Math 1496 - Calc 1

Summary of curve sketching

- x & y intercepts
- symmetry
- Domain, Range
- continuity
- VA & HA (Infinite Limits
- Differentability
- Extrema
- Increasing / Decreasing
- concavity
- T

Ex. 1 $y = x^3 - 6x^2 - 15x = x(x^2 - 6x - 15)$

x intercepts $x = 0$, $x = \frac{6 \pm \sqrt{36 + 60}}{2} = 3 \pm 2\sqrt{6}$

y intercept (0, 0)

no symmetry

Domain all x Range all y

cents for all x

no VA or HA

it is differentiable for all x

$y' = 3x^2 - 12x - 15$

$= 3(x^2 - 4x - 5) = 3(x+1)(x-5)$

$y' = 0$ when $x = -1$, $x = 5$

$y'' = 6x - 12 = 6(x-2)$ $\quad y'' = 0$ $x = 2$ Possible PI

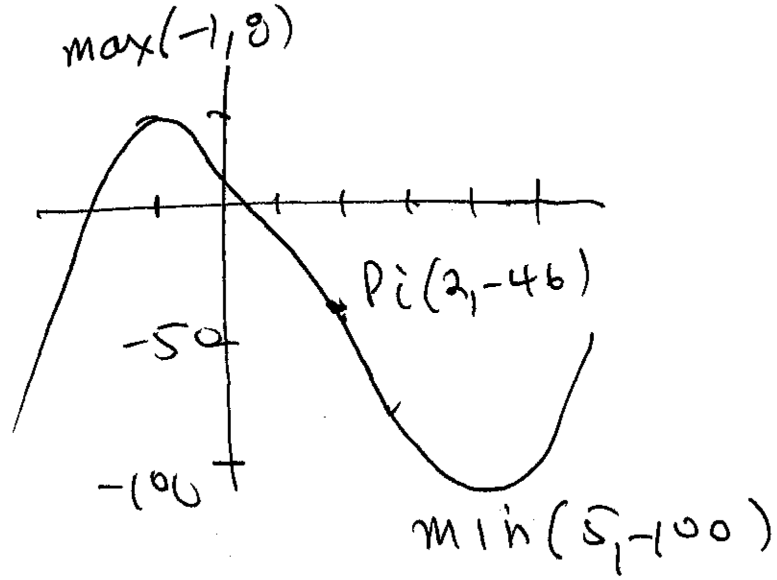
x		-1		2		5	
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$
$x-5$	$-$	$-$	$-$	$-$	$-$	0	$+$
$(x+1)(x-5)$	$+$	0	$-$	$-$	$-$	0	$+$
slope	/	—	\	\	\	—	/
$6(x-2)$	$-$	$-$	$-$	0	$+$	$+$	$+$
∩/∪		∩		PI	∪		

increasing $(-\infty, -1)$ $(5, \infty)$ $\quad$ max ~~min~~ $(-1, 8)$

decreasing $(-1, 5)$ $\quad$ min ~~max~~ $(5, -100)$

concave ↑ $(2, \infty)$ $\quad$ PI $(2, -46)$

concave ↓ $(-\infty, 2)$



Ex 2 $\quad y = x(x-2)^3$

x intercept $\quad x = 0, 2$

y intercepts $\quad x = 0$

$y' = (x-2)^3 + 3x(x-2)^2$

$= (x-2)^2(x-2+3x) = (x-2)^2(4x-2) = 2(x-2)^2(2x-1)$

$y' = 0$ when $x = 2, \frac{1}{2}$

$y'' = 4(x-2)(2x-1) + 2(x-2)^2 \cdot 2$

$= 4(x-2)(2x-1+x-2)$

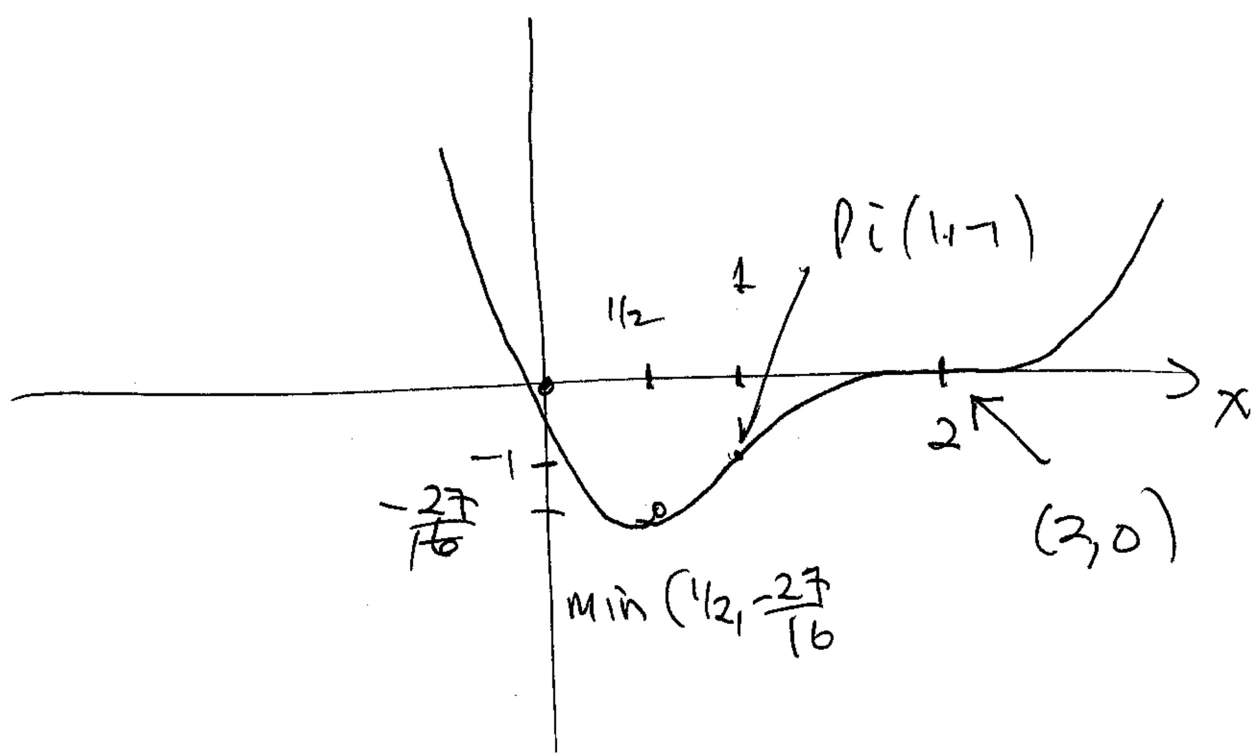
$= 4(x-2)(3x-3) = 12(x-2)(x-1)$

$y'' = 0$ when $x = 1, 2$

poss. PI

sign chart

x		$1/2$		1		2	
$(x-2)^2$	+	+	+	+	+	0	+
$2x-1$	−	0	+	+	+	+	+
$(2x-1)(x-2)^2$	−	0	+	+	+	0	+
slope	\	—	/	/	/	—	/
$x-1$	−	−	−	0	+	+	+
$x-2$	−	−	−	−	−	0	+
$(x-1)(x-2)$	+	+	+	0	−	0	+
h/v		∪		Pi	∩	Pi	∪



Ex 2 $\quad f(x) = \dfrac{x^3}{x(x^2-1)}$

$$f'(x) = \frac{3x^2(x^2-1) - 2x\cdot x^3}{(x^2-1)^2} = \frac{x^4-3x^2}{(x^2-1)^2} = \frac{x^2(x^2-3)}{(x^2-1)^2}$$

$f' = 0$ when $x = 0$ $\quad x = \pm\sqrt{3}$

x		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$	
x^2	+	+	+	+	+	0	+	+	+	+	
$x-\sqrt{3}$	−	−	−	−	−	−	−	−	−	0	+
$x+\sqrt{3}$	−	0	+	+	+	+	+	+	+	+	+
$(x^2-1)^2$	+	+	+	0	+	+	+	0	+	+	+
$\dfrac{x^2(x+\sqrt{3})(x-\sqrt{3})}{(x^2-1)^2}$	+	0	−	∞	−	0	−	∞	−	0	+
slope	/	—	\	|	\	—	\	|	\	—	/
		max		VA				VA		min	

$$\begin{array}{r} x \\ x^2-1 \overline{) x^3} \\ \underline{x^3 - x} \\ x \end{array}$$

$$f = \frac{x^3}{x^2-1} = x + \frac{x}{x^2-1}$$

As $x \to \infty$ $f \to x$ slant asymptote