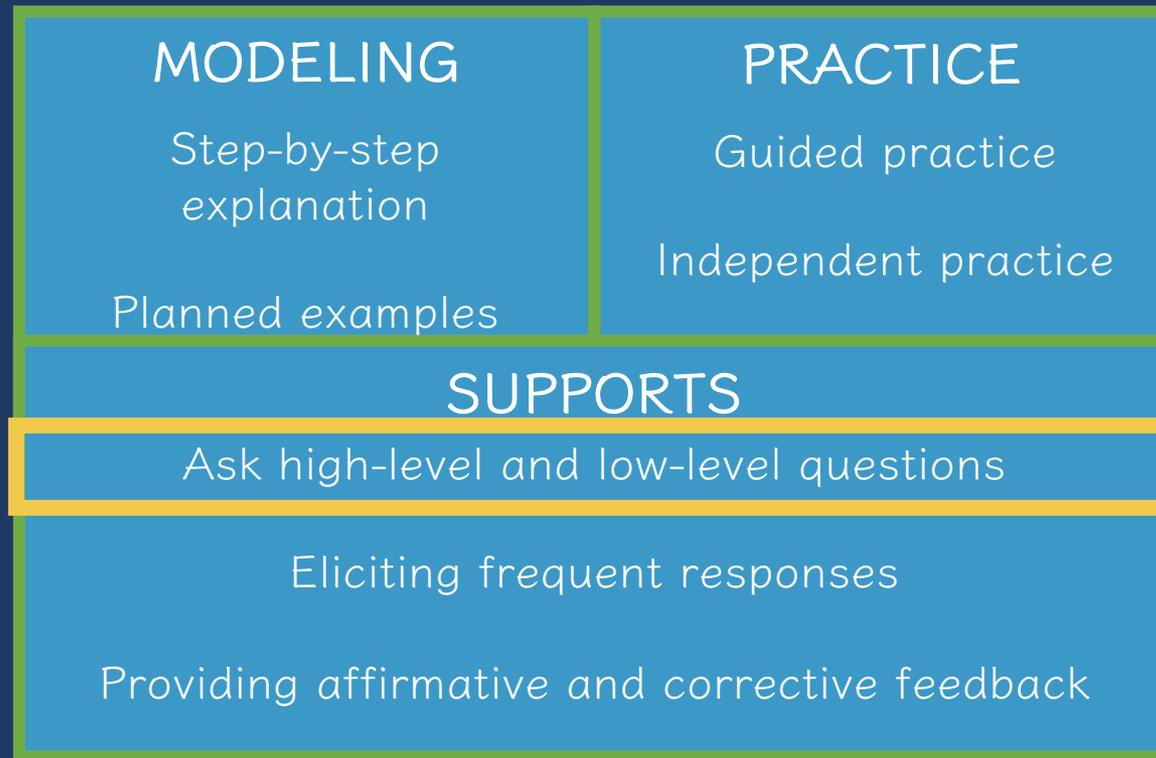
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

These **Supports** should be used in  
both **Modeling** and **Practice**.





During **Modeling** and **Practice**, it is essential to engage students and check for understanding.



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



“What is 7 times 9?”

“63.”



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

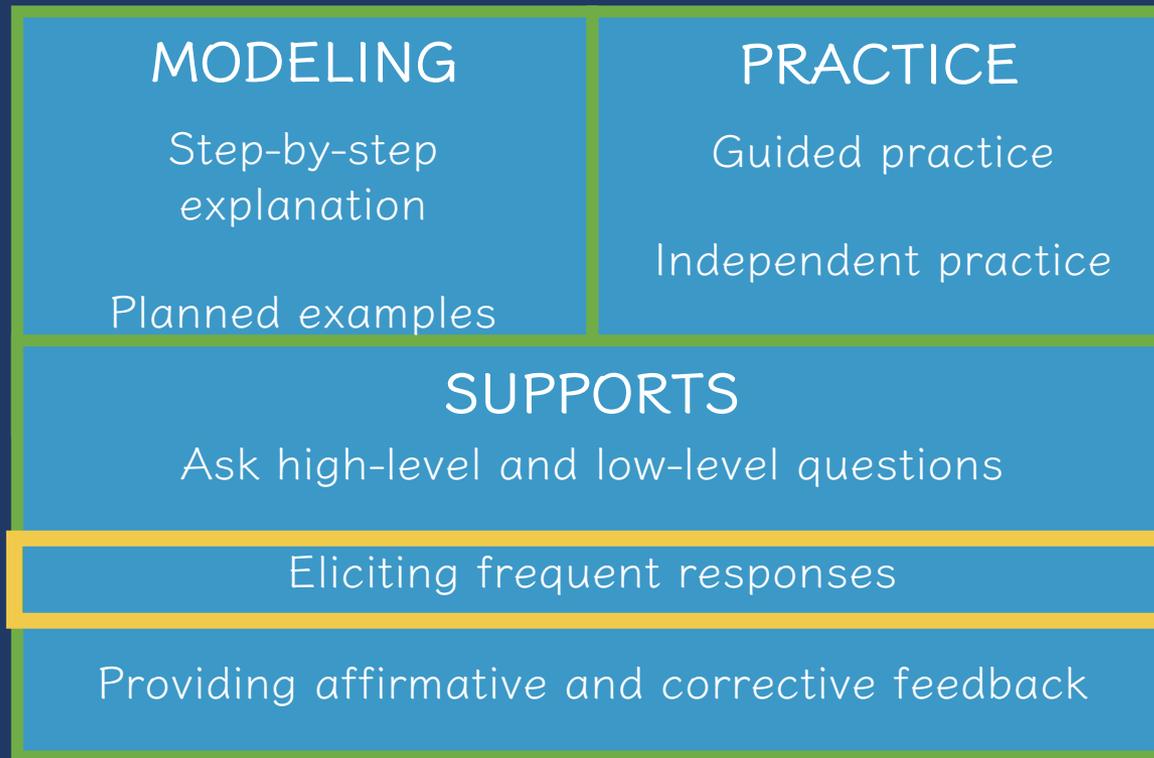
Providing affirmative and corrective feedback



“Why do you use  
zero pairs?”

“Because a positive 1  
and a negative 1 equal  
0. I use the zero pair  
to help me subtract.”





During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

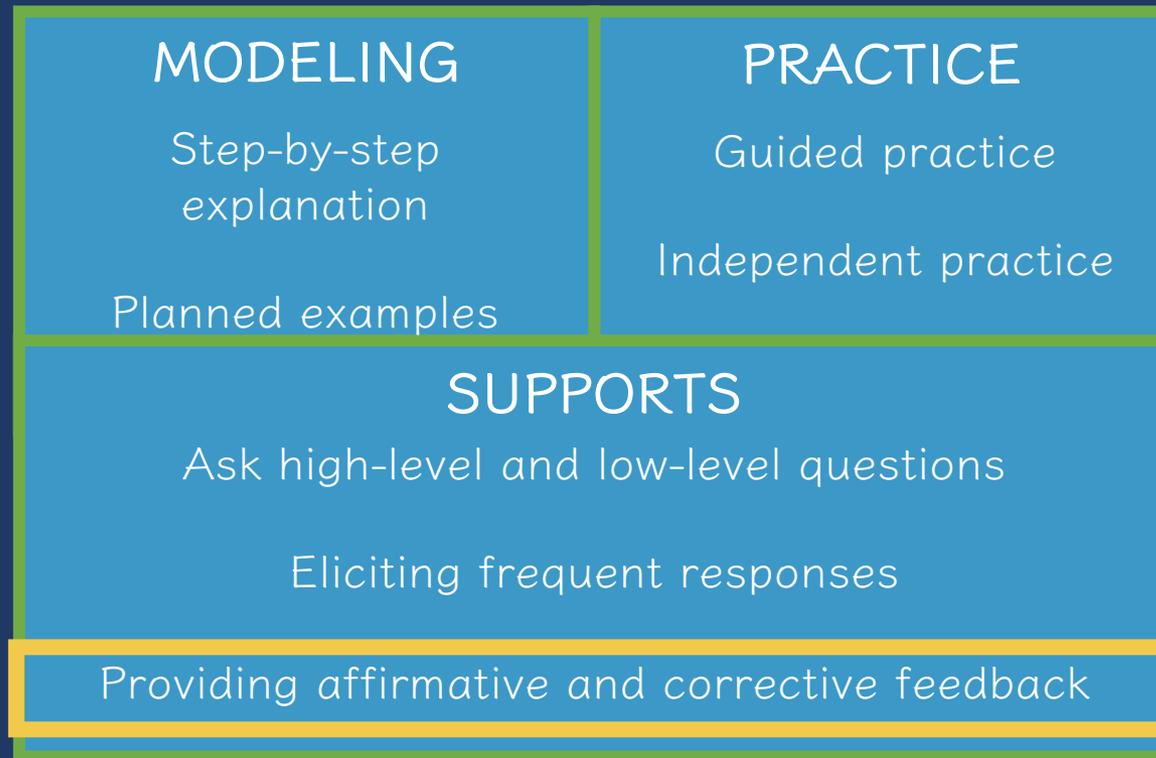
Eliciting frequent responses

Providing affirmative and corrective feedback



- Oral
- Written
- With manipulatives
- With drawings
- With gestures





During **Modeling** and **Practice**, students should receive immediate feedback on their responses.



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



“Nice work using your  
word problem attack  
strategy.”



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



“Let’s look at that again. Tell me how you added in the hundreds column.”



## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



## MODELING

Step-by-step explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback



What are your strengths with explicit instruction?

What are opportunities for growth with explicit instruction?



# Mathematical Language



# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

## INSTRUCTIONAL STRATEGIES

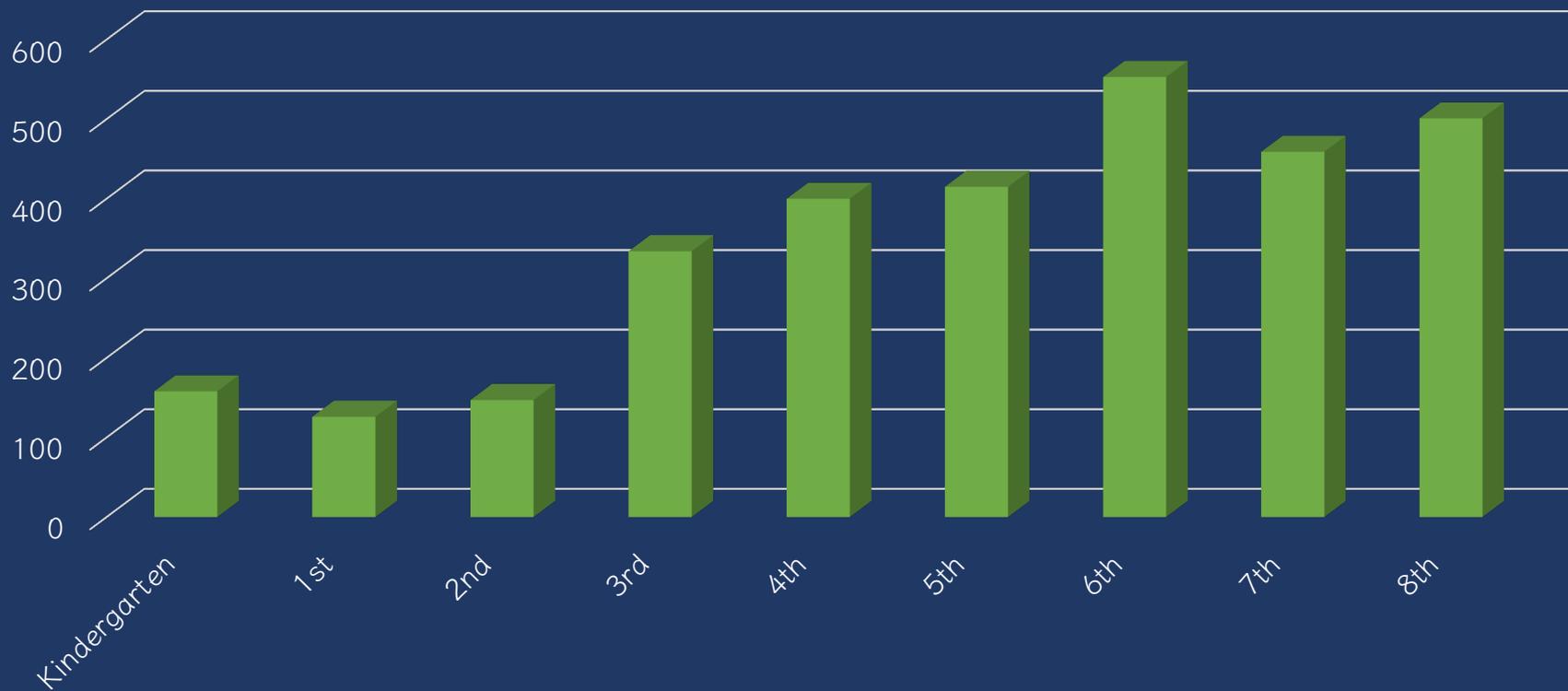


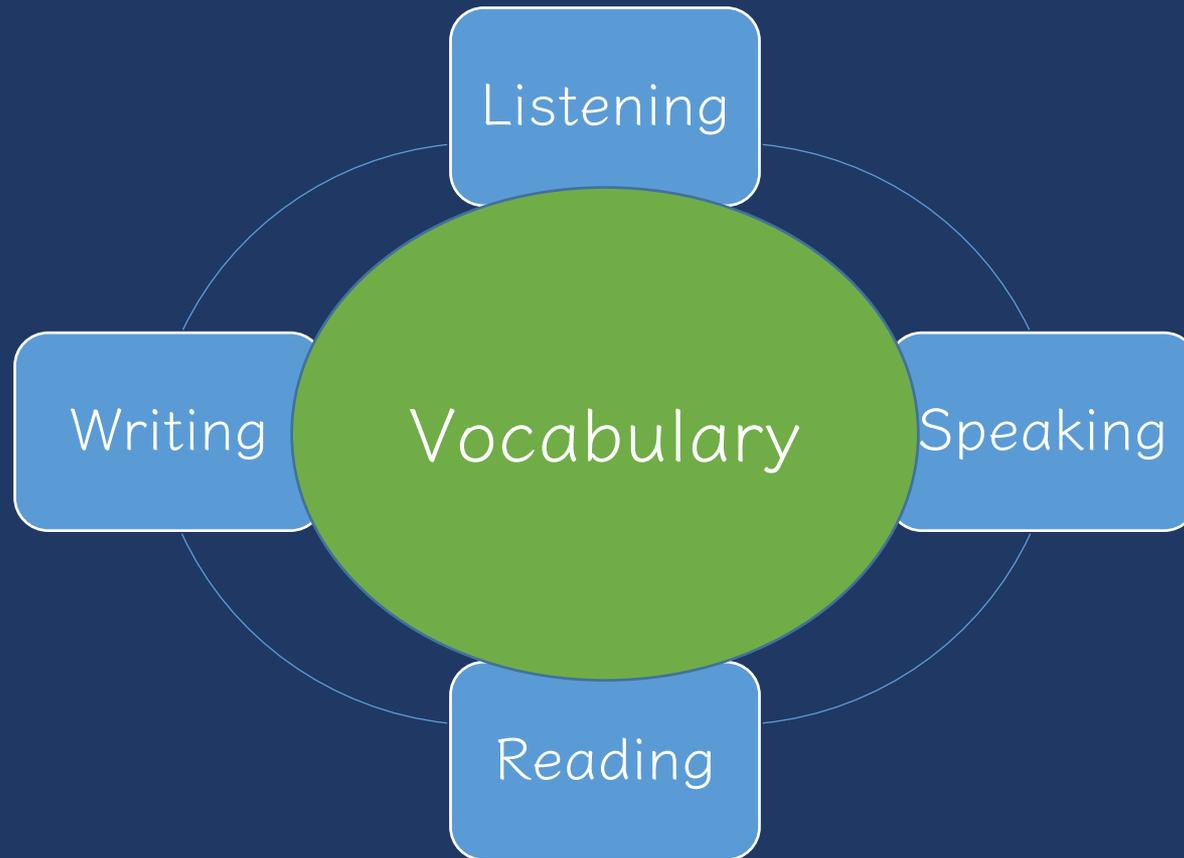
Chapter 5: Mathematical Language

Instead of that...	Say this...

Page 77







Use formal math language

Use terms precisely





What number is in the tens place?



What digit is in the tens place?  
What is the value of the digit in the tens place?

135

**Why this is important...**

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.



The alligator eats the  
bigger number



is less than  
OR  
is greater than

**Why this is important...**

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because  $<$  and  $>$  are two distinct symbols.





carry OR borrow



regroup OR  
trade OR  
exchange

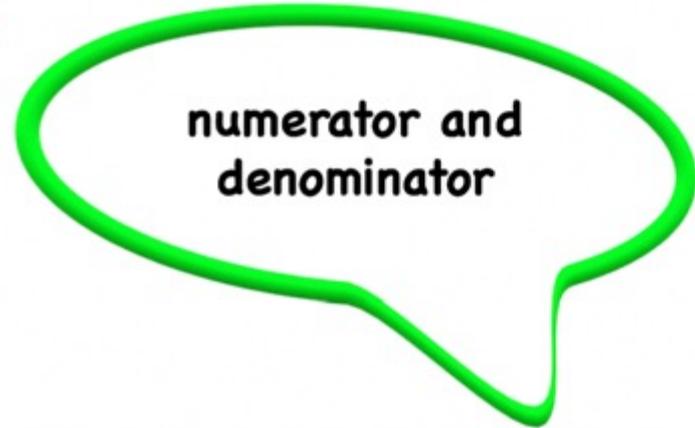
$$\begin{array}{r} 167 \\ + 294 \\ \hline \end{array}$$

**Why this is important...**

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding of regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



top number and  
bottom number

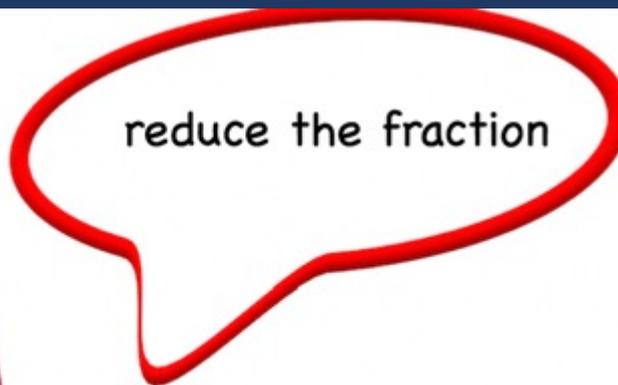


numerator and  
denominator

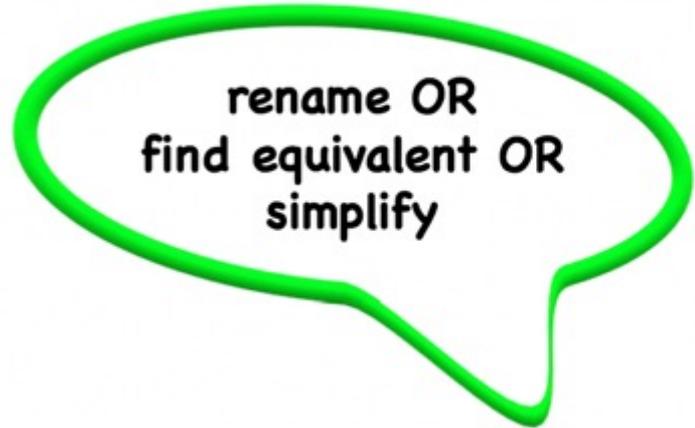
**Why this is important...**

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.





reduce the fraction



rename OR  
find equivalent OR  
simplify

**Why this is important...**

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.



Four point seven  
Four point oh seven

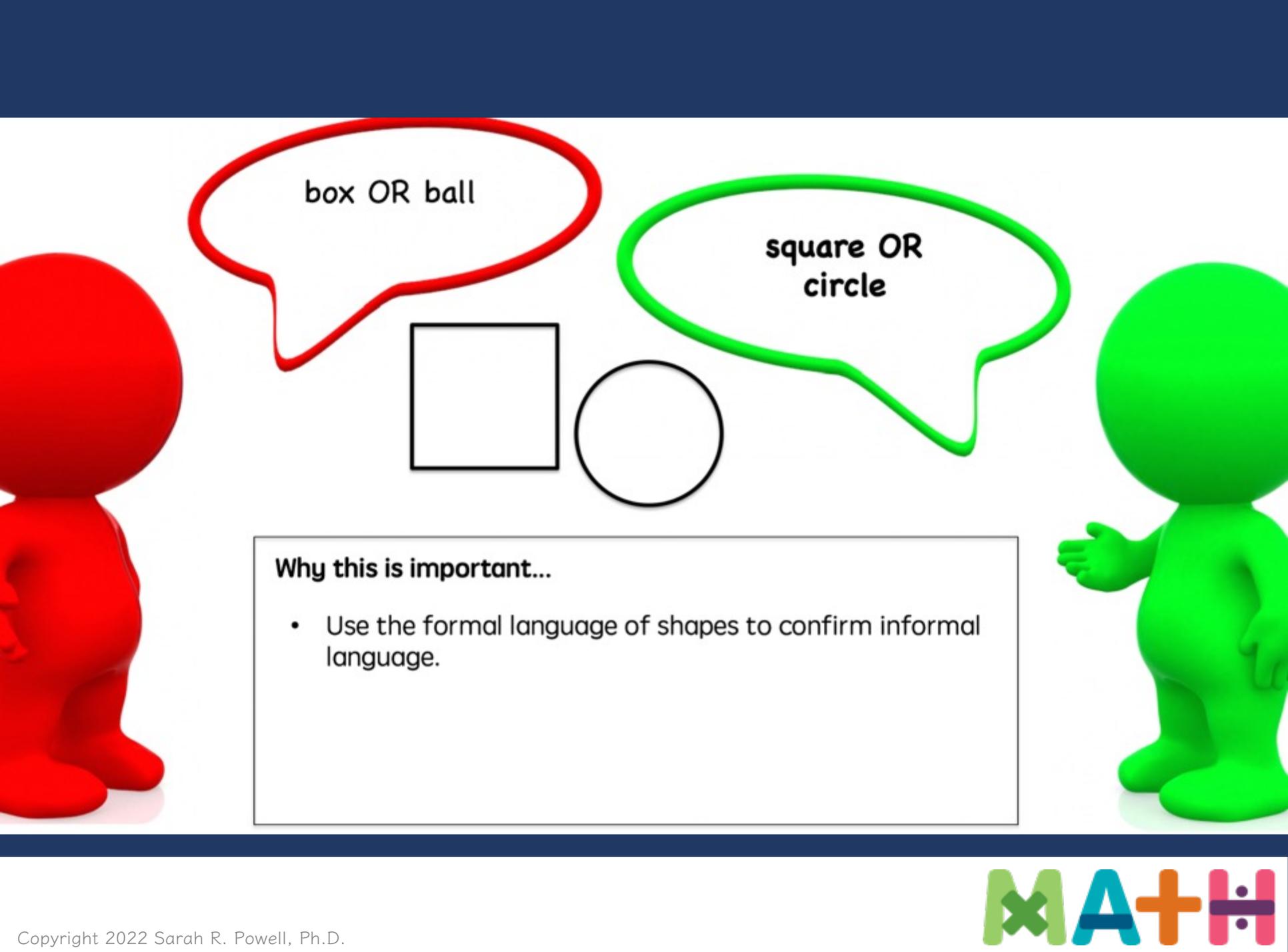


Four and seven tenths  
Four and seven hundredths

4.7  
4.07

**Why this is important...**

- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



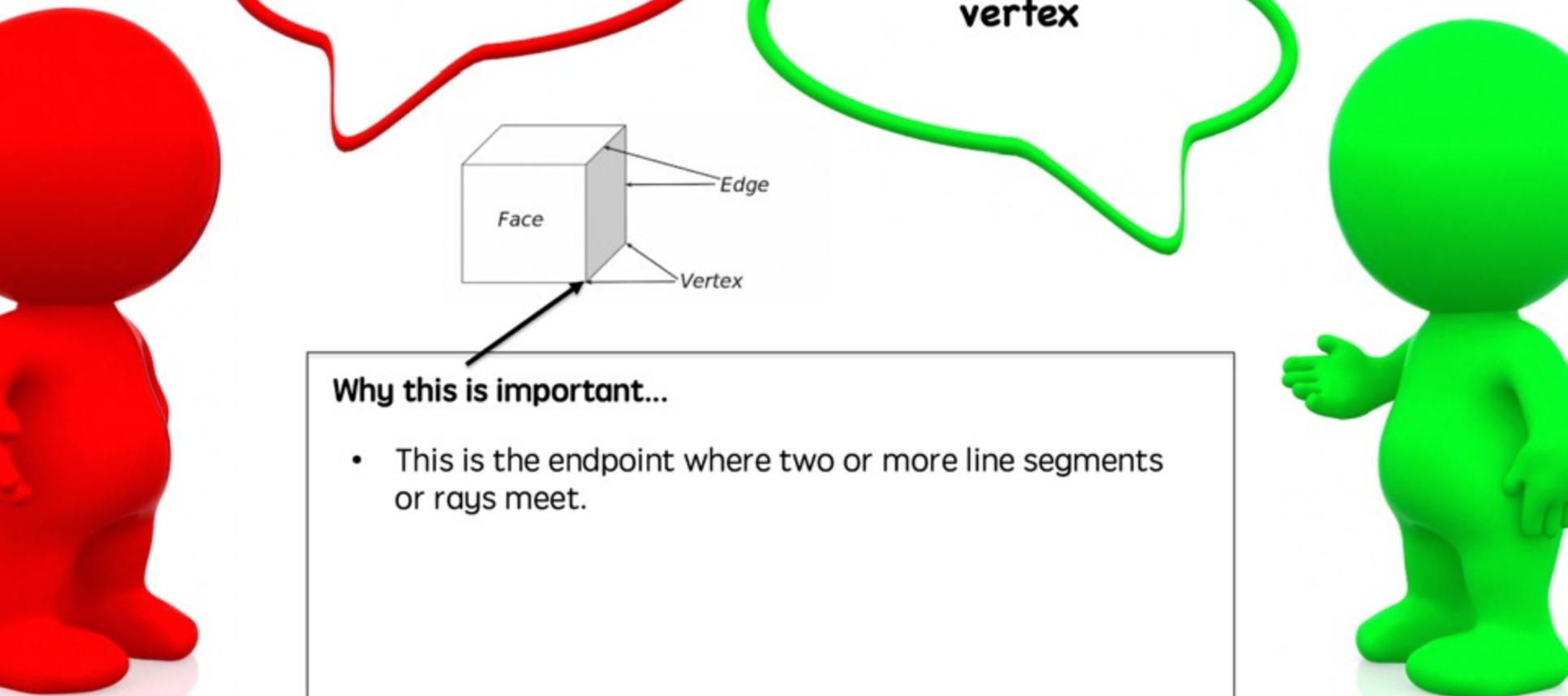
box OR ball

square OR  
circle



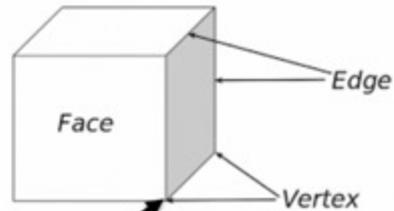
**Why this is important...**

- Use the formal language of shapes to confirm informal language.



point

vertex



**Why this is important...**

- This is the endpoint where two or more line segments or rays meet.



long hand and  
short hand



minute hand and  
hour hand

**Why this is important...**

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.

## Chapter 5: Mathematical Language

Instead of that...	Say this...



Identify examples of  
“Instead of \_\_\_\_\_, say  
\_\_\_\_\_.”

Use formal math language

Use terms precisely



## **Factor**

$$1 \times 8 = 8$$

$$2 \times 4 = 8$$

factor factor

## **Multiple**

$$8 \times 1 = 8$$

$$8 \times 2 = 16$$

multiples of 8

E

## **Improper fraction**

$$\frac{8}{5}$$

## **Mixed number**

$$1\frac{3}{5}$$

## **Proper fraction**

$$\frac{2}{9}$$

## **Proportion**

$$\frac{2}{5} = \frac{8}{20}$$

## **Ratio**

$$4:3$$

## **Unit fraction**

$$\frac{1}{6}$$

D



**Equation**  $9x - 4 = 7x$

**Expression**  $9x - 4$

**Formula**  $a^2 + b^2 = c^2$

**Function**  $f(x)$

**Inequality**  $9x - 4 > 6x$

C

**Coefficient**

**Constant**

**Term**

**Variable**

term

term

term

$2x^2$

+

$x$

-

$3$

variable  
coefficient

variable

constant

A



# Quadrilaterals

**Kite**



**Parallelogram**



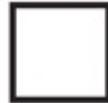
**Rectangle**



**Rhombus**



**Square**



**Trapezoid**

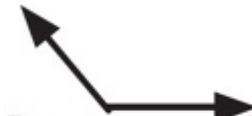


A

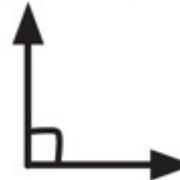
**Acute angle**



**Obtuse angle**



**Right angle**

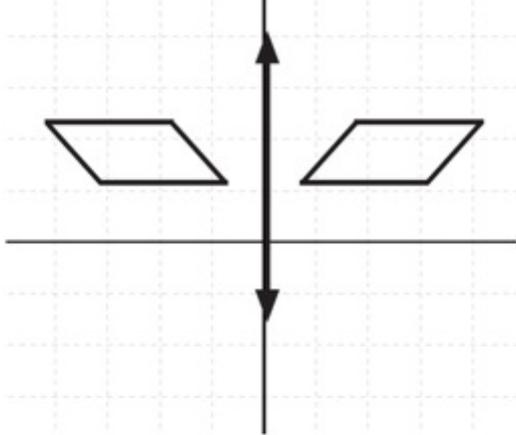


**Straight angle**

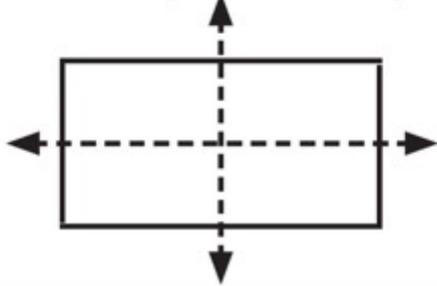


B

## Line of reflection

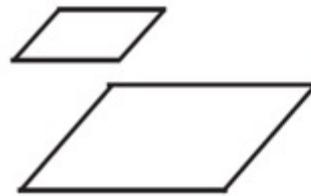


## Line of symmetry



F

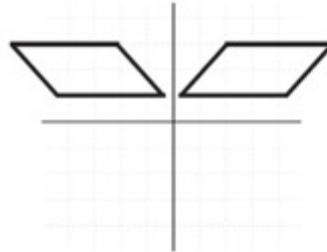
## Dilation



## Scale factor

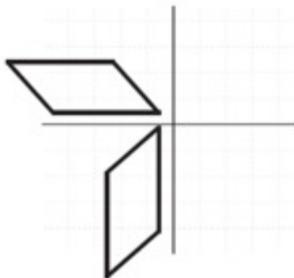
← scale factor  
is 1:2

## Reflection

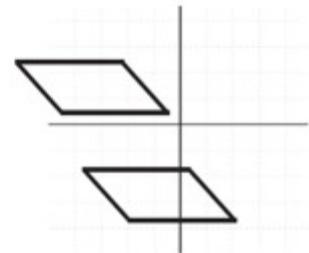


## Transformation

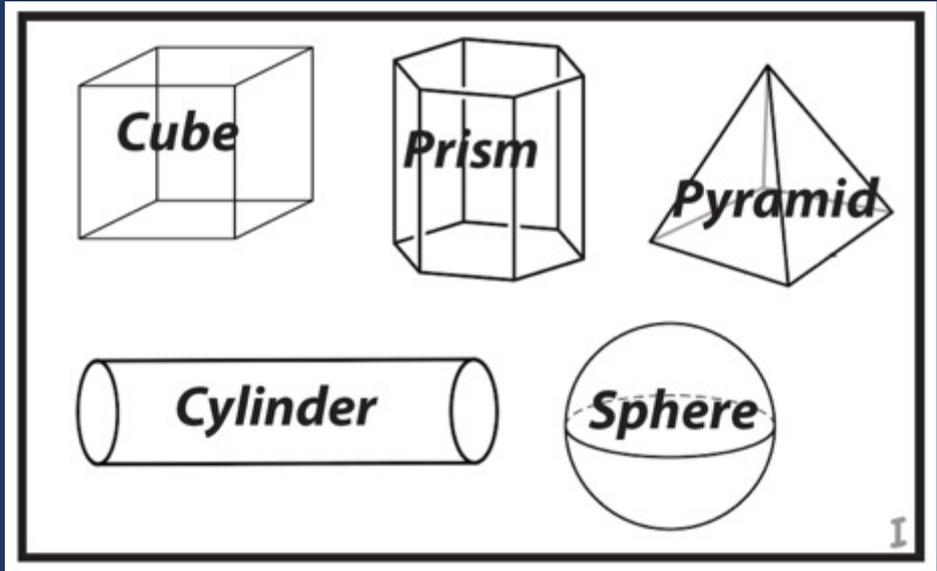
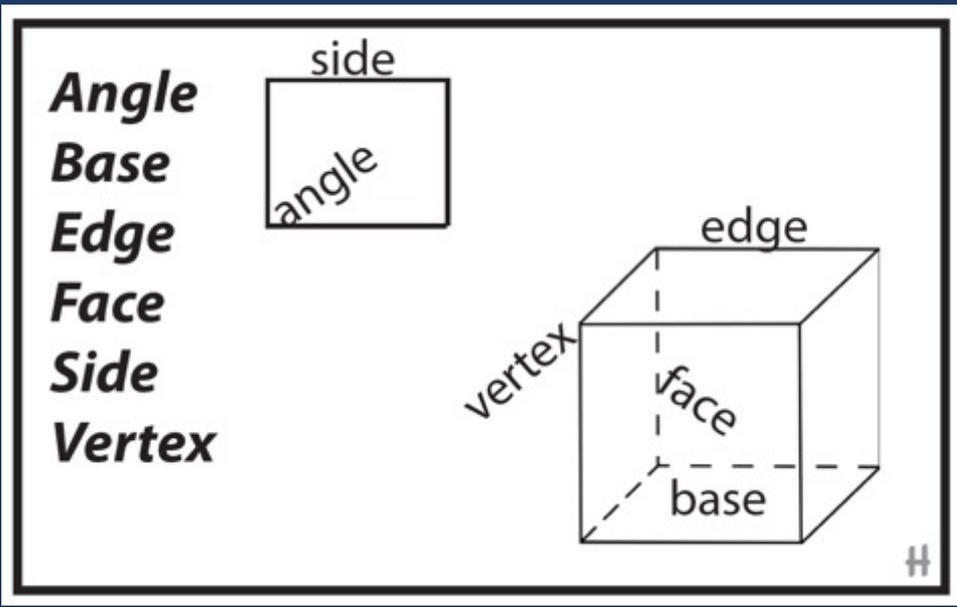
## Rotation



## Translation



G



Use formal math language

Use terms precisely



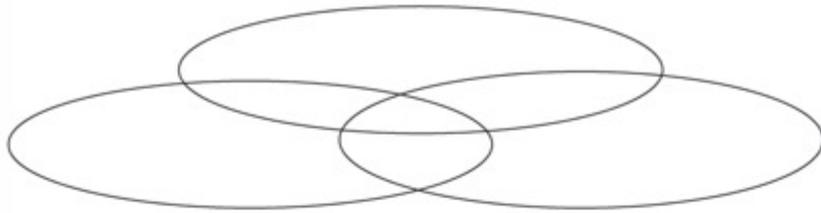
What are your strategies for focusing on math language?



# Multiple Representations



Chapter 5: Multiple Representations



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# Instructional Platform

## INSTRUCTIONAL DELIVERY

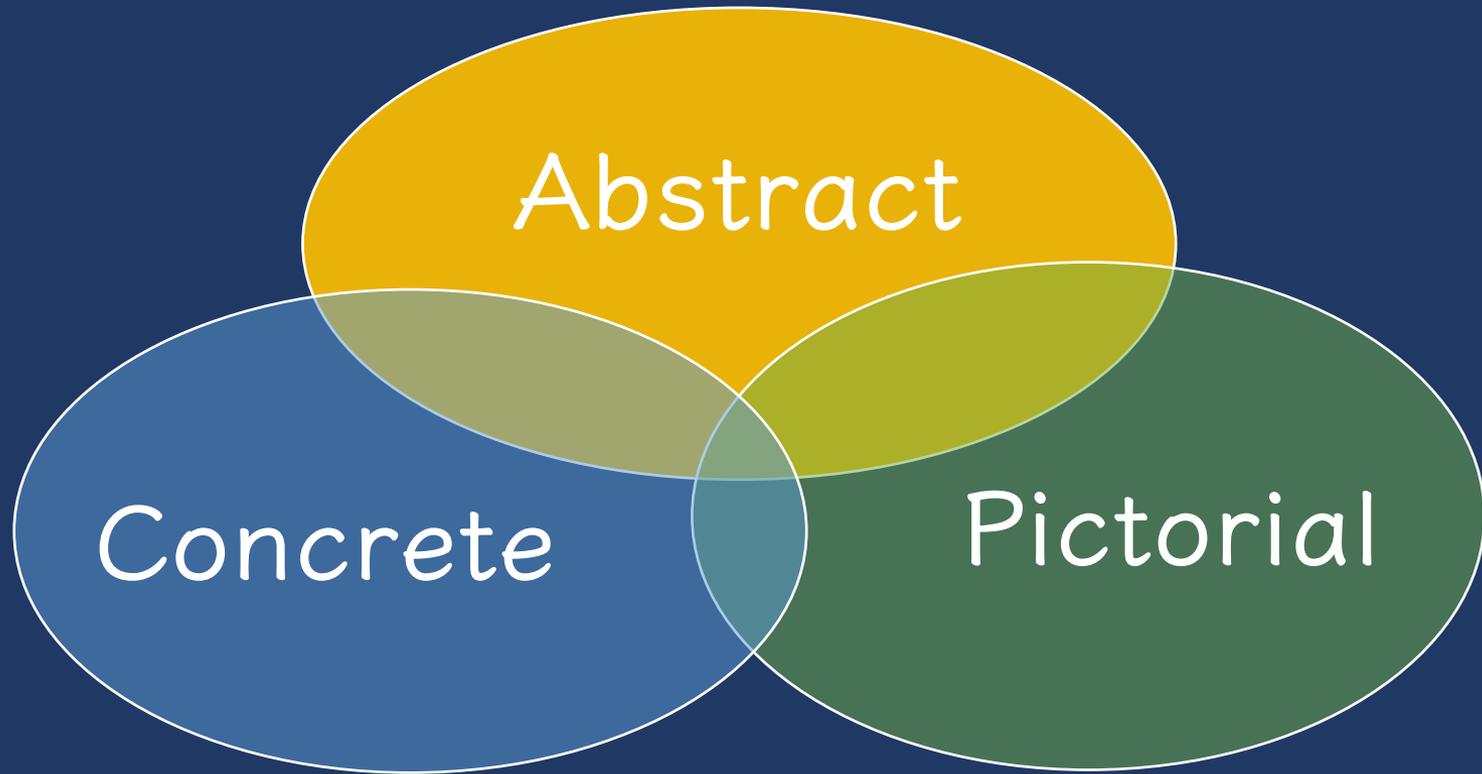
Explicit  
instruction

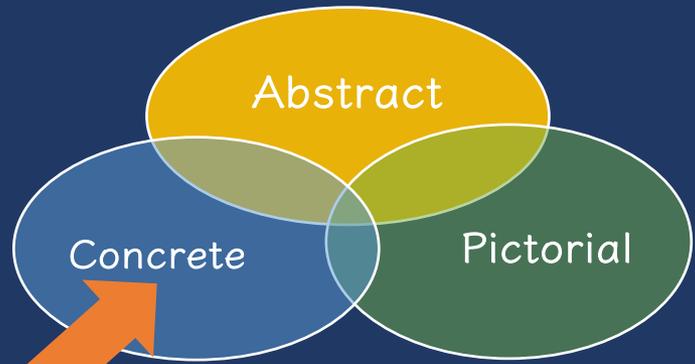
Precise  
language

Multiple  
representations

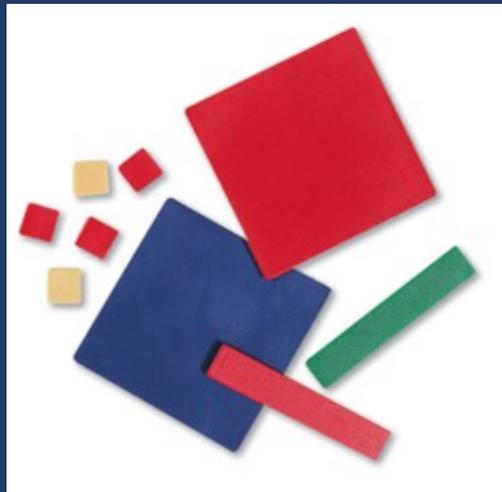
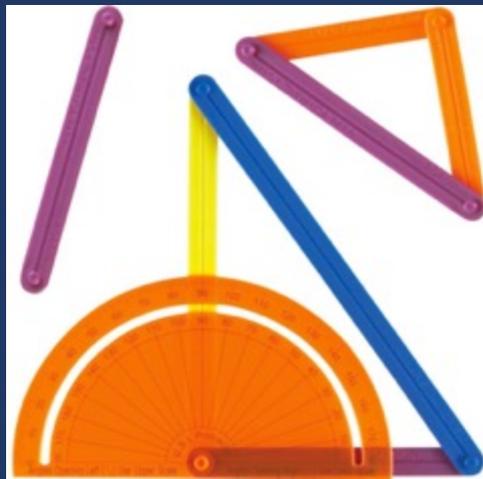
## INSTRUCTIONAL STRATEGIES

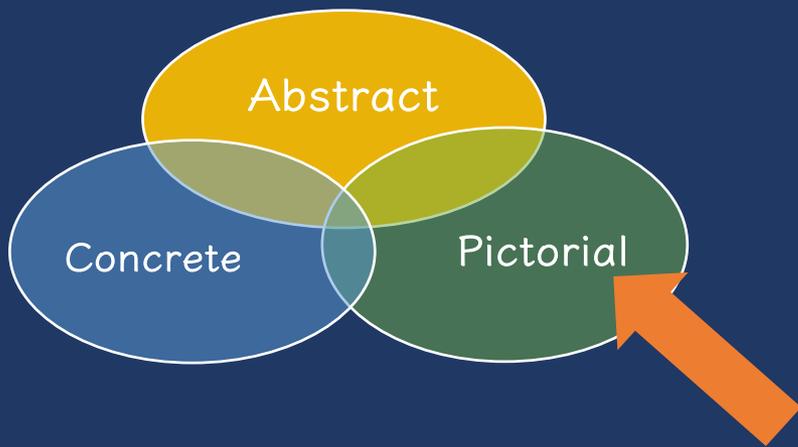




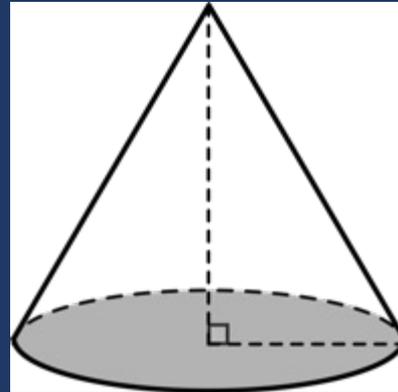
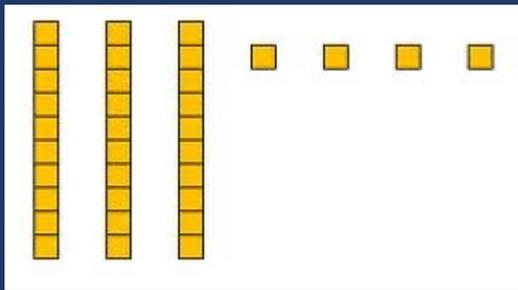


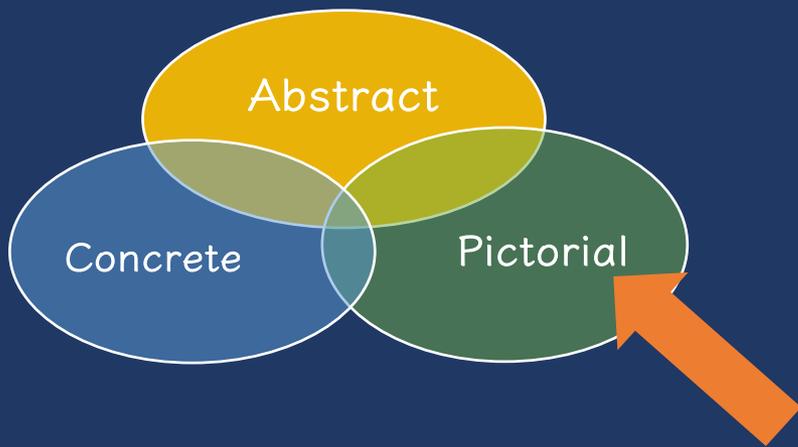
Three-dimensional objects



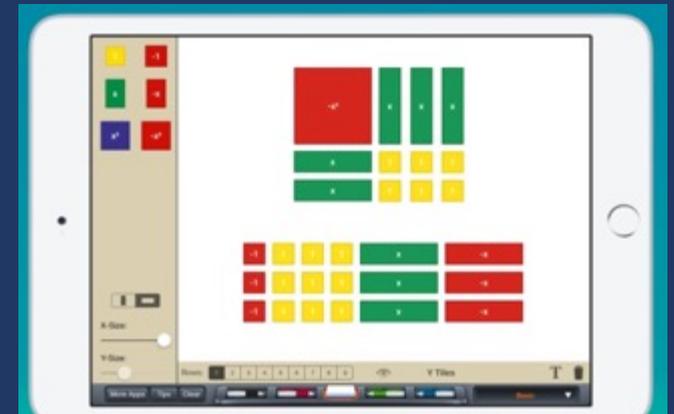
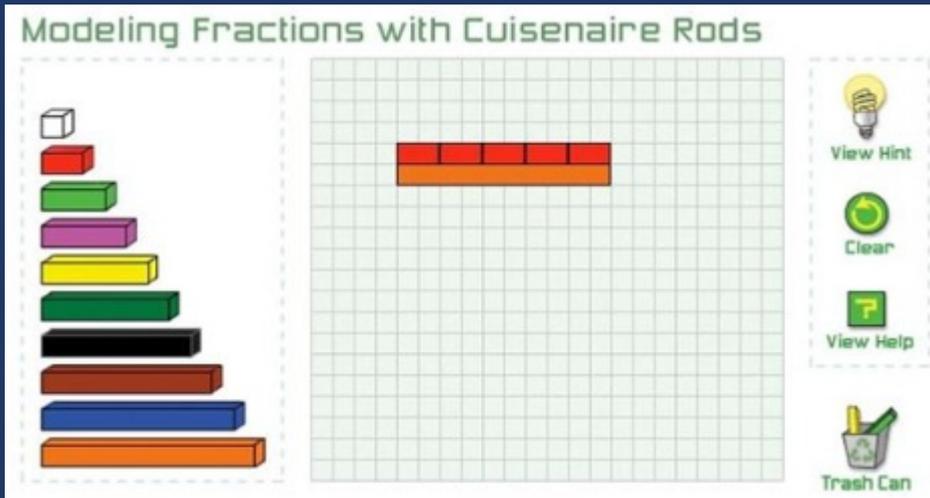
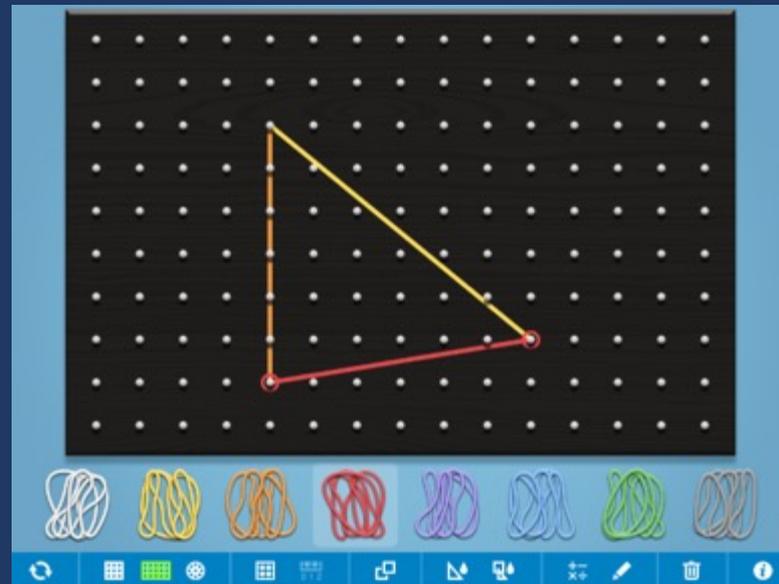


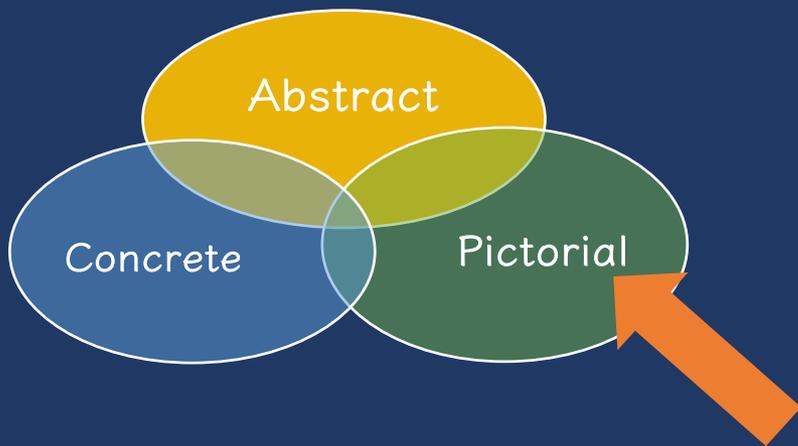
## Two-dimensional images





Two-dimensional images





Two-dimensional images



[bit.ly/srpowell](https://bit.ly/srpowell)

## Virtual Manipulatives

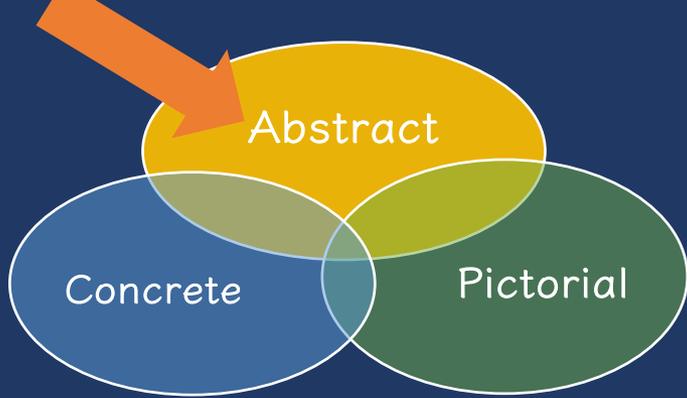
Help students see and learn math using different tools!

Number & Operations	Place Value
Fractions & Decimals	Integers & Algebra
Geometry	Time & Money
Data & Probability	Extras

Sarah R. Powell, Ph.D.  
srpowell@utexas.edu  
www.sarahpowellphd.com  
@sarahpowellphd

Fractions & Decimals				
	fraction strips	fraction strips	fraction strips	Cuisenaire rods
fraction circles	geoboard	geoboard	geoboard	
two-color counters	decimal strips	place value disks	percentage strips	





Numerals and symbols and words

$$2 + 8 = 10$$

$$34 = 3 \text{ tens and } 4 \text{ ones}$$

$$x - 6 = 8$$

$$\begin{array}{r} 4,179 \\ + \quad 569 \\ \hline \end{array}$$





Explore 3 virtual manipulatives.

Share with a partner.



# Building Fluency





# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

Fluency building



# Building Fluency

Fluency is doing mathematics easily and accurately.

Fluency in mathematics makes mathematics easier.

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both conceptual learning and procedural learning.



Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Identifying shapes

Knowing multiples

Knowing formulas



Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 56 \\ \div 8 \\ \hline \end{array}$$



Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ - 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div 15 \\ \hline \end{array}$$



Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ + 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div 0.14 \\ \hline \end{array}$$



Addition	Subtraction
Multiplication	Division

Build fluency with integer computation

$$\begin{array}{r} 1.4 \\ + -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times -12 \\ \hline \end{array}$$

$$-14 - (-7) =$$

$$-135 \div 2 =$$



Addition	Subtraction
Multiplication	Division



Describe the fluency needs of your students.



# Teaching Math in Middle School

Using MTSS to Meet All Students' Needs



Leanne R. Ketterlin-Geller, Sarah R. Powell,  
David J. Chard, & Lindsey Perry

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Chapter 1 Laying the Foundation for Algebra .....	3
Chapter 2 Supporting All Students Through Multitiered Instruction .....	23
Chapter 3 Supporting All Students Through Differentiation, Accommodation, and Modification .....	35
<b>Section II: Designing and Delivering Effective Mathematics Instruction</b> .....	49
Chapter 4 Aims for Effective Mathematics Instruction .....	51
<b>Chapter 5 Evidence-Based Practices for Instruction and Intervention</b> .....	65
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Chapter 7 Designing Interventions .....	95
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<b>Section III: Using Data to Make Decisions</b> .....	119
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Chapter 17 Implementing MTSS: Voices From the Field .....	241



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# Word-Problem Solving



# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving  
instruction



# How Students Solve Word Problems



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

71

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

71

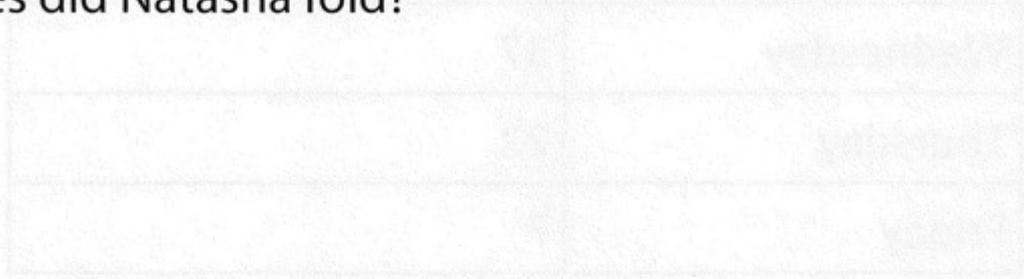
43

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

Donna



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



96 paper fold

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 25 \\ \hline 121 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 25 \\ \hline 121 \end{array}$$

✓

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 25 \\ \hline 121 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 22 \\ \hline 118 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ - 25 \\ \hline 71 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} + 1 \\ + 96 \\ + 25 \\ \hline 121 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 25 \\ \hline 121 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

---

$$\begin{array}{r} 1 \\ 96 \\ + 25 \\ \hline 121 \end{array}$$

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ - 25 \\ \hline 71 \end{array}$$



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ - 25 \\ \hline 71 \end{array}$$

Correct



Undetermined



Repeated information



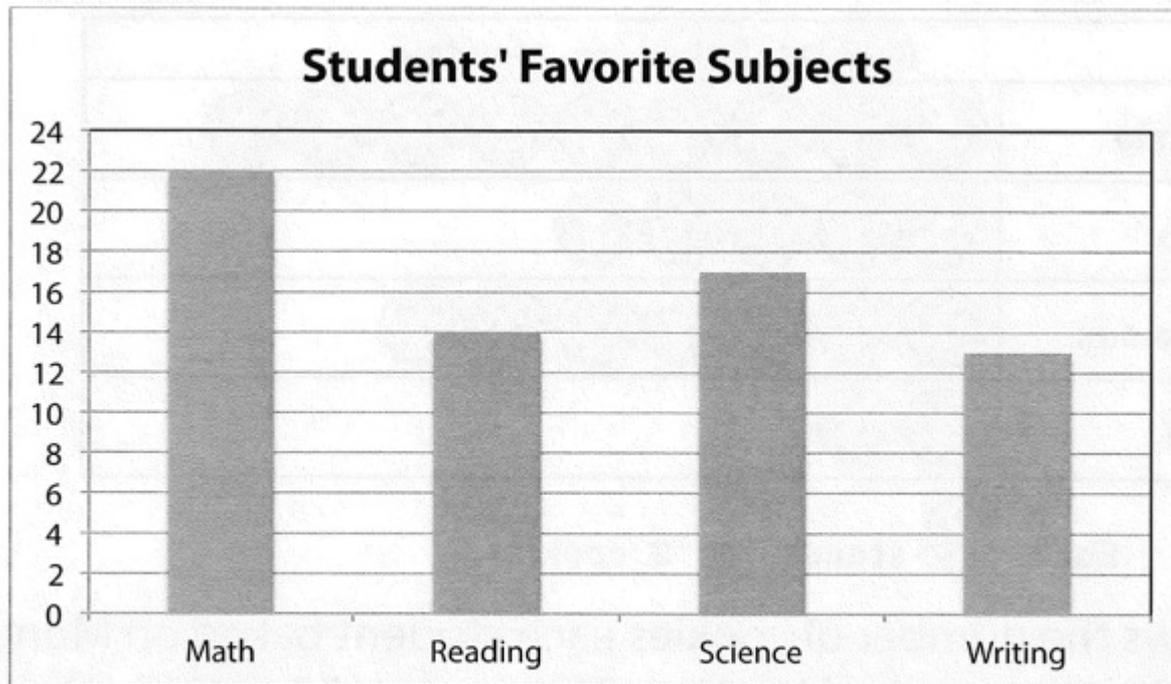
Wrong information



Wrong (misapplied) operation



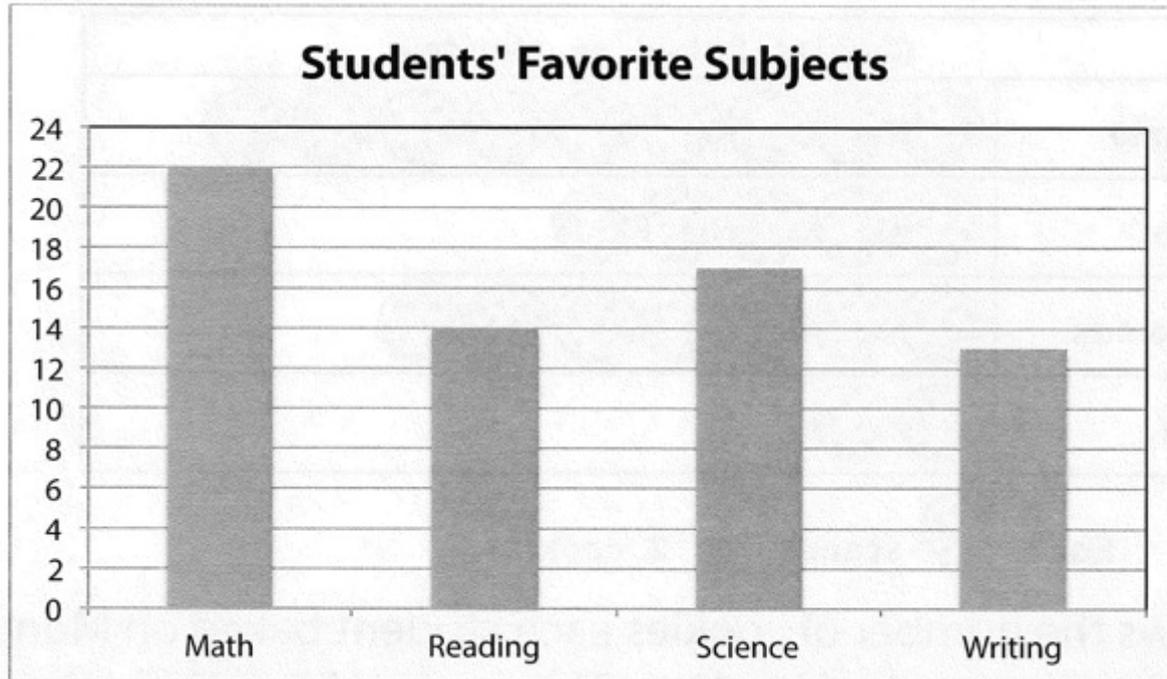
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?



J.



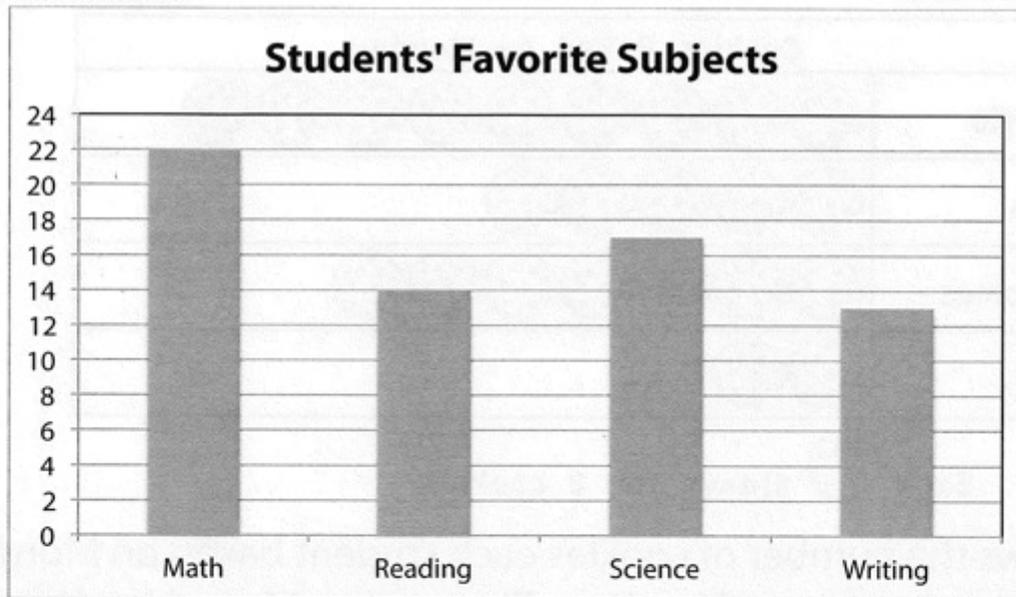
The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

60

|||||

|||||

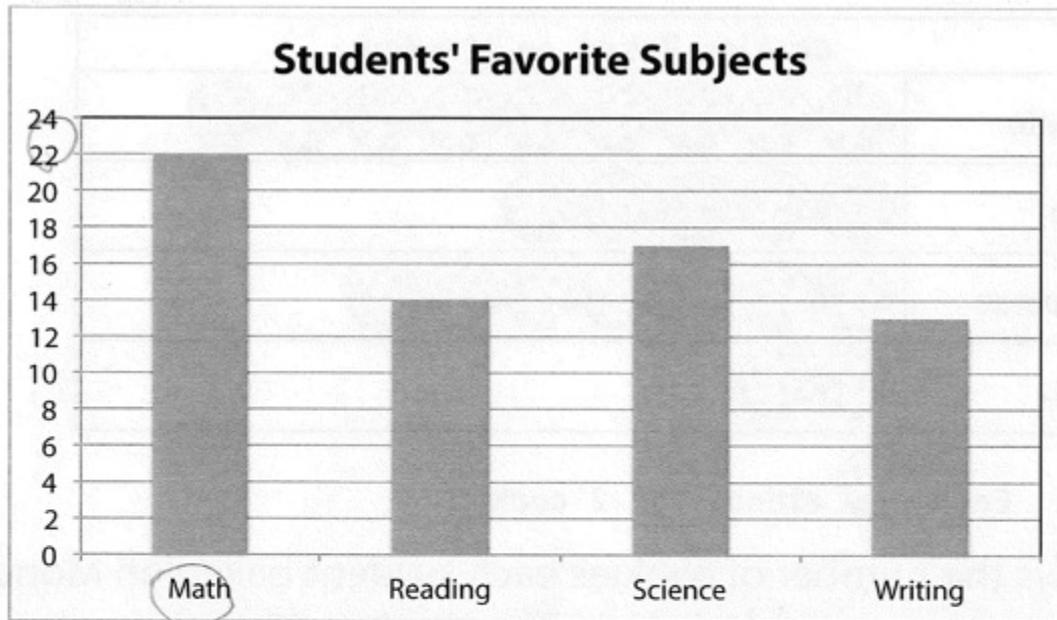
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

They choose 8% more than reading

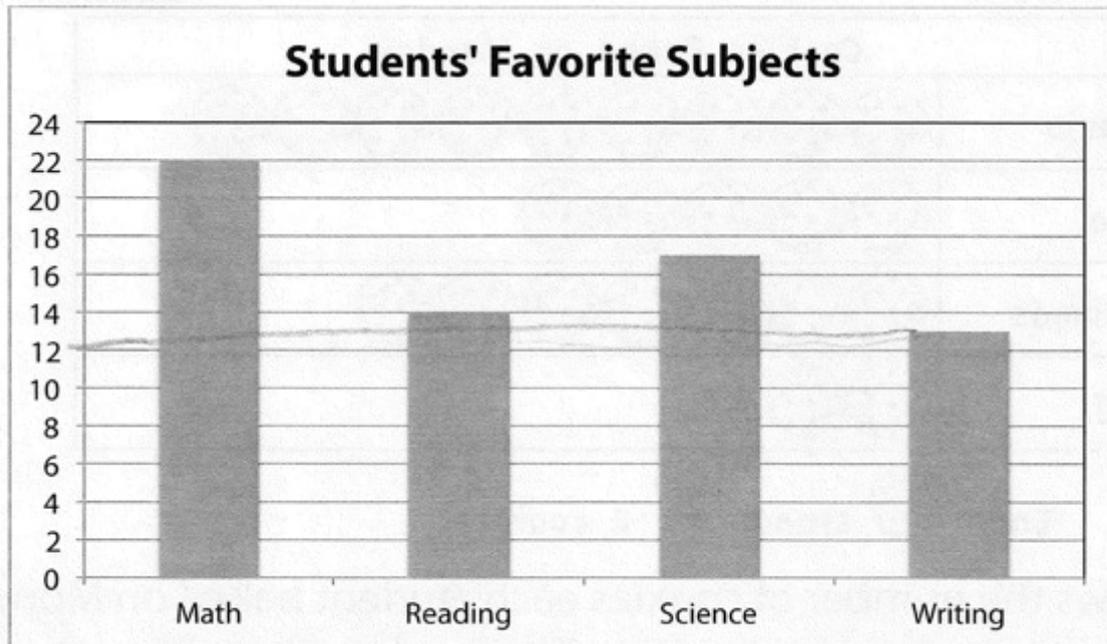
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

22  
Math

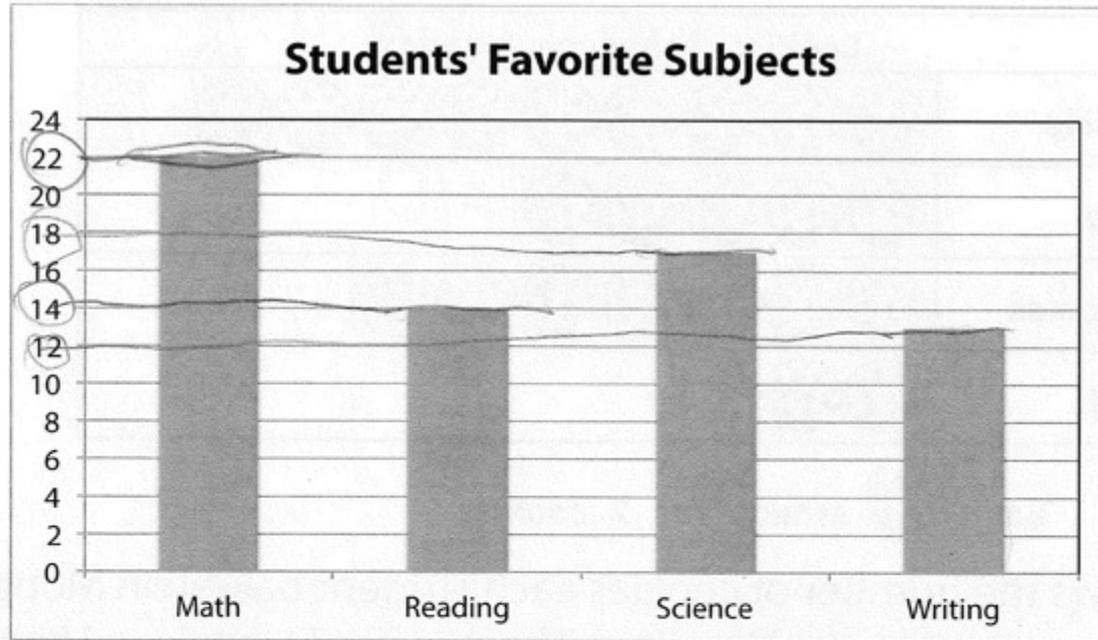
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

(22) math      (13) writing

J.

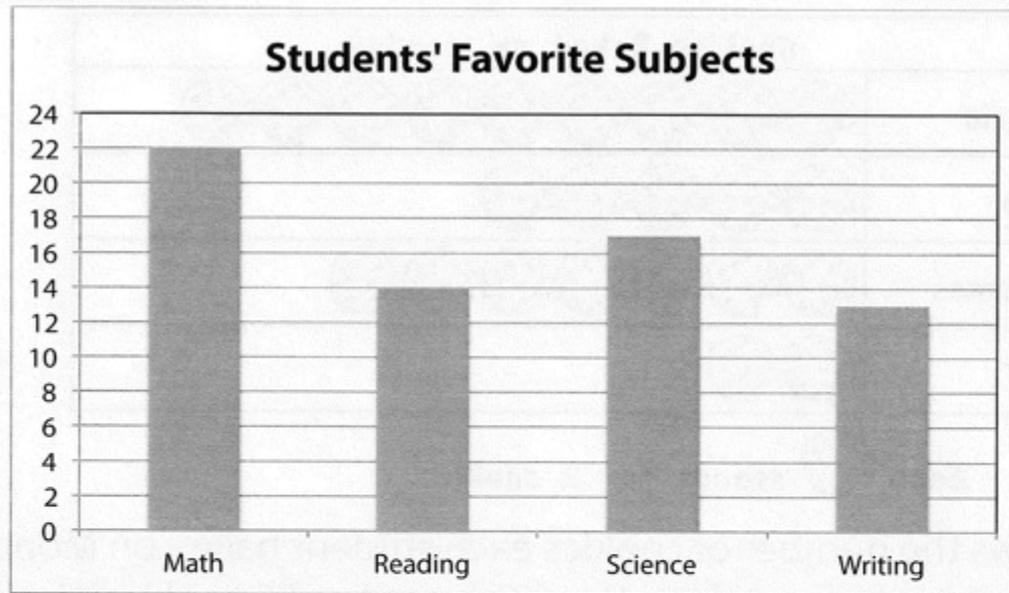


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

22  
Math

14  
writing

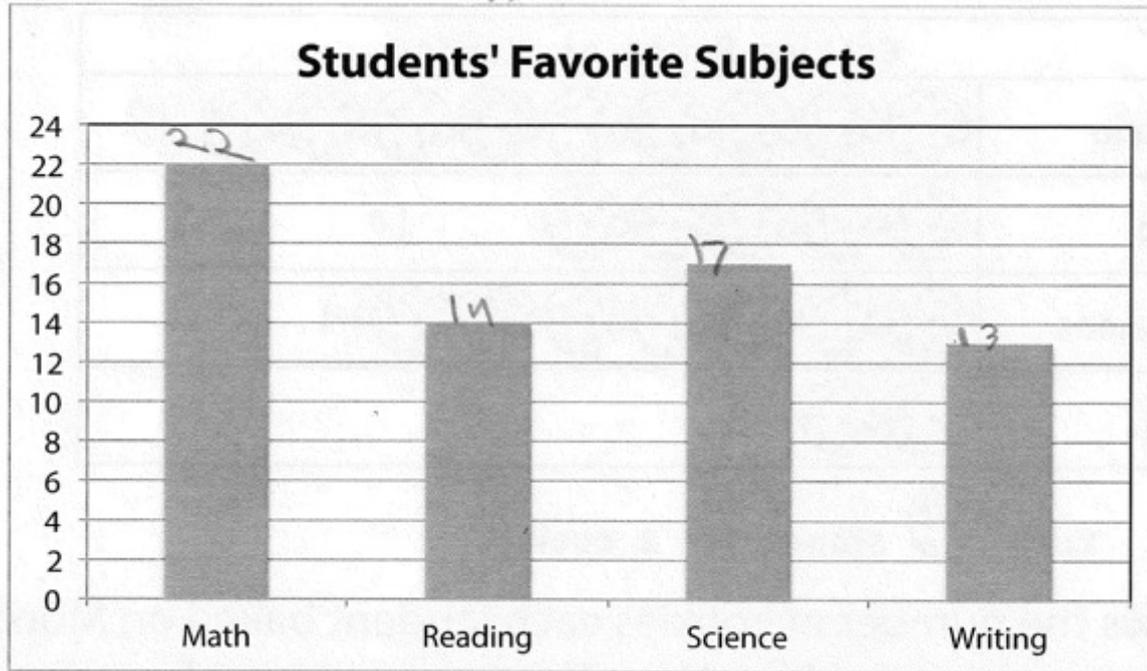
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

the students chooses they  
love more math d'can es  
⑤ to 22 math

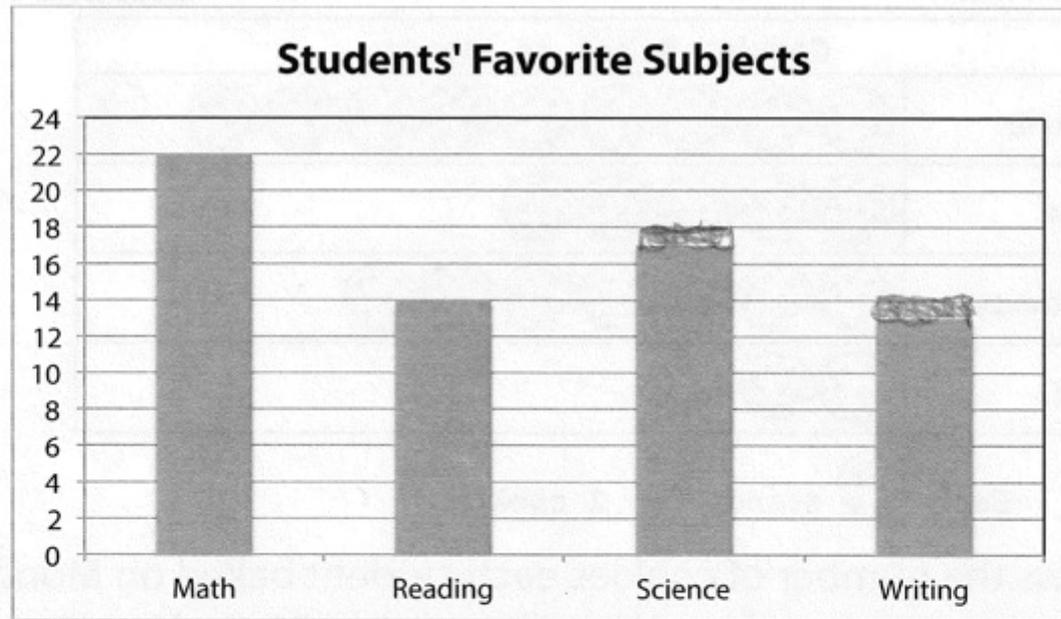
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 14 \\ + 13 \\ \hline 27 \end{array}$$

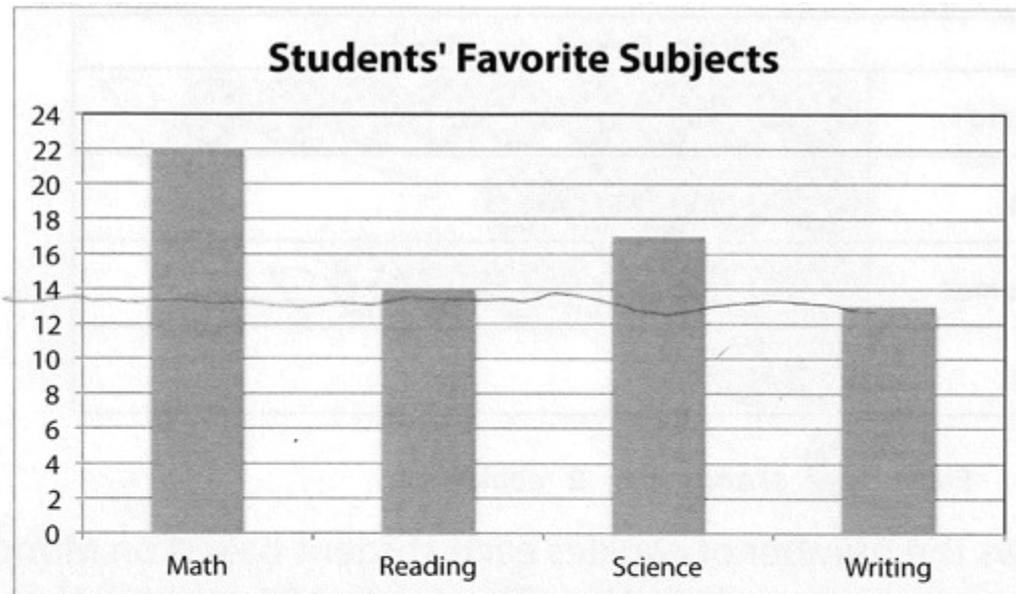
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ + 14 \\ \hline 36 \end{array}$$

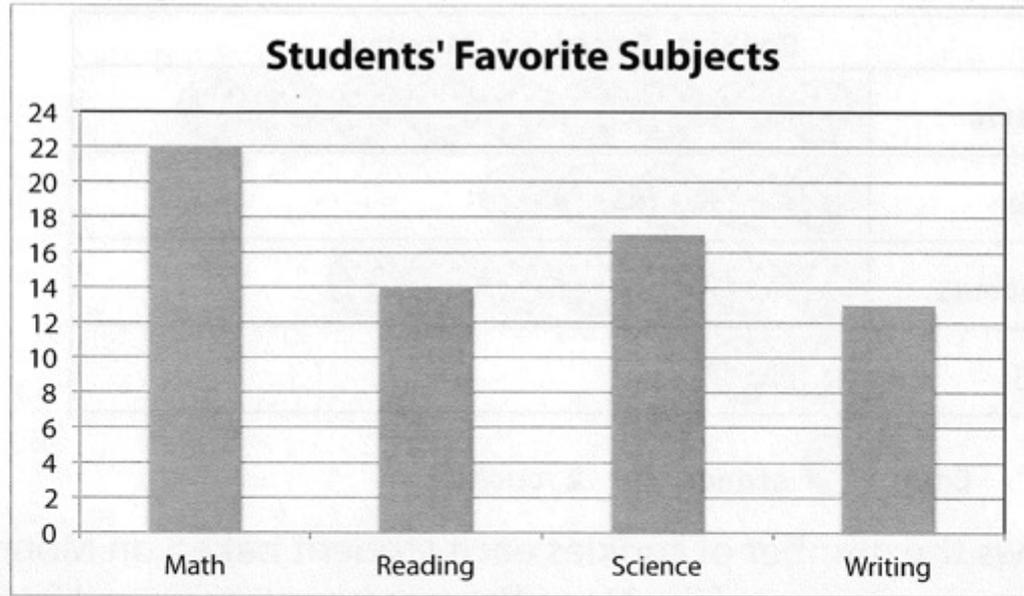
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 24 \\ + 13 \\ \hline 37 \end{array}$$

J.

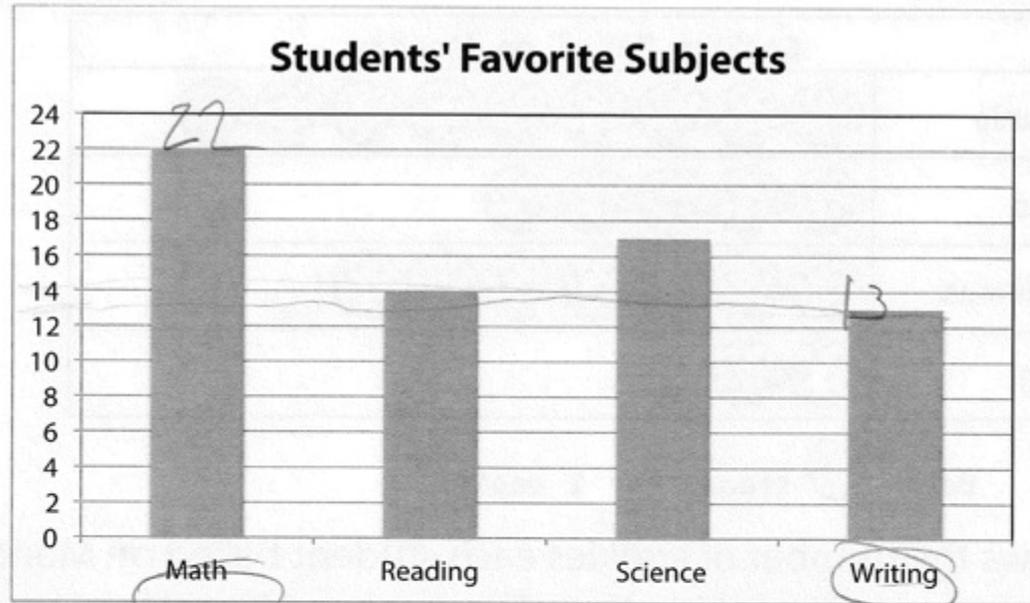


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ + 12 \\ \hline 34 \end{array}$$

34

J.

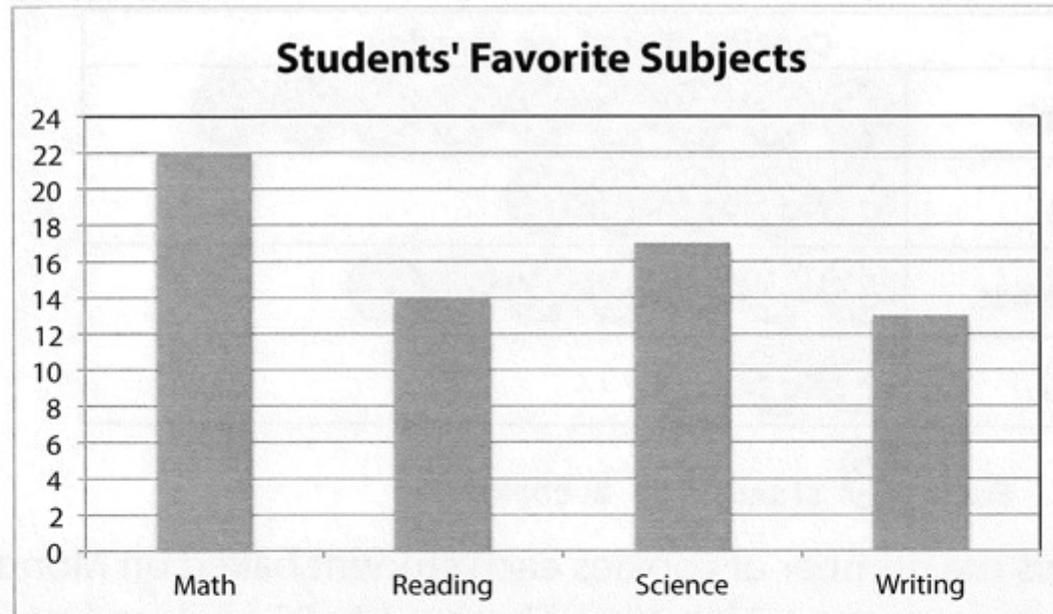


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ + 13 \\ \hline 35 \end{array}$$

14

J.

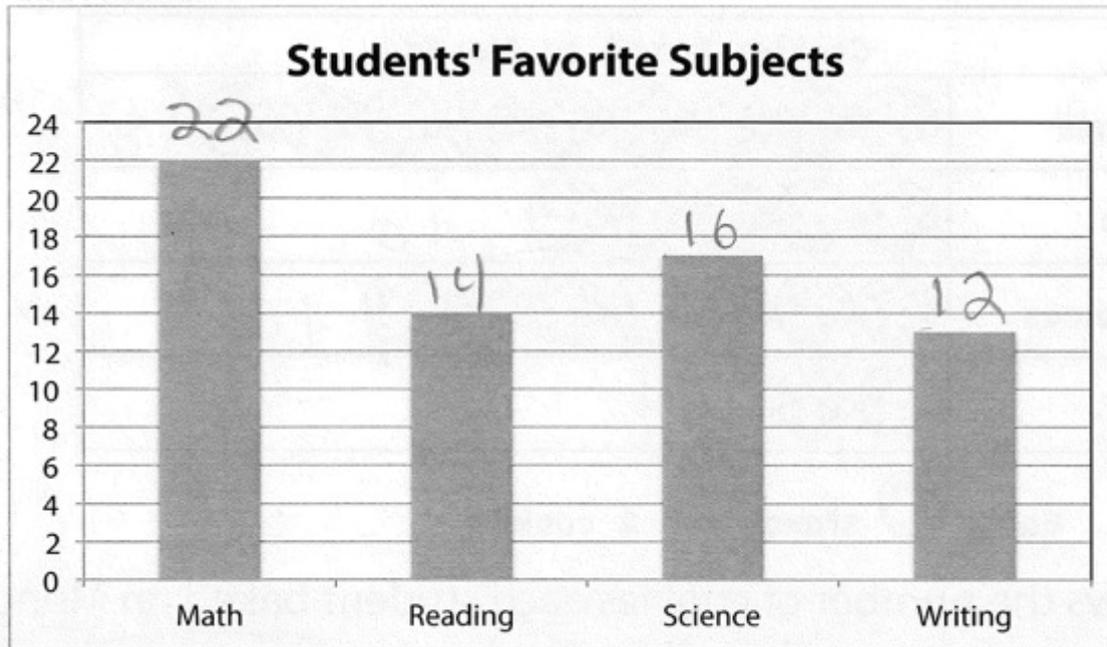


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$



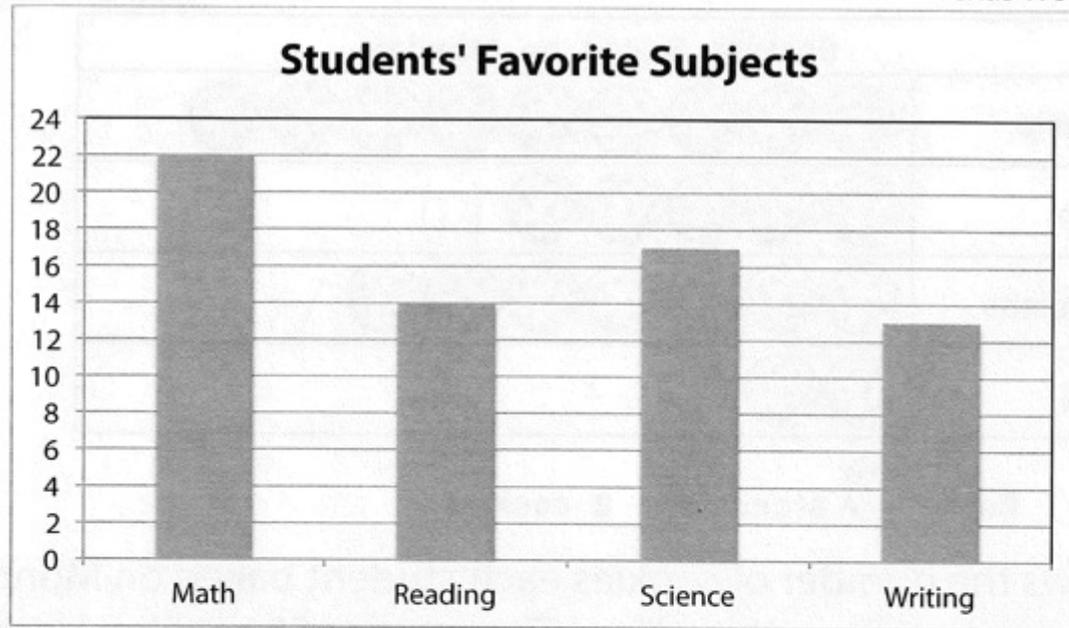
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 12 \\ \hline 10 \end{array}$$

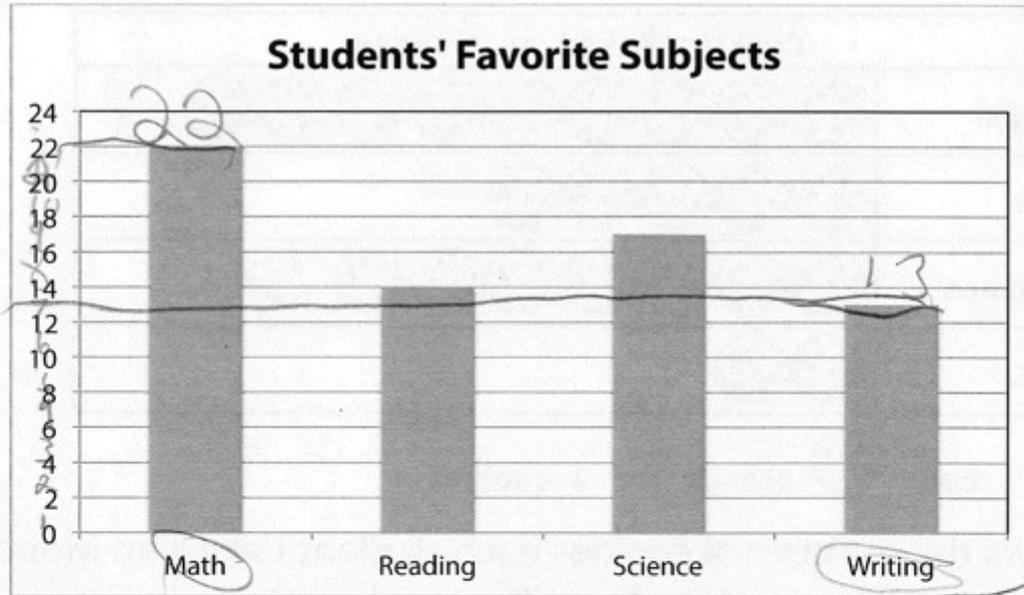
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$

J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$

Correct



Undetermined



Didn't provide answer



Wrong (misapplied) operation



Wrong information



Computation error



# COMMON

Undetermined

Repeated  
information from  
problem

Didn't provide  
answer

Wrong  
information

Wrong  
(misapplied)  
operation

# UNCOMMON

Addition error

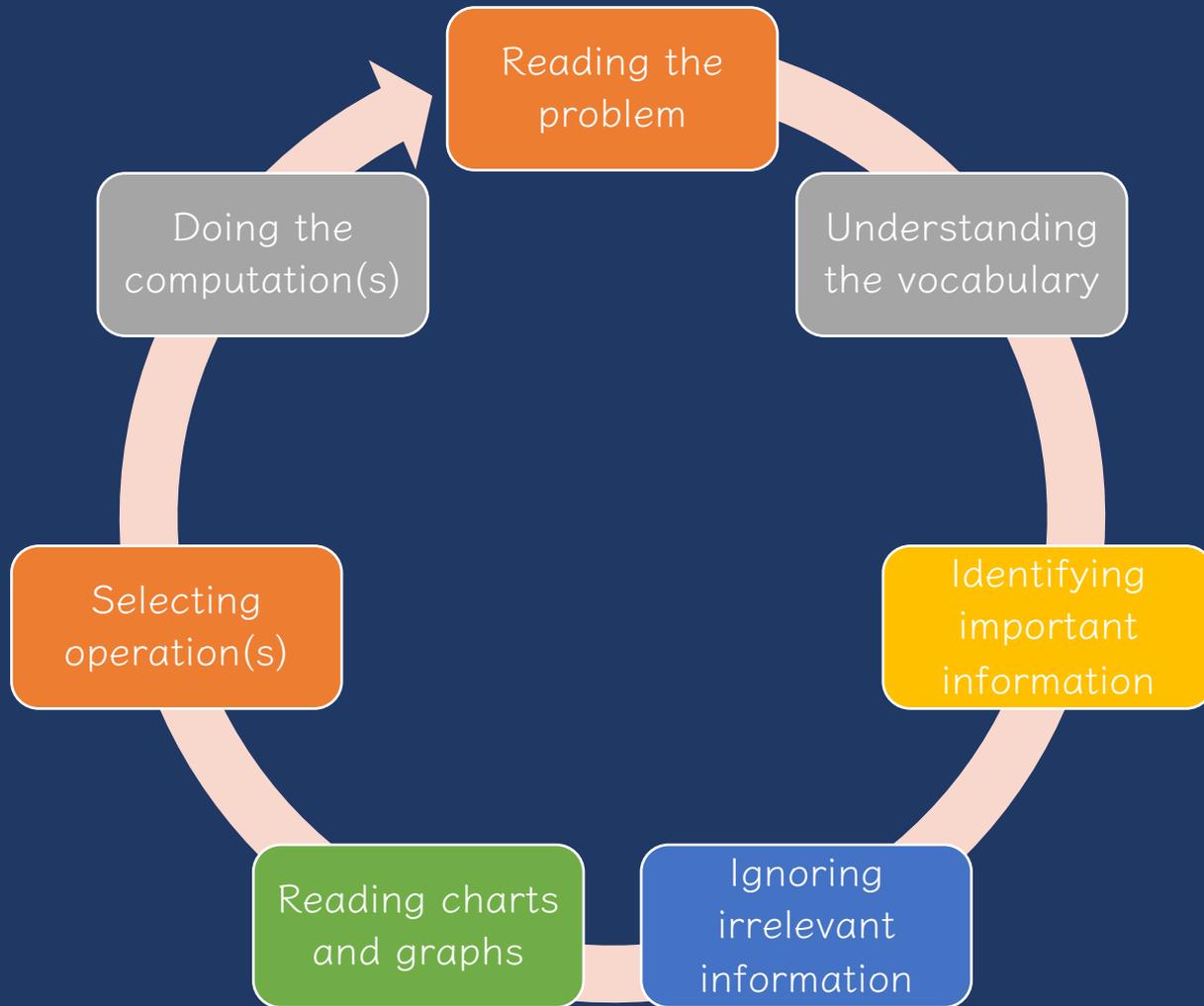
Subtraction error



Chapter 6: Word-Problem Solving

Teaching Problem Solving





# Ineffective Strategies





1. Keywords tied to operations





Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?

### Key Words Used In Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> <li>add</li> <li>all together or altogether</li> <li>and</li> <li>both</li> <li>combined</li> <li>how many in all</li> <li>how much</li> <li>in all</li> <li>increased by</li> <li>plus</li> <li>sum</li> <li>together</li> <li>total</li> </ul>	<ul style="list-style-type: none"> <li>change</li> <li>decreased by</li> <li>difference</li> <li>fewer or fewer than</li> <li>how many are left (or have left)</li> <li>how many did not have</li> <li>how many</li> <li>how much taller, heavier or less</li> <li>less or less</li> <li>lost</li> <li>minus</li> <li>need to</li> <li>reduce</li> <li>remain</li> <li>subtract</li> <li>take away</li> </ul>
Multiplication Words	Division Words
<ul style="list-style-type: none"> <li>by (dimension)</li> <li>double</li> <li>each group</li> <li>every</li> <li>factor of</li> <li>increased by</li> <li>multiplied by</li> <li>of</li> <li>product</li> <li>times</li> <li>triple</li> </ul>	<ul style="list-style-type: none"> <li>as much</li> <li>cut up</li> <li>each group</li> <li>equal share</li> <li>half (or of)</li> <li>how many parts</li> <li>per</li> <li>percent</li> <li>quotient</li> <li>ratio of</li> <li>separated</li> <li>share</li> <li>split</li> <li>total</li> <li>together</li> <li>total</li> <li>in all</li> <li>end</li> </ul>

### OPERATION cue words

ADDITION	SUBTRACTION
<ul style="list-style-type: none"> <li>and</li> <li>total</li> <li>join</li> <li>more than</li> <li>in all</li> <li>sum</li> <li>increased</li> </ul>	<ul style="list-style-type: none"> <li>less than</li> <li>decreased</li> <li>remaining</li> <li>left</li> <li>fewer</li> <li>take away</li> <li>difference</li> <li>minus</li> </ul>
MULTIPLICATION	DIVISION
<ul style="list-style-type: none"> <li>product</li> <li>times</li> <li>also</li> <li>by</li> <li>of</li> <li>equal groups</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>each</li> <li>broken into</li> <li>per</li> <li>distributed evenly</li> <li>parts</li> </ul>

### KEY WORDS

ADDITION	MULTIPLICATION
<ul style="list-style-type: none"> <li>sum</li> <li>total</li> <li>more than</li> <li>plus</li> <li>both</li> <li>combined</li> <li>increased by</li> <li>perimeter</li> </ul>	<ul style="list-style-type: none"> <li>product</li> <li>double</li> <li>area</li> <li>each</li> <li>times</li> <li>per</li> <li>every</li> <li>each</li> <li>evenly</li> <li>equal groups</li> <li>half</li> </ul>
SUBTRACTION	DIVISION
<ul style="list-style-type: none"> <li>difference</li> <li>remain</li> <li>left</li> <li>less than</li> <li>minus</li> <li>how many more</li> <li>fewer than</li> <li>decrease</li> <li>give away</li> <li>reduce</li> <li>discount</li> <li>how many more</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>divide by</li> <li>into</li> <li>split</li> <li>out of</li> <li>shared</li> <li>per</li> <li>every</li> <li>each</li> <li>evenly</li> <li>equal groups</li> <li>half</li> </ul>



### Problem Solving Key Words

Addition	Subtraction
<ul style="list-style-type: none"> <li>add</li> <li>together</li> </ul>	<ul style="list-style-type: none"> <li>are not</li> <li>decrease</li> <li>difference</li> <li>fewer, larger, shorter</li> <li>How much more?</li> <li>left</li> <li>less than</li> <li>minus</li> <li>remain</li> <li>take away</li> </ul>

### key words

<ul style="list-style-type: none"> <li>addition</li> <li>sum</li> <li>both</li> <li>more than</li> </ul>	<ul style="list-style-type: none"> <li>in all</li> <li>together</li> <li>perimeter</li> </ul>	<ul style="list-style-type: none"> <li>total</li> <li>plus</li> <li>add</li> </ul>
<ul style="list-style-type: none"> <li>abstract</li> <li>main</li> <li>is than</li> </ul>	<ul style="list-style-type: none"> <li>decrease</li> <li>take away</li> <li>how many more...</li> </ul>	<ul style="list-style-type: none"> <li>fewer</li> <li>minus</li> </ul>

### Math Operation - Key Words

Addition	Subtraction
<ul style="list-style-type: none"> <li>add</li> <li>altogether</li> <li>and</li> <li>both</li> <li>in all</li> <li>sum</li> <li>total</li> <li>increase</li> </ul>	<ul style="list-style-type: none"> <li>difference</li> <li>fewer than</li> <li>gone/take away</li> <li>decreased by</li> <li>how many more</li> <li>show much longer/smaller/shorter</li> <li>minus</li> <li>remaining</li> </ul>
Multiplication	Division
<ul style="list-style-type: none"> <li>area</li> <li>product</li> <li>Each</li> <li>by - of - per</li> <li>Times</li> <li>double, twice, triple</li> <li>total</li> <li>increase</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>divide</li> <li>into</li> <li>equal parts/share equally</li> <li>per</li> <li>amount of each</li> </ul>

### Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> <li>plus</li> <li>sum</li> <li>add</li> <li>total</li> <li>all together</li> <li>increase</li> <li>more</li> <li>combine</li> </ul>	<ul style="list-style-type: none"> <li>subtract</li> <li>minus</li> <li>difference</li> <li>left</li> <li>left over</li> <li>decrease</li> <li>take away</li> <li>fewer</li> </ul>	<ul style="list-style-type: none"> <li>times</li> <li>product</li> <li>factor</li> <li>double</li> <li>groups</li> <li>each</li> <li>area</li> <li>rows</li> </ul>	<ul style="list-style-type: none"> <li>quotient</li> <li>split</li> <li>share</li> <li>divide</li> <li>separate</li> <li>each</li> <li>average</li> <li>equal groups</li> </ul>



# Math Words Poster Set

★★★★★ (4.1)

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*Description of Single-Step Word Problems (n = 132)*

Schema	Occurrence of schema		Any keyword		Schema-specific keywords <sup>a</sup>		Multiple keywords <sup>a</sup>		Keyword(s) led to correct solution <sup>a</sup>	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

<sup>a</sup>When a problem featured a keyword.





*Description of Multi-Step Word Problems (n = 84)*

Schema	Occurrence of schema <sup>a</sup>		Any keyword		Keyword(s) led to correct solution <sup>b</sup>	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

<sup>a</sup>Sum across schemas does not equal 100 because each word problem featured more than one schema.

<sup>b</sup>When a problem featured a keyword.



Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

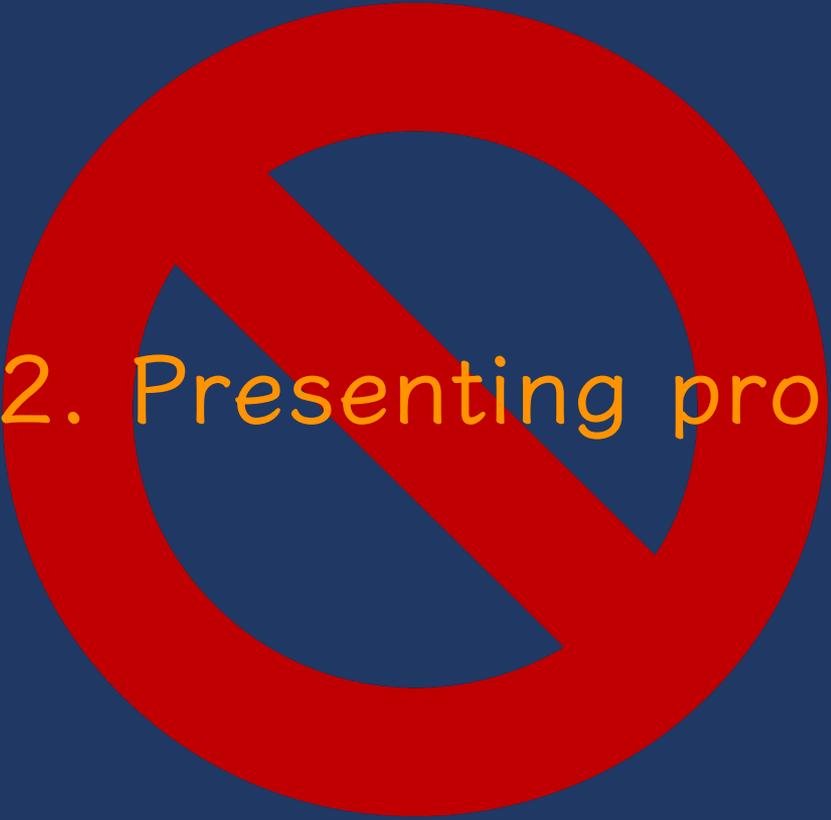
- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by  $24^{\circ}\text{C}$  per minute for 3 minutes. What was the overall change of the temperature of the substance?







## 2. Presenting problems by operation



Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books did Noah have in all?
2. Bonnie found 8 rocks on her front yard and 7 rocks in her backyard. How many rocks did she find in all?
3. Edward had 5 toy cars. He got 7 more toy cars. How many toy cars did Edward have in all?
4. Mariela collected 11 feathers. She found 3 more feathers. How many feathers did she have in all?
5. LaMonte made 14 cookies. He made 8 more cookies. How many cookies did LaMonte have in all?

## Division Word Problems

1. Zookeeper Al had 567 bananas. He gave an equal number of bananas to 9 monkeys in the zoo and 567 bananas. How many bananas did each monkey get? And how many are left over?
2. Betty has 427 oranges. She wants to pack them up equally in 23 boxes. How many oranges will she have in each box and how much does she have left over?
3. Mr. King has 1376 pages of paper. He wants to give 32 pages to each student. How many extra pages will he have left over?
4. Mr. King has 1376 pages of paper. He wants to give 32 pages to each student. He instead gives 33 pages to each student. Will there be enough paper for all the students. How much more scrap paper does he need?

# Effective Strategies



Teach an attack strategy

Teach about schemas



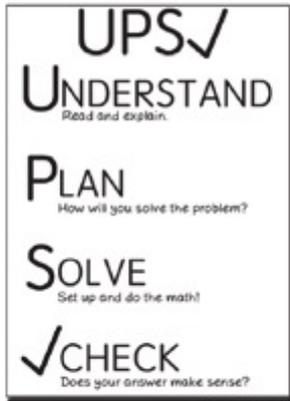
## Chapter 6: Attack Strategies

### SOLVE

Study the problem.  
Organize the facts.  
Line up the plan.  
Verify the plan with computation.  
Examine the answer.

### R-CUBES

Read the problem.  
Circle key numbers.  
Underline the question.  
Box action words.  
Evaluate steps.  
Solve and check.



Page 82



# RIDE

**R**ead the problem.

**I**dentify the relevant information.

**D**etermine the operation and unit for the answer.

**E**nter the correct numbers and calculate, then check the answer.

# RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



## STAR

**S**top and read the problem carefully.

**T**hink about your plan and the strategy you will use.

**A**ct. Follow your plan and solve the problem.

**R**eview your answer.

## RICE

**R**ead and record the problem.

**I**llustrate your thinking.

**C**ompute.

**E**xplain your thinking.



## SUPER

Slowly read the story problem twice.

Underline the question and circle the numbers you need.

Picture it. Draw the scenario to show what is happening.

Explain the problem with a number sentence.

Rewrite the answer in a sentence.

## SHINES

Slowly and carefully read the problem.

Highlight or underline key information.

Identify the question by drawing a circle around it.

Now solve the problem. Show your work.

Examine your work for precision, accuracy, and clarity.

Share your answer by writing a sentence.



## SOLVE

**S**tudy the problem.

**O**rganize the facts.

**L**ine up the plan.

**V**erify the plan with computation.

**E**xamine the answer.

## R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

Evaluate steps.

Solve and check.



# UPS✓

## UNDERSTAND

Read and explain.

## PLAN

How will you solve the problem?

## SOLVE

Set up and do the math!

## ✓CHECK

Does your answer make sense?

Created by Sarah Powell (spowell@ustin.utexas.edu)





Share your favorite attack strategy.



Teach an attack strategy

Teach about schemas



Total

Difference

Change

Page 84

Page 88

Equal Groups

Comparison

Ratios/Proportions



Total

Part-part-whole  
Combine

**Parts** put together into a **total**

Daniela saw **3** canoes and **8** kayaks.  
How many boats did Daniela see?

Total

Daniela saw **11** boats. If **3** of the boats  
were canoes, how many were kayaks?

Part

Daniela saw **11** boats. **8** of the boats  
were kayaks, how many were canoes?

Part



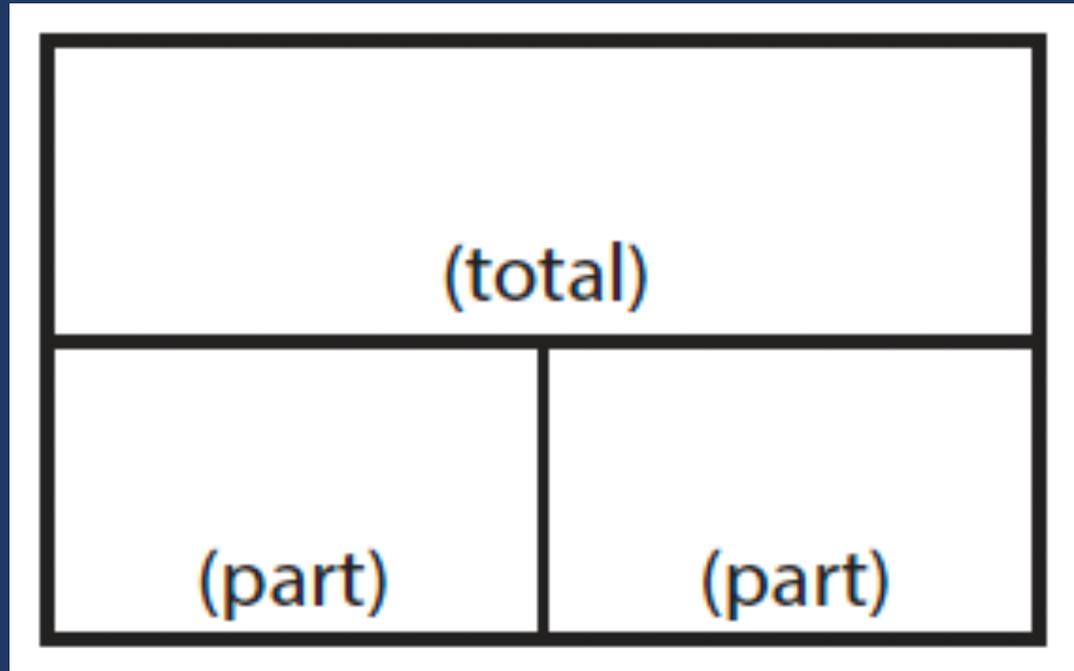
Total

“Are parts put together for a total?”



Total

$$P1 + P2 = T$$



# Total

## Chapter 6: Additive Word Problems

A.  
Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.  
In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.  
Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?

D.  
Damian's dog eats  $9\frac{1}{2}$  cups of dog food each week. Monte's dog eats  $4\frac{1}{4}$  cups less each week than Damian's dog. How much does Monte's dog eat in a week?



# Total



Write a Total problem.

## Chapter 6: Additive Word Problems

E.  
A plant was  $3\frac{3}{4}$  inches tall at the beginning of June. By the end of July, the plant was  $9\frac{1}{8}$  inches tall. How many inches did the plant grow in 2 months?

F.  
Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?




# Difference Compare

**Greater** and **lesser** amounts compared for a **difference**

Adrianna has **10** pencils. Tracy has **4** pencils. How many more pencils does Adrianna have?

Difference

Adrianna has **6** more pencils than Tracy. If Tracy has **4** pencils, how many does Adrianna have?

Greater amount

Tracy has **6** fewer pencils than Adrianna. Adrianna has **10** pencils. How many pencils does Tracy have?

Lesser amount



Total

“Are parts put together for a total?”

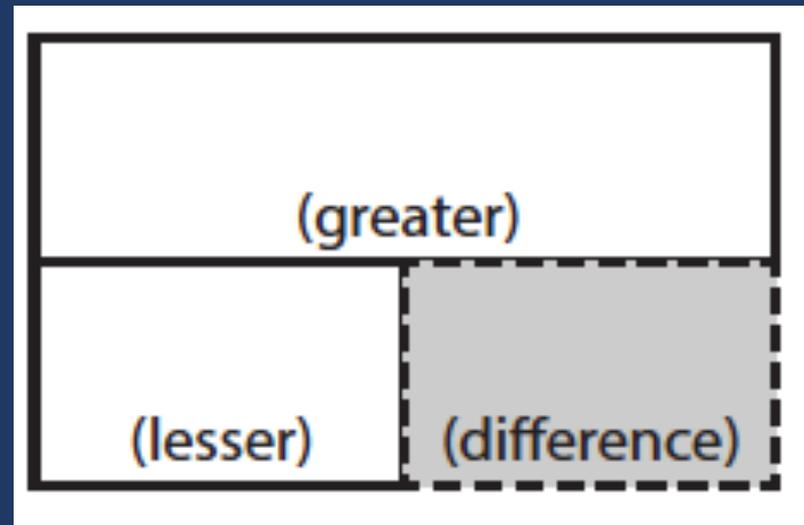
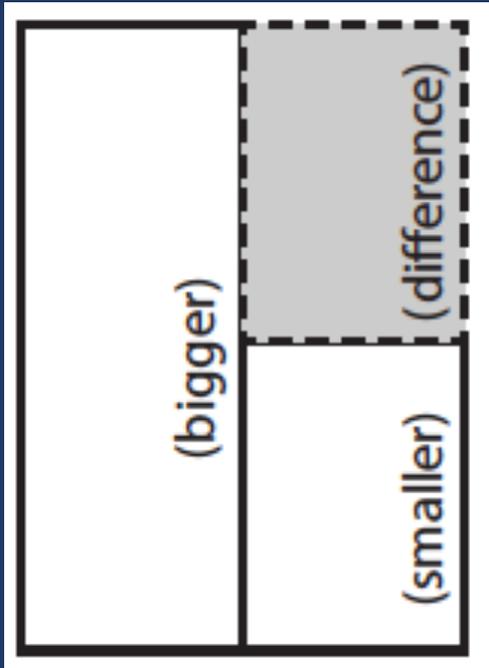
Difference

“Are amounts compared for a difference?”



# Difference

$$G - L = D$$



# Difference

## Chapter 6: Additive Word Problems

A.  
Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

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# Difference



Write a  
Difference  
problem.

## Chapter 6: Additive Word Problems

E.  
A plant was  $3\frac{3}{4}$  inches tall at the beginning of June. By the end of July, the plant was  $9\frac{1}{8}$  inches tall. How many inches did the plant grow in 2 months?

F.  
Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?


# Change

Join

An amount that **increases** or **decreases**

Nickole had **6** notebooks. Then, she bought **3** notebooks. How many notebooks does Nickole have now?

End amount

Nickole had **6** notebooks. Then, she bought a few more notebooks. Now, Nickole has **9** notebooks. How many notebooks did she buy?

Change amount

Nickole had some notebooks. Then, she bought **3** notebooks. Now, Nickole has **9** notebooks. How many notebooks did she have to start with?

Start amount



# Change

Separate

An amount that increases or **decreases**

Samantha baked **20** cookies. Then, she ate **3** of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked **20** cookies. Then, she ate some of the cookies. Now, she has **17** cookies. How many cookies did Samantha eat?

Change amount

Samantha baked some cookies. She ate **3** of the cookies and has **17** cookies left. How many cookies did Samantha bake?

Start amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

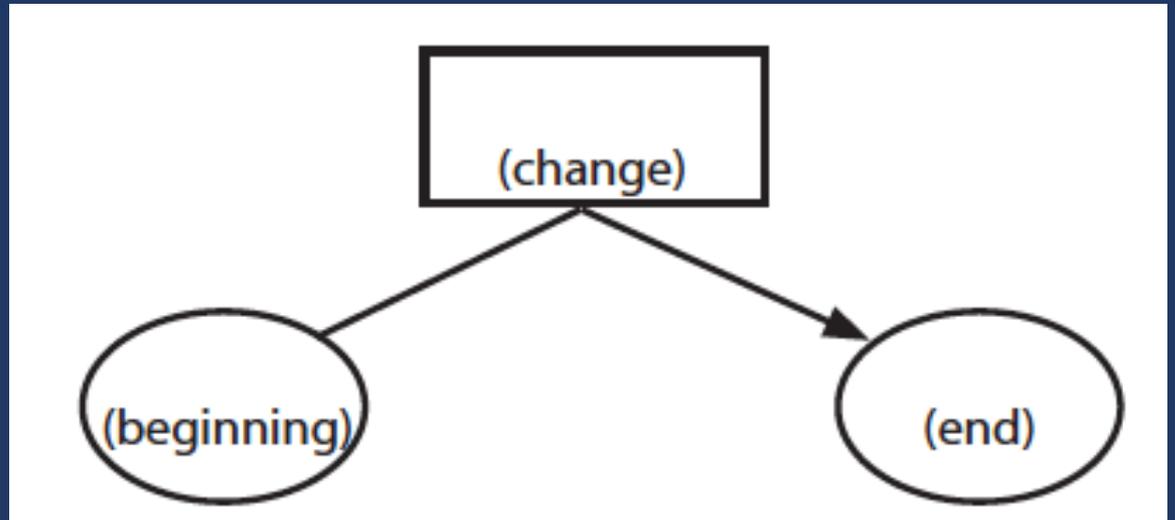
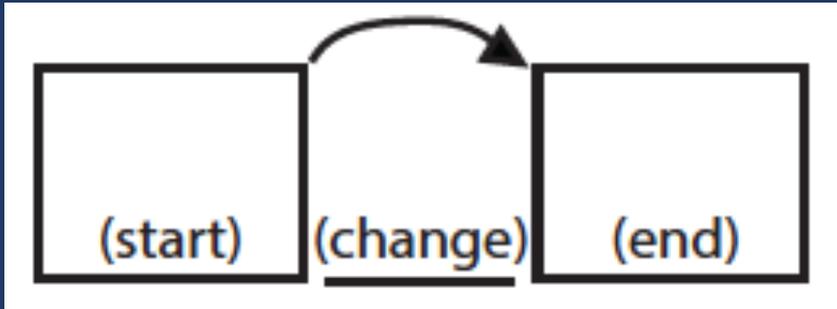
Change

“Does an amount increase or decrease?”



# Change

$$ST \quad + / - \quad C \quad = \quad E$$



# Change

## Chapter 6: Additive Word Problems

E.  
A plant was  $3\frac{3}{4}$  inches tall at the beginning of June. By the end of July, the plant was  $9\frac{1}{8}$  inches tall. How many inches did the plant grow in 2 months?

F.  
Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?



# Change



Write a Change problem.

## Chapter 6: Additive Word Problems

E.  
A plant was  $3\frac{3}{4}$  inches tall at the beginning of June. By the end of July, the plant was  $9\frac{1}{8}$  inches tall. How many inches did the plant grow in 2 months?

F.  
Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?




# Schema Check!



# Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

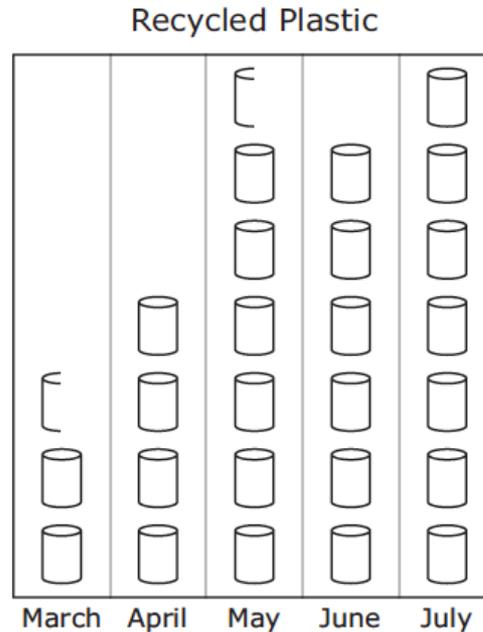
## 26. Part A

The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?

# Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Each  means 20 pounds.

Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

# Total

Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.



## 10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

- Ⓐ 300
- Ⓑ 340
- Ⓒ 350
- Ⓓ 360

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



# Equal Groups

Array  
Vary

**Groups** multiplied by **number in each group** for a **product**

Toni has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Toni have altogether?

Product

Toni has **24** crayons. They want to place them equally into **2** boxes. How many crayons will Toni place in each box?

Number in each group

Toni has **24** crayons. They put them into boxes with **12** crayons each. How many boxes did Toni use?

Groups



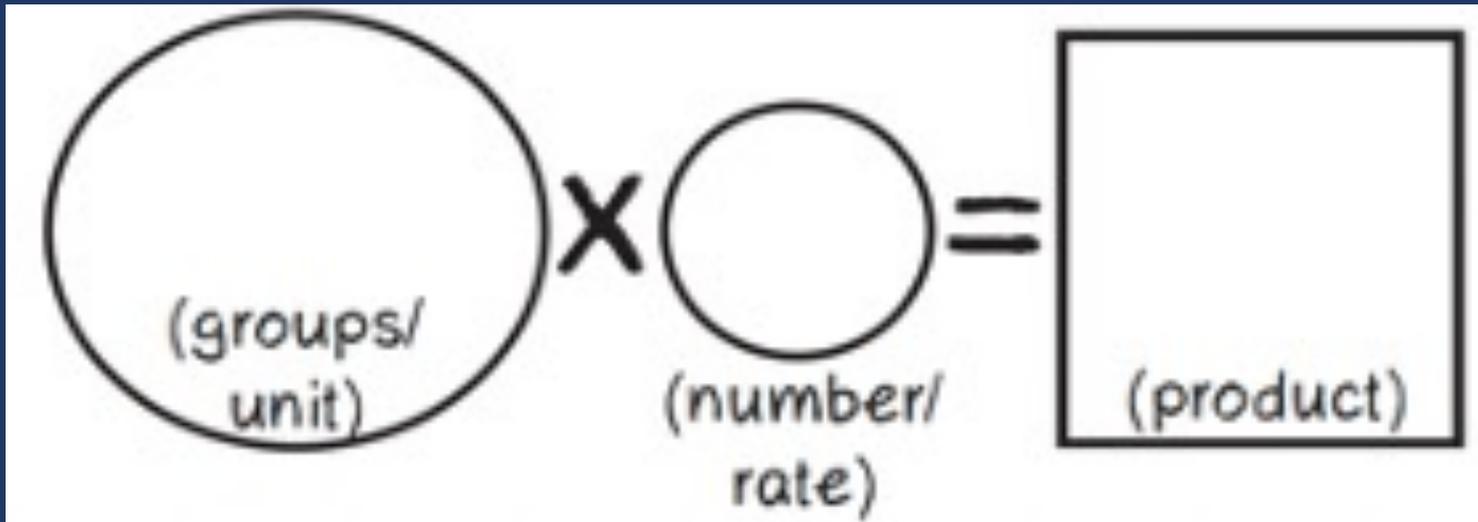
# Equal Groups

“Are there groups with an equal number in each group?”



# Equal Groups

$$GR \times N(E) = P$$



# Equal Groups

## Chapter 6: Multiplicative Word Problems

A.  
Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?

B.  
Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?

C.  
Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

D.  
Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?



# Equal Groups



Write an Equal Groups problem.

## Chapter 6: Multiplicative Word Problems

E. The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

F. Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.


# Comparison

**Set** multiplied by a number of **times**  
for a **product**

Brooke ran **6** minutes. Shaleeni ran **4**  
times longer than Brooke. How many  
minutes did Shaleeni run?

Set

Number of  
times

Product



## Equal Groups

“Are there groups with an equal number in each group?”

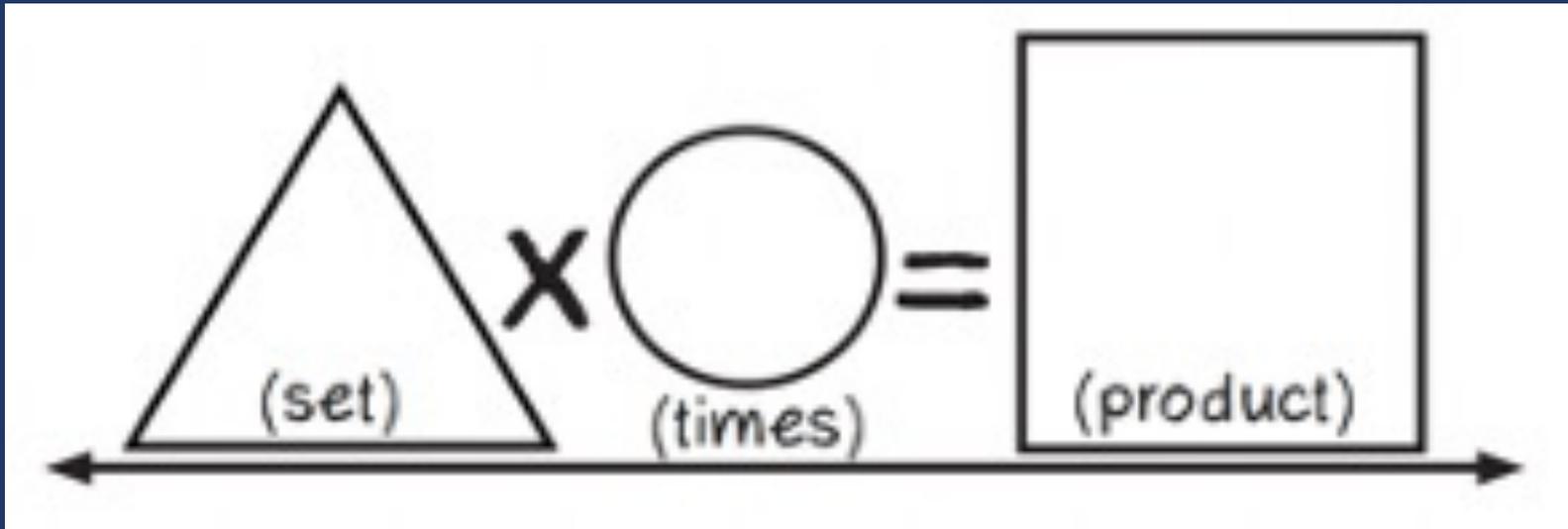
## Comparison

“Is a set compared a number of times?”



# Comparison

$$S \times T = P$$



# Comparison

## Chapter 6: Multiplicative Word Problems

A.  
Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?

B.  
Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?

C.  
Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

D.  
Susan has 7 times as many books as Mo. Mo has 18 books. How many books Susan has?



# Comparison



Write a  
Comparison  
problem.

## Chapter 6: Multiplicative Word Problems

E.  
The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

F.  
Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.


# Ratios/Proportions

Description of **relationships** among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was **3:5**. If she baked **25** brownies, how many cookies did she bake?

Emma typed **56** words in **2** minutes. At this rate, how many words could Emma type in **7** minutes?

## Equal Groups

“Are there groups with an equal number in each group?”

## Comparison

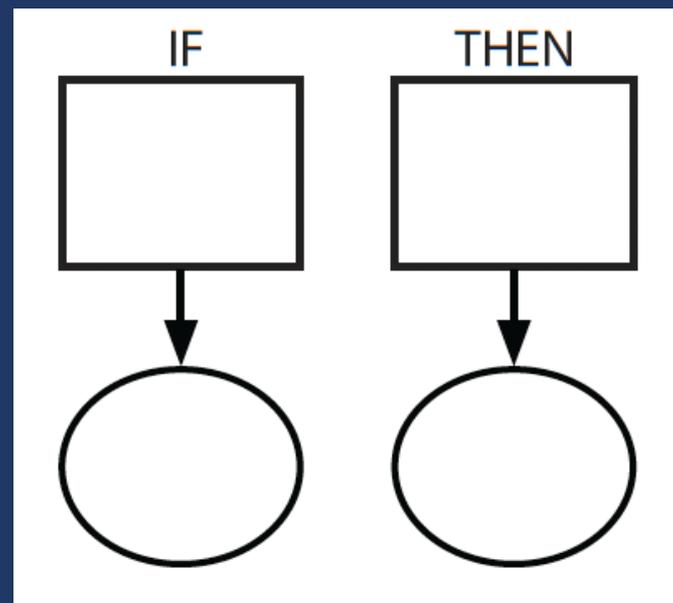
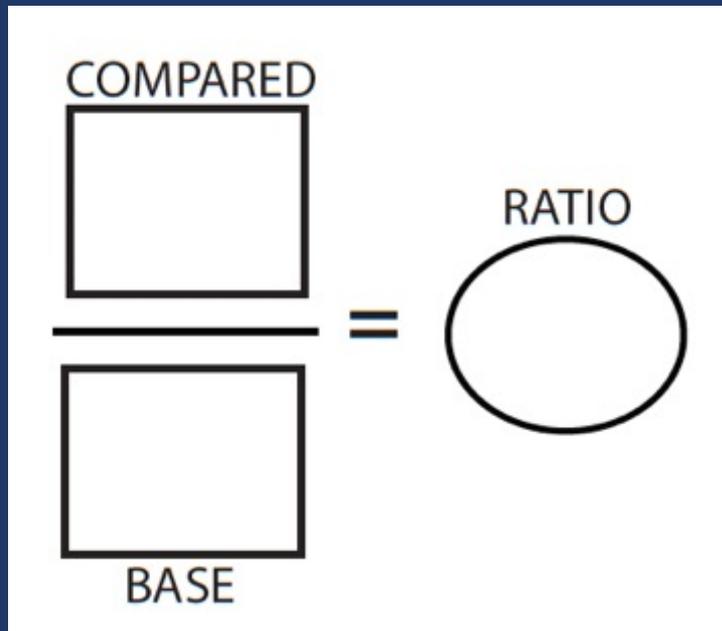
“Is a set compared a number of times?”

## Ratios/Proportions

“Are there relationships among quantities - if this, then this?”



# Ratios/Proportions



# Ratios/Proportions

## Multiplicative Word Problems

H.  
The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

I.  
Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.

J.  
Margarita baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

NOTES:



# Ratios/Proportions



Write a  
Proportions  
problem.

## Chapter 6: Multiplicative Word Problems

E.  
The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

F.  
Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.



# Schema Check!



# Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

# Ratios/Proportions

A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?



# Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Total

Difference

Change

Equal  
Groups

Comparison

Ratio/  
Proportion



Teach an attack strategy

Teach about schemas





# Pirate Math Equation Quest

About

Research

Individual

Small Group

STAAR

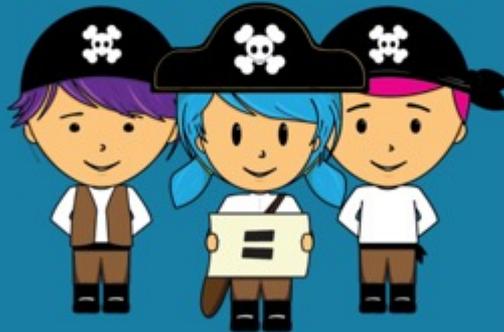
Videos

## Welcome to Pirate Math Equation Quest!

### Individual Word-Problem Intervention



### Small-Group Word-Problem Intervention



### Small-Group Word-Problem Intervention for STAAR



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## Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.



### MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES



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Chapter 7: Designing Interventions

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Chapter 8: Implementing Interventions

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continuum of mathematics learning



Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...

Fluently multiply multi-digit whole numbers using the standard algorithm.

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Fluently add and subtract within 5.

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.



Where student IS

Where student NEEDS TO BE

Fluently add and subtract within 5.

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fluently multiply multi-digit whole numbers using the standard algorithm.

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.



Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and  $\frac{1}{10}$  of what it represents in the place to its left.

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right...

Compose and decompose numbers from 11 to 19 into ten ones and some further ones...

Use place value understanding to round whole numbers to the nearest 10 or 100.





Compose and decompose numbers from 11 to 19 into ten ones and some further ones...

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Use place value understanding to round whole numbers to the nearest 10 or 100.

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right...

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.



Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20...

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve real-world and mathematical problems leading to two linear equations in two variables.

Use multiplication and division within 100 to solve word problems...

Solve addition and subtraction word problems, and add and subtract within 10...

Use addition and subtraction within 100 to solve one- and two-step word problems...

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators...



Solve addition and subtraction word problems, and add and subtract within 10...

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20...

Use addition and subtraction within 100 to solve one- and two-step word problems...

Use multiplication and division within 100 to solve word problems...

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Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators...

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions...

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve real-world and mathematical problems leading to two linear equations in two variables.



Where student IS

Use addition and subtraction within 100 to solve one- and two-step word problems...

Where student NEEDS TO BE

Use multiplication and division within 100 to solve word problems...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations...



Where student IS

Where student NEEDS TO BE

Explain addition and subtraction strategies using place value and properties of operations.

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Apply properties of operations as strategies to multiply and divide...

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...

Use multiplication and division within 100 to solve word problems...

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Understand that three digits of a three-digit number represent hundreds, tens, and ones.

Find whole number quotients: remainders with up to four-digit dividends: one-digit divisors using strategies based on place value, the properties of operations, and/or relationships between multiplication and division.

Fluently multiply multi-digit whole numbers using the standard algorithm.

Use multi-digit whole numbers with place value and whole-number operations to solve problems...



Kindergarten	Grade 1	Grade 2	Grade 3
<b>Comparing and Ordering Numbers</b>			
(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:
(G) compare sets of objects up to at least 20 in each set using comparative language.	(E) use place value to compare whole numbers up to 120 using comparative language.	(D) use place value to compare and order whole numbers up to 1,200 using comparative language, numbers, and symbols (>, <, or =).	(D) compare and order whole numbers up to 100,000 and represent comparisons using the symbols >, <, or =.
(H) use comparative language to describe two numbers up to 20 presented as written numerals.	(F) order whole numbers up to 120 using place value and open number lines.		
	(G) represent the comparison of two numbers to 100 using the symbols >, <, or =.		

<https://www.texasgateway.org/resource/vertical-alignment-charts-revised-mathematics-teks>



An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence	Represent and solve problems involving addition and subtraction	Represent and solve problems involving addition and subtraction	Represent & solve problems involving multiplication and division	Use the four operations with whole numbers to solve problems	Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Add and subtract within 20	Understand properties of multiplication and the relationship between multiplication and division	Generalize place value understanding for multi-digit whole numbers	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Analyze proportional relationships and use them to solve real-world and mathematical problems	Understand the connections between proportional relationships, lines, and linear equations**
Compare numbers	Add and subtract within 20	Use place value understanding and properties of operations to add and subtract	Multiply & divide within 100	Use place value understanding and properties of operations to perform multidigit arithmetic	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems	Use properties of operations to generate equivalent expressions	Analyze and solve linear equations and pairs of simultaneous linear equations
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Work with addition and subtraction equations	Measure and estimate lengths in standard units	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understandings of arithmetic to algebraic expressions	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Define, evaluate, and compare functions
Work with numbers 11-19 to gain foundations for place value	Extend the counting sequence	Relate addition and subtraction to length	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Reason about and solve one-variable equations and inequalities		Use functions to model relationships between quantities
	Understand place value		Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects	Understand decimal notation for fractions, and compare decimal fractions	Graph points in the coordinate plane to solve real-world and mathematical problems*	Represent and analyze quantitative relationships between dependent and independent variables		
	Use place value understanding and properties of operations to add and subtract		Geometric measurement: understand concepts of area and relate area to multiplication and to addition					
	Measure lengths indirectly and by iterating length units							

\* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.

\*\* Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

<https://achievethecore.org/category/774/mathematics-focus-by-grade-level>



Table A.3. Grades 6–8 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*



**Curriculum Focal Points and Connections**

**Expectations of the Content Standards**

**Grade 6 Curriculum Focal Points**

**Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals**

Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.

**Number and Operations: Connecting ratio and rate to multiplication and division**

Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., “If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.

**Algebra: Writing, interpreting, and using mathematical expressions and equations**

Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that

*Number and Operations, Grades 6–8*

- ● ● Work flexibly with fractions, decimals, and percents to solve problems
- ● ● Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line
- ● Develop meaning for percents greater than 100 and less than 1
- ● Understand and use ratios and proportions to represent quantitative relationships
- ● Develop an understanding of large numbers [identified in Grades 4 and 5 Curriculum Focal Points] and recognize and appropriately use exponential, scientific, and calculator notation
- Use factors, multiples, prime factorization, and relatively prime numbers to solve problems
- Develop meaning for integers and represent and compare quantities with them
- ● Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
- ● Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals
- ● Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems
- ● Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

<https://www.nctm.org/curriculumfocalpoints/>





What are your resources for planning around a continuum of math?

How can your district or school engage in vertical planning?



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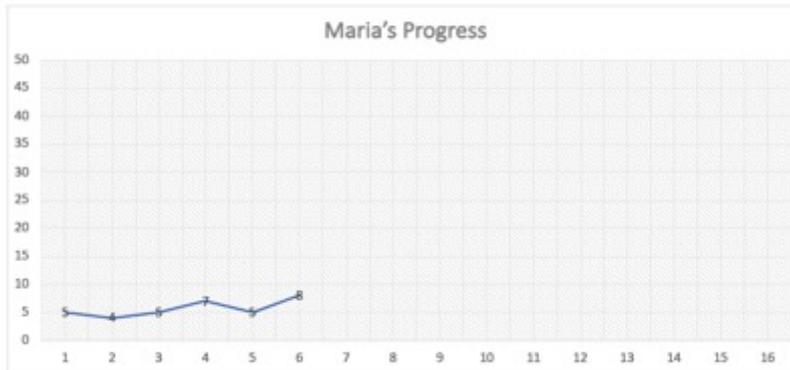


## Chapter 12: Progress Monitoring

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### Goal Setting: Benchmark

1. Identify appropriate grade-level benchmark
2. Mark benchmark on student graph with an X
3. Draw goal-line from baseline progress monitoring scores to X



# What is Progress Monitoring?

Tests/measures/probes administered frequently

Compare scores to understand mathematics growth

Must be reliable and  
valid

Must have alternate  
forms



# Where to Find Progress Monitoring Measures?

National Center on Intensive Intervention

The screenshot shows the website header with the logo "National Center on INTENSIVE INTERVENTION at the American Institutes for Research®". A search bar and social media icons are on the right. The navigation menu includes "About DBI -", "Tools Charts -" (circled in yellow), "Implementation & Intervention -", "Training -", "Special Topics -", "Resource by Audience -", and "News & Events". Below the menu, there are three main content areas: 1) A blue box titled "What is DBI? Learn about NCII's approach to intensive intervention!". 2) A diagram showing a flow from "Validated Intervention Program" to "Progress Monitor", which then leads to "Diagnosis Data" and "Implementation Adaptation". 3) A diagram showing a "Validated Intervention Program" leading to a "Progress Monitor" which can be "NONRESPONSIVE" or "RESPONSIVE". Below these are three smaller promotional boxes: "Register for our Upcoming Webinar on Building Social and Emotional Competencies Among Students with Intensive Needs", "Recommendations for Building State Capacity to Support DBI Implementation", and "New Self-Paced Module: Using Teaming to Implement DBI".



[www.intensiveintervention.org](http://www.intensiveintervention.org)



# Progress Monitoring Considerations

- Skills to be measured—age and grade appropriate
- Cost and training requirements
- Administration and scoring time
- Data management
- Technical rigor (consider population)
  - Reliability
  - Validity
  - Evidence of being sensitive to change
  - Alternate/parallel forms



# Number Identification

6 16 23 10 17

38 97 20 15 24

14 33 11 79 8

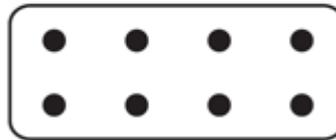
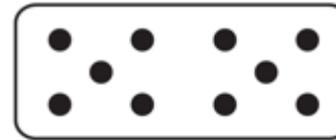
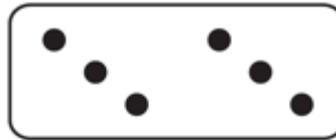
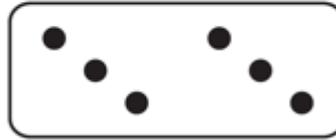
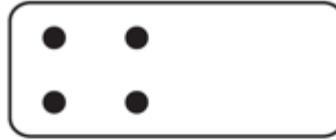
21 19 93 3 49

4 30 12 9 1

28 7 27 2 13



# Quantity Discrimination



# Missing Number

13 \_\_\_ 15 16

50 60 \_\_\_ 80

40 45 \_\_\_ 55

50 51 \_\_\_ 53

23 \_\_\_ 25 26

15 20 \_\_\_ 30

27 28 \_\_\_ 30

38 48 \_\_\_ 68

75 \_\_\_ 85 90

83 \_\_\_ 85 86

Acadience® Math / Computation Grade 4  
Benchmark 1 / Form A

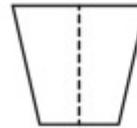
Total: \_\_\_\_\_

1. $\begin{array}{r} 527 \\ +320 \\ \hline \end{array}$	2. $\begin{array}{r} 4778 \\ +2242 \\ \hline \end{array}$	3. $8\frac{4}{5} - 6\frac{2}{5} =$	4. $\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$	5. $4\overline{)573}$
6. $\begin{array}{r} 197 \\ - 74 \\ \hline \end{array}$	7. $\frac{5}{8} + \frac{2}{8} =$	8. $\begin{array}{r} 7273 \\ - 387 \\ \hline \end{array}$	9. $\begin{array}{r} 19 \\ \times 11 \\ \hline \end{array}$	10. $9\frac{7}{12} - 1\frac{4}{12} =$
11. $8\overline{)642}$	12. $7\overline{)49}$	13. $\begin{array}{r} 99 \\ \times 72 \\ \hline \end{array}$	14. $\frac{1}{4} + \frac{2}{4} =$	15. $\begin{array}{r} 526 \\ \times 6 \\ \hline \end{array}$
16. $8\frac{9}{10} - 1\frac{5}{10} =$	17. $\frac{1}{3} + \frac{1}{3} =$	18. $\frac{9}{12} - \frac{2}{12} =$	19. $\begin{array}{r} 829 \\ \times 7 \\ \hline \end{array}$	20. $6\overline{)939}$
21. $3\overline{)397}$	22. $\begin{array}{r} 65 \\ \times 23 \\ \hline \end{array}$	23. $\begin{array}{r} 2414 \\ - 668 \\ \hline \end{array}$	24. $\begin{array}{r} 7568 \\ +1638 \\ \hline \end{array}$	25. $\begin{array}{r} 34 \\ \times 12 \\ \hline \end{array}$

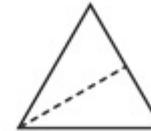
Computation

Total: \_\_\_\_\_

1. Is the dotted line a line of symmetry for each shape? Write "yes" or "no" in the space provided below each shape.



\_\_\_\_\_



\_\_\_\_\_



\_\_\_\_\_

2. Compare the number in Box 1 with the number in Box 2. Fill in the blank with > (greater than), = (equal to), or < (less than):

Box 1	>, =, <	Box 2
835		751
333		613
131		168

3. List three numbers that are multiples of 4:

\_\_\_\_\_

4. Jake read 17 books over the summer that were nonfiction and 43 books that were fiction. His friend Ross read 38 books total. How many more books did Jake read than Ross? \_\_\_\_\_ books.

5. Compare the decimal in Box 1 with the decimal in Box 2. Fill in the blank with > (greater than), = (equal to), or < (less than):

Box 1	>, =, <	Box 2
0.47		0.25
0.39		0.68
0.89		0.91

6. We rented a movie that was 2 hours and 15 minutes long. How many minutes total was the movie? \_\_\_\_\_ minutes.

# Concepts and Applications





Next Number /  
Counting

Grades K-1

Geometry /  
Measurement

Grades 1-6

Proportional  
Reasoning / Quantity  
Discrimination /  
Number Properties

Middle School

Algebra

High School

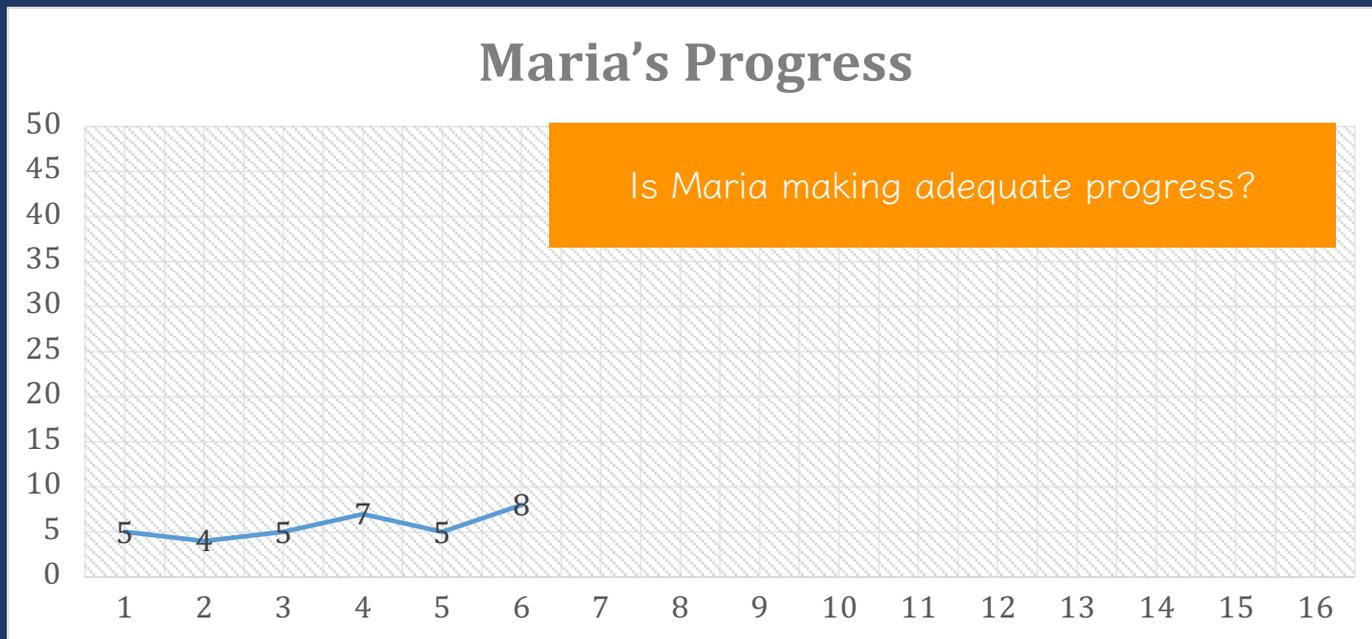




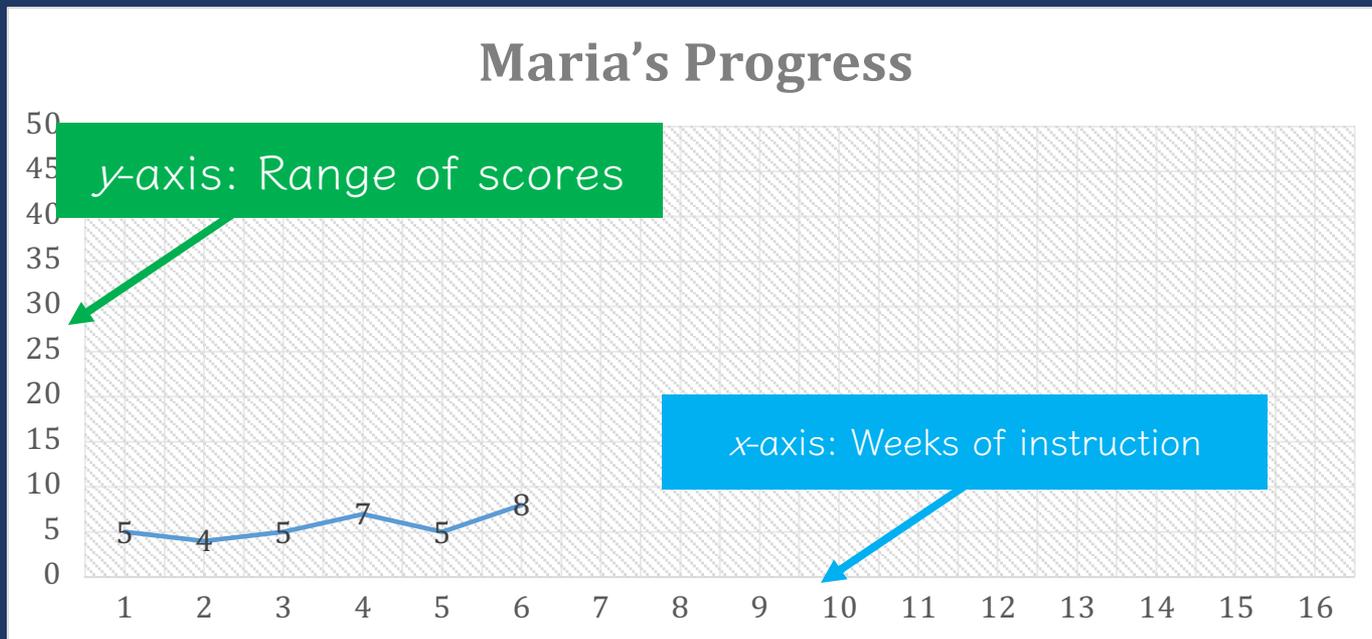
Which progress monitoring measures do you use (or plan to use) at middle school?



# Goal Setting and Decision Making



# Graphing



# Setting Goals

Benchmark

Slope (ROI)

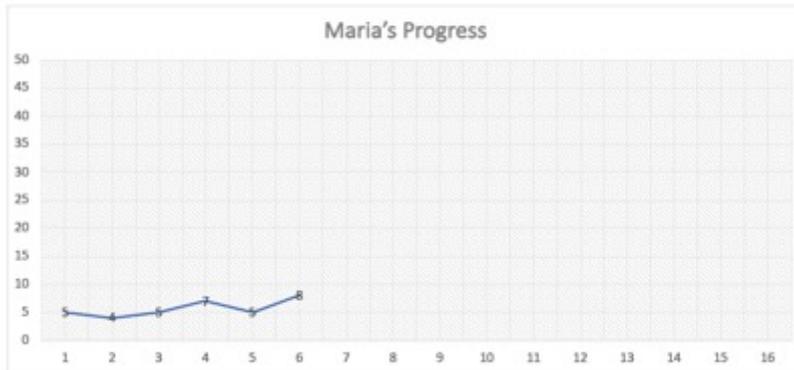
Intra-  
individual



## Chapter 12: Progress Monitoring

### Goal Setting: Benchmark

1. Identify appropriate grade-level benchmark
2. Mark benchmark on student graph with an X
3. Draw goal-line from baseline progress monitoring scores to X



## Benchmark

1. Identify appropriate grade-level benchmark
2. Mark benchmark on student graph with an X
3. Draw goal-line from baseline progress monitoring scores to X



# Benchmark

## 1. Identify appropriate grade-level benchmark

Grade	Computation	Concepts and Applications
1	20 digits	20 points
2	20 digits	20 points
3	30 digits	30 points
4	40 digits	30 points
5	30 digits	15 points
6	35 digits	15 points

# Benchmark

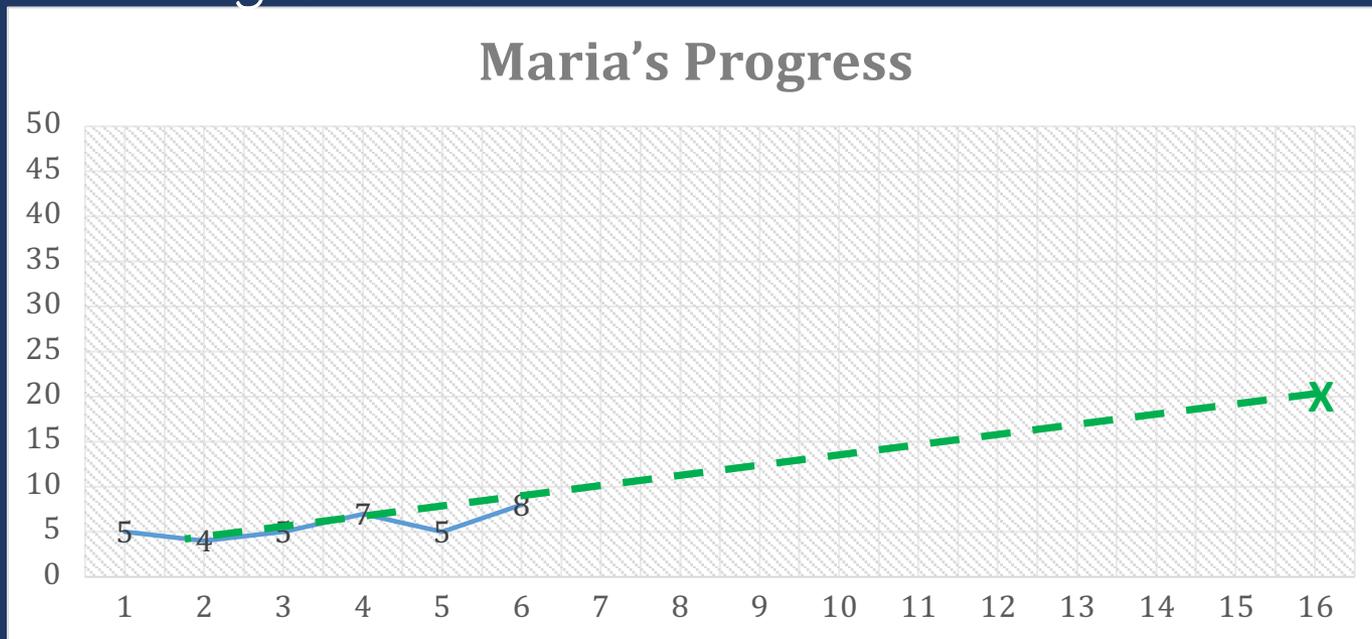
## 1. Identify appropriate grade-level benchmark

Grade	Computation	Concepts and Applications
1	20 digits	20 points
2	20 digits	20 points
3	30 digits	30 points
4	40 digits	30 points
5	30 digits	15 points
6	35 digits	15 points

Maria: 2<sup>nd</sup>-  
grade  
student  
using  
Computation

# Benchmark

1. Identify appropriate grade-level benchmark
2. Mark benchmark on student graph with an X
3. Draw goal-line from baseline progress monitoring scores to X



# Setting Goals

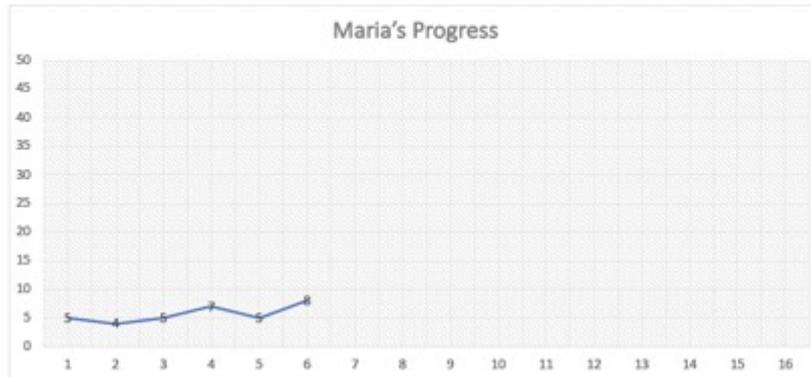
Benchmark

Slope (ROI)



### Goal Setting: Slope (Rate of Improvement)

1. Locate slope (i.e., rate of improvement – ROI)
2. Multiply ROI by number of weeks left in intervention
3. Add to baseline of progress monitoring scores
4. Mark goal on student graph with an X
5. Draw goal-line from baseline progress monitoring scores to X



## Slope (ROI)

1. Locate slope (i.e., rate of improvement – ROI)
2. Multiply ROI by number of weeks left in intervention
3. Add to baseline of progress monitoring scores
4. Mark goal on student graph with an X
5. Draw goal-line from baseline progress monitoring scores to X



# Slope (ROI)

1. Locate slope (i.e., rate of improvement – ROI)

Grade	Computation—Slope for Digits Correct	Concepts and Applications — Slope for Points
1	0.35	No data available
2	0.30	0.40
3	0.30	0.60
4	0.70	0.70
5	0.70	0.70
6	0.40	0.70

# Slope (ROI)

1. Locate slope (i.e., rate of improvement – ROI)

Grade	Computation—Slope for Digits Correct	Concepts and Applications — Slope for Points
1	0.35	No data available
2	0.30	0.40
3	0.30	0.60
4	0.70	0.70
5	0.70	0.70
6	0.40	0.70

Maria: 2<sup>nd</sup>-  
grade  
student using  
Computation

# Slope (ROI)

1. Locate slope (i.e., rate of improvement – ROI)

0.30



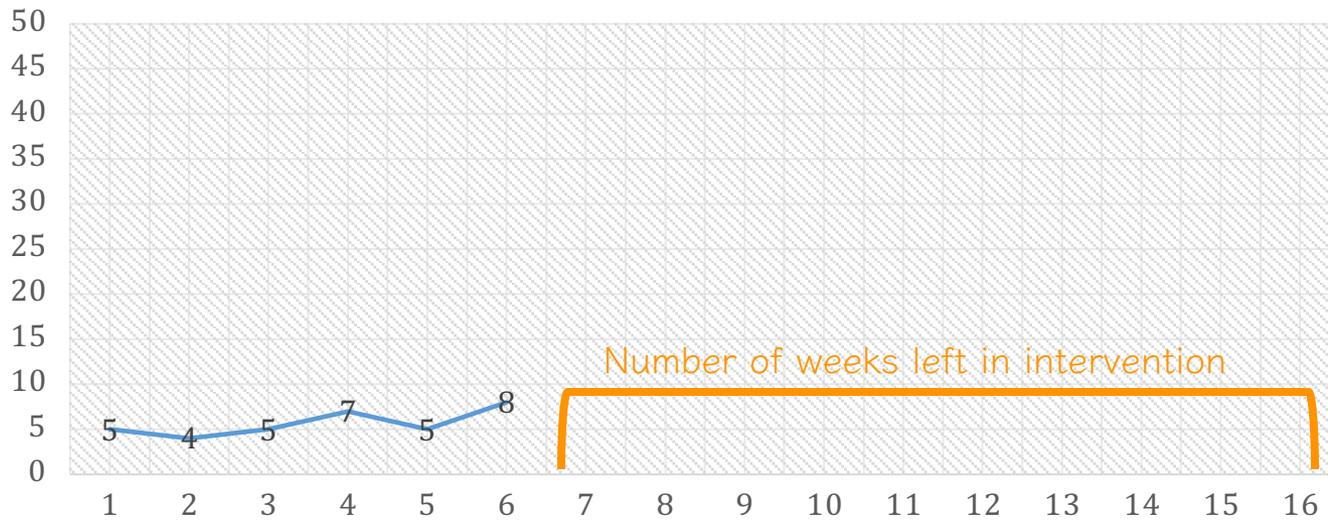
## Slope (ROI)

1. Locate slope (i.e., rate of improvement – ROI) 0.30
2. Multiply ROI by number of weeks left in intervention 0.30 ×



# Slope (ROI)

## Maria's Progress



## Slope (ROI)

1. Locate slope (i.e., rate of improvement – ROI) 0.30
2. Multiply ROI by number of weeks left in intervention  $0.30 \times 10 = 3$



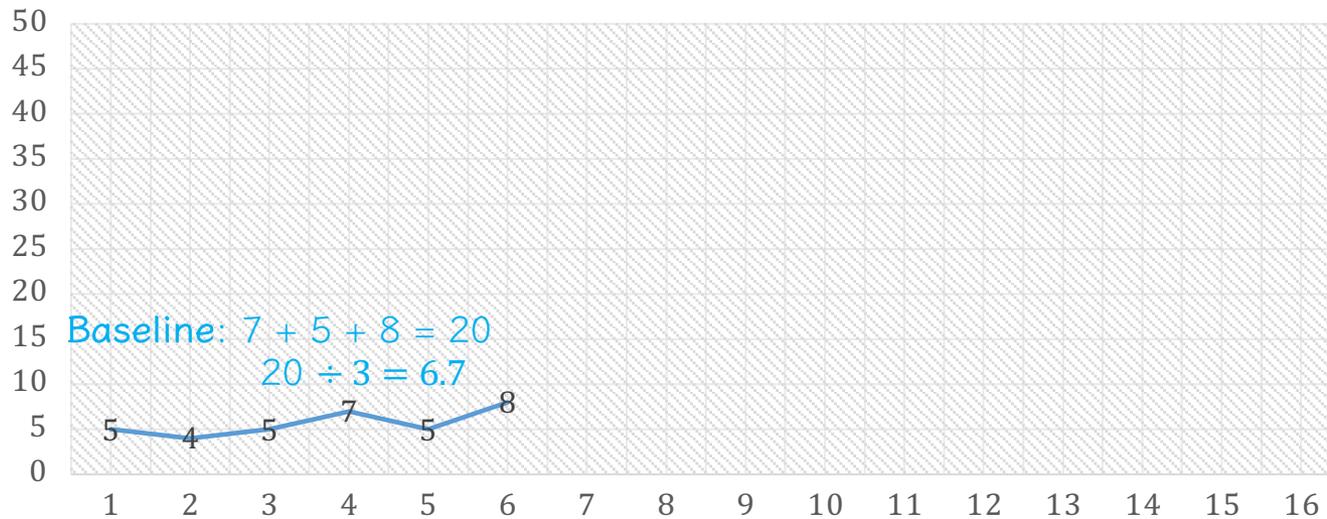
## Slope (ROI)

1. Locate slope (i.e., rate of improvement – ROI) 0.30
2. Multiply ROI by number of weeks left in intervention  $0.30 \times 10 = 3$
3. Add to baseline of progress monitoring scores  $3 +$



# Slope (ROI)

## Maria's Progress



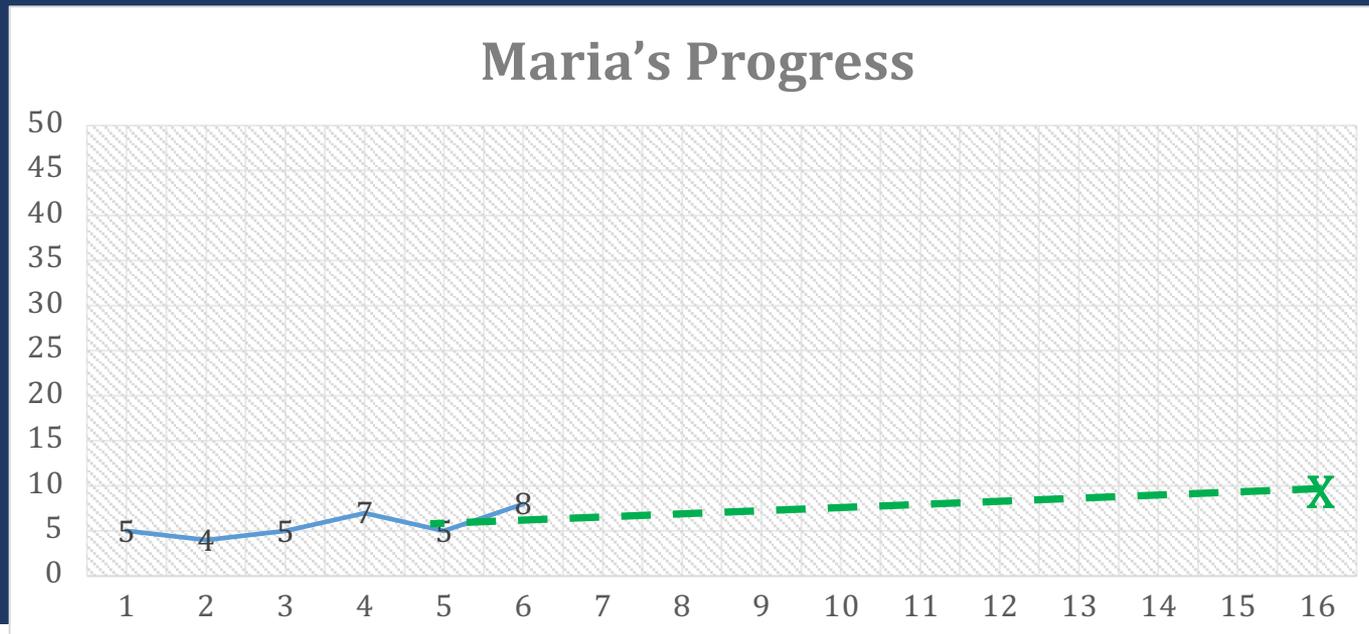
## Slope (ROI)

1. Locate slope (i.e., rate of improvement – ROI)  $0.30$
2. Multiply ROI by number of weeks left in intervention  $0.30 \times 10 = 3$
3. Add to baseline of progress monitoring scores  $3 + 6.7 = 9.7$



# Slope (ROI)

1. Locate slope (i.e., rate of improvement – ROI)  $0.30$
2. Multiply ROI by number of weeks left in intervention  $0.30 \times 10 = 3$
3. Add to baseline of progress monitoring scores  $3 + 6.7 = 9.7$
4. Mark goal on student graph with an X
5. Draw goal-line from baseline progress monitoring scores to X



# Setting Goals

Benchmark

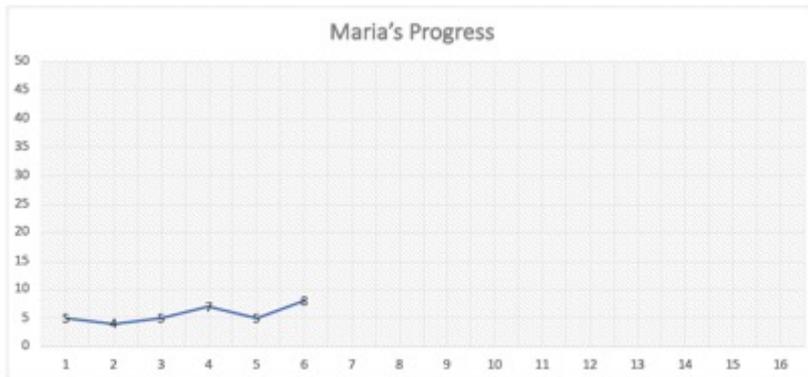
Slope (ROI)

Intra-  
individual



### Goal Setting: Intra-Individual Framework

1. Identify student's slope
2. Multiply slope by 1.5
3. Multiply by number of weeks until end of intervention
4. Add to student's baseline score
5. Mark goal on student graph with an X
6. Draw goal-line from baseline progress monitoring scores to X



## Intra- individual

1. Identify student's slope
2. Multiply slope by 1.5
3. Multiply by number of weeks until end of intervention
4. Add to student's baseline score
5. Mark goal on student graph with an X
6. Draw goal-line from baseline progress monitoring scores to X



# Intra- individual

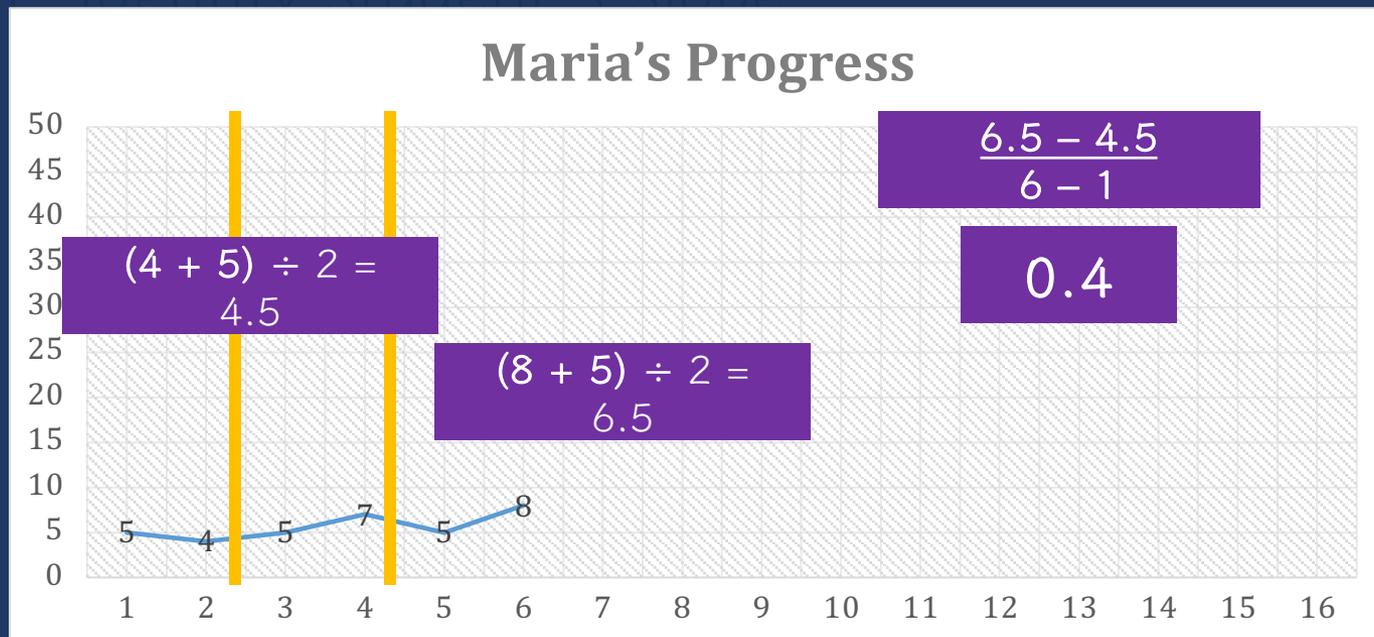
1. Identify student's slope

$$\text{SLOPE CALCULATION:}$$
$$\frac{3^{\text{rd}} \text{ median} - 1^{\text{st}} \text{ median}}{\# \text{data points} - 1}$$



# Intra- individual

SLOPE CALCULATION:  
 $\frac{3^{\text{rd}} \text{ median} - 1^{\text{st}} \text{ median}}{\# \text{data points} - 1}$



# Intra- individual

1. Identify student's slope

0.4



## Intra- individual

1. Identify student's slope
2. Multiply slope by 1.5

0.4

$$0.4 \times 1.5 = 0.6$$



## Intra- individual

1. Identify student's slope

0.4

2. Multiply slope by 1.5

$$0.4 \times 1.5 = 0.6$$

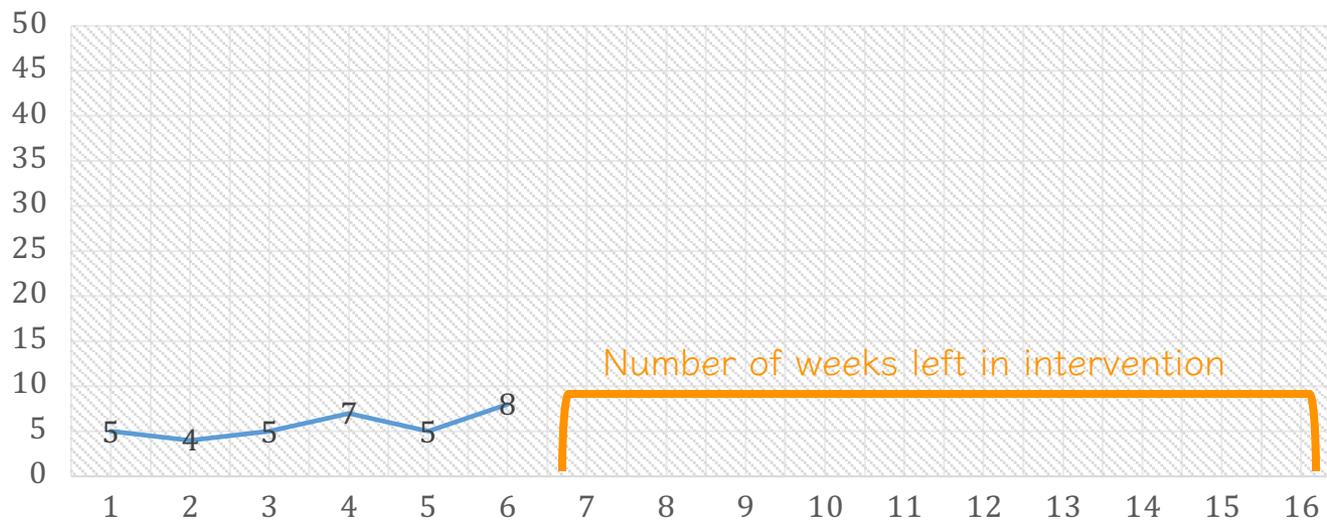
3. Multiply by number of weeks until end of intervention

0.6 ×



# Intra- individual

## Maria's Progress



## Intra- individual

1. Identify student's slope
2. Multiply slope by 1.5
3. Multiply by number of weeks until end of intervention

$$0.4$$

$$0.4 \times 1.5 = 0.6$$

$$0.6 \times 10 = 6$$



## Intra- individual

1. Identify student's slope
2. Multiply slope by 1.5
3. Multiply by number of weeks until end of intervention
4. Add to student's baseline score

0.4

$$0.4 \times 1.5 = 0.6$$

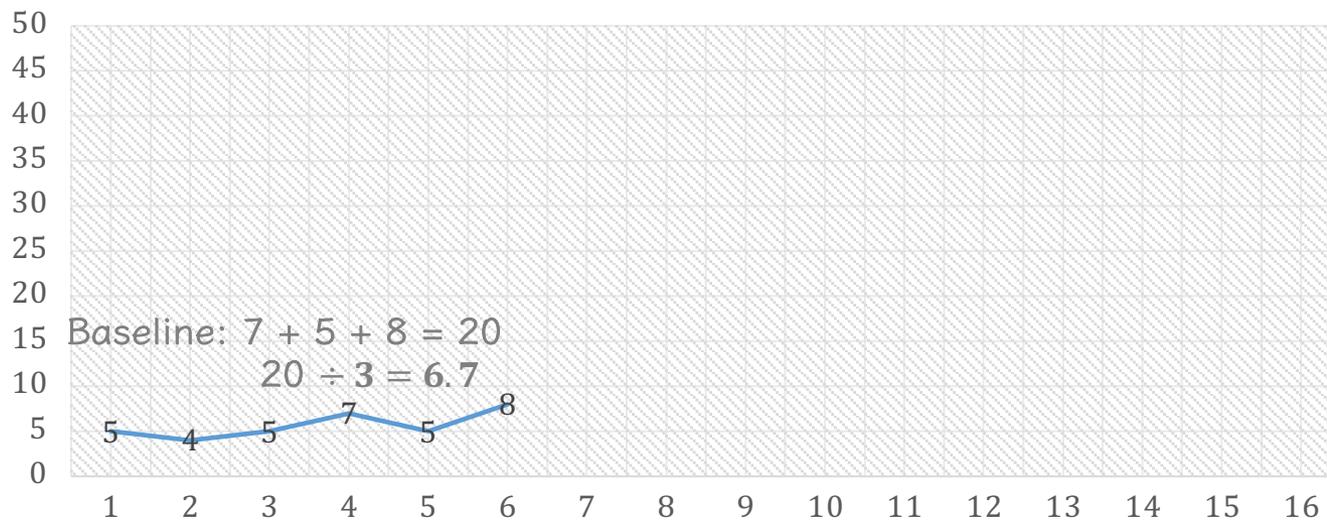
$$0.6 \times 10 = 6$$

6



# Intra- individual

## Maria's Progress



## Intra- individual

1. Identify student's slope
2. Multiply slope by 1.5
3. Multiply by number of weeks until end of intervention
4. Add to student's baseline score

$$0.4$$

$$0.4 \times 1.5 = 0.6$$

$$0.6 \times 10 = 6$$

$$6 + 6.7 = 12.7$$



# Intra- individual

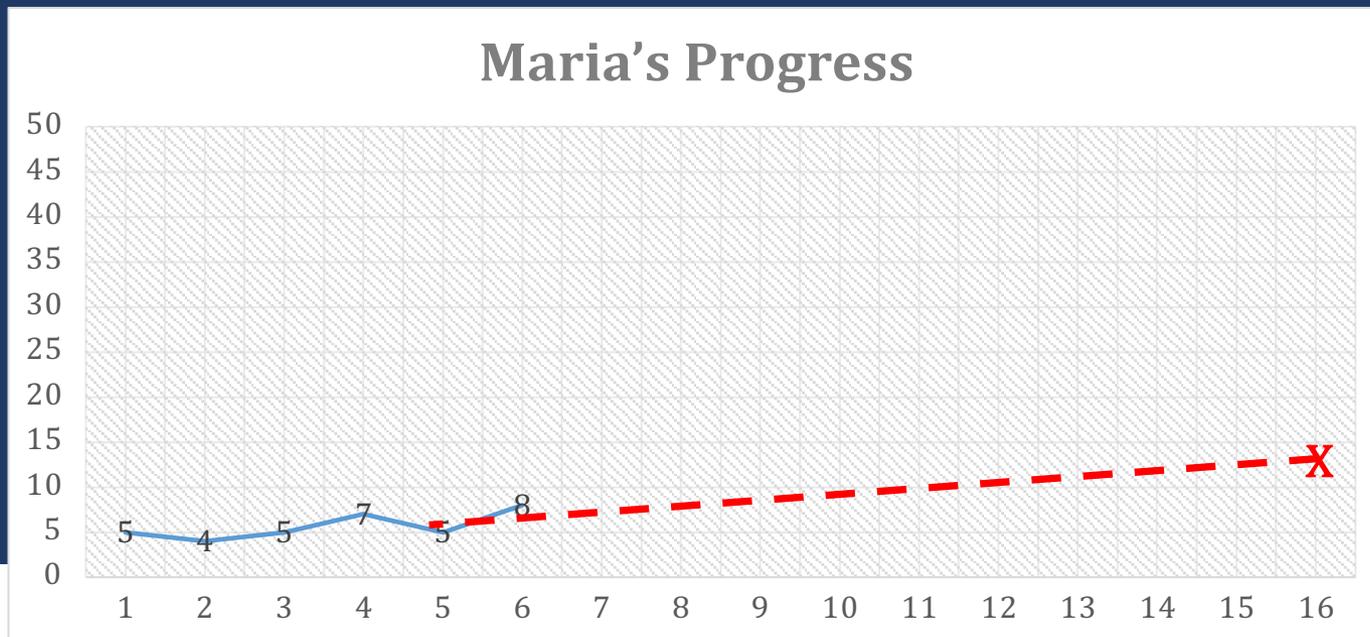
1. Identify student's slope
2. Multiply slope by 1.5
3. Multiply by number of weeks until end of intervention
4. Add to student's baseline score
5. Mark goal on student graph with an X
6. Draw goal-line from baseline progress monitoring scores to X

$$0.4$$

$$0.4 \times 1.5 = 0.6$$

$$0.6 \times 10 = 6$$

$$6 + 6.7 = 12.7$$



# To Review

Benchmark

Slope (ROI)

Intra-  
individual





Which goal setting method(s) might you use?

# Determining Response

Four most recent, consecutive scores



Trendline



## Determining Response

Four most recent, consecutive scores

Trendline



# Determining Response

Four most recent, consecutive scores



If at least 6 weeks of instruction have occurred:

- If all four most recent scores fall **above** the goal-line, increase the goal.

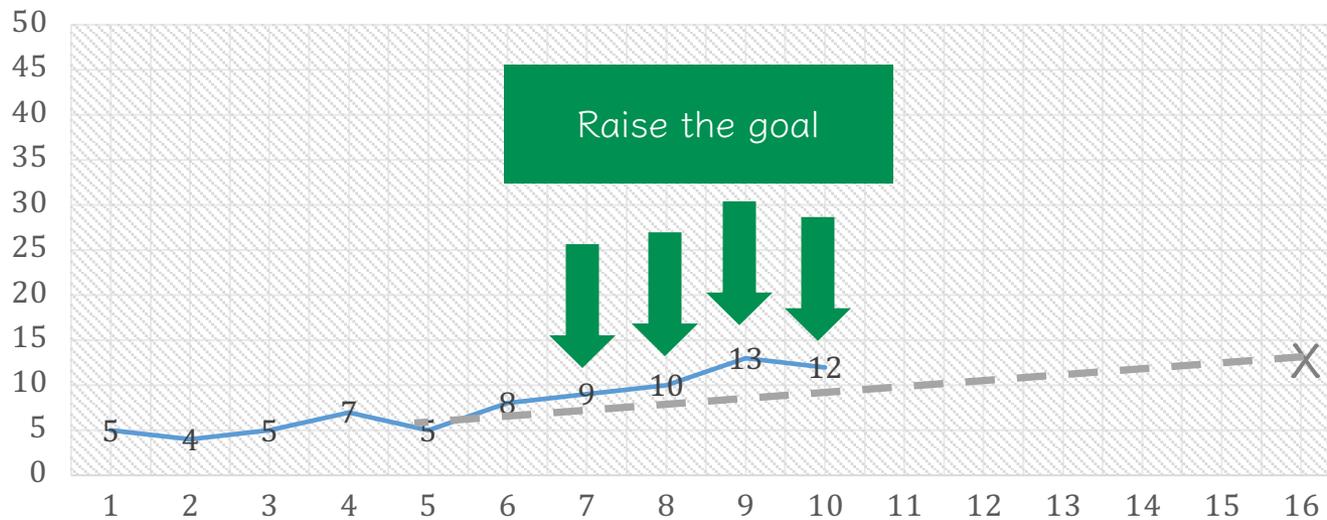


# Determining Response

Four most recent, consecutive scores



## Maria's Progress



# Determining Response

Four most recent, consecutive scores



If at least 6 weeks of instruction have occurred:

- If all four most recent scores fall **above** the goal-line, increase the goal.
- If all four most recent scores fall **below** the goal-line, adapt the intervention.

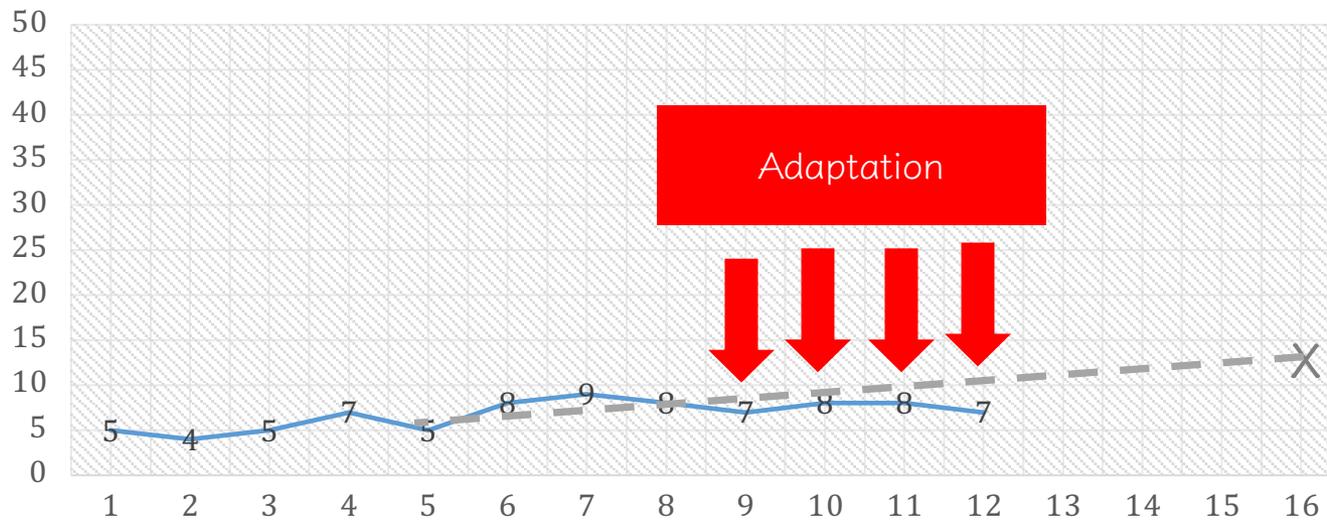


# Determining Response

Four most recent, consecutive scores



## Maria's Progress



# Determining Response

Four most recent, consecutive scores

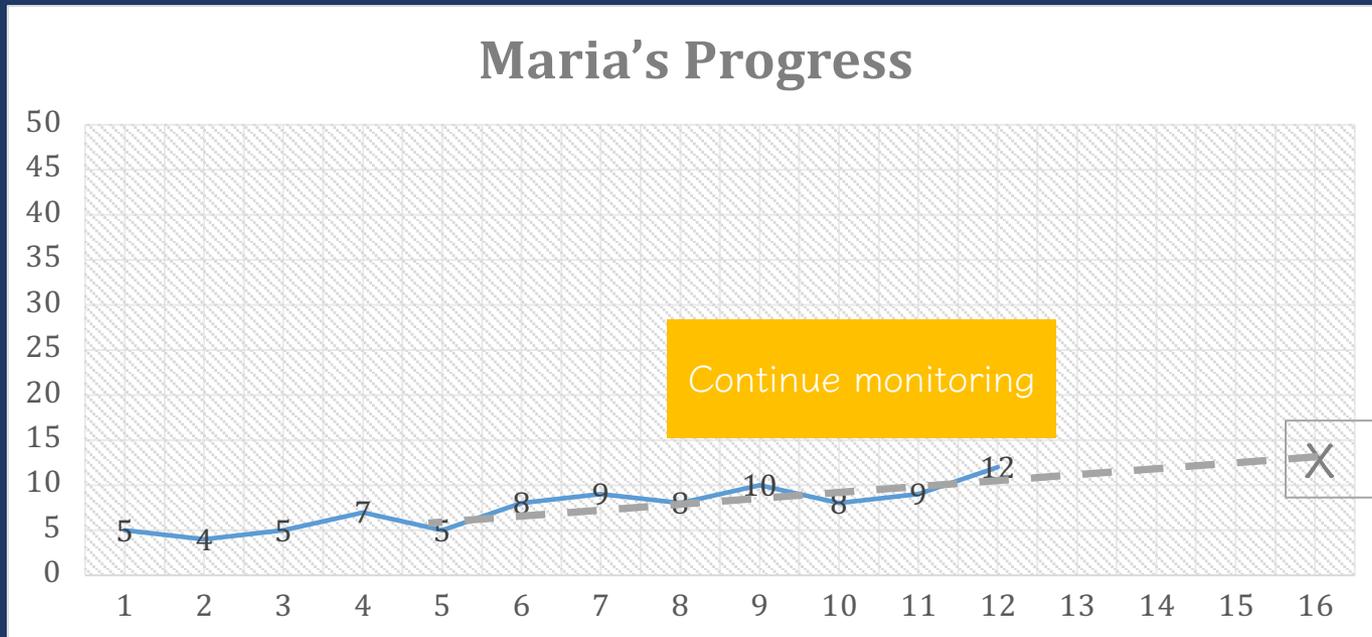


If at least 6 weeks of instruction have occurred:

- If all four most recent scores fall **above** the goal-line, increase the goal.
- If all four most recent scores fall **below** the goal-line, adapt the intervention.
- If the four most recent scores fall both **above and below** the goal-line, continue monitoring data.

# Determining Response

Four most recent, consecutive scores



# Determining Response

Four most recent, consecutive scores



Trendline

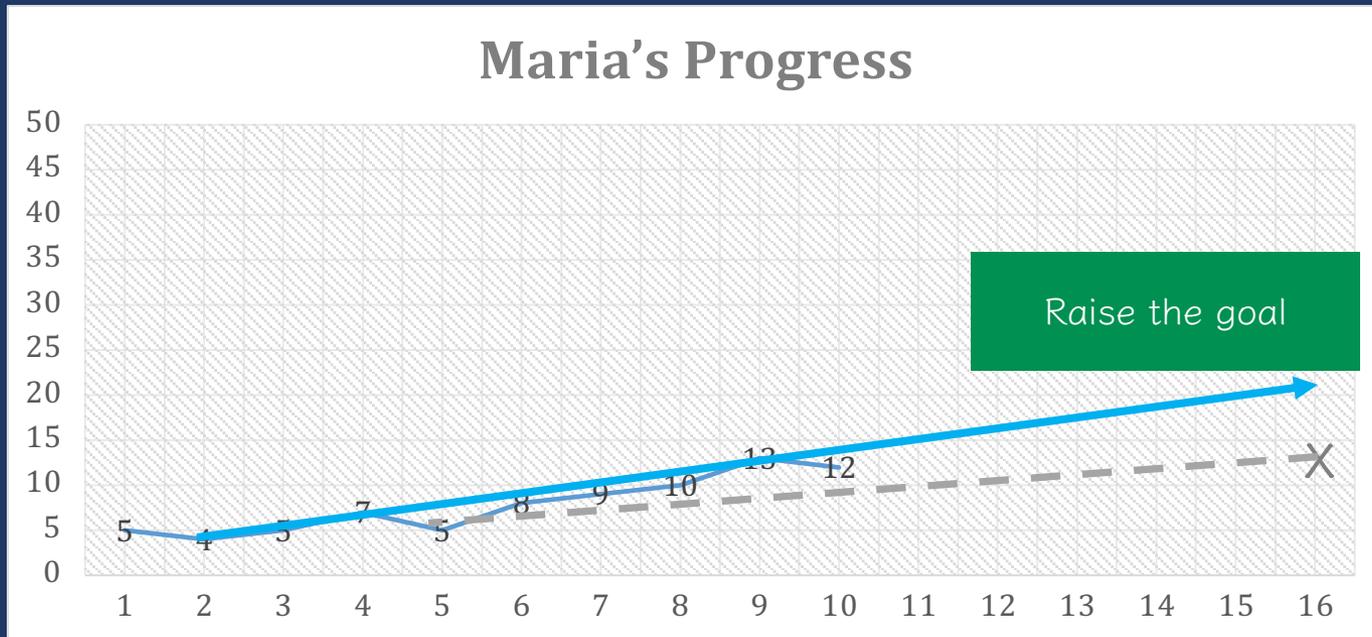


# Determining Response



- If the trend-line is **steeper** than the goal line, then increase the goal.

# Determining Response

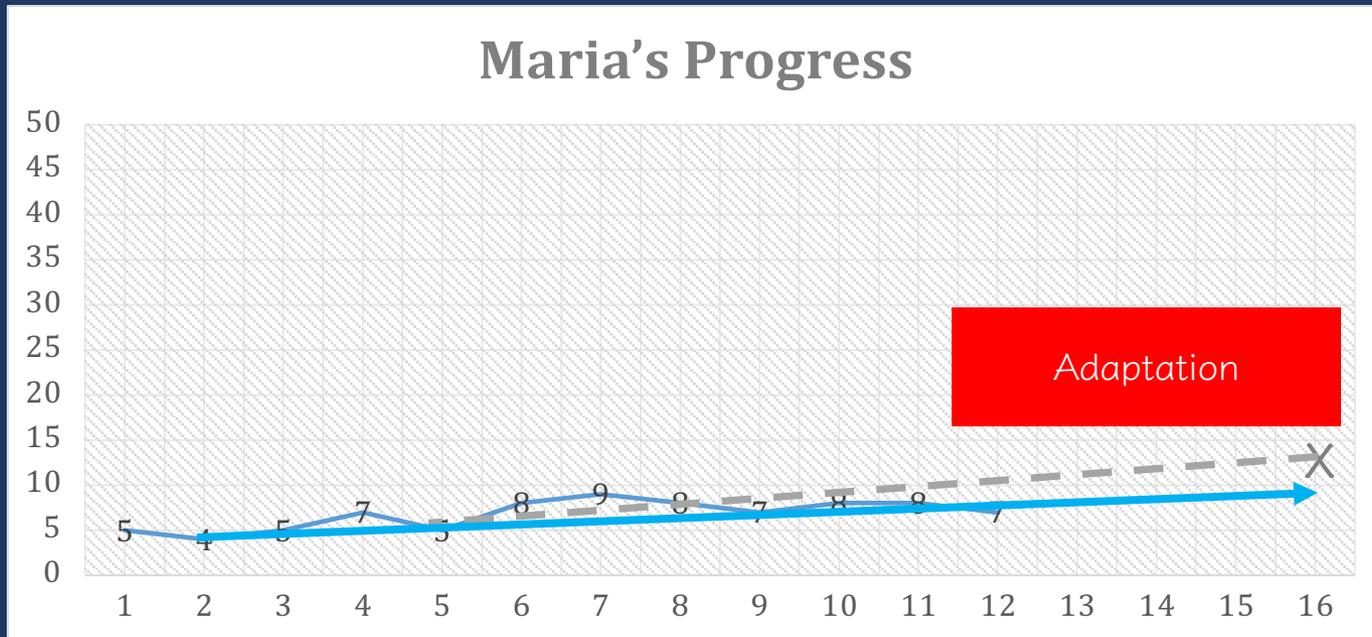


# Determining Response



- If the trend-line is **steeper** than the goal line, then increase the goal.
- If the trend-line is **flatter** than the goal line, then adapt the intervention.

# Determining Response

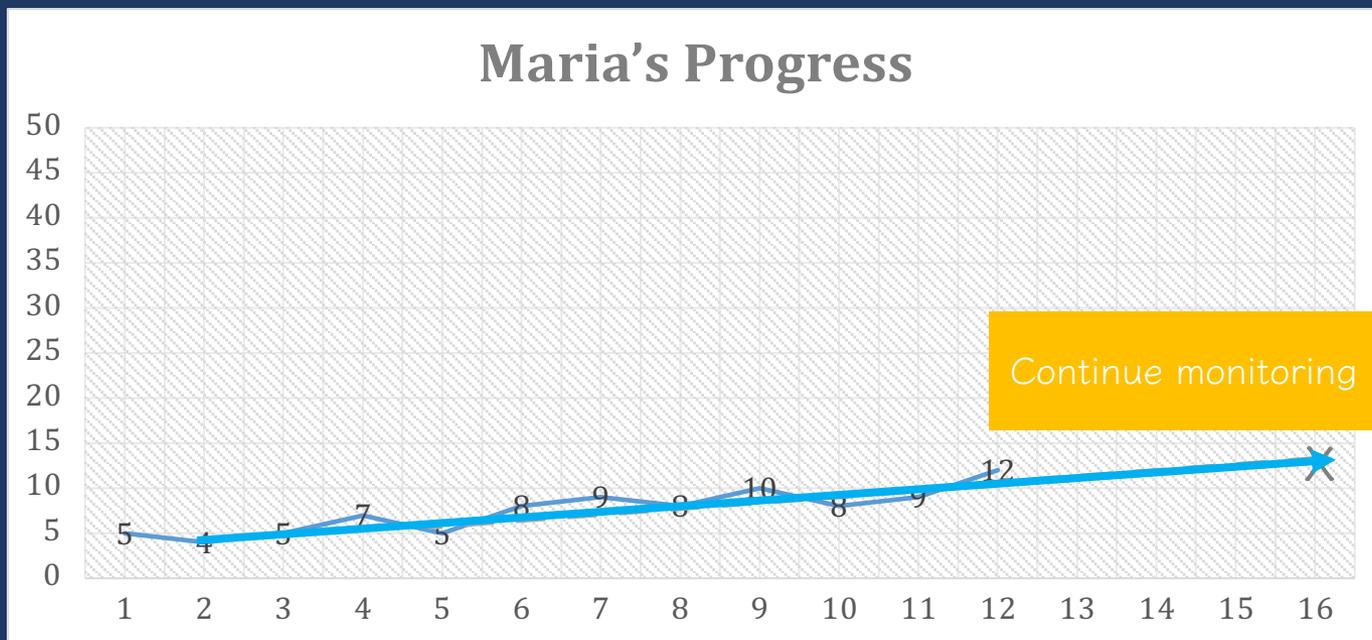


# Determining Response



- If the trend-line is **steeper** than the goal line, then increase the goal.
- If the trend-line is **flatter** than the goal line, then adapt the intervention.
- If the trend-line and goal-line are **fairly equal**, continue monitoring progress.

# Determining Response





Which decision making method(s)  
might you use?

# Teaching Math in Middle School

Using MTSS to Meet All Students' Needs



Leanne R. Ketterlin-Geller, Sarah R. Powell,  
David J. Chard, & Lindsey Perry

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Chapter 1 Laying the Foundation for Algebra .....	3
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<b>Section II: Designing and Delivering Effective Mathematics Instruction</b> .....	49
Chapter 4 Aims for Effective Mathematics Instruction .....	51
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Chapter 6 Instructional Practices to Support Problem Solving .....	81
Chapter 7 Designing Interventions .....	95
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Chapter 16 Collaboration as the Foundation for Implementing MTSS .....	227
Chapter 17 Implementing MTSS: Voices From the Field .....	241



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Chapter 15: Readiness Checklist - Campus Administration

	Is this practice, action, or belief already established?	If yes, what is working well? What changes could be made to improve this practice, action, or belief?	If no, what are the current conditions that exist? What is needed to change this practice, action, or belief?
There is campus-level support at the highest levels, including agreement to adopt an MTSS model and allocate required resources.			
There is an understanding of and commitment to a long-term change process (3 or more years).			
There is long-term commitment of resources for administering assessments and implementing tiered instructional support.			
The district leadership team has a basic level of knowledge of the research related to RtI and the desire to learn more.			
There is expertise at the district- and campus-level with respect to research-based practices for academic success and positive behavioral outcomes.			

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Chapter 15: Readiness Checklist - MTSS Team

	Is this practice, action, or belief already established?	If yes, what is working well? What changes could be made to improve this practice, action, or belief?	If no, what are the current conditions that exist? What is needed to change this practice, action, or belief?
There is campus-wide commitment to distributed leadership.			
Key stakeholders* are willing to work together.			
There is a content specialist who is communicative and considered a leader on campus.			
There is a data specialist who is communicative and considered a leader on campus.			
There is someone who could serve as the MTSS coordinator who is communicative and considered a leader on campus.			
There is a common planning time for educators to make instructional plans and review assessment data.			

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## Chapter 15: Readiness Checklist - Assessment

	Is this practice, action, or belief already established?	If yes, what is working well? What changes could be made to improve this practice, action, or belief?	If no, what are the current conditions that exist? What is needed to change this practice, action, or belief?
We administer a universal screener in mathematics to all students.			
We administer diagnostic assessments to students who are struggling document their strengths and areas of improvement.			
We monitor progress of students who are struggling or at-risk for mathematics difficulties.			
We have structured conversations around assessment results to inform instructional decisions.			
We provide ongoing professional learning and growth opportunities for interpreting assessment results to guide instructional decisions.			
We have a data management system in place.			

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Chapter 15: Readiness Checklist - Instruction

	Is this practice, action, or belief already established?	If yes, what is working well? What changes could be made to improve this practice, action, or belief?	If no, what are the current conditions that exist? What is needed to change this practice, action, or belief?
We use a research-validated Tier 1 instructional program in mathematics.			
We use (or are able to acquire) research-based supplemental intervention materials for Tier 2 support.			
We use (or are able to acquire) research-based supplemental intervention materials for Tier 3 support.			
We have highly trained educators to provide Tier 1 instruction with fidelity.			
We have highly trained educators to provide Tier 2 instruction with fidelity.			
We have highly trained educators to provide Tier 3 instruction with fidelity.			
We have systems in place to evaluate the fidelity of implementation of instruction in Tiers 1-3.			
We provide ongoing professional learning and growth opportunities for implementing evidence-based instructional practices.			
We provide ongoing professional learning and growth opportunities that focus on deepening teachers' content knowledge in mathematics.			

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## Chapter 15: Readiness Checklist - Professional Learning

	Is this practice, action, or belief already established?	If yes, what is working well? What changes could be made to improve this practice, action, or belief?	If no, what are the current conditions that exist? What is needed to change this practice, action, or belief?
Professional learning and growth opportunities exist for all staff and across all roles within the school community.			
Professional learning and growth opportunities include ongoing support such as coaching, peer feedback, or professional learning communities.			
Professional learning and growth opportunities are aligned with the goals of MTSS.			
All stakeholders believe in the value of professional learning and growth.			
Opportunities to learn about MTSS exist for parents and other stakeholders within the school community.			
Professional learning and growth opportunities address relevant aspects of implementing MTSS.			
Professional learning and growth opportunities focus on improving learning by supporting the needs of all students.			

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## MTSS: The First Four Years

	YEAR 1	YEAR 2	YEAR 3	YEAR 4
<b>MTSS SET UP</b>				
<b>MTSS team</b>	Select members for MTSS team; Determine personnel for delivery for Tier 1	Determine personnel for delivery of Tier 2	Determine personnel for delivery of Tier 3	
<b>MTSS plan</b>	Determine MTSS plan for middle school	Implement plan at Tier 1	Continue implementation of Tier 1; Implement plan at Tier 2	Continue implementation of Tiers 1 and 2; Implement plan at Tier 3
<b>Scheduling</b>	Determine whether time is devoted to math instruction at Tier 1	Schedule Tier 2 intervention time within school day	Schedule Tier 3 intervention time within school day	
<b>Training</b>	Train all staff on MTSS structure with a focus on Tier 1	Train all staff on MTSS structure at Tier 2	Train all staff on MTSS structure at Tier 3	
<b>TIER 1</b>				
<b>Math screener</b>	Choose screener	Implement screener		
<b>Tier 1 math instruction</b>	Review current evidence-based practices	Implement Tier 1 evidence-based practices with fidelity		
<b>Math progress monitoring</b>	Choose progress monitoring measure	Implement progress monitoring with "at-risk" students		
<b>Decision making at Tier 1</b>	Determine decision making process	Implement decision making at Tier 1		
<b>TIER 2</b>				
<b>Tier 2 math instruction</b>		Review and select evidence-based Tier 2 interventions	Implement Tier 2 interventions with fidelity	
<b>Math progress monitoring</b>		Determine whether additional progress monitoring measures are necessary for Tier 2	Implement progress monitoring with Tier 2 students	
<b>Decision making at Tier 2</b>		Determine decision making process	Implement decision making at Tier 2	
<b>TIER 3</b>				
<b>Diagnostic assessments</b>		Select appropriate math diagnostics	Pilot diagnostic assessments with select Tier 2 students	Implement diagnostic assessments with Tier 3 students
<b>Tier 3 math instruction</b>			Review and select evidence-based Tier 3 interventions	Implement Tier 3 interventions with fidelity; Make adaptations based on diagnostic data
<b>Math progress monitoring</b>				Implement progress monitoring with Tier 3 students
<b>Decision making at Tier 3</b>				Implement decision making at Tier 3

Chapter 15: Four-Year Timeline

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# Teaching Math in Middle School

Using MTSS to Meet All Students' Needs



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