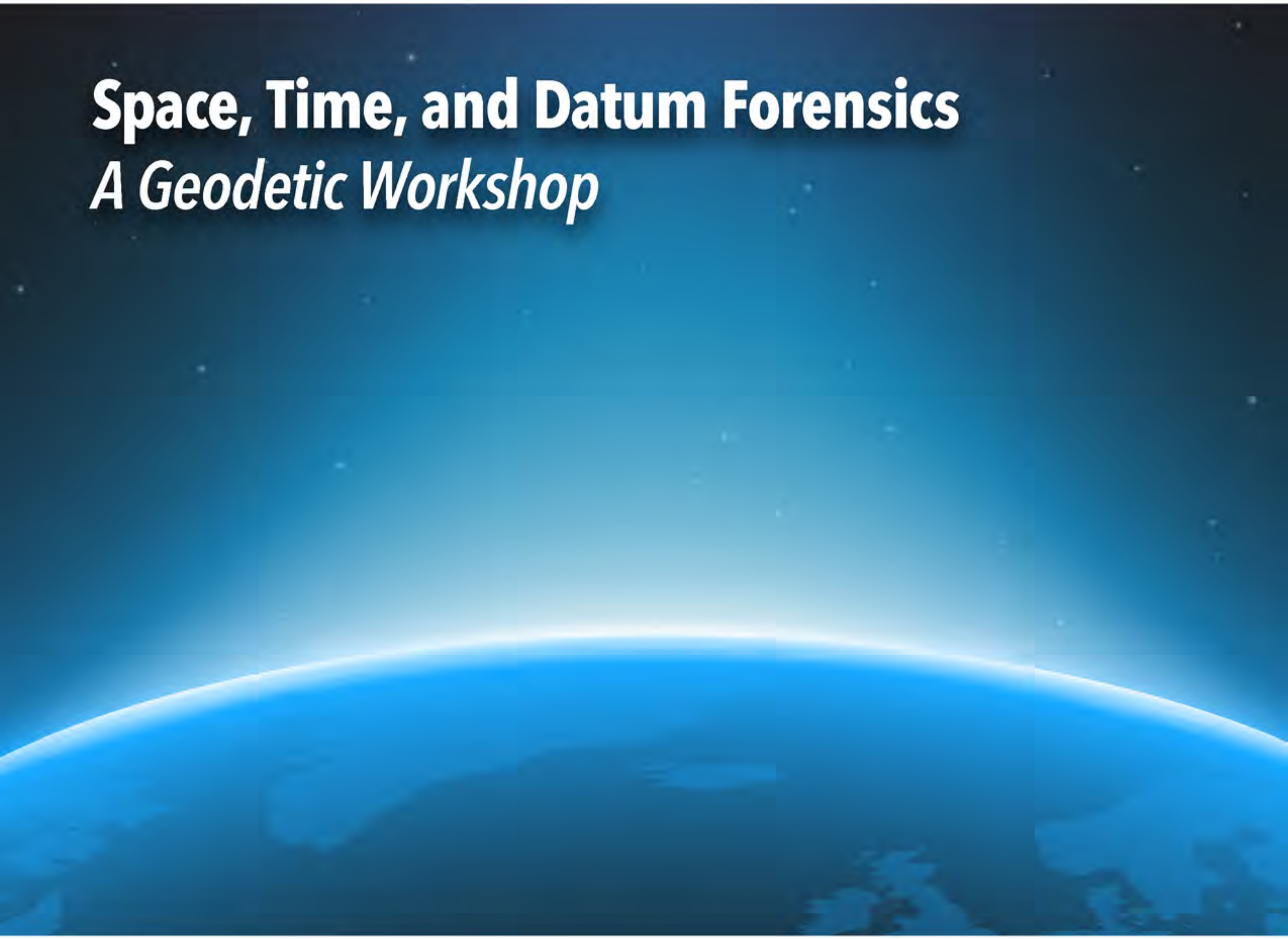
Space, Time, and Datum Forensics
A Geodetic Workshop



Michael L. Dennis, RLS, PE

Sponsored by Oregon GPS User's Group

Workshop location:

NW Eola Viticulture Center
Chemeketa Community College
215 Doaks Ferry Rd., Salem, OR 97304

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WORKSHOP ABSTRACT

What do you do when your horizontal control monuments are NAD 83 (1998), your field work is NAD 83 (2011) epoch 2010.00, and the client requires NAD 83 (1991) deliverables? Which geoid model should be used with the different horizontal datums? How do you set up a long-term project in light of the fact that new horizontal (geometric) and vertical datums will be defined and adopted by NGS within the next 8 years? How does all this work with the Oregon Coordinate Reference System low-distortion projections? What is the difference between NAD 83, IGS08, ITRF2008, and WGS 84? What software tools are available?

SPEAKER BIOGRAPHY

Michael L. Dennis, RLS, PE is owner of Geodetic Analysis, LLC. His firm provides geodetic consulting services, including coordinate system design; GNSS control survey planning and processing (including NGS "Bluebook" Height Modernization projects), spatial data management, survey and GIS data integration, development of field and office procedures for surveying and mapping, providing educational seminars throughout the US, and creation of custom geodetic and GIS computer algorithms. Mr. Dennis is on the board of the American Association for Geodetic Surveying (AAGS) and is Chair of the AAGS Geodetic Education and Certification Committee. He is also a member of the American Society of Civil Engineers Geomatics Division, the American Society for Photogrammetry and Remote Sensing, and the National Society of Professional Surveyors. In addition, Mr. Dennis is a geodesist at NGS currently on a long-term leave of absence while pursuing a PhD in Geomatics Engineering and GIS at Oregon State University.

Today, GPS has thrust surveyors into the thick of geodesy, which is no longer the exclusive realm of distant experts. Thankfully, in the age of the microcomputer, the computational drudgery can be handled with software packages. Nevertheless, it is unwise to venture into GPS believing that knowledge of the basics of geodesy is, therefore, unnecessary. It is true that GPS would be impossible without computers, but blind reliance on the data they generate eventually leads to disaster.

Jan Van Sickle (2015, p. 130)

Note: This document is intended to accompany a workshop. Therefore some of the material may appear incomplete or be unclear if it used without attending the workshop.

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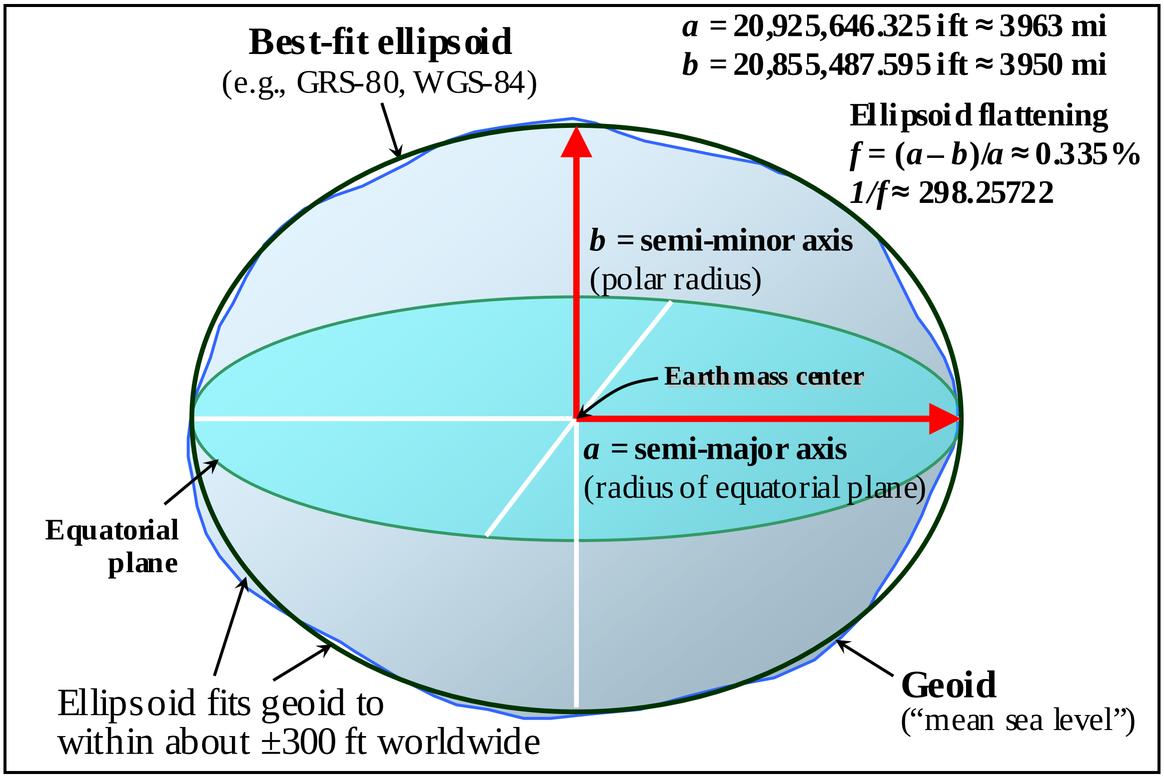
Section 1

GEODESY FUNDAMENTALS

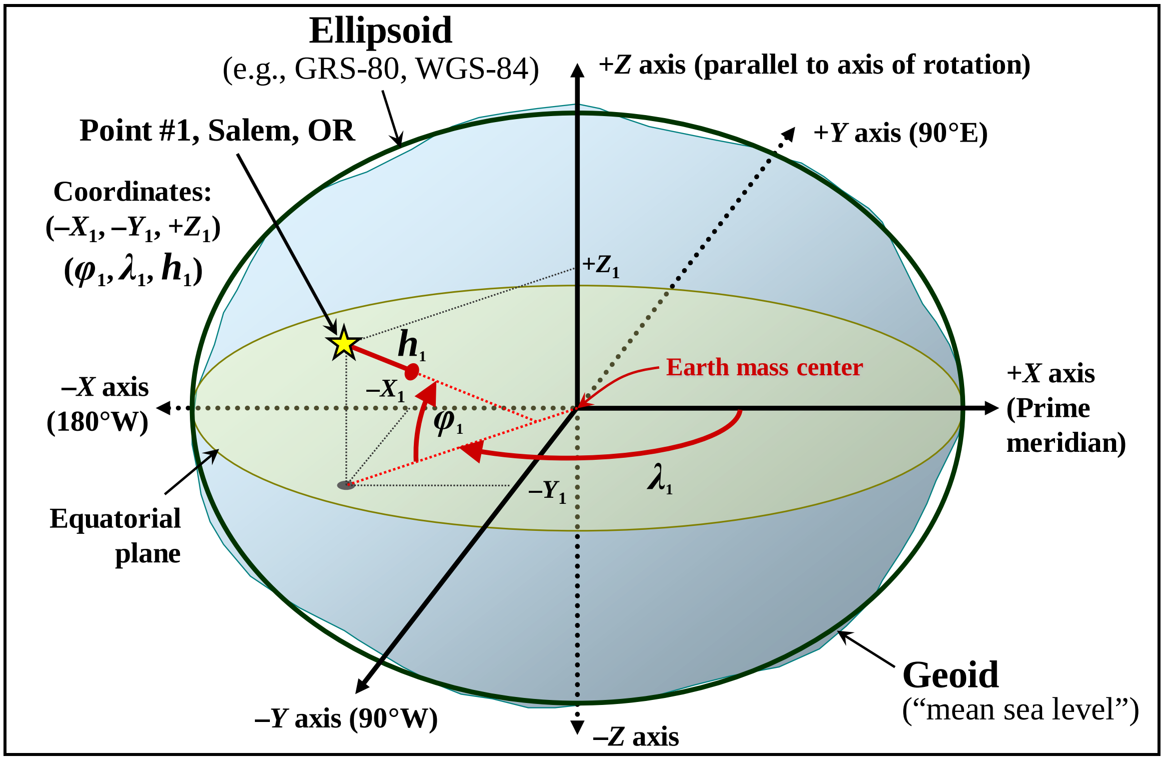
Table 1.1 Useful numerical values for this document.

Symbol	Description	Numerical values
a	GRS-80 ellipsoid semi-major axis (identical to WGS-84 value)	6,378,137 m (*exact*) = 20,925,646.325 459 ift = 20,925,604.474 167 sft
f	GRS-80 geometrical flattening WGS-84 geometrical flattening	$298.257\ 222\ 101^{-1}$ (*published value*) $298.257\ 223\ 563^{-1}$ (*published value*)
b	GRS-80 ellipsoid semi-minor axis WGS-84 ellipsoid semi-minor axis	6,356,752.314 140 m = 20,855,486.594 949 ift = 20,855,444.883 876 sft 6,356,752.314 245 m = 20,855,486.595 293 ift = 20,855,444.884 319 sft
e^2	GRS-80 first eccentricity squared WGS-84 first eccentricity squared	0.006 694 380 022 901 0.006 694 379 990 141
ift	international foot	1 ift ≡ 0.3048 m (2 ppm shorter than sft)
sft	US survey foot	1 sft ≡ 1200/3937 m (2 ppm longer than ift)
ppm	Parts per million	Value multiplied by one million (analogous to “percent” which is “parts per hundred”)
rad	Radian (angular measure)	1 rad = 180° / π (i.e., 1 rad ≈ 57.295 779 513°)
π	Pi (irrational number)	π = 3.141 592 653 589 793 238 462 643 383…
γ_0	Normal gravity on the GRS 80 ellipsoid at 45° latitude	9.806199 m/s^2 32.172569 ift/s^2 = 32.172505 sft/s^2

The geodetic reference ellipsoid



Earth-Centered, Earth-Fixed (ECEF) Cartesian coordinates



Geodetic ellipsoid parameters and computations

The geodetic ellipsoid of revolution is completely defined by two numbers. By convention, these are usually a, the semi-major axis, and $1/f$, the inverse geometric flattening. These can be used to compute other commonly used ellipsoid parameters, such as the following two:

Equation 1.1 Ellipsoid semi-minor axis

$$\boxed{b = a\left(1 - f\right)}$$

Equation 1.2 Ellipsoid first eccentricity squared

$$\boxed{e^2 = 2f - f^2}$$

Example computations

Given: The following parameters for the GRS-80, WGS-84, and Clarke 1866 ellipsoids:

Ellipsoid	GRS-80	WGS-84	Clarke 1866
Semi-major axis, *a*	6,378,137 m (exact)	6,378,137 m (exact)	20,925,832.164 sft
Inverse flattening, 1/*f*	298.257 222 101	298.257 223 563	294.978 698 214

Find: The semi-minor axis (in international feet) of these ellipsoids.

Computations:

GRS-80: $b = 6{,}378{,}137 \text{ m} \times \left[1 - \dfrac{1}{298.257222101}\right] \times \left(\dfrac{1\,\text{ift}}{0.3048\text{ m}}\right) =$ **<u>20,855,486.5949 ift</u>**

WGS-84: $b = 6{,}378{,}137 \text{ m} \times \left[1 - \dfrac{1}{298.257223563}\right] \times \left(\dfrac{1\,\text{ift}}{0.3048\text{ m}}\right) =$ **<u>20,855,486.5953 ift</u>**

Clarke 1866: $b = 20{,}925{,}832.164 \text{ sft} \times \left[1 - \dfrac{1}{294.978698214}\right] \times \dfrac{1.000002\text{ ift}}{1\,\text{sft}} =$ **<u>20,854,933.727 ift</u>**

Computation of Earth radius

The Radii of curvature at a point in the *meridian* (north-south) and *prime vertical* (east-west) are frequently used in geodesy:

Equation 1.3 Meridian radius (north-south)

$$\boxed{R_M = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\varphi\right)^{3/2}}}$$

Equation 1.4 Prime vertical radius (east-west)

$$\boxed{R_N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}}$$

where φ is the geodetic latitude at the point where the radius is computed.

a is the ellipsoid semi-major axis (= 20,925,646.325 459 ift for the GRS-80 ellipsoid)

e^2 is the ellipsoid first eccentricity squared (= 0.006 694 380 022 901 for GRS-80)

R_M and R_N are used to compute other commonly used Earth radii, such as the following two:

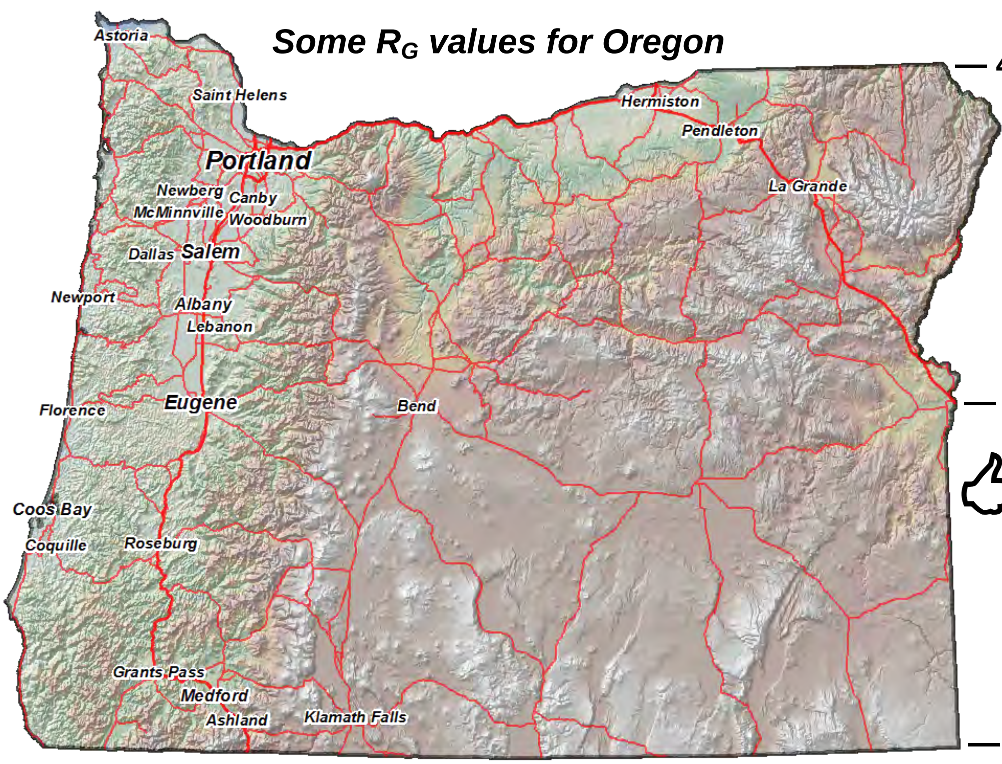
Equation 1.5 Radius of curvature in a specific azimuth, α

$$\boxed{R_\alpha = \frac{R_M\ R_N}{R_M\sin^2\alpha + R_N\cos^2\alpha}}$$

Equation 1.6 Geometric mean radius of curvature

$$\boxed{R_G = \sqrt{R_M\ R_N} = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}}$$

R_G is the essentially the "average" radius of curvature at a point on the ellipsoid, and is the one we will use for radius computations in this workshop.



— 46° N, R_G = 20,927,981 ift

— 44° N, R_G = 20,923,076 ift

— 42° N, R_G = 20,918,185 ift

Rule of thumb:

Geometric mean radius *increases* by about 35 ft per mile north (between latitudes 40° and 50°)

Example computation

Given: Point at latitude φ = 44°15'21.22736" N (midway between RDM A and REDM CORS).

Find: The radii of curvature in the meridian, prime vertical, at an azimuth of α = 7°17'11" (from RDM A to REDM CORS), and the geometric mean radius (for the GRS-80 ellipsoid).

Computations: First convert latitude and azimuth to decimal degrees:

$\varphi = 44 + 15/60 + 21.22736/3600 = 44.2558964889°$

$\alpha = 7 + 17/60 + 11/3600 = 7.28637°$

Now compute following function of latitude (since it appears in most of the equations):

$1 - e^2 \sin^2\varphi = 1 - 0.006694380023 \times [\sin(44.2558964889°)]^2 = 0.996739740503$

Now compute the various radii:

$$R_M = \frac{20{,}925{,}646.325 \times (1 - 0.006694380023)}{(0.996739740503)^{3/2}} \qquad = \mathbf{\underline{20{,}887{,}627.422\ ift}}$$

$$R_N = \frac{20{,}925{,}646.325}{\sqrt{0.996739740503}} \qquad = \mathbf{\underline{20{,}959{,}841.481\ ift}}$$

$$R_\alpha = \frac{20{,}887{,}627.422 \quad \times \quad 20{,}959{,}841.481}{20{,}887{,}627.422 \times [\sin(7.28637°)]^2 + 20{,}959{,}841.481 \times [\cos(7.28637°)]^2}$$

$$= \mathbf{\underline{20{,}888{,}785.085\ ift}}$$

$$R_G = \frac{20{,}925{,}646.325 \times \sqrt{1 - 0.006694380023}}{0.996739740503} \qquad = \mathbf{\underline{20{,}923{,}703.297\ ift}}$$

Check: $R_G = \sqrt{R_M \; R_N} = \sqrt{20{,}852{,}873 \times 20{,}948{,}210} = \underline{20{,}923{,}703.297 \text{ ift}}$ ✓

The NGS Datasheet (*page 1 of 3*)

```
1         National Geodetic Survey,   Retrieval Date = FEBRUARY 26, 2016
 AF8301 ***********************************************************************
 AF8301  PACS         -  This is a Primary Airport Control Station.
 AF8301  DESIGNATION -  RDM A
 AF8301  PID         -  AF8301
 AF8301  STATE/COUNTY-  OR/DESCHUTES
 AF8301  COUNTRY     -  US
 AF8301  USGS QUAD   -  REDMOND (1975)
 AF8301
 AF8301                         *CURRENT SURVEY CONTROL
 AF8301  ______________________________________________________________________
 AF8301* NAD 83(2011) POSITION- 44 15 07.30805(N) 121 08 57.28517(W)   ADJUSTED
 AF8301* NAD 83(2011) ELLIP HT-   915.319 (meters)        (06/27/12)   ADJUSTED
 AF8301* NAD 83(2011) EPOCH   -  2010.00
 AF8301* NAVD 88 ORTHO HEIGHT -   936.440 (meters)     3072.30  (feet) ADJUSTED
 AF8301  ______________________________________________________________________
 AF8301  NAD 83(2011) X  - -2,367,351.543 (meters)                     COMP
 AF8301  NAD 83(2011) Y  - -3,916,788.058 (meters)                     COMP
 AF8301  NAD 83(2011) Z  -  4,428,832.050 (meters)                     COMP
 AF8301  LAPLACE CORR    -          4.39  (seconds)                    DEFLEC12B
 AF8301  GEOID HEIGHT    -        -21.118 (meters)                     GEOID12B
 AF8301  DYNAMIC HEIGHT  -        936.120 (meters)     3071.25  (feet) COMP
 AF8301  MODELED GRAVITY -    980,245.4   (mgal)                       NAVD 88
 AF8301
 AF8301  VERT ORDER      -  FIRST     CLASS II
 AF8301
 AF8301  Network accuracy estimates per FGDC Geospatial Positioning Accuracy
 AF8301  Standards:
 AF8301         FGDC (95% conf, cm)     Standard deviation (cm)     CorrNE
 AF8301            Horiz  Ellip           SD_N   SD_E   SD_h      (unitless)
 AF8301  -------------------------------------------------------------------
 AF8301  NETWORK    0.20   0.65           0.09   0.07   0.33      0.06976836
 AF8301  -------------------------------------------------------------------
 AF8301  Click here for local accuracies and other accuracy information.
 AF8301
 AF8301
 AF8301.This mark is at Roberts Field Airport (RDM)
 AF8301
 AF8301.This is a reference station for the REDMOND
 AF8301.National Continuously Operating Reference Station (REDM).
 AF8301
 AF8301.The horizontal coordinates were established by GPS observations
 AF8301.and adjusted by the National Geodetic Survey in June 2012.
 AF8301
 AF8301.NAD 83(2011) refers to NAD 83 coordinates where the reference
 AF8301.frame has been affixed to the stable North American tectonic plate. See
 AF8301.NA2011 for more information.
 AF8301
 AF8301.The horizontal coordinates are valid at the epoch date displayed above
 AF8301.which is a decimal equivalence of Year/Month/Day.
 AF8301
 AF8301.The orthometric height was determined by differential leveling and
 AF8301.adjusted by the NATIONAL GEODETIC SURVEY
 AF8301.in July 2002.
 AF8301
 AF8301.No vertical observational check was made to the station.
 AF8301
 AF8301.Significant digits in the geoid height do not necessarily reflect accuracy.
 AF8301.GEOID12B height accuracy estimate available here.
 AF8301
```

The NGS Datasheet (*page 2 of 3*)

```
AF8301.The X, Y, and Z were computed from the position and the ellipsoidal ht.
AF8301
AF8301.The Laplace correction was computed from DEFLEC12B derived deflections.
AF8301
AF8301.The ellipsoidal height was determined by GPS observations
AF8301.and is referenced to NAD 83.
AF8301
AF8301.The dynamic height is computed by dividing the NAVD 88
AF8301.geopotential number by the normal gravity value computed on the
AF8301.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
AF8301.degrees latitude (g = 980.6199 gals.).
AF8301
AF8301.The modeled gravity was interpolated from observed gravity values.
AF8301
AF8301. The following values were computed from the NAD 83(2011) position.
AF8301
AF8301;                   North         East     Units Scale Factor Converg.
AF8301;SPC OR S     -   287,409.917 1,448,144.087   MT  1.00007385   -0 26 39.0
AF8301;SPC OR S     -   942,945.92  4,751,128.89   iFT  1.00007385   -0 26 39.0
AF8301;UTM  10      - 4,901,531.409   647,753.722   MT  0.99986849   +1 17 30.2
AF8301
AF8301!             -  Elev Factor  x  Scale Factor =   Combined Factor
AF8301!SPC OR S     -   0.99985650  x   1.00007385  =   0.99993034
AF8301!UTM  10      -   0.99985650  x   0.99986849  =   0.99972501
AF8301
AF8301:                Primary Azimuth Mark                     Grid Az
AF8301:SPC OR S     -  RDM AP STA G                             093 43 42.0
AF8301:UTM  10      -  RDM AP STA G                             091 59 32.8
AF8301
AF8301|---------------------------------------------------------------------|
AF8301| PID    Reference Object                     Distance      Geod. Az  |
AF8301|                                                           dddmmss.s |
AF8301| AA8007 RDM AP STA G                        419.133 METERS 0931703.0 |
AF8301| AC7367 RDM ARP 2 1972                      355.248 METERS 31252     |
AF8301|---------------------------------------------------------------------|
AF8301
AF8301                          SUPERSEDED SURVEY CONTROL
AF8301
AF8301  NAD 83(2007)-   44 15 07.30657(N)    121 08 57.28689(W) AD(2007.00) 0
AF8301  ELLIP H (02/10/07)  915.339  (m)                        GP(2007.00)
AF8301  NAD 83(1998)-   44 15 07.30626(N)    121 08 57.28684(W) AD(       ) B
AF8301  ELLIP H (06/21/01)  915.341  (m)                        GP(       ) 4 2
AF8301  NAD 83(1998)-   44 15 07.30482(N)    121 08 57.28613(W) AD(       ) B
AF8301  ELLIP H (05/28/98)  915.377  (m)                        GP(       ) 1 1
AF8301  NAVD 88 (05/28/98)  936.44   (m)  GEOID96 model used    GPS OBS
AF8301
AF8301.Superseded values are not recommended for survey control.
AF8301
AF8301.NGS no longer adjusts projects to the NAD 27 or NGVD 29 datums.
AF8301.See file dsdata.txt to determine how the superseded data were derived.
AF8301
AF8301_U.S. NATIONAL GRID SPATIAL ADDRESS: 10TFQ4775301531(NAD 83)
AF8301
AF8301_MARKER: DH = HORIZONTAL CONTROL DISK
AF8301_SETTING: 66 = SET IN ROCK OUTCROP
AF8301_STAMPING: RDM A 1997
AF8301_MARK LOGO: NGS
AF8301_MAGNETIC: N = NO MAGNETIC MATERIAL
```

The NGS Datasheet (*page 3 of 3*)

```
AF8301_STABILITY: A = MOST RELIABLE AND EXPECTED TO HOLD
AF8301+STABILITY: POSITION/ELEVATION WELL
AF8301_SATELLITE: THE SITE LOCATION WAS REPORTED AS SUITABLE FOR
AF8301+SATELLITE: SATELLITE OBSERVATIONS - February 16, 2000
AF8301
AF8301  HISTORY     - Date     Condition        Report By
AF8301  HISTORY     - 1997     MONUMENTED       NGS
AF8301  HISTORY     - 20000216 GOOD             NGS
AF8301
AF8301                          STATION DESCRIPTION
AF8301
AF8301'DESCRIBED BY NATIONAL GEODETIC SURVEY 1997 (AJL)
AF8301'THE STATION IS LOCATED AT ROBERTS FIELD ON THE EAST SIDE OF REDMOND
AF8301'ALONG THE SOUTHWEST EDGE OF THE PARALLEL TAXIWAY FOR RUNWAY 10-28 AND
AF8301'SOUTH OF THE INTERSECTION OF THE 2 RUNWAYS.  OWNERSHIP--CITY OF
AF8301'REDMOND, P O BOX, REDMOND, OR.  THE AIRPORT MANAGER IS CAROLYN NOVICK.
AF8301'THE PHONE NUMBER IS () .  TO REACH THE STATION FROM THE JUNCTION OF US
AF8301'HIGHWAY 97 AND STATE HIGHWAY 126 IN REDMOND GO EASTERLY FOR 1.65 MI
AF8301'(2.66 KM) ON THE STATE HIGHWAY TO THE US FORESTRY SERVICE REDMOND AIR
AF8301'CENTER ENTRANCE ROAD RIGHT, TURN RIGHT AND GO SOUTHERLY ALONG THE
AF8301'PAVED ROAD FOR 0.2 MI (0.3 KM) TO GATE NUMBER 26 (LOCKED) AND THE
AF8301'NATIONAL INTERAGENCY FIRE SUPPORT CACHE BUILDING ON THE RIGHT.
AF8301'CONTINUE SOUTH PASSING THROUGH THE GATE AND ACROSS A US FOREST SERVICE
AF8301'RAMP TO TAXIWAY B, TURN LEFT AND GO EASTERLY FOR 0.4 MI (0.6 KM) ALONG
AF8301'THE TAXIWAY TO PARALLEL TAXIWAY F FOR RUNWAY 4-22 ON THE RIGHT, TURN
AF8301'RIGHT AND GO SOUTHWEST FOR 0.7 MI (1.1 KM) ALONG THE PARALLEL TAXIWAY
AF8301'AND CROSSING RUNWAY 10-28 TO THE INTERSECTION OF PARALLEL TAXIWAY G
AF8301'FOR RUNWAY 10-28, TURN LEFT AND GO SOUTHEAST FOR 0.2 MI (0.3 KM) ON
AF8301'THE PARALLEL TAXIWAY CROSSING RUNWAY 4-22 TO THE STATION ON THE RIGHT
AF8301'IN THE NORTHERN END OF A SLIGHT RIDGE CONSISTING OF A BASALT OUTCROP.
AF8301'THE STATION IS SET IN A DRILL HOLE IN AN EXPOSED AREA OF BASALT
AF8301'OUTCROP, LOCATED 159.9 FT (48.7 M) SOUTH OF TAXIWAY LIGHT NUMBER G 26,
AF8301'59.8 FT (18.2 M) SOUTHWEST OF THE SOUTHWEST EDGE OF THE TAXIWAY, 59.0
AF8301'FT (18.0 M) WEST OF TAXIWAY LIGHT NUMBER G 25, 4.0 FT (1.2 M)
AF8301'NORTHEAST OF THE CENTER OF A ROCK CAIRN, AND THE STATION IS ABOUT
AF8301'4-FEET ABOVE THE LEVEL OF THE TAXIWAY. NOTE--THIS STATION IS A PACS.
AF8301
AF8301                          STATION RECOVERY (2000)
AF8301
AF8301'RECOVERY NOTE BY NATIONAL GEODETIC SURVEY 2000 (GAS)
AF8301'2.7 KM (1.65 MI) EASTERLY ALONG STATE HIGHWAY 126 FROM THE MOST
AF8301'NORTHERLY OF 2 JUNCTIONS OF U.S.  HIGHWAY 97 IN REDMOND, THENCE 0.3 KM
AF8301'(0.20 MI) SOUTHERLY ALONG THE ENTRANCE ROAD TO THE U.S.  FOREST
AF8301'SERVICE REDMOND AIR CENTER AND GATE 26 TO ROBERTS FIELD AIRPORT,
AF8301'THENCE 0.1 KM (0.05 MI) SOUTHERLY ACROSS AN APRON, THENCE 0.6 KM (0.35
AF8301'MI) EASTERLY ALONG TAXIWAY B, THENCE 1.1 KM (0.70 MI) SOUTHWESTERLY
AF8301'ALONG TAXIWAY F CROSSING RUNWAY 10-28, THENCE 0.2 KM (0.10 MI)
AF8301'SOUTHEASTERLY ALONG TAXIWAY G CROSSING RUNWAY 4-22, IN THE NORTH END
AF8301'OF A SMALL RIDGE OF BASALT OUTCROP, 48.7 M (159.8 FT) SOUTH OF TAXIWAY
AF8301'LIGHT NUMBER G 26, 18.2 M (59.7 FT) SOUTHWEST OF THE SOUTHWEST EDGE OF
AF8301'THE TAXIWAY, 18.0 M (59.1 FT) WEST OF TAXIWAY LIGHT NUMBER G 25, 1.2 M
AF8301'(3.9 FT) NORTHEAST OF THE CENTER OF A ROCK CAIRN, AND 1.1 M (3.6 FT)
AF8301'ABOVE THE LEVEL OF THE TAXIWAY.  NOTE--THIS IS A PRIMARY AIRPORT
AF8301'CONTROL STATION.  THIS IS A CORS SITE REFERENCE STATION.  THE MONUMENT
AF8301'IS ON PROPERTY OWNED BY THE CITY OF REDMOND, ROBERTS FIELD-REDMOND
AF8301'MUNICIPAL AIRPORT, CAROLYN NOVICK MANAGER, P.O.  BOX 726, REDMOND, OR
AF8301'97756-0100, PHONE NUMBER (541) 548-3496.
```

The NGS CORS position and velocity datasheet (*page 1 of 2*)

```
                             ***IGS 08***
               REDMOND (REDM),  OREGON

 Created on 31Aug2011 at 09:30:54.
 ___________________________________________________________________________
|                                                                           |
|        Antenna Reference Point(ARP): REDMOND CORS ARP                     |
|        ----------------------------------------------                     |
|                            PID = AH2507                                   |
|                                                                           |
| IGS08 POSITION (EPOCH 2005.0)                                             |
| Computed in Aug 2011 using data through gpswk 1631.                       |
|     X =  -2366949.550 m     latitude    =  44 15 35.16273 N               |
|     Y =  -3916333.706 m     longitude   = 121 08 52.37145 W               |
|     Z =   4429451.140 m     ellipsoid height =  919.926   m               |
|                                                                           |
| IGS08 VELOCITY                                                            |
| Computed in Aug 2011 using data through gpswk 1631.                       |
|     VX =  -0.0137 m/yr      northward =  -0.0083 m/yr                     |
|     VY =   0.0025 m/yr      eastward  =  -0.0130 m/yr                     |
|     VZ =  -0.0067 m/yr      upward    =  -0.0011 m/yr                     |
|                                                                           |
|                                                                           |
| NAD_83 (2011) POSITION (EPOCH 2010.0)                                     |
| Transformed from IGS08 (epoch 2005.0) position in Aug 2011.               |
|     X =  -2366948.814 m     latitude    =  44 15 35.14667 N               |
|     Y =  -3916334.905 m     longitude   = 121 08 52.31510 W               |
|     Z =   4429451.077 m     ellipsoid height =  920.345   m               |
|                                                                           |
| NAD_83 (2011) VELOCITY                                                    |
| Transformed from IGS08 velocity in Aug 2011.                              |
|     VX =   0.0046 m/yr      northward =   0.0046 m/yr                     |
|     VY =   0.0031 m/yr      eastward  =   0.0023 m/yr                     |
|     VZ =   0.0015 m/yr      upward    =  -0.0026 m/yr                     |
|___________________________________________________________________________|
|                                                                           |
| L1 Phase Center of the current GPS antenna: REDMOND CORS L1 PC C          |
| ----------------------------------------------------------------          |
| The D/M element, chokerings, -radome antenna                              |
| (Antenna Code = TRM29659.00     NONE) was installed on 26Jul1998.         |
|        The L2 phase center is 0.029 m above the L1 phase center.          |
|                            PID = AH2508                                   |
|                                                                           |
| IGS08 POSITION (EPOCH 2005.0)                                             |
| Computed in Aug 2011 using data through gpswk 1631.                       |
|     X =  -2366949.583 m     latitude    =  44 15 35.16274 N               |
|     Y =  -3916333.762 m     longitude   = 121 08 52.37144 W               |
|     Z =   4429451.204 m     ellipsoid height =  920.017   m               |
|                                                                           |
| The IGS08 VELOCITY of the L1 PC is the same as that for the ARP.          |
|                                                                           |
|                                                                           |
| NAD_83 (2011) POSITION (EPOCH 2010.0)                                     |
| Transformed from IGS08 (epoch 2005.0) position in Aug 2011.               |
|     X =  -2366948.848 m     latitude    =  44 15 35.14668 N               |
|     Y =  -3916334.961 m     longitude   = 121 08 52.31509 W               |
|     Z =   4429451.141 m     ellipsoid height =  920.436   m               |
|                                                                           |
| The NAD_83 (2011) VELOCITY of the L1 PC is the same as that for the ARP.  |
|___________________________________________________________________________|
```

The NGS CORS position and velocity datasheet (*page 2 of 2*)

```
 ____________________________________________________________________________
|                                                                            |
|         Monument: REDMOND CORS MONUMENT                                    |
|         -------------------------------                                    |
|                             PID = AH2509                                   |
|         Inscribed: NONE                                                    |
|                                                                            |
|                                                                            |
| IGS08 POSITION (EPOCH 2005.0)                                              |
| Computed in Aug 2011 using data through gpswk 1631.                        |
|     X =  -2366949.520 m     latitude    =  44 15 35.16273 N                |
|     Y =  -3916333.657 m     longitude   = 121 08 52.37145 W                |
|     Z =   4429451.085 m     ellipsoid height =  919.847   m                |
|                                                                            |
| The IGS08 VELOCITY of the monument is the same as that for the ARP.        |
|                                                                            |
|                                                                            |
| NAD_83 (2011) POSITION (EPOCH 2010.0)                                      |
| Transformed from IGS08 (epoch 2005.0) position in Aug 2011.                |
|     X =  -2366948.785 m     latitude    =  44 15 35.14667 N                |
|     Y =  -3916334.857 m     longitude   = 121 08 52.31510 W                |
|     Z =   4429451.022 m     ellipsoid height =  920.266   m                |
|                                                                            |
| The NAD_83 (2011) VELOCITY of the monument is the same as that for the ARP. |
|____________________________________________________________________________|


 * Latitude, longitude and ellipsoid height are computed from their
   corresponding cartesian coordinates using dimensions for the
   GRS 80 ellipsoid: semi-major axis = 6,378,137.0 meters
                         flattening = 1/298.257222101...


 * WARNING: Mixing of antenna types can lead to errors of up to 10 cm.
   in height unless antenna-phase-center variation and antenna-phase-center
   offset are properly modeled.  See next comment.

 * The coordinates shown on this page were computed using absolute antenna
   calibrations.  CORS coordinates began using absolute antenna calibrations
   beginning with IGS08 and NAD 83 (2011, MA11, PA11).  For additional
   information on the derivation of these positions and velocities and
   antenna calibrations consult:
   http://geodesy.noaa.gov/CORS/coords.shtml
   http://geodesy.noaa.gov/ANTCAL


 * For more site specific information on the equipment history and
   monumentation type consult:
   ftp://geodesy.noaa.gov/cors/station_log/redm.log.txt
   http://geodesy.noaa.gov/cgi-cors/corsage_2.prl?site=redm


 * The NAD_83 position of this site was revised in Sep. 1998.
 * The NAD_83 position & velocity were revised in Mar. 2002.
 * The ITRF00 position & velocity were revised in Apr. 2006.
 * The position & velocity were revised in Aug 2011.
```

The NGS Geodetic Toolkit


NGS Geodetic Tool Kit

National Geodetic Survey

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NGS Geodetic Tool Kit

On-line interactive computation of geodetic values

See the text version of an **article** about the NGS Geodetic Toolkit that appeared in the *Professional Surveyor* magazine, May 2003 Volume 23, Number 4.

See all the Professional Surveyor Articles about the NGS Geodetic Toolkit.

These utilities require Internet Explorer version 6.0+ and Netscape version 6.0+

To learn more about a particular online program, click on its link for a description:

- DEFLEC99
- DEFLEC09
- DEFLEC12A
- DYNAMIC_HT
- GEOCON
- GEOCON11
- GEOID12A
- GEOID12
- GEOID09
- GEOID06
- GEOID03
- GEOID99
- G99SSS
- USGG2012
- USGG2009
- USGG2003
- HTDP
- IGLD85
- Inverse/Forward/Invers3D/Forwrd3D
- Leveling Online Computations User Service (LOCUS)
- LVL_DH
- Magnetic Declination
- NADCON
- NAVD 88 Modelled Gravity
- Online Adjustment User Services
- Online Adjustment Utilities User Services
- OPUS
- State Plane Coordinates
- Surface Gravity Prediction
- Tidal and Orthometric Elevations
- U.S. National Grid
- Universal Transverse Mercator Coordinates
- VERTCON
- XYZ Coordinate Conversion

Or... Know what you want to do?

Select a function from this list below:

Select a Toolkit Shortcut

For more information contact NGS Information Services:
by email or call (301) 713-3242, Monday - Friday, 7:00 AM - 4:30 PM eastern time.

Website Owner: National Geodetic Survey / Last modified by Joe Whitsitt Aug 12 2014

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Gaining geodetic insight through GPS

A comparison between GPS and terrestrial measurements using a total station

Both GPS and total stations determine three-dimensional coordinates, but they differ in virtually every other respect, to wit:

- **Observations**
  - Total stations are used to directly observe slope distance, horizontal angle, and zenith angle
    - Total station EDM sends and receives the signal that it uses for computing distance
  - GPS observes the pseudorange, carrier phase (fractional wavelength), and Doppler shift of the signals transmitted from the satellites
    - GPS only receives signals from the satellites (a one-way ranging system)
- **Measurements**
  - The vector components from a total station to the prism are directly measured
    - Total station measures both distance and angles
  - The vector components between GPS antennas are computed, NOT observed
    - This has implications for error propagation and control network design
    - GPS does NOT measure angles
- **Computations**
  - Coordinates can be determined from total station observations using simple plane trigonometry
  - Geodetic methods MUST be used to compute coordinates from GPS vectors
- **Reference frame**
  - Total stations are referenced to the gravity vector (plumbline) passing through the vertical axis of the instrument
  - GPS is referenced to a world-wide coordinate system (in common with the satellites) with its origin located at the Earth's center of the mass

Geodesy: The science of positioning

Geodesy is a quantitative scientific field dealing with the size and shape of the Earth (or other planetary bodies), precise determination of coordinates and relationship between coordinates on the Earth, and includes study of the Earth's gravity field. It is the science behind surveying, mapping, and navigation, and it is essential for using GPS.

The bottom line: GPS is a geodetic tool that *requires* geodesy to perform computations and it is explicitly referenced to the *entire* Earth.

Section 2

GEODETIC DATUM DEFINITIONS AND REFERENCE COORDINATES

How are the data connected to the Earth?

Examples of georeferencing errors for Oregon

Table 2.1 Examples of various positioning error sources and their magnitudes for Oregon due to geodetic datum definition and reference coordinate problems (abbreviations and technical terms are defined in the Glossary).

Positioning error examples for Oregon	Error magnitudes
Using NAD 27 when NAD 83 required	Varies from ~250 to 330 feet (horizontal)
Using "WGS 84" when NAD 83 required (e.g., by using WAAS corrections or CORS IGS08 coordinates)	~4 to 5 feet (horizontal) ~1 to 2 feet (vertical)
Using published three-parameter datum transformation between NAD 27 and "WGS 84" for NAD 83 projects	Up to ~20 feet (horizontal)
Using NADCON to transform coordinates between NAD 27 and NAD 83	~1 foot (horizontal)
Using NADCON to transform coordinates between NAD 83(1986) "original" and NAD 83(1991) "HARN"	~0.5 foot (horizontal)
Using NAD 83(1986) "original" when NAD 83(1991) "HARN" required	Up to 4.5 feet (horizontal)
Using NAD 83(1998) when NAD 83(2007/CORS96) coordinates required	Up to 0.5 ft (horizontal) Up to 0.6 ft (vertical)
Using NAD 83(2007/CORS96) when NAD 83(2011) epoch 2010.00 coordinates required	Up to 0.5 ft (horizontal) Up to 0.6 ft (vertical)
Using only 7 of the 14 published transformation parameters between WGS 84/IGS08/ITRF08 and NAD 83(2011) and neglecting tectonic velocities	~0.2 to 0.3 ft (horizontal) ~0.06 ft (vertical)
Using published NGS 14-parameter transformation between WGS 84/IGS08/ITRF08 and NAD 83(2011) but neglecting tectonic velocities for 5 year epoch time difference	Up to ~0.2 ft (horizontal)
Autonomous (uncorrected) GPS single-point positioning precision (at 95% confidence)	~10 to 20 ft (horizontal) ~20 to 50 ft (vertical)

The NGS Datasheet as a geodetic reference coordinate source

Recommend using Datasheets with GPS-derived coordinates, because they give ellipsoid height (as well as ECEF coordinates).

```
AF8301 ***********************************************************************
AF8301  PACS         -  This is a Primary Airport Control Station.
AF8301  DESIGNATION -  RDM A
AF8301  PID          -  AF8301
AF8301  STATE/COUNTY-  OR/DESCHUTES
AF8301  COUNTRY      -  US
AF8301  USGS QUAD    -  REDMOND (1975)
AF8301
AF8301                        *CURRENT SURVEY CONTROL
AF8301
AF8301* NAD 83(2011) POSITION- 44 15 07.30805(N) 121 08 57.28517(W)   ADJUSTED   = φ and λ
AF8301* NAD 83(2011) ELLIP HT-    915.319 (meters)        (06/27/12)   ADJUSTED   = h
AF8301* NAD 83(2011) EPOCH    -   2010.00
AF8301* NAVD 88 ORTHO HEIGHT -    936.440 (meters)     3072.30  (feet) ADJUSTED   = H
AF8301
AF8301  NAD 83(2011) X  - -2,367,351.543 (meters)                      COMP       = X
AF8301  NAD 83(2011) Y  - -3,916,788.058 (meters)                      COMP       = Y
AF8301  NAD 83(2011) Z  -  4,428,832.050 (meters)                      COMP       = Z
AF8301  LAPLACE CORR    -          4.39  (seconds)                     DEFLEC12B
AF8301  GEOID HEIGHT    -        -21.118 (meters)                      GEOID12B
AF8301  DYNAMIC HEIGHT  -        936.120 (meters)     3071.25  (feet) COMP
AF8301  MODELED GRAVITY -    980,245.4   (mgal)                        NAVD 88
AF8301
AF8301  VERT ORDER      -  FIRST     CLASS II
AF8301
AF8301  Network accuracy estimates per FGDC Geospatial Positioning Accuracy
AF8301  Standards:
AF8301         FGDC (95% conf, cm)     Standard deviation (cm)     CorrNE
AF8301            Horiz  Ellip            SD_N   SD_E   SD_h      (unitless)
AF8301  -------------------------------------------------------------------
AF8301  NETWORK    0.20   0.65           0.09   0.07   0.33      0.06976836
AF8301  -------------------------------------------------------------------
AF8301  Click here for local accuracies and other accuracy information.
```

Some things to note about NGS Datasheets:

- Units of “feet” for orthometric heights on NGS Datasheets are in US survey feet.
  - For above Datasheet, **NAVD 88 *H* = 936.440 m = 3072.30 sft = 3072.31 ift**
- Many conventional stations do not have accurate orthometric heights, so the orthometric height cannot be used with geoid model to determine accurate ellipsoid heights.
- Conventionally (optically) determined control is almost always less accurate than survey-grade GPS, so using such control for surveys is not advised
  - Only GPS stations have positional accuracies given as linear “network” and “local” values in centimeters (relative “order” system not used)
- The epoch date of 2010.00 (Jan 1, 2010) gives the coordinates of the station at that date.
  - Coordinates change with time, and that change can be substantial in tectonically active areas (in northwest Oregon, horizontal NAD 83 velocities can exceed ***1.5 cm/year***).

OPUS output as a geodetic reference coordinate source

The *Online Positioning User Service*

This is an excellent alternative to the NGS Datasheets if there are no high-quality GPS-derived NGS control stations locally available.

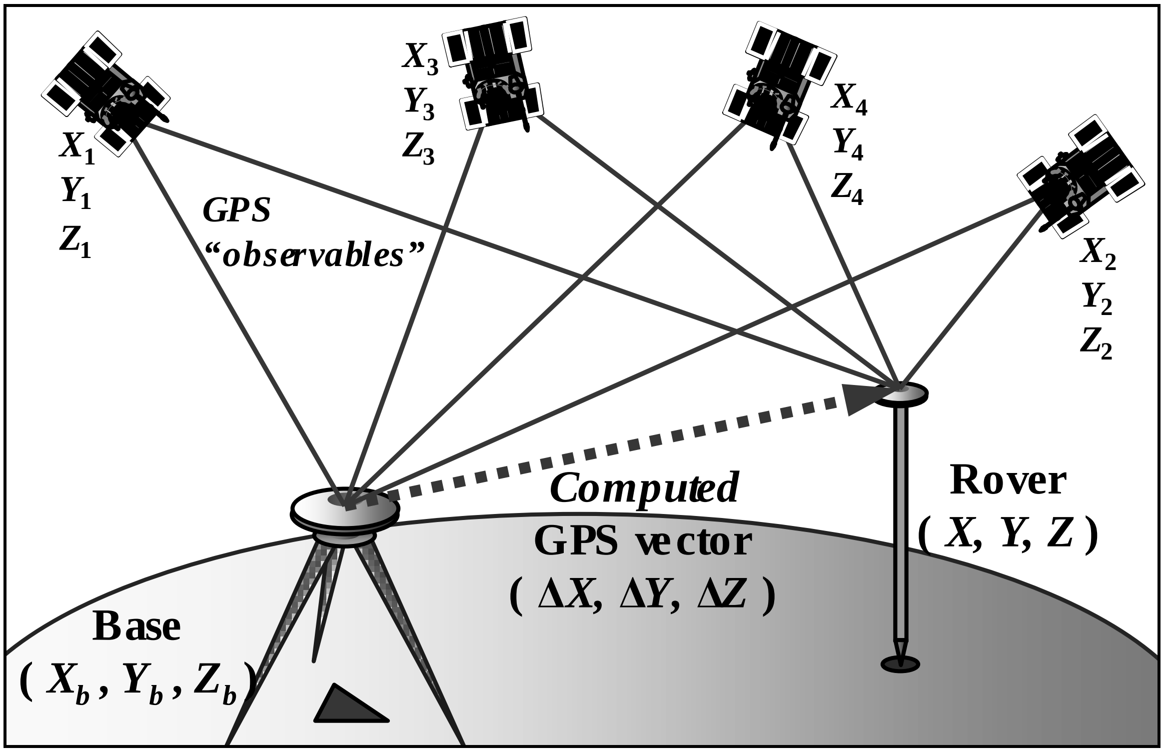
- More accurate than conventional (optical) control
- Requires logging raw GPS data (observables) at the receiver for at least 2 hours (or as little as 15 minutes using the "Rapid Static" option)
    - o This can easily be done at a GPS base while performing a survey

```
                              NGS OPUS SOLUTION REPORT
                              ========================

All computed coordinate accuracies are listed as peak-to-peak values.
For additional information: http://www.ngs.noaa.gov/OPUS/about.jsp#accuracy

      USER: mld@geodetic.xyz                          DATE: February 26, 2016
RINEX FILE: redm032s.16o                              TIME: 22:54:01 UTC

  SOFTWARE: page5  1209.04 master53.pl 022814      START: 2016/02/01  18:00:00
 EPHEMERIS: igs18821.eph [precise]                  STOP: 2016/02/01  22:00:00
  NAV FILE: brdc0320.16n                        OBS USED:  9762 / 10032   :  97%
  ANT NAME: TRM29659.00     NONE             # FIXED AMB:    47 /    49   :  96%
ARP HEIGHT: 0.000                            OVERALL RMS: 0.011(m)

 REF FRAME: NAD_83(2011)(EPOCH:2010.0000)              IGS08 (EPOCH:2016.0870)

         X:     -2366948.819(m)   0.005(m)          -2366949.707(m)   0.005(m)
         Y:     -3916334.905(m)   0.003(m)          -3916333.680(m)   0.003(m)
         Z:      4429451.080(m)   0.009(m)           4429451.076(m)   0.009(m)

       LAT:   44 15 35.14668      0.006(m)        44 15 35.15991      0.006(m)
     E LON:  238 51  7.68470      0.005(m)       238 51  7.62188      0.005(m)
     W LON:  121  8 52.31530      0.005(m)       121  8 52.37812      0.005(m)
    EL HGT:          920.348(m)   0.008(m)               919.924(m)   0.008(m)
 ORTHO HGT:          941.471(m)   0.020(m) [NAVD88 (Computed using GEOID12B)]

                        UTM COORDINATES    STATE PLANE COORDINATES
                         UTM (Zone 10)         SPC (3602 OR S)
Northing (Y) [meters]     4902392.828           288268.367
Easting (X)  [meters]      647844.553          1448260.996
Convergence  [degrees]     1.29286129          -0.44323538
Point Scale                0.99986882           1.00007642
Combined Factor            0.99972455           0.99993212

US NATIONAL GRID DESIGNATOR: 10TFQ4784402392(NAD 83)


                              BASE STATIONS USED
PID       DESIGNATION                        LATITUDE    LONGITUDE DISTANCE(m)
DP8352 P063 SHANIKO___OR2005 CORS ARP      N445521.784 W1205646.093   75402.1
DP9278 PLNA PAULINA CORS ARP               N440755.421 W1195800.382   95492.5
DG5352 STAY STAYTON COOP CORS ARP          N444950.532 W1224915.034  147317.5

                 NEAREST NGS PUBLISHED CONTROL POINT
AH2507      REDMOND CORS ARP               N441535.146 W1210852.315       0.0
```

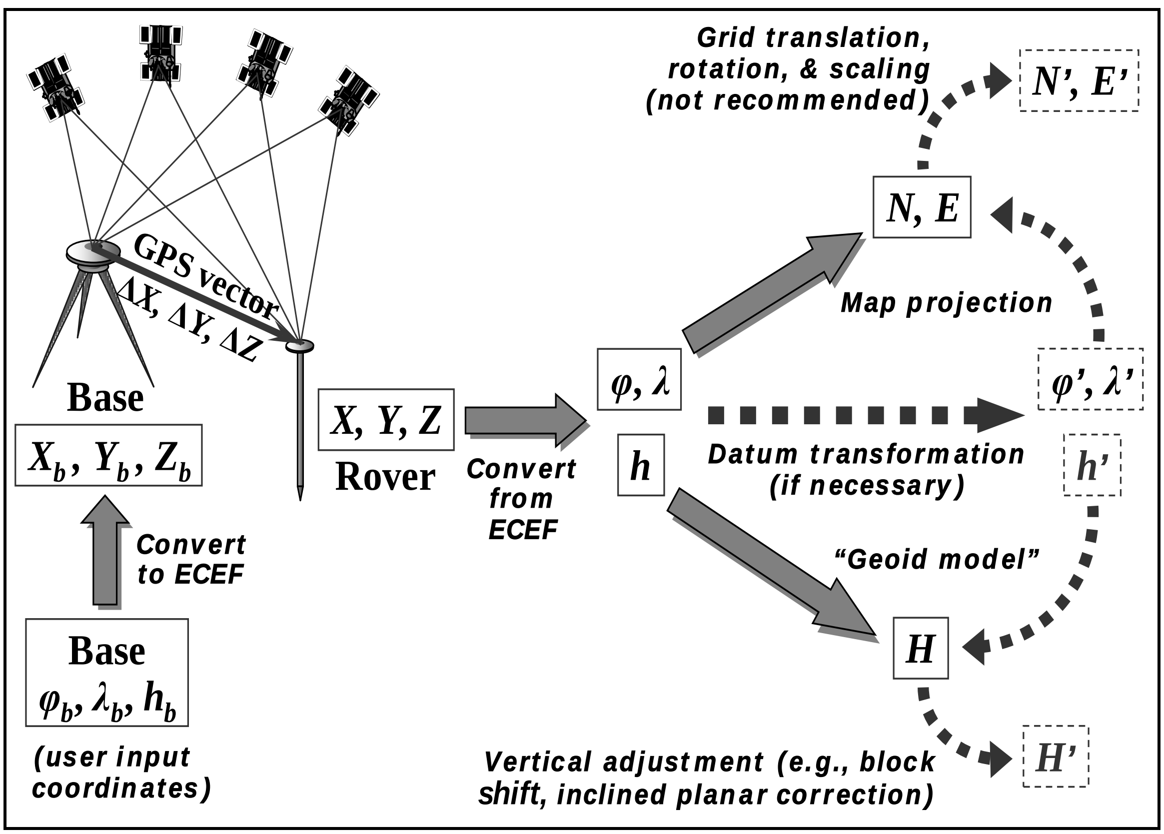
Some things to note about OPUS output:

- Gives both NAD 83(2011) and IGS 08 coordinates.
  - NAD 83(2011) is for epoch 2010.00, and is the same in ALL areas of the NSRS for both passive marks and CORS. In the 2007 adjustment an epoch of 2007.00 was used for passive marks in AZ, CA, NV, OR, WA, and AK, and 2002.00 was use everywhere else (but 2002.00 was used for all CONUS CORS and 2003.00 was used for AK CORS).
  - IGS08 is for day of observation (e.g., date 2016.09 = Feb 1, 2016 for this example).
    - This is NOT the same as the current version of WGS 84 (G1762), which was computed at epoch 2005.0, so the coordinates will differ by the date difference times the IGS08 station velocity (about 0.035 ft/year to the NW in OR, so for this case nearly 0.4 ft in 11 years).
    - However, IGS08 (≈ ITRF 08) and WGS 84 (G1762) can be considered equivalent to within about 1 cm (0.03 ft) if both refer to the same epoch.
- Slightly different results will be obtained depending on which GPS orbits were used.
  - Final orbits available after about 12 days.
  - "Rapid" orbits available in 17 hours, and are nearly as accurate as final orbits.
- Values to right of coordinates are accuracy estimates in meters, e.g., 0.008 (m).
  - These are based on the maximum difference between the 3 positions computed by OPUS.
  - Can also estimate accuracy (or at least precision) yourself if have multiple OPUS solutions on a single point.
- Detailed ("extended") output also available
  - Gives additional information such as CORS details, coordinate transformations, velocities, actual vector components, GPS solution statistics, internal precision estimates, and approximate orthometric height with respect to proposed new vertical datum (approximately 3 feet LOWER than NAVD 88 in Pacific Northwest).
- Three versions of OPUS now available: OPUS-S ("Static"), OPUS-RS ("Rapid Static"), and OPUS-Projects (OP)
  - OPUS-S was formerly simply known as "OPUS" and requires a dataset duration of at least 2 hours.
  - OPUS-RS will process shorter datasets (duration from 15 minutes to 2 hours).
    - Accuracy of OPUS-RS results varies by location and is best in areas with dense CORS coverage and with rover inside a polygon of CORS
    - If poor results are achieved with OPUS-RS, use of OPUS-S is recommended (for dataset durations of more than 2 hours).
  - OP can process and adjust GPS data from multiple receivers and occupations.
    - OPUS-S is used to submit data to OP
    - Attending a (free) training is required to get access to OP

Relative positioning with "survey-grade" GPS



GPS computation flowchart



Computation of coordinates using GPS vector components

Below are equations for computing geodetic coordinates of a new station using the GPS vector from a base station of known geodetic coordinates.

Equation 2.1 Converting latitude, longitude, and height to ECEF coordinates

$$\boxed{\begin{aligned} X &= (R_N + h)\cos\varphi\ \cos\lambda \\ Y &= (R_N + h)\cos\varphi\ \sin\lambda \\ Z &= \left[R_N(1-e^2)+h\right]\sin\varphi \end{aligned}}$$ (Leick, 2004, p. 371)

where X, Y, and Z are the ECEF coordinates of a point

φ, λ, and h are the latitude, longitude, and ellipsoid height of the point, respectively

$\boxed{R_N = a\left(1-e^2\sin^2\varphi\right)^{-1/2}}$ is the prime vertical radius of curvature (Leick, 2004, p. 369)

a is the ellipsoid semi-major axis (= 20,925,646.325 459 ift for the GRS-80 ellipsoid)

e^2 is the ellipsoid first eccentricity squared (= 0.006 694 380 022 901 for GRS-80)

Equation 2.2 Computing coordinates from GPS vector components

$$\boxed{X = X_b + \Delta X \qquad Y = Y_b + \Delta Y \qquad Z = Z_b + \Delta Z}$$

where X, Y, and Z are the ECEF coordinates to be determined

X_b, Y_b, and Z_b are the ECEF coordinates of the GPS base

ΔX, ΔY, and ΔZ are the delta ECEF components of the GPS vector

Equation 2.3 Converting ECEF coordinates to latitude, longitude, and height

$$\boxed{\begin{aligned} \varphi &= \tan^{-1}\left[\frac{Z}{\sqrt{X^2+Y^2}}\left(1+\frac{e^2 R_N \sin\varphi_0}{Z}\right)\right] \\ \lambda &= \tan^{-1}\left(\frac{Y}{X}\right) \\ h &= \frac{\sqrt{X^2+Y^2}}{\cos\varphi} - R_N \end{aligned}}$$ (Leick, 2004, pp. 371-372)

where φ_0 is a latitude that can be initially approximated as $\varphi_0 = \tan^{-1}\left(\dfrac{Z}{(1-e^2)\sqrt{X^2+Y^2}}\right)$.

The approximate latitude value is then substituted into the right side of the first line of Equation 2.3, and then the resulting value of φ is substituted as φ_0, and the process repeated until the change in φ is negligible.

Given: A GPS base station located at midpoint between points RDM A and REDM CORS, with NAD 83 coordinates of φ = 44°15'21.22736" N, λ = 121°08'54.80014" W, and h = 3011.130 ift. The following GPS vector components were determined from this base to point RDM A:

$\Delta X = -660.684$ ift $\quad \Delta Y = -743.404$ ift $\quad \Delta Z = -1015.406$ ift

Find: The NAD 83 coordinates of point RDM A.

Computations:

Step 1. Convert GPS base latitude, longitude, and ellipsoid height to ECEF coordinates.

The prime vertical radius of curvature for this station was computed in Exercise 1.3:

R_N = 20,959,841.481 ift

Now compute the ECEF values for the GPS base:

$$X_b = (R_N + h) \times \cos\varphi \times \cos\lambda$$

$$X_b = (20{,}959{,}841.481 + 3011.130) \times \cos(44.25589648889°) \times \cos(-121.14855559306°)$$

$$= \mathbf{-7{,}766{,}240.703\ ift}$$

$$Y_b = (R_N + h) \times \cos\varphi \times \sin\lambda$$

$$Y_b = (20{,}959{,}841.481 + 3011.130) \times \cos(44.25589648889°) \times \sin(-121.14855559306°)$$

$$= \mathbf{-12{,}849{,}611.118\ ift}$$

$$Z_b = [R_N \times (1- e^2) + h] \times \sin\varphi$$

$$Z_b = [20{,}959{,}841.481 \times (1- 0.006694380023) + 3011.130] \times \sin(44.25589648889°)$$

$$= \mathbf{14{,}531{,}304.285\ ift}$$

Step 2. Compute ECEF coordinates of new GPS station (RDM A).

$$X = X_b + \Delta X = (-7{,}766{,}240.703\ \text{ift}) + (-660.684\ \text{ift}) = \mathbf{-7{,}766{,}901.387\ ift}$$

$$Y = Y_b + \Delta Y = (-12{,}849{,}611.118\ \text{ift}) + (-743.404\ \text{ift}) = \mathbf{-12{,}850{,}354.522\ ift}$$

$$Z = Z_b + \Delta Z = (14{,}531{,}304.285\ \text{ift}) + (-1015.406\ \text{ift}) = \mathbf{14{,}530{,}288.879\ ift}$$

Step 3. Convert ECEF coordinates of new station to latitude, longitude, and ellipsoid height.

Equation 2.3 was used to compute the following results for station RDM A (compare to those computed using the NGS Geodetic Toolkit).

Latitude, φ = 44° 15' 07.30805'' N
Longitude, λ = 121° 08' 57.28518'' W
Ellipsoid height, h = 3003.015 ift

✓ *These results were computed using Equation 2.3 (required only 2 iterations in Excel for accuracy shown)*

Non-iterative methods for calculating φ are available. One below is from You (2000).

Equation 2.4 Non-iterative method for computing latitude from ECEF coordinates

$$\boxed{\varphi = tan^{-1}\left(\frac{a}{b} tan\, \beta_0\right)}$$

where

$$\beta_0 = tan^{-1}\left[\frac{Z}{R}\sqrt{\frac{R^2 + a^2 - b^2}{X^2 + Y^2}}\right]$$

$$R = \sqrt{\frac{1}{2}\left[X^2 + Y^2 + Z^2 - a^2 + b^2 + \sqrt{(X^2 + Y^2 + Z^2 - a^2 + b^2)^2 + 4(a^2 - b^2)\, Z^2}\right]}$$

The equations for longitude and ellipsoid height are the same as in Equation 2.3. The longitude is completely unaffected, but ellipsoid height is not since it is a function of latitude.

The accuracy of Equation 2.4 depends on *h*. It gives values for latitude and ellipsoid height accurate to within about 0.3 mm for $h <$ ~1000 and about 1 mm for $h <$ ~2000 m. For the example problem, the latitude based on Equation 2.4 is the same as given above. The ellipsoid height is slightly greater, h = 3003.016 ift versus the correct value of h = 3003.015 ift.

For more accurate results, the value of beta can be refined as $\beta_1 = \beta_0 + \Delta\beta$, where

$$\Delta\beta = \frac{\left(bR - a\sqrt{R^2 + a^2 - b^2} + a^2 - b^2\right) sin\, \beta_0}{\dfrac{a\sqrt{R^2 + a^2 - b^2}}{cos\, \beta_0} - (a^2 - b^2)\, cos\, \beta_0}$$

Using β_1 in place of β_0 in Equation 2.4 gives results accurate to better than 0.1 mm for $h <$ ~200,000 m.

An approximate method for computing ellipsoidal distance

This gives a method for computing an approximate ellipsoidal distance between two points with geodetic coordinates (latitude, longitude, and ellipsoid height). For the GRS-80, WGS-84, Clarke 1866, and most other Earth ellipsoids, note the following:

Rules of Thumb

1 arc-second of latitude ≈ 101.4 ft (*long by 0 to 0.5 ft in CONUS*)

1 arc-second of longitude ≈ 101.4 ft × cos(latitude) (*short by 0.1 to 0.2 ft in CONUS*)

Based on these relationships, we can compute an approximate distance, to wit:

Equation 2.5 Approximate ellipsoidal distance between a pair of geodetic coordinates

$$\boxed{s \approx 101.4\sqrt{(\Delta\varphi'')^2 + (\Delta\lambda''\cos\bar{\varphi})^2}} \text{ feet}$$

This equation is accurate to within about ±0.5% everywhere on the Earth

where $\Delta\varphi''$ is change in latitude between two points in arc-seconds

$\Delta\lambda''$ is change in longitude between two points in arc-seconds

$\bar{\varphi}$ is average latitude of the two points

Example computation

Given: Points RDM A and REDM CORS.

Find: The approximate ellipsoidal distance between the points RDM A and REDM CORS.

Computations:

The average latitude of RDM A and REDM is $\bar{\varphi} = 44.2558964889°$

$$s \approx 101.4\sqrt{(\Delta\varphi'')^2 + (\Delta\lambda''\cos\bar{\varphi})^2}$$

$$= 101.4 \times \sqrt{(35.14667'' - 07.30805'')^2 + [(52.31510'' - 57.28517'') \times \cos(44.2558964889°)]^2}$$

$$= 101.4 \times \sqrt{(27.83862)^2 + (-3.55971)^2} = \underline{\mathbf{2846\ ft}}$$

Check using NGS Inverse tool:

Actual ellipsoid distance (geodesic) = 866.3086 m = <u>2842.220 ift</u>

Approximate geodetic inverse error = +3.8 ft = +0.1% ✓

Local geodetic horizon method for computing geodetic distance

Computation of accurate geodetic distances is difficult, but the *local geodetic horizon* (LGH) method based on ECEF coordinates is very accurate over short to moderate distances.

Equation 2.6 Computing local geodetic horizon distance from delta ECEF coordinates

Horizontal components from point A to B:

$$\Delta N = -\Delta X \sin \varphi_A \cos \lambda_A - \Delta Y \sin \varphi_A \sin \lambda_A + \Delta Z \cos \varphi_A$$

$$\Delta E = -\Delta X \sin \lambda_A - \Delta Y \cos \lambda_A$$

Up and height components from point A to B:

$$\Delta U = \Delta X \cos \varphi_A \cos \lambda_A + \Delta Y \cos \varphi_A \sin \lambda_A + \Delta Z \sin \varphi_A$$

$$\Delta h = h_B - h_A$$

Given: Points REDM CORS and RDM A from the previous problem.

Find: The approximate ellipsoidal distance between from REDM CORS to RDM A.

Computations: Compute distance from REDM CORS to RDM A.

REDM CORS: φ_A = 44°15'35.14667"N, λ_A = 121°08'52.31510"W, h_A = 3019.505 ift

$\Delta X = -1321.288$ ift, $\Delta Y = -1486.722$ ift, $\Delta Z = -2030.928$ ift

$$\Delta N = 1321.288 \times \sin(44°15'35.14667''\text{N}) \cos(121°08'52.31510''\text{W}) + 1486.722 \times \sin(44°15'35.14667''\text{N}) \sin(121°08'52.31510''\text{W}) - 2030.928 \times \cos(44°15'35.14667''\text{N}) = -2819.510 \text{ ift}$$

$$\Delta E = 1321.288 \times \sin(121°08'52.31510''\text{W}) + 1486.722 \times \cos(121°08'52.31510''\text{W}) = -361.800 \text{ ift}$$

Horizontal distance = $\sqrt{\Delta N^2 + \Delta E^2} = \sqrt{2819.510^2 + 361.800^2}$ = 2842.628 ift

Note: In general distance A → B does not equal B → A, and the horizontal difference increases as the height difference increases. For zero height difference, distance A → B differs from B → A by less than 1 mm for distances of less than 100 km. In this example, distance RDM A → REDM = 2842.631 ift, for a difference of 0.003 ift.

Check using NGS Inverse tool: Actual ellipsoid distance = 866.3086 m = 2842.220 ift

Approximate geodetic inverse error = +0.408 ft = +0.014%. Most of this difference is because the LGH distance is at the topographic surface; if the LGH distance were computed on the ellipsoid (i.e., with $h = 0$ for both points) it would be 2,842.220 ift, the same as the ellipsoidal distance to the significance shown. For LGH computations on the ellipsoid, LGH distance will be shorter than ellipsoidal distance by less than 1 mm for distances of less than about 6 km.

Note that in general $\Delta U < \Delta h$, but they differ by less than 1 mm for distances of less than 100 m. The difference increases rapidly, to 8 cm at a distance of 1 km and about 7.8 m at 10 km. In this case, $\Delta h = -16.490$ ft and $\Delta U = -16.683$ ft, and the difference is 0.193 ft (5.9 cm).

Datums and datum transformations

Datum. Any quantity or set of quantities used as a reference or basis for determining other quantities.

Geodetic (geometric) datum. A set of (at least 8) constants specifying the coordinate system for geodetic control (latitude, longitude, height).

2 for reference ellipsoid size and shape (usually semi-major axis and flattening)

3 to specify location of origin (at or near center of Earth for modern datums)

3 to specify the orientation of coordinate axes

Vertical datum. A set of fundamental "elevations" to which other "elevations" are referred.

Datum transformation. Mathematical method for converting one geodetic or vertical datum to another (there are several types, and they vary widely in accuracy).

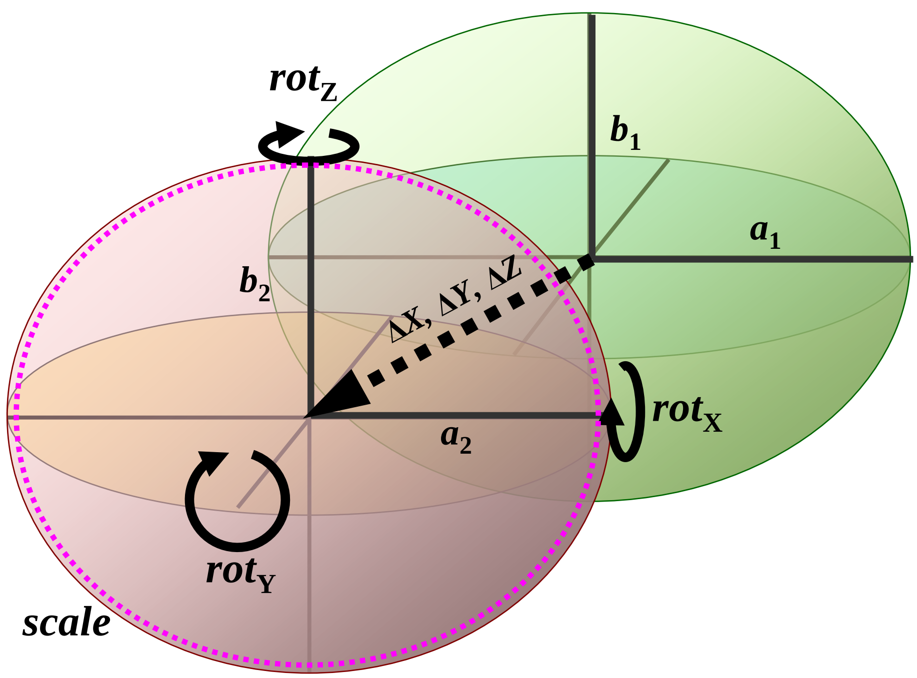
Types of datum transformations

Datum transformations are typically either equation-based or grid-based, although some can be a combination of both types, such the NGS program Horizontal Time Dependent Positioning (HTDP).

Equation-based datum transformations. Consist of closed-form equations.

Parametric transformations. Latitude, longitude, and ellipsoid height are converted to *X*, *Y*, *Z*, then one of the following transformations are performed. Then the new *X*, *Y*, *Z* values are converted to new latitude, longitude, and ellipsoid height values. In figure below, the dimensions of the reference ellipsoid (*a* and *b* axes) may or may not change in the transformation.

Equation-based datum transformations



3-parameter: 3-dimensional translation of origin as ΔX, ΔY, ΔZ (*like a GPS vector*)

7-parameter: 3 translations *plus* 3 rotations (one about each of the axes) *plus* a scale

14-parameter: A 7-parameter transformation where each parameter changes with time (each has a *velocity*)

Molodensky transformations. Converts directly between datums without using X, Y, Z coordinates. The ΔX, ΔY, ΔZ values are incorporated directly into the transformation equations.

Grid-based datum transformations. Consists of a grid of coordinate differences where differences at locations between grid points are interpolated. They are used when the differences are too irregular for simple equations. There are many types that can be horizontal only, 3-D, or vertical only.

NADCON (horizontal only). Transforms between NAD 27 and NAD 83. NADCON HPGN/HARN transforms between the original (1986) realization of NAD 83 and the first GPS-based (HPGN/HARN) realizations.

GEOCON (3D). Transforms between the HPGN/HARN realizations of NAD 83 and the NSRS2007 realization. GEOCON11 transforms between NSRS2007 and the 2011 epoch 2010.00 realization. Also outputs estimated (worst-case) accuracy of the transformation.

VERTCON (vertical only). Transforms between NGVD 29 and NAVD 88 vertical datums. Incorporates gravitational effects of topography used for NAVD 88 (not used for NGVD 29).

NGS hybrid geoid models (vertical only). Transforms between NAD 83 ellipsoid heights and NAVD 88 orthometric heights ("elevations"). Each model associated with a specific realization of NAD 83 (e.g., GEOID12B is for NAD 83 (2011) epoch 2010.00). Created from gravimetric geoid models and observed ellipsoid heights on NAVD 88 benchmarks.

VDatum (vertical only). Transforms between vertical/ellipsoidal datums and local tidal datums, as well as International Great Lakes Datum of 1985 (IGLD 85). Incorporates both VERTCON and hybrid geoid models along with local tidal transformation models and Great Lakes hydraulic correctors.

Combined equation- and grid-based transformations.

HTDP (3D + time = 4D). Uses both 14-parameter transformations and a horizontal crustal deformation velocity grid. If the input and output dates (epochs) are the same, it only performs 14-parameter transformations (i.e., the velocity grids are notes used).

NOTE: HTDP can only be used to transform between "generic" NAD 83 and specific realizations of the global reference frame, all of which are considered identical (i.e., WGS 84 = ITRF = IGS). HTDP does ***not*** transform between the various realizations of NAD 83. Such transformations must be performed with NADCON HPGN/HARN or GEOCON.

Helmert similarity datum transformations

The most common equation-based datum transformations are some form of the Helmert similarity transformation. These types of transformations preserve shape because the scale is constant (i.e., does not vary with direction).

Equation 2.7 Helmert 7-parameter similarity transformation

$$
\begin{aligned}
X_{out} = T_X &+ [\, X_{in} \cos R_Y \cos R_Z \\
&+ \; Y_{in} \cos R_Y \sin R_Z \\
&- \; Z_{in} \sin R_Y \,] \, (1 + S) \\[1em]
Y_{out} = T_Y &+ [\, X_{in} \, (\sin R_X \sin R_Y \cos R_Z \; - \; \cos R_X \sin R_Z \,) \\
&+ \; Y_{in} \, (\sin R_X \sin R_Y \sin R_Z \; + \; \cos R_X \cos R_Z \,) \\
&+ \; Z_{in} \sin R_X \cos R_Y \,] \, (1 + S) \\[1em]
Z_{out} = T_Z &+ [\, X_{in} \, (\cos R_X \sin R_Y \cos R_Z \; + \; \sin R_X \sin R_Z \,) \\
&+ \; Y_{in} \, (\cos R_X \sin R_Y \sin R_Z \; - \; \sin R_X \cos R_Z \,) \\
&+ \; Z_{in} \cos R_X \cos R_Y \,] \, (1 + S)
\end{aligned}
$$

where X_{out} , Y_{out} , Z_{out} are the output coordinates
X_{in} , Y_{in} , Z_{in} are the input coordinates
T_X , T_Y, T_Z are translations along the X, Y, and Z axes
R_X , R_Y , R_Z are rotations about the X, Y, and Z axes
S is the (fractional part of the) scale factor

If the relationship between the datums changes with time, then translation, rotation, and/or scale also change with time. The change with time is a *velocity* and is denoted by placing a dot over the symbol, e.g., $\Delta\dot{X}$, $\dot{R}_X$, $\dot{S}$. At time t for a time span of Δt with respect to some reference time t_0 (i.e., $\Delta t = t - t_0$), the parameters are:

Equation 2.8 Time-varying Helmert similarity transformation parameters

Translation	Rotation	Scale
$T_X(t) = T_X(t_0) + \dot{T}_X \Delta t$	$R_X(t) = R_X(t_0) + \dot{R}_X \Delta t$	$S(t) = S(t_0) + \dot{S} \Delta t$
$T_Y(t) = T_Y(t_0) + \dot{T}_Y \Delta t$	$R_Y(t) = R_Y(t_0) + \dot{R}_Y \Delta t$	
$T_Z(t) = T_Z(t_0) + \dot{T}_Z \Delta t$	$R_Z(t) = R_Z(t_0) + \dot{R}_Z \Delta t$	

A note on terminology: Different names are given to this transformation depending on the sign convention used for the rotations. In the US and Australia, rotations are positive counterclockwise and it as sometimes called a *coordinate frame* transformation. In Europe the rotations are positive clockwise and it is sometimes called a *position vector* transformation. One method can be converted to the other by changing the sign of the rotations.

Equation 2.7 and 2.8 together represent the general a 14-parameter transformation because it has 3 translations + 3 translation velocities + 3 rotations + 3 rotation velocities + 1 scale + 1 scale velocity = 14 parameters. If Δt is zero or none of the parameters vary with time, it reduces to a 7-parameter transformation. If the rotations are also zero, it reduces to a 3-parameter transformation.

The 14 Helmert transformation parameters from IGS08 to the 2011, PA11, and MA11 realizations of NAD 83 are given in Table 2.2.

Table 2.2 Examples Helmert transformation parameters from IGS08 to NAD 83 (Note that a transformation using only these parameters will not give correct results if the IGS08 and NAD 83 epochs differ. In such cases the NGS program HTDP should be used, which combines these transformations with crustal motion models. Values in parentheses are translations in feet)

IGS08 to NAD 83 Helmert transformation parameters (from HTDP version 3.2.5)*			
NAD 83 realization	**NAD 83 (2011)**	**NAD 83 (PA11)**	**NAD 83 (MA11)**
Tectonic plate	**North America**	**Pacific**	**Mariana**
Parameters at reference epoch t_0 = 1997.00			
Translation *X* (m)	0.99343 (3.25928 ft)	0.90803 (2.97910 ft)	
Translation *Y* (m)	–1.90331 (–6.24446 ft)	–2.01611 (–6.61453 ft)	
Translation *Z* (m)	–0.52655 (–1.72753 ft)	–0.56525 (–1.85449 ft)	
Rotation about *X*-axis (mas*)	25.91458	27.74067	28.97167
Rotation about *Y*-axis (mas*)	9.42655	13.46845	10.42045
Rotation about *Z*-axis (mas*)	11.59929	2.71235	8.92835
Scale (parts per billion)	1.71504	1.10000	
Change in parameters with respect to reference epoch t_0 = 1997.00			
Translation *X* (m / year)	0.00079 (0.00259 ft/yr)	0.00010 (0.00033 ft/yr)	
Translation *Y* (m / year)	–0.00060 (–0.00197 ft/yr)	0.00010 (0.00033 ft/yr)	
Translation *Z* (m / year)	–0.00134 (–0.00440 ft/yr)	–0.00180 (–0.00591 ft/yr)	
Rotation about *X*-axis (mas / year)	0.06669	-0.38353	–0.01953
Rotation about *Y*-axis (mas / year)	–0.75749	1.00686	0.10486
Rotation about *Z*-axis (mas / year)	–0.05129	–2.18573	–0.34673
Scale (parts per billion / year)	–0.10201	0.08000	

***NOTE:** Some HTDP transformation parameters in this table differ slightly from those given on NGS web page www.geodesy.noaa.gov/CORS/coords.shtml due to rounding. HTDP parameters should be considered definitive.
**mas = milliarc-seconds, where 1 mas = 4.84814×10^{-9} radian

Given: The published IGS08 and NAD 83 coordinates of the REDM CORS ARP.

	Latitude	Longitude	Ellipsoid height
IGS08 epoch 2005.00	44°15'35.16273"N	121°08'52.37145"W	3018.130 ift
NAD 83 (2011) epoch 2010.00	44°15'35.14667"N	121°08'52.31510"W	3,019.505 ift

Find: The NAD 83 (2011) epoch 2010.00 coordinates from the IGS08 coordinates using the transformation parameters in Table 2.2 for the following five cases:

1. 3 parameters only (i.e., also assuming rotations are zero)
2. 7 parameters (i.e., also assuming velocities are zero)
3. 14 parameters with input and output epoch = 2010.00 (Δt = 2010 – 1997 = 13 years)
4. 14 parameters with input and output epoch = 2005.00 (Δt = 2005 – 1997 = 8 years)
5. HTDP (all 14 parameters plus crustal motion model with input epoch = 2005.00 and output epoch = 2010.00)

Computations:

First compute the IGS08 epoch 2005.00 ECEF coordinates from Equation 2.1:

X_{in} = –7,765,582.512 ift

Y_{in} = –12,848,863.864 ift

Z_{in} = 14,532,320.013 ift

Case 1. 3 translation parameters only

$X_{out} = T_X + X_{in}$ = 3.259 + (–7,765,582.512) ift = –7,765,579.253 ift → **Δ*X* = 3.259 ift**

$Y_{out} = T_Y + Y_{in}$ = –6.244 + (–12,848,863.864) ift = –12,848,870.109 ift → **Δ*Y* = –6.244 ift**

$Z_{out} = T_Z + Z_{in}$ = –1.728 + 14,532,320.013 ift = 14,532,318.286 ift → **Δ*Z* = –1.728 ift**

Case 2. 7 parameters: 3 translation + 3 rotation + 1 scale (using *r* to generically symbolize rotations)

$X_{out} = T_X + [X_{in}\, r + Y_{in}\, r - Z_{in}\, r]\,(1 + S)$ = –7,765,580.653 ift → **Δ*X* = 1.859 ift**

$Y_{out} = T_Y + [X_{in}\, r + Y_{in}\, r + Z_{in}\, r]\,(1 + S)$ = –12,848,867.868 ift → **Δ*Y* = –4.004 ift**

$Z_{out} = T_Z + [X_{in}\, r + Y_{in}\, r + Z_{in}\, r]\,(1 + S)$ = 14,532,319.570 ift → **Δ*Z* = –0.443 ift**

Case 3. 14 parameters: (3 translation + 3 rotation + 1 scale) + (rate of change for each). Use input and output epoch = 2010.00 (Δt = 2010.00 – 1997.00 = **13.00 years**)

$X_{out} = [T_X(1997) + 13\dot{T}_X] + \{X_{in}\,[r(1997) + 13\dot{r}] + Y_{in}\,[r(1997) + 13\dot{r}) - Z_{in}\,(r(1997) + 13\dot{r})\}$

$\times\,[1 + S(1997) + 13\dot{S}]$ = –7,765,579.874 ift → **Δ*X* = 2.639 ift**

$Y_{out} = [T_Y(1997) + 13\dot{T}_Y] + \{X_{in}\,[r(1997) + 13\dot{r}] + Y_{in}\,[r(1997) + 13\dot{r}) + Z_{in}\,(r(1997) + 13\dot{r})\}$

$\times\,[1 + S(1997) + 13\dot{S}]$ = –12,848,867.841 ift → **Δ*Y* = –3.977 ift**

$Z_{out} = [T_Z(1997) + 13\dot{T}_Z] + \{X_{in}\,[r(1997) + 13\dot{r}] + Y_{in}\,[r(1997) + 13\dot{r}) + Z_{in}\,(r(1997) + 13\dot{r})\}$

$\times\,[1 + S(1997) + 13\dot{S}]$ = 14,532,319.918 ift → **Δ*Z* = –0.095 ift**

Case 4. 14 parameters: (3 translation + 3 rotation + 1 scale) + (rate of change for each). Use input and output epoch = 2010.00 (Δt = 2005.00 – 1997.00 = **8.00 years**)

$$X_{out} = [T_X\,(1997) + 8\dot{T}_X\,] + \{X_{in}\,[r(1997) + 8\dot{r}] + Y_{in}\,[r(1997) + 8\dot{r}) - Z_{in}\,(r(1997) + 8\dot{r})\}$$
$$\times\,[1 + S(1997) + 8\dot{S}] = –7{,}765{,}580.173 \text{ ift} \rightarrow \qquad \mathbf{\Delta X = 2.339\ ift}$$

$$Y_{out} = [T_Y\,(1997) + 8\dot{T}_Y\,] + \{X_{in}\,[r(1997) + 8\dot{r}] + Y_{in}\,[r(1997) + 8\dot{r}) + Z_{in}\,(r(1997) + 8\dot{r})\}$$
$$\times\,[1 + S(1997) + 8\dot{S}] = –12{,}848{,}867.851 \text{ ift} \rightarrow \qquad \mathbf{\Delta Y = –3.987\ ift}$$

$$Z_{out} = [T_Z\,(1997) + 8\dot{T}_Z\,] + \{X_{in}\,[r(1997) + 8\dot{r}] + Y_{in}\,[r(1997) + 8\dot{r}) + Z_{in}\,(r(1997) + 8\dot{r})\}$$
$$\times\,[1 + S(1997) + 8\dot{S}] = 14{,}532{,}319.784 \text{ ift} \rightarrow \qquad \mathbf{\Delta Z = –0.229\ ift}$$

Case 5. HTDP with IGS08 input epoch of 2005.00 and NAD 83 output epoch of 2010.00. Computed as 14 parameters with input and output epoch = 2010.00 (Δt = 2005.00 – 1997.00 = 8.00 years) ***plus*** 5.00 years of NAD 83 differential tectonic velocities ($\dot{X}_{out}$, $\dot{Y}_{out}$, $\dot{Z}_{out}$):

$$X_{out} = [T_X\,(1997) + 8\dot{T}_X\,] + \{X_{in}\,[r(1997) + 8\dot{r}] + Y_{in}\,[r(1997) + 8\dot{r}) - Z_{in}\,(r(1997) + 8\dot{r})\}$$
$$\times\,[1 + S(1997) + 8\dot{S}] + 5\dot{X}_{out} = –7{,}765{,}580.099 \text{ ift} \rightarrow \qquad \mathbf{\Delta X = 2.413\ ift}$$

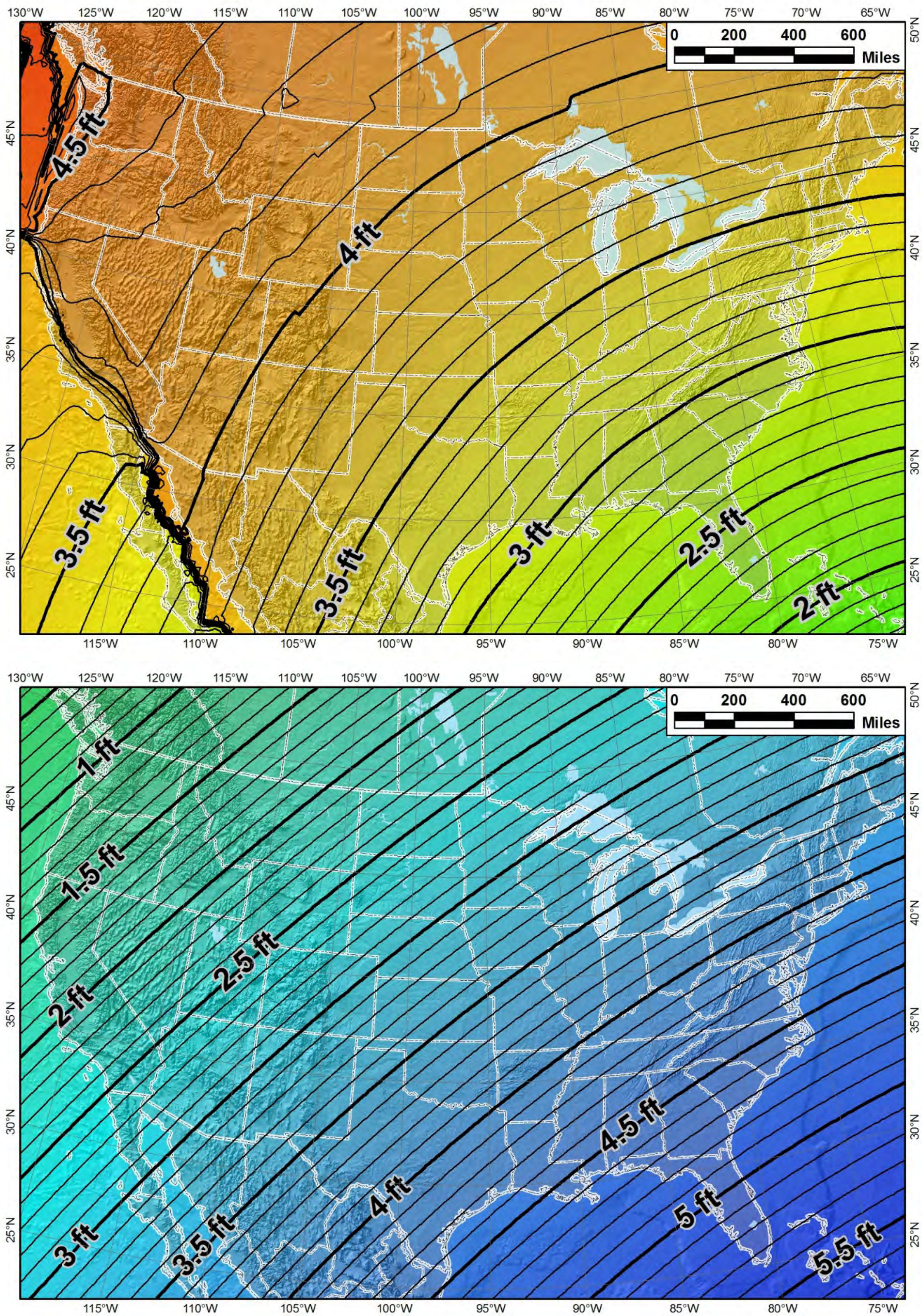
$$Y_{out} = [T_Y\,(1997) + 8\dot{T}_Y\,] + \{X_{in}\,[r(1997) + 8\dot{r}] + Y_{in}\,[r(1997) + 8\dot{r}) + Z_{in}\,(r(1997) + 8\dot{r})\}$$
$$\times\,[1 + S(1997) + 8\dot{S}] + 5\dot{Y}_{out} = –12{,}848{,}867.806 \text{ ift} \rightarrow \qquad \mathbf{\Delta Y = –3.942\ ift}$$

$$Z_{out} = [T_Z\,(1997) + 8\dot{T}_Z\,] + \{X_{in}\,[r(1997) + 8\dot{r}] + Y_{in}\,[r(1997) + 8\dot{r}) + Z_{in}\,(r(1997) + 8\dot{r})\}$$
$$\times\,[1 + S(1997) + 8\dot{S}] + 5\dot{Z}_{out} = 14{,}532{,}319.830 \text{ ift} \rightarrow \qquad \mathbf{\Delta Z = –0.183\ ift}$$

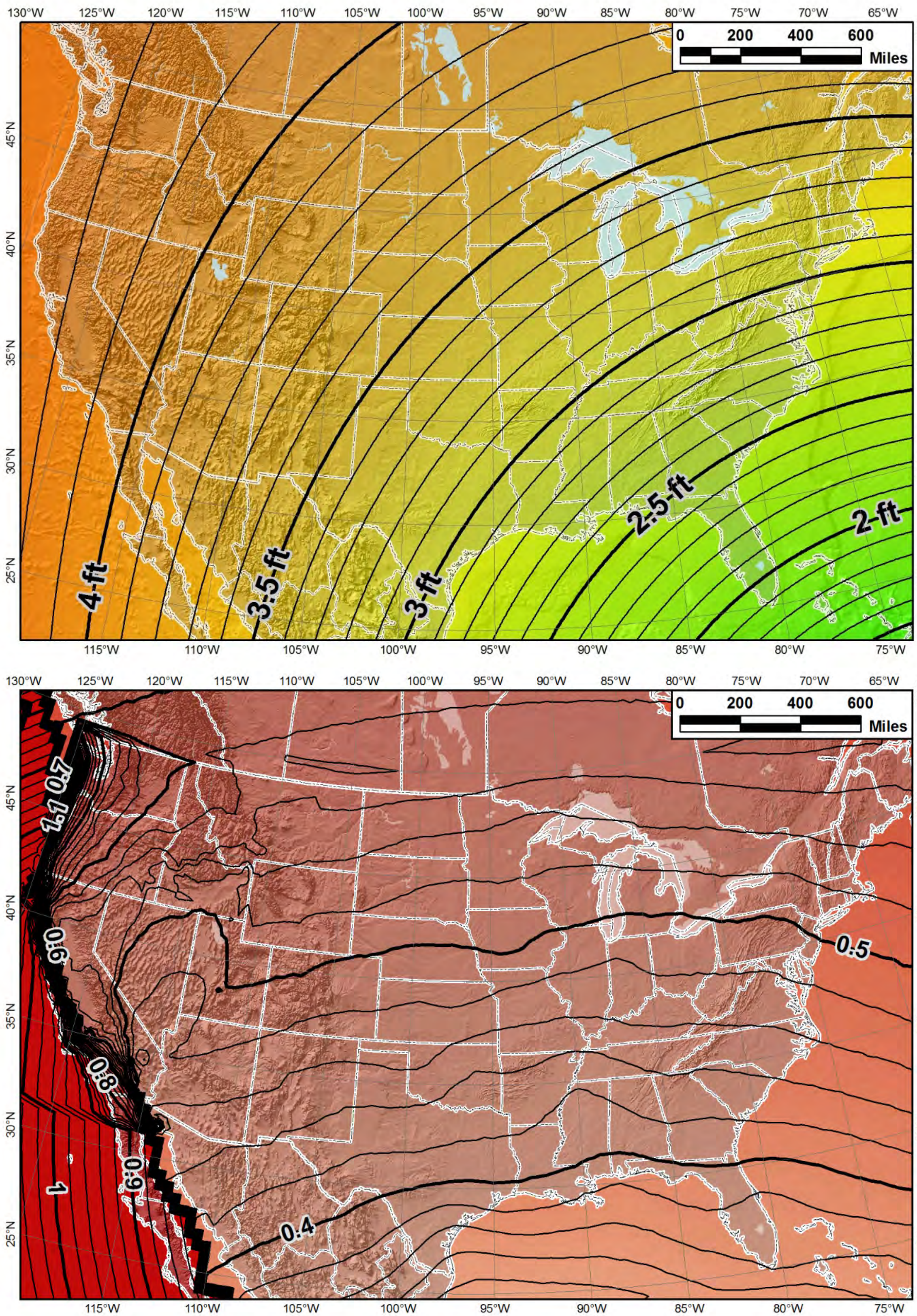
Compare the five cases to the actual published coordinate difference. The ΔX, ΔY, and ΔZ values are converted to ΔN, ΔE, and ΔU values using Equation 2.6 ($\Delta U = \Delta h$ because total distance much less than 100 m).

Source of coordinate deltas IGS08 → NAD 83 (2011)	**Coordinate change (ft)**			**Error with respect to published (ft)**			
	Δ*N*	**Δ*E***	**Δ*h***	**Δ*N* error**	**Δ*E* error**	**Horiz error**	**Δ*h* error**
Published coordinate deltas	**–1.627**	**4.102**	**1.375**				
3-parameter (no rot or scale)	-3.790	6.019	1.414	–2.164	1.918	**2.891**	**0.040**
7-parameter (time = 1997)*	-2.038	3.662	1.456	–0.411	–0.439	**0.602**	**0.081**
14-parameter, in = out = 2010	-1.491	4.315	1.394	0.136	0.214	**0.253**	**0.019**
14-parameter, in = out = 2005	-1.701	4.064	1.418	–0.075	–0.038	**0.083**	**0.043**
HTDP, in = 2005, out = 2010	-1.614	4.104	1.394	0.012	0.003	**0.012**	**0.020**

*Transformation coded into most commercial software for NAD 83 (2011) ←→ IGS08/ITRF08/WGS84.



Horizontal (top) and height (bottom) change, in feet, from IGS08 epoch 2005.00 to NAD 83 (2011) epoch 2010.00. 14-parameter transformation plus crustal deformation model.



Horizontal change (top) and its error (bottom), in feet, from IGS08 to NAD 83 (2011). 7-parameter transformation with no time dependence (nominally 1997.0), as implemented in most software.

Section 3

GRID COORDINATE SYSTEMS AND COMPUTATIONS

How are the data displayed? How are the data used?

Examples of grid coordinate errors for Oregon

Table 3.1 Examples of various positioning error sources and their magnitudes for Oregon due to grid coordinate system and computation problems (abbreviations and technical terms are defined in the Glossary).

Positioning error examples for Oregon	Error magnitudes (horizontal)
Using SPCS 27 projection parameters for SPCS 83 projects	North zone: 1,175 ***miles*** South zone: 553 ***miles***
Determining State Plane coordinates in US survey feet when international feet are required	North zone: Up to 18 feet South zone: Up to 12 feet
Determining Oregon Statewide Lambert coordinates in US survey feet when international feet are required	Up to 5.5 feet
Using linear coordinates from a geographic “projection” to compute distances	Up to ~1600 feet horizontal per mile (at 46° latitude)
Using SPCS grid distances when “ground” distances are required (example here is for South Zone)	~0.8 foot horizontal per mile (in Bend, elev ≈ 3600 ft)
Using Oregon Statewide Lambert grid distances when “ground” distances are required	~2.1 feet horizontal per mile (in Bend, elev ≈ 3600 ft)

Grid coordinate system information in NGS Datasheets and OPUS output

Both NGS Datasheets and OPUS output use the geodetic coordinates of the point to compute grid (map projection) coordinates in the State Plane and Universal Transverse Mercator coordinate systems. They also provide the convergence angle, grid point scale factor, and combined scale factor for both systems.

Portion of NGS Datasheet for station RDM A (AF8301)

```
AF8301  PACS         -  This is a Primary Airport Control Station.
AF8301  DESIGNATION -  RDM A
AF8301  PID          -  AF8301
AF8301  STATE/COUNTY-  OR/DESCHUTES
AF8301  COUNTRY      -  US
AF8301  USGS QUAD    -  REDMOND (1975)
AF8301
AF8301                       *CURRENT SURVEY CONTROL
AF8301  ______________________________________________________________________
AF8301* NAD 83(2011) POSITION- 44 15 07.30805(N) 121 08 57.28517(W)   ADJUSTED
AF8301* NAD 83(2011) ELLIP HT-   915.319 (meters)        (06/27/12)   ADJUSTED
AF8301* NAD 83(2011) EPOCH    -  2010.00
AF8301* NAVD 88 ORTHO HEIGHT -   936.440  (meters)    3072.30  (feet) ADJUSTED
AF8301  ______________________________________________________________________
AF8301  NAD 83(2011) X  - -2,367,351.543 (meters)                     COMP
AF8301  NAD 83(2011) Y  - -3,916,788.058 (meters)                     COMP
AF8301  NAD 83(2011) Z  -  4,428,832.050 (meters)                     COMP
.
.                                                          = k          = γ
AF8301;                    North         East     Units Scale Factor Converg.
AF8301;SPC OR S     -   287,409.917 1,448,144.087   MT  1.00007385  -0 26 39.0
AF8301;SPC OR S     -   942,945.92  4,751,128.89   iFT  1.00007385  -0 26 39.0
AF8301;UTM  10      - 4,901,531.409   647,753.722   MT  0.99986849  +1 17 30.2
AF8301
AF8301!             -  Elev Factor  x  Scale Factor =   Combined Factor
AF8301!SPC OR S     -   0.99985650  x   1.00007385  =   0.99993034
AF8301!UTM  10      -   0.99985650  x   0.99986849  =   0.99972501   = δ + 1
```

Portion of OPUS output for CORS CORV (monument)

```
                             NGS OPUS SOLUTION REPORT
                             ========================

REF FRAME: NAD_83(2011)(EPOCH:2010.0000)              IGS08 (EPOCH:2016.0870)

         X:      -2366948.819(m)   0.005(m)          -2366949.707(m)   0.005(m)
         Y:      -3916334.905(m)   0.003(m)          -3916333.680(m)   0.003(m)
         Z:       4429451.080(m)   0.009(m)           4429451.076(m)   0.009(m)

       LAT:    44 15 35.14668      0.006(m)        44 15 35.15991      0.006(m)
     E LON:   238 51  7.68470      0.005(m)       238 51  7.62188      0.005(m)
     W LON:   121  8 52.31530      0.005(m)       121  8 52.37812      0.005(m)
    EL HGT:             920.348(m)   0.008(m)               919.924(m)   0.008(m)
 ORTHO HGT:             941.471(m)   0.020(m) [NAVD88 (Computed using GEOID12B)]

                        UTM COORDINATES    STATE PLANE COORDINATES
                         UTM (Zone 10)         SPC (3601 OR N)
Northing (Y) [meters]     4902392.828           288268.367
Easting (X)  [meters]      647844.553          1448260.996
Convergence  [degrees]     1.29286129          -0.44323538 = γ
Point Scale                0.99986882           1.00007642   = k
Combined Factor            0.99972455           0.99993212      = δ + 1
```

Map projection types and conformality

When a map projection is associated with a specific geodetic datum (i.e., geometric reference frame or geographic coordinate system), it is called a *projected coordinate systems* (PCS). A PCS must always include a projection type, geodetic datum, and linear unit.

Thousands of map projection types have been developed, and about a hundred are commonly used for a wide range of geospatial applications. Fortunately, the list of projections appropriate for surveying engineering is much shorter, because they should satisfy three criteria:

1. Appropriate for large-scale mapping (i.e., not just for covering large portions of the Earth)
2. Widely available and well-defined in commercial geospatial software packages
3. Conformal

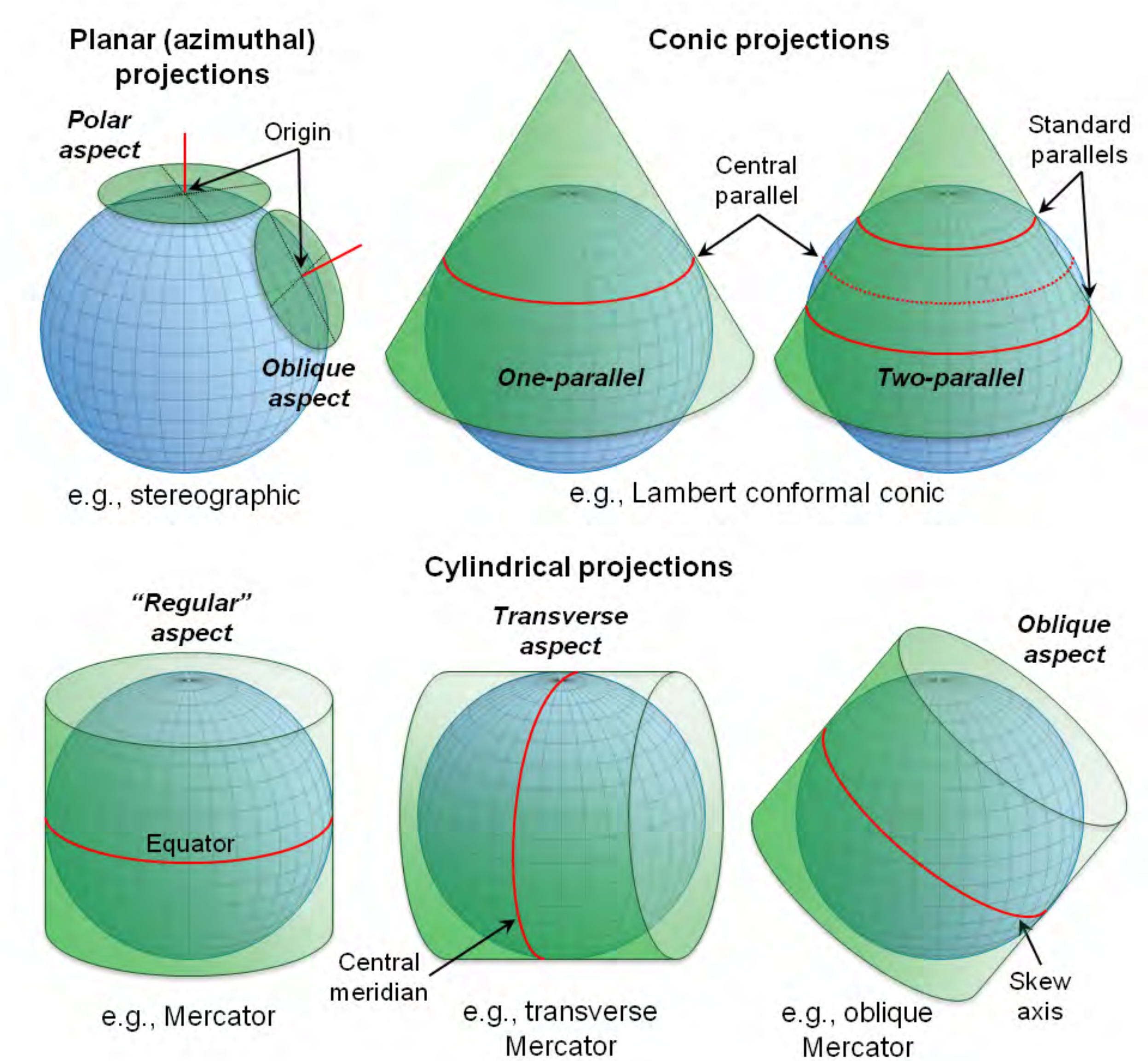
Based on these three criteria, the number of conformal map projections appropriate for survey engineering applications reduces to the four listed in Table 3.2: The transverse Mercator (TM), Lambert conformal conic (LCC), oblique Mercator (OM), and stereographic. Table 3.2 also indicates which of the projections are used in the following well-known PCSs: State Plane Coordinate System (SPCS), Universal Transverse Mercator (UTM), and Universal Polar Stereographic (UPS) systems.

Table 3.2 Conformal projections used for large-scale engineering and surveying applications

Projection	Type	Usage*	Comments
Transverse Mercator (TM)	Cylindrical	SPCS, UTM	Often used for areas elongate in north-south direction. Perhaps the most widely used projection for large-scale mapping. Also called the Gauss-Krüger projection.
Lambert conformal conic (LCC)	Conical	SPCS	Often used for areas elongate in east-west direction. Also widely used for both large- and small-scale mapping. Includes both the one-parallel and two-parallel versions (which are mathematically identical).
Oblique Mercator (OM)	Cylindrical	SPCS	Often used for areas elongate in oblique direction. Not used as often as the TM and LCC projections, but widely available in commercial software. A common implementation is the Hotine OM (also called Rectified Skew Orthomorphic).
Stereographic (oblique and polar aspects)	Planar (azimuthal)	UPS	Suitable for small areas, but for large areas scale error almost always be greater than TM, LCC, or OM because it does not conform to Earth curvature in any direction. Also known as Double Stereographic projection. Polar aspect (origin at Earth's poles) used for polar regions.

*SPCS = State Plane Coordinate System; UTM = Universal Transverse Mercator; UPS = Universal Polar Stereographic

The "flat" surface upon which coordinates are projected is called the *developable surface*. There are three types – plane, cylinder, and cone – as shown below. Each of these is "flat" in the sense that it can be represented as a plane without distortion, because it has an infinite radius of curvature in at least one direction. Conceptually, the cylinder and cone can be "cut" parallel to their central axis (which is the direction of infinite curvature) and laid flat without changing the relationship between the projected coordinates. Another way to think of this is that there is only one developable surface, the cone. A cone of infinite height is a cylinder, and a cone of zero height is a plane.



Map projection developable surfaces and their projection axes.

Each of the projection types listed in Table 3.2 has a specific set of five to seven defining parameters. One is k_0, the projection scale (factor) on the *projection axis*. The projection axis is the line along which projection scale error is minimum and constant with respect to the reference ellipsoid, as shown on previous page. It is the *central meridian* (λ_C) for the TM, the *central parallel* (φ_C) for the LCC, and the *skew axis* for the OM. Actually the scale is not quite constant along the OM skew axis but is minimum at a single point (the local origin) and increases slowly along the axis with distance from the origin. The stereographic projection does not have a projection axis *per se* but rather a single point of minimum scale at its origin.

Map projection distortion

Map projection distortion is an *unavoidable* consequence of attempting to represent a curved surface on a flat surface. It can be thought of as a change in the "true" relationship between points located on the surface of the Earth and the *representation* of their relationship on a plane. Distortion cannot be eliminated — it is a ***F***act of ***L***ife. The best we can do is decrease the effect.

There are two general types of map projection distortion:

1. Linear distortion. Difference in distance between a pair of grid (map) coordinates when compared to the true ("ground") distance, denoted here by $\boldsymbol{\delta}$.
   - Can express as a ratio of distortion length to ground length:
     - E.g., feet of distortion per mile; parts per million (= mm per km)
     - *Note:* 1 foot / mile = 189 ppm = 189 mm / km
   - Linear distortion can be positive or negative:
     - NEGATIVE distortion means the grid (map) length is SHORTER than the "true" horizontal (ground) length.
     - POSITIVE distortion means the grid (map) length is LONGER than the "true" horizontal (ground) length.
2. Angular distortion. For conformal projections (e.g., Transverse Mercator, Lambert Conformal Conic, Stereographic, Oblique Mercator, etc.), it equals the *convergence* (*mapping*) *angle*, $\boldsymbol{\gamma}$. The convergence angle is the difference between grid (map) north and true (geodetic) north.
   - Convergence angle is zero on the projection central meridian, positive east of the central meridian, and negative west of the central meridian
   - Magnitude of the convergence angle increases with distance from the central meridian, and its rate of change increases with increasing latitude:

Latitude	**Convergence angle 1 mile from CM**	**Latitude**	**Convergence angle 1 mile from CM**
0°	0° 00' 00"	50°	±0° 01' 02"
10°	±0° 00' 09"	60°	±0° 01' 30"
20°	±0° 00' 19"	70°	±0° 02' 23"
30°	±0° 00' 30"	80°	±0° 04' 54"
40°	±0° 00' 44"	89°	±0° 49' 32"

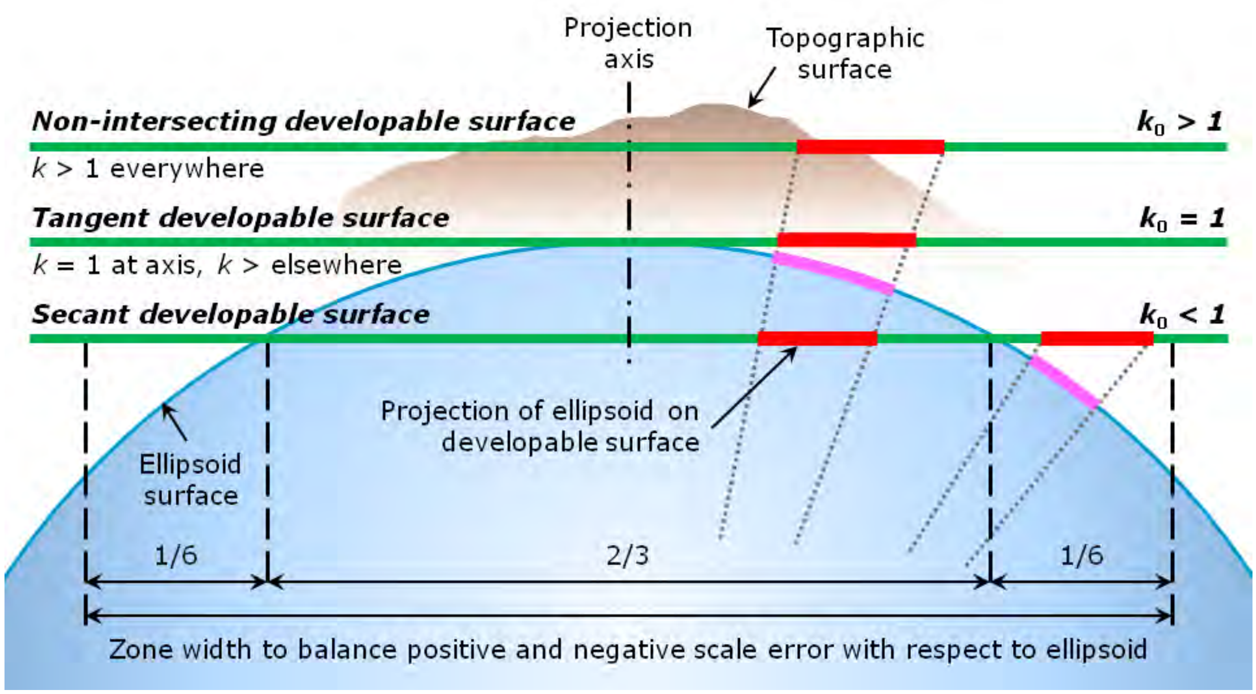
- Usually convergence is not as much of a concern as linear distortion, and it can only be minimized by staying close to the projection central meridian (or the Equator).

Total linear distortion of grid (map) coordinates is a combination of distortion due to Earth curvature and distortion due to ground height above the ellipsoid. In many areas, distortion due to variation in ground height is greater than that due to curvature. This is illustrated in the diagrams and tables on the following pages.

Table 3.3 Horizontal distortion of grid coordinates due to Earth curvature

Maximum zone width for secant projections	Maximum linear horizontal distortion, δ		
	Parts per million	Feet per mile	Ratio (absolute value)
16 miles (26 km)	±1 ppm	±0.005 ft/mile	1 : 1,000,000
35 miles (57 km)	±5 ppm	±0.026 ft/mile	1 : 200,000
50 miles (81 km)	±10 ppm	±0.053 ft/mile	1 : 100,000
71 miles (114 km)	**±20 ppm**	**±0.11 ft/mile**	**1 : 50,000**
112 miles (180 km)	±50 ppm	±0.26 ft/mile	1 : 20,000
158 miles (255 km) e.g., SPCS)*	±100 ppm	±0.53 ft/mile	1 : 10,000
317 miles (510 km) e.g., UTM)†	±400 ppm	±2.11 ft/mile	1 : 2500

*State Plane Coordinate System; zone width shown is valid between ~0° and 45° latitude
†Universal Transverse Mercator; zone width shown is valid between ~30° and 60° latitude



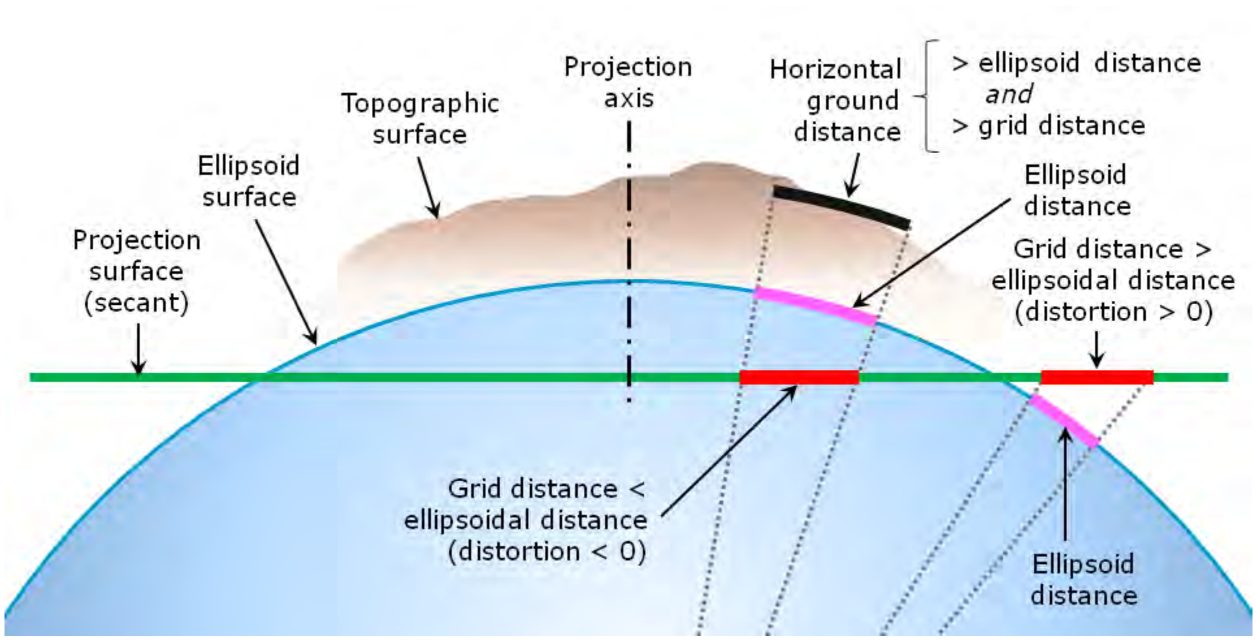
Secant, tangent, and non-intersecting projection developable surfaces

Table 3.4 Horizontal distortion of grid coordinates due to ground height above the ellipsoid

Height below (–) and above (+) projection surface	Maximum linear horizontal distortion, δ		
	Parts per million (mm/km)	Feet per mile	Ratio (absolute value)
±100 feet (30 m)	±4.8 ppm	±0.03 ft/mile	~1 : 209,000
±400 feet (120 m)	**±19 ppm**	**±0.1 ft/mile**	**~1 : 52,000**
±1000 feet (300 m)	±48 ppm	±0.3 ft/mile	~1 : 21,000
+2000 feet (610 m)	–96 ppm	–0.5 ft/mile	~1 : 10,500
+3500 feet* (1100 m)	–167 ppm	–0.9 ft/mile	~1 : 6000
+11,200 feet† (3400 m)	–535 ppm	–2.8 ft/mile	~1 : 1900

*Approximate mean topographic height in Oregon (and the city of Bend)
† Approximate maximum topographic height in Oregon

Rule of Thumb:
A 100-ft change in height causes a 4.8 ppm change in distortion



Linear distortion of secant map projection with respect to ellipsoid and topography

Distortion computations

Linear distortion is the ratio of grid distance to horizontal ground distance. One way to estimate distortion is to compute the distance between a pair of points based on the grid coordinates determined by the GPS software. This grid distance can then be divided by the true ground distance between these points measured using a (properly calibrated) tape or EDM.

Equation 3.1 Approximating distortion at a point using measured grid and ground distances

$$\boxed{\delta \approx \left(\frac{\sqrt{\Delta N^2 + \Delta E^2}}{\text{true horizontal ground distance}} \right) - 1}$$

Distortion can be computed more accurately (and conveniently) at a single point using the familiar "combined scale factor" approach:

Equation 3.2 Computing distortion at a point using Earth radius

$$\boxed{\delta = k\left(\frac{R_G}{R_G + h} \right) - 1}$$

Example computation

Given: Points RDM A and REDM CORS ARP from the previous examples. The ellipsoid heights (h) of these points are listed below, along with the grid coordinates and grid point scale factors (k) derived from the adjusted geodetic coordinates (given in Exercise 2.2). The true horizontal ground distance between these points is 2842.629 ift.

RDM A: NAD 83(2011) epoch 2010.00 ellipsoid height, h = 3003.015 ift

Coordinate system	Northing, *N* (ift)	Easting, *E* (ift)	Grid scale factor, *k*
Oregon Statewide Lambert	912,676.730	1,142,257.986	0.999 762 836
SPCS 83, Oregon South (3602)	942,945.923	4,751,128.893	1.000 073 846
OCRS (Bend-Redmond-Prineville)	275,341.592	288,871.362	1.000 146 037

REDM CORS ARP: NAD 83(2011) epoch 2010.00 ellipsoid height, h = 3019.505 ift

Coordinate system	Northing, *N* (ift)	Easting, *E* (ift)	Grid scale factor, *k*
Oregon Statewide Lambert	915,492.225	1,142,641.877	0.999 762 839
SPCS 83, Oregon South (3602)	945,762.356	4,751,512.467	1.000 076 423
OCRS (Bend-Redmond-Prineville)	278,161.558	289,229.629	1.000 145 076

Find: The linear distortion (in parts per million) at the midpoint between points RDM A and REDM CORS ARP in OR Lambert, UTM, and OCRS coordinates using both Equations 3.1 and 3.2 (geometric mean radius R_G = 20,923,703 ift was determined at the midpoint previously).

Computations: For midpoint, use the mean grid scale factor and mean ellipsoid height = 3017 ft.

Oregon Statewide Lambert

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(915{,}492.225-912{,}676.730)^2+(1{,}142{,}641.877-1{,}142{,}257.986)^2}}{2842.629}\right)-1=\left(\frac{2{,}841.546}{2842.629}\right)-1$$

= 0.9996190 – 1 ➔ *in parts per million* ➔ – 0.0003810 × 1,000,000 = **–381.0 ppm**

Using Equation 3.2:

$$\delta = \frac{0.999762836+0.999762839}{2}\left(\frac{20{,}923{,}703}{20{,}923{,}703+3017}\right)-1 = 0.9996190 - 1 \rightarrow \mathbf{-381.0\ ppm}$$

(= –2.01 ft/mile)

SPCS 83 Oregon South Zone

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(945{,}762.356-942{,}945.923)^2+(4{,}751{,}512.467-4{,}751{,}128.893)^2}}{2842.629}\right)-1=\left(\frac{2{,}842.433}{2842.629}\right)-1$$

= 0.9999310 – 1 ➔ *in parts per million* ➔ – 0.0000690 × 1,000,000 = **–69.0 ppm**

Using Equation 3.2:

$$\delta = \frac{1.000073846+1.000076423}{2}\left(\frac{20{,}923{,}703}{20{,}923{,}703+3017}\right)-1 = 0.9999300 - 1 \rightarrow \mathbf{-70.0\ ppm}$$

(= –0.37 ft/mile)

OCRS (Bend-Redmond-Prineville Zone)

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(278{,}161.558-275{,}341.592)^2+(289{,}229.629-288{,}871.362)^2}}{2842.629}\right)-1=\left(\frac{2{,}842.633}{2842.629}\right)-1$$

= 1.0000014 – 1 ➔ *in parts per million* ➔ 0.0000014 × 1,000,000 = **+1.4 ppm**

Using Equation 3.2:

$$\delta = \frac{1.000146037+1.000145076}{2}\left(\frac{20{,}923{,}703}{20{,}923{,}703+3017}\right)-1 = 1.0000021 - 1 \rightarrow \mathbf{+2.1\ ppm}$$

(= +0.01 ft/mile)

Methods for creating low-distortion grid coordinate systems

1. Design a Low Distortion Projection (LDP) for a specific project geographic area.

Use a conformal projection referenced to the existing geodetic datum.

Described in detail in next section.

2. Scale the reference ellipsoid "to ground".

A map projection referenced to this new "datum" is then designed for the project area.

<u>Problems</u>:

- Requires a new ellipsoid (datum) for every coordinate system, which makes it more difficult to implement than an LDP.
- New datum makes it more complex than an LDP, yet it does not perform any better.
- *Generates new set of latitudes that can be substantially different from original latitudes.*
    - Change in latitude can exceed 3 feet per 1000 ft of topographic height, depending on method used for scaling the ellipsoid (this case is for scaling with constant flattening).
    - Can lead to confusion over which latitude values are correct; requires use of datum transformation for correct implementation.

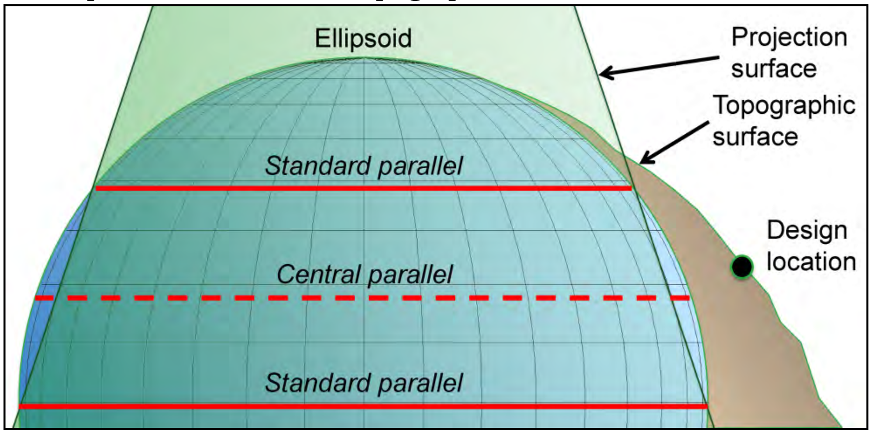
3. Scale an existing published map projection "to ground".

Referred to as "modified" State Plane when an existing SPCS projection definition is used.

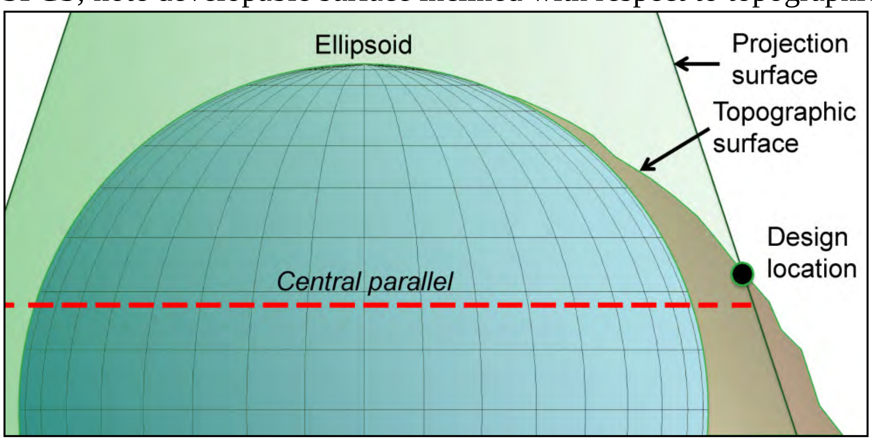
<u>Problems</u>:

- Generates coordinates with values similar to "true" State Plane (can cause confusion).
    - Can eliminate this problem by translating grid coordinates to get smaller values.
- Often yields "messy" parameters when a projection definition is back-calculated from the scaled coordinates (in order to import the data into a GIS).
    - More difficult to implement in a GIS, and may cause problems due to rounding or truncating of "messy" projection parameters (especially for large coordinate values).
    - Can reduce this problem through judicious selection of "scaling" parameters.
- Does ***not*** reduce the convergence angle (it is same as that of original SPCS definition).
    - In addition *arc-to-chord correction* may be significant if projection axis distant (used along with convergence angle for converting grid azimuths to geodetic azimuths).
- ***MOST IMPORTANT:*** **Usually does not minimize distortion over as large an area as the other two methods.**
    - Extent of low-distortion coverage generally *decreases* as distance *increases* from projection axis (i.e., central meridian for TM and central parallel for LCC projection).
    - State Plane axis usually does NOT pass through the project area and may be oriented perpendicular to the long dimension of a project, decreasing area of coverage.
    - *Sketches illustrating this problem with "modified" SPCS are shown on the next page.*

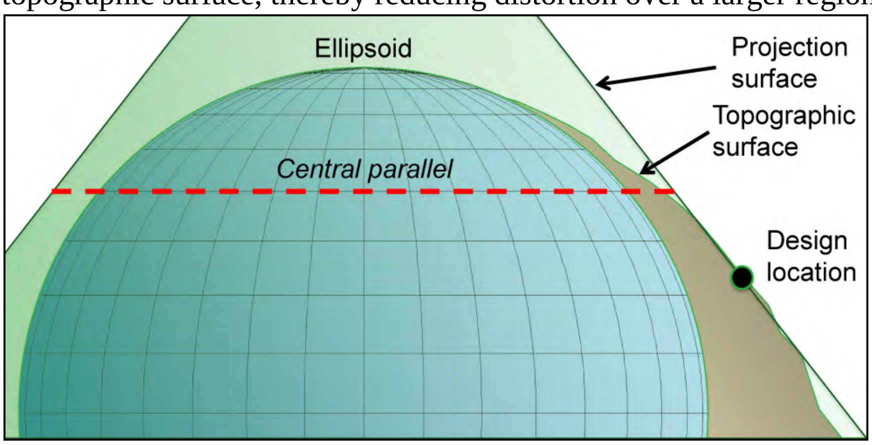
(a) Typical SPCS situation (for LCC projection). Projection is secant to ellipsoid, with developable surface below topographic surface.



(b) SPCS scaled "to ground" at design location. Central parallel in same location as original SPCS; note developable surface inclined with respect to topographic surface.



(c) LDP design. Note central parallel moved north to align developable surface with topographic surface, thereby reducing distortion over a larger region.

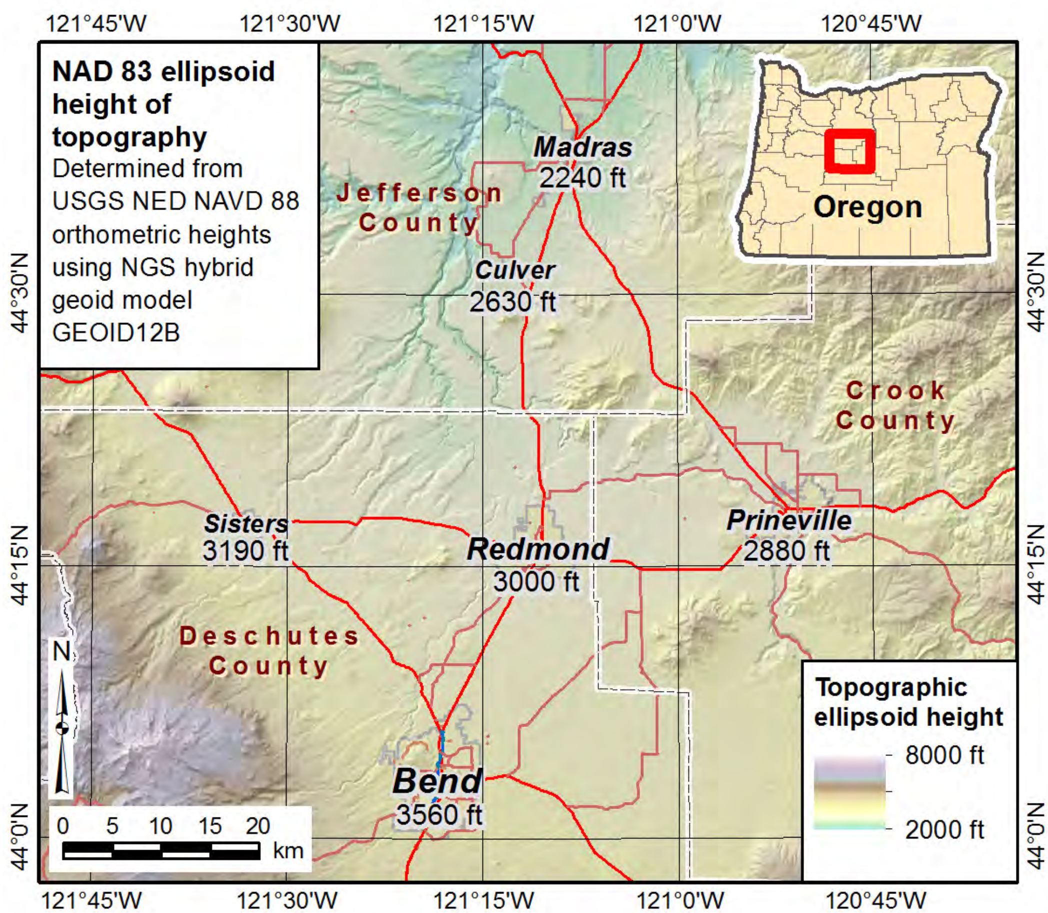


Comparison of (a) SPCS, (b) "modified" SPCS, and (c) LDP.

Six steps for designing a low-distortion projection coordinate system

LDP design example is the southern Deschutes River valley of central Oregon (shown in map below). This example follows the design of the Bend-Redmond-Prineville zone in the Oregon Coordinate Reference System (OCRS). The design process is illustrated in the six steps below.

- First three steps are mainly to initiate the design; step 4 is where the design is optimized to minimize distortion over the largest area possible.
- Overall design objective is ±20 ppm for the region and ±10 ppm within the three largest towns (Bend, Redmond, and Prineville).
- Towns of Sisters, Culver, and Madras are also used for evaluation.



LDP design area, showing topographic ellipsoid heights of towns.

Step 1. Determine *representative* ellipsoid height, h_0 (*not* elevation).

- Start design process using ellipsoid heights at arbitrary locations in the six towns
  - NAD 83 ellipsoid heights from USGS DEM with GEOID12B hybrid geoid
  - Mean ellipsoid height taken as "representative" for initial design, $\boldsymbol{h_0}$ **= 2860 ft**
- Design area is about 45 miles north-south and about 35 miles east-west
  - Distortion can be limited to ±10 ppm for a zone width of 50 miles, so it appears distortion criterion can be achieved (with respect to Earth curvature)

- Height range is 1380 ft, i.e., ±690 ft
  - Range corresponds to ±33 ppm based on the ±4.8 ppm per ±100 ft, possibly problematic since the design objective is ±20 ppm overall

Step 2. Choose projection type and place projection axis near center of area.

- Design area somewhat longer north-south than east-west → use a TM projection?
  - But topographic height overall decreases from south to north, and such slope tends to favor the LCC projection as illustrated schematically.
  - In actual design for this OCRS zone, both projection types were evaluated
  - LCC gives low distortion over larger area, so only the LCC evaluated here
- *Initial design:* LCC central parallel placed near center of area, $\boldsymbol{\varphi_0}$ **= 44°20'00"N**

Step 3. Scale projection axis to representative ground height, $\boldsymbol{h_o}$.

- Bring projection developable surface to the topographic surface
  - Compute initial projection axis scale as $k_0 = 1 + h_0 / R_G$
  - Use initial central parallel of φ_0 = 44°20'N gives R_G = 20,923,900 ft
- Projection axis scale factor is $\boldsymbol{k_0}$ **= 1.00013** (rounded to five decimal places)

Step 4. Compute distortion throughout project area and refine design.

- Equation 3.2 used to compute total linear distortion at specific points for a given LDP (distortion values for initial LDP design in the left column of table below).
  - Distortion for Bend (the largest town in the region) = –28.5 ppm.
  - Distortion exceeds ±10 ppm criterion for Bend and overall target of ±20 ppm.
  - Could fix in Bend by increasing projection scale by 30 ppm, to k_0 = 1.00016, which would change its distortion to +1.5 ppm.
  - But this would increase distortion at all other points by 30 ppm, yielding max in Madras of +69.9 ppm, much greater than target max of 20 ppm.

Distortion performance for six different LCC projection alternatives.

LCC axis scale	*Initial* 1.00013	1.00013	1.00012	***Final*** **1.00012**	1.00011	1.00010
Axis latitude	44°20' N	44°30' N	44°35' N	**44°40' N**	44°45' N	44°50' N
Location	**Linear distortion (parts per million)**					
Bend	-28.5	-10.4	-8.2	**6.1**	12.4	20.9
Redmond	-9.5	-2.2	-5.4	**3.5**	4.5	7.6
Prineville	-4.2	1.7	-2.2	**6.0**	6.3	8.7
Sisters	-18.7	-12.3	-16.0	**-7.6**	-7.0	-4.4
Culver	13.2	7.6	-2.0	**0.6**	-4.8	-8.1
Madras	39.9	28.9	16.6	**16.4**	8.3	2.3
Mean	**-1.3**	**2.2**	**-2.9**	**4.2**	**3.3**	**4.5**
Range	**68.4**	**41.2**	**32.5**	**23.9**	**19.5**	**29.0**
Std deviation	**±24.6**	**±15.0**	**±10.8**	**±7.8**	**±7.6**	**±10.4**

- Changing projection scale has essentially no effect on distortion variability
  - Range and standard deviation will be about the same regardless of the scale

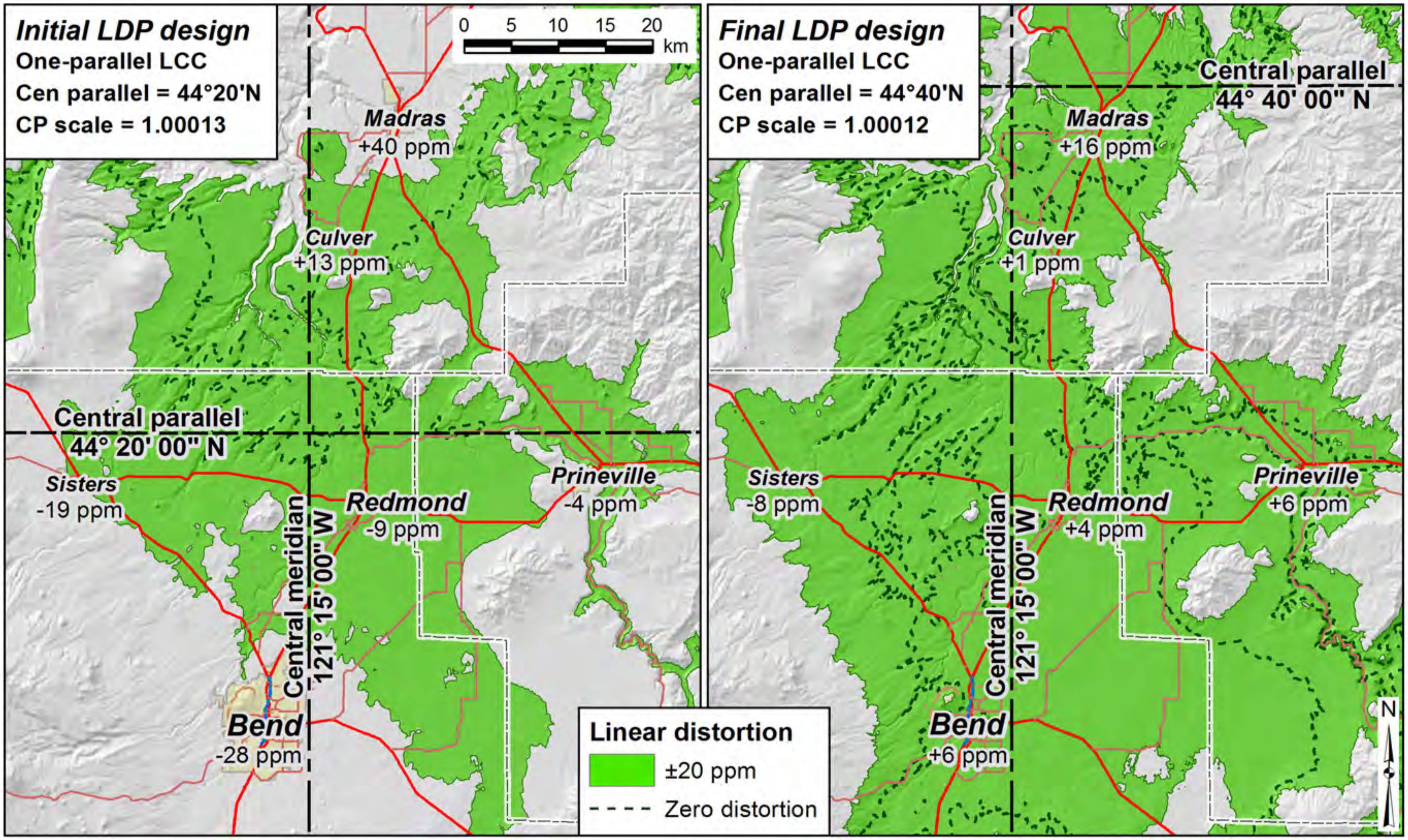
- For a given projection, variability can only be changed by changing the location of the projection axis
  - Result of doing that is shown in table above (the axis scale was also changed so that mean distortion was within ±10 ppm for all cases)
  - Range and standard deviation decrease from 68.4 and ±24.6 ppm, respectively, to minimums of 19.5 and ±7.6 ppm for φ_0 = 44°45'N
  - Design with φ_0 = 44°40'N has distortion less than 10 ppm in Bend, Redmond, and Prineville, and variability is also less for these towns.
- Evaluating distortion at discrete points typically not sufficient to optimize design
  - More comprehensive evaluation done by computing distortion on regular grid
  - Distortion can be visualized and analyzed everywhere, as shown in maps on the next two pages for both the initial and final LDP designs.
    - All areas within ±20 ppm shaded green, and zero distortion contour shown
    - Substantial improvement in performance achieved by moving central parallel north by 20 arc-minutes.
    - Final design: $\boldsymbol{\varphi_0}$ **= 44° 40' N** and $\boldsymbol{k_0}$ **= 1.00012** (highlighted in table on previous page)

Step 5. Keep the definition simple and clean.

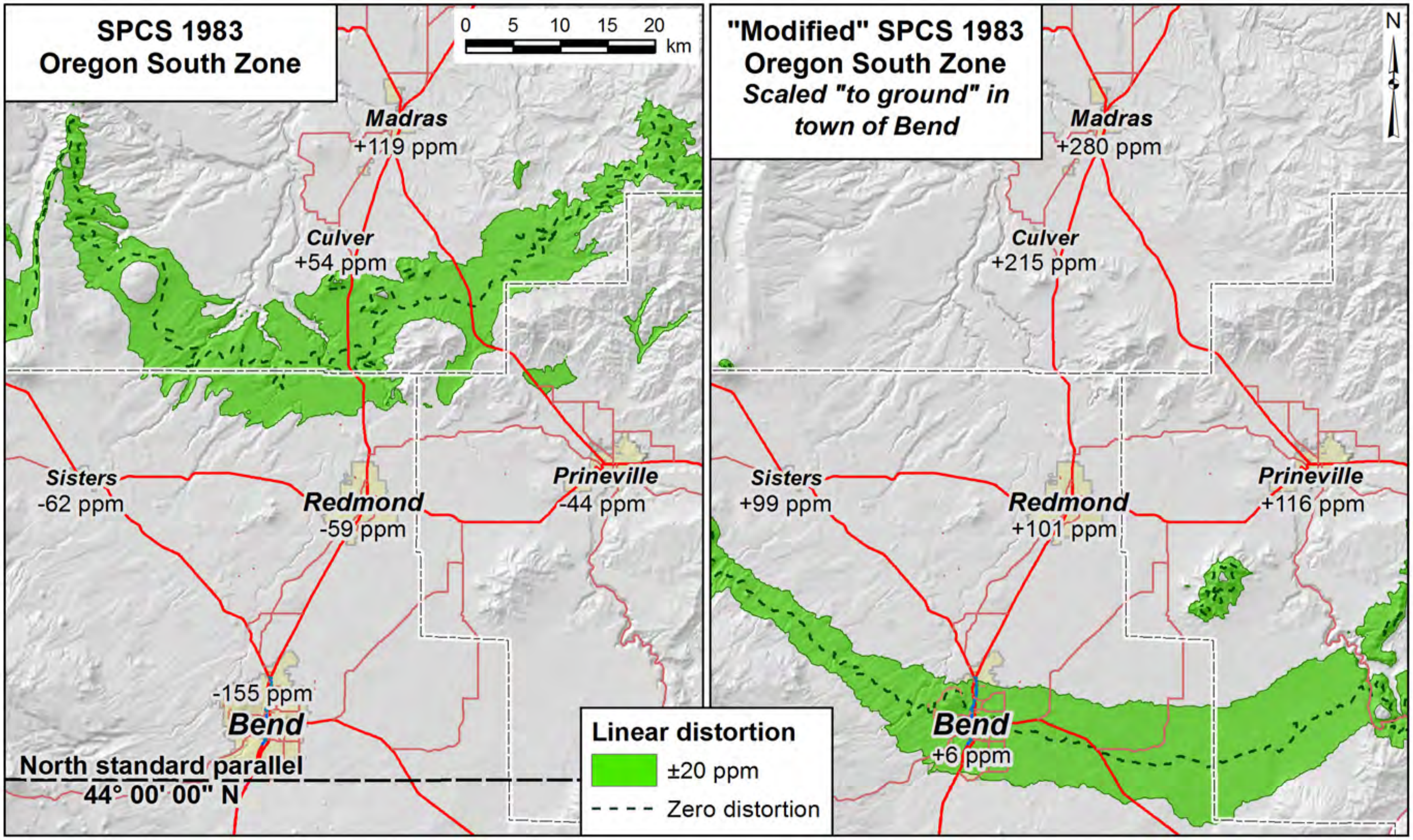
- Good practice to use simple and "clean" values for the defining parameters (consistent with how SPCS and UTM were defined)
- Only values that affect distortion are the projection axis scale and location (and axis orientation for the OM projection)
- Other parameters for geodetic origin and false northing and easting have no effect on distortion, they still must be specified; below are recommendations:
  - Define the projection axis scale using no more than six decimal places (five decimal places were used in this example).
  - Define the geodetic origin (e.g., central parallel, central meridian) to nearest whole (or nearest five) arc-minutes. Values with non-repeating decimal equivalents are also recommended, if it does not compromise performance.
  - Use whole numbers for the grid origin (false northing and easting) in the defining linear unit such that projected coordinates are distinct from other systems in the design area (such as SPCS and UTM). Many other options for the grid origin can be used, based on preference and convenience.

Step 6. Explicitly define linear unit and geometric reference system.

- Specify linear unit; if feet are used identify type (international or US survey)
- An LDP is not a coordinate system unless it is associated with a geometric reference system (i.e., geodetic datum, geographic coordinate system)
  - For OCRS, datum specified as North American Datum of 1983 (NAD 83)
  - Specific NAD 83 realization is *not* be specified (e.g., 2011, 2007, HARN, etc.)
    - Realization affects only coordinates, not the coordinate system definition
    - Ellipsoid parameters are the same for all realizations of NAD 83
    - SPCS 83 follows convention of *not* specifying the realization; all are simply generic "NAD 83"



Areas with ±20 ppm linear distortion in example for initial and final LDP designs.



Areas with ±20 ppm linear distortion in design example for original and "modified" SPCS 83 OR South Zone.

Comparison to State Plane and “modified” State Plane

As mentioned previously, despite the popularity of “modified” SPCS, the performance is almost always inferior to a carefully designed LDP. This is illustrated in the maps on the previous page for the SPCS 83 Oregon South Zone, both original and “modified” by scaling “to ground” such that it gives the same distortion in Bend as the final LDP design (+6 ppm). The difference in performance with LDPs is striking, even though all are based on the LCC projection. Even the initial LDP design covers a far larger area with low distortion than does SPCS scaled to ground.

For both original and scaled SPCS, low distortion (±20 ppm) is only achieved in a narrow band more-or-less parallel to the projection axis (located 60 miles south of Bend). Scaling SPCS has essentially no effect on the width of the band; it merely shifts it so that it is centered on Bend. This is a vivid example of how changing the projection scale has virtually no impact on variability. Indeed, the range and standard deviation for both original and scaled SPCS are 274 and ±97 ppm, respectively (versus range of 24 ppm and standard deviation of ±8 ppm for the final LDP design).

Design optimization and the importance of cooperation

The design objective is usually to minimize linear distortion over the largest area possible. These goals are at odds with one another, so LDP design is an ***optimization problem***. The six steps given previously are intended to address commonly encountered situations. However, often the most important part is not technical – getting concurrence among the many stakeholders impacted by the design, such as surveyors, engineers, GIS professionals, and both public and private organizations that make use of geospatial data in the design area.

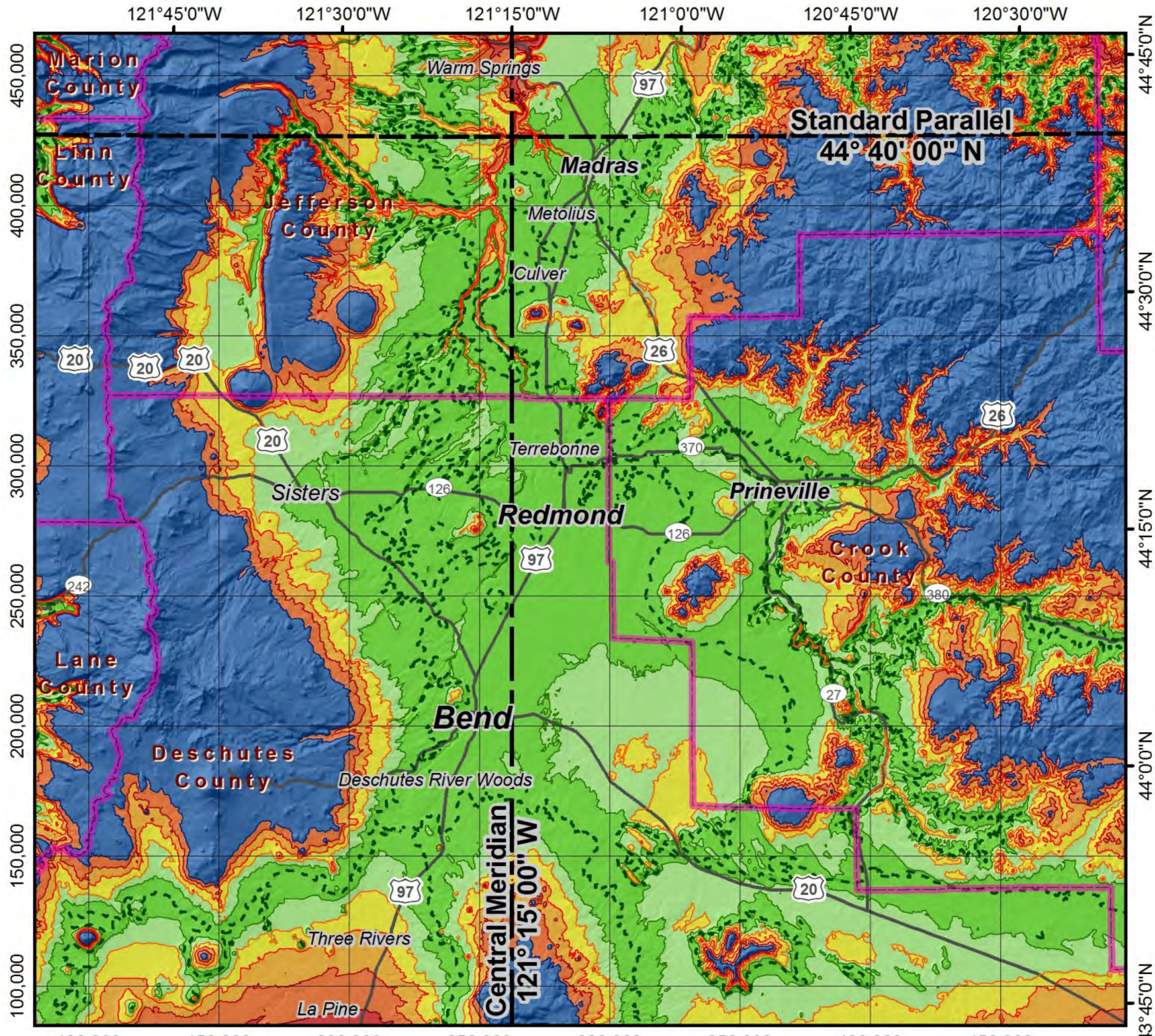
Compatibility of design with multiple software platforms

The projection parameters, linear unit, and geodetic datum can be used directly to create a coordinate system definition that is compatible with most surveying, engineering, GIS, and other geospatial software. For example, this can be done for Esri software by creating a projection file (*.prj), or for Trimble software by using Coordinate System Manager to augment the coordinate system database file (*.csd).

The final design projection parameters are shown on the next page, which are the values adopted for this as the Bend-Redmond-Prineville Zone of the Oregon Coordinate Reference System (OCRS). See www.oregon.gov/ODOT/HWY/GEOMETRONICS/Pages/ocrs.aspx for more information.

Note on computation of grid point scale factor

Not all geospatial software computes the grid point scale factor, k, which is essential for computing total distortion. So equations to compute k for the TM and LCC projections are given on the following pages (equations 3.3 through 3.6).

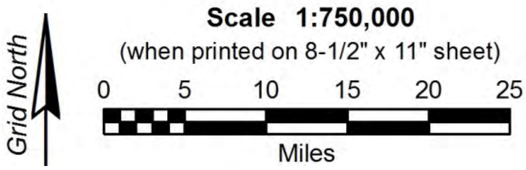


Oregon Coordinate Reference System
Bend-Redmond-Prineville Zone

Lambert Conformal Conic projection (single parallel)
North American Datum of 1983

Stnd parallel & grid origin:	44° 40' 00" N
Central meridian:	121° 15' 00" W
False northing:	130 000.000 m
False easting:	80 000.000 m
Standard parallel scale:	1.000 120 (exact)

Scale 1:750,000
(when printed on 8-1/2" x 11" sheet)



Projected map grid is shown in units of international feet

Linear distortion
- - - Zero distortion
- < -50 ppm (< -0.25 ft/mi)
- ±10 ppm = ±0.05 ft/mi
- ±(10-20) ppm = ±(0.05-0.1) ft/mi
- ±(20-30) ppm = ±(0.1-0.15) ft/mi
- ±(30-40) ppm = ±(0.15-0.2) ft/mi
- ±(40-50) ppm) = ±(0.2-0.25) ft/mi
- > +50 ppm (> +0.25 ft/mi)

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Projection grid scale factor and convergence angle computation

For the ***Transverse Mercator*** projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 32-35):

Equation 3.3 Transverse Mercator projection grid scale factor formula

$$k = k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1 - e^2} \right) \left[1 + \frac{(\Delta\lambda \cos\varphi)^2}{12} \left(5 - 4\tan^2\varphi + \frac{e^2 \cos^2\varphi}{1 - e^2} \left(9 - 24\tan^2\varphi \right) \right) \right] \right\}$$

where $\Delta\lambda = \lambda_0 - \lambda$ (in radians, for negative west longitude)

λ = geodetic longitude of point

λ_0 = central meridian longitude

and all other variables are as defined previously.

The following shorter equation can be used to approximate k for the Transverse Mercator projection. It is accurate to better than 0.02 part per million (at least 7 decimal places) if the computation point is within about ±1° of the central meridian (about 50 to 60 miles between latitudes of 30° and 45°):

Equation 3.4 Approximate Transverse Mercator projection grid scale factor formula

$$k \approx k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1 - e^2} \right) \right\}$$

Note that this equation may not be sufficiently accurate for computing k throughout a UTM system zone (at the zone width of ±3° from the central meridian the error can exceed 1 ppm).

An even simpler equation can be used to approximate the grid scale factor, which utilizes the grid coordinate easting value and is about twice as accurate as the previous equation (i.e., better than 0.01 part per million if the computation point is within about ±1° of the central meridian):

Equation 3.5 Another approximate Transverse Mercator projection grid scale factor formula

$$k \approx k_0 + \frac{(E_0 - E)^2}{2(k_0 R_G)^2}$$

where E = Easting of the point where k is computed (in same units as R_G)

E_0 = False easting (on central meridian) of projection definition (in same units as R_G)

R_G = Earth geometric mean radius of curvature (can use 20,920,000 feet for Oregon)

For the ***Lambert Conformal Conic*** projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 26-29):

Equation 3.6 Lambert Conformal Conic projection grid scale factor formula

$$\boxed{k = k_0 \frac{\cos\varphi_C}{\cos\varphi}\sqrt{\frac{1-e^2\sin^2\varphi}{1-e^2\sin^2\varphi_C}}\exp\left\{\frac{\sin\varphi_C}{2}\left[\ln\frac{1+\sin\varphi_C}{1-\sin\varphi_C}-\ln\frac{1+\sin\varphi}{1-\sin\varphi}+e\left(\ln\frac{1+e\sin\varphi}{1-e\sin\varphi}-\ln\frac{1+e\sin\varphi_C}{1-e\sin\varphi_C}\right)\right]\right\}}$$

Where k_0 = projection grid scale factor applied to central parallel (tangent to ellipsoid if $k_0 = 1$)

φ_C = geodetic latitude of central parallel = standard parallel for one-parallel LCC

$e = \sqrt{e^2} = \sqrt{2f - f^2}$ = first eccentricity of the reference ellipsoid

and all other variables are as defined previously. In order to use this equation for a two-parallel LCC, the two-parallel LCC must first be converted to an equivalent one-parallel LCC by computing φ_C and k_0. The equations to do this are long, but are provided here for the sake of completeness. For a two-parallel LCC, the central parallel is

$$\varphi_C = \sin^{-1}\left[\frac{2\ln\left(\dfrac{\cos\varphi_S}{\cos\varphi_N}\sqrt{\dfrac{1-e^2\sin^2\varphi_N}{1-e^2\sin^2\varphi_S}}\right)}{\ln\left(\dfrac{1+\sin\varphi_N}{1-\sin\varphi_N}\right)-\ln\left(\dfrac{1+\sin\varphi_S}{1-\sin\varphi_S}\right)+e\left[\ln\left(\dfrac{1+e\sin\varphi_S}{1-e\sin\varphi_S}\right)-\ln\left(\dfrac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right)\right]}\right],$$

and the central parallel scale factor is

$$k_0 = \frac{\cos\varphi_N}{\cos\varphi_C}\sqrt{\frac{1-e^2\sin^2\varphi_0}{1-e^2\sin^2\varphi_N}}$$

$$\times\ \exp\left\{\frac{\sin\varphi_C}{2}\left[\ln\left(\frac{1+\sin\varphi_N}{1-\sin\varphi_N}\right)-\ln\left(\frac{1+\sin\varphi_C}{1-\sin\varphi_C}\right)+e\left(\ln\left[\frac{1+e\sin\varphi_C}{1-e\sin\varphi_C}\right]-\ln\left[\frac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right]\right)\right]\right\},$$

where φ_N and φ_S = geodetic latitude of northern and southern standard parallels, respectively, and all other variables are as defined previously.

Convergence angles. For the TM, the convergence angle can be approximated as $\gamma = -\Delta\lambda \sin\varphi$ (where all variables are as defined previously; the units of γ are the same as the units of $\Delta\lambda$). This equation is accurate to better than ±00.2" if the computation point is within ~1° of the central meridian. For any LCC, the convergence angle is exactly $\gamma = -\Delta\lambda \sin\varphi_C$.

Two methods for computing horizontal "ground" distance

This exercise gives two simple methods for computing horizontal "ground" distances using geodetic information. The first method is done by scaling the ellipsoid distance (geodesic) using the average of the ellipsoid heights at the endpoints, as follows:

Equation 3.7 Approximate geodetic "ground" distance based on ellipsoid distance (geodesic)

$$\boxed{D_{grnd} = s\left(1 + \frac{\bar{h}}{\bar{R}_G}\right)}$$

where s is the ellipsoid distance (geodesic)

$\bar{h}$ is the average ellipsoid height of the two points

$\bar{R}_G$ is the geometric mean radius of curvature at the midpoint latitude of the two points

The second method for computing a horizontal ground distance can be done by using a GPS (GNSS) vector directly. Neglecting Earth curvature, this distance can be computed as:

Equation 3.8 Approximate "ground" distance based on GPS (GNSS) vector components

$$\boxed{D_{grnd} = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 - \Delta h^2}}$$

where ΔX, ΔY, ΔZ are the GPS vector components (as ECEF Cartesian coordinate deltas)

Δh = change in ellipsoid height between vector end points

Note that this method can also be used with end point coordinates (rather than a GPS vector), by converting the latitude, longitude, and ellipsoid heights to X, Y, Z ECEF coordinates using Equation 2.1, and then using the difference in ECEF coordinates in Equation 3.8.

Accounting for curvature increases this horizontal ground distance, but for distances of less than 20 miles (about 30 km), the increase is less than 1 part per million (ppm), i.e., less than 0.1 ft (3 cm). The horizontal distance can be multiplied by the following curvature correction factor to get the approximate curved horizontal ground distance (error is less than about 0.01 ft for distances under 50 miles):

Equation 3.9 Correction factor applied to horizontal distance to account for curvature

$$\boxed{C_C = \frac{2\bar{R}_G \sin^{-1}\left(\dfrac{D_{grnd}}{2\bar{R}_G}\right)}{D_{grnd}}}$$

where all variables are as defined previously. An Earth radius of 20,920,000 ft is sufficiently accurate in Oregon for distances of less than about 100 miles (causes less than 0.01 ft error).

Example computation

Given: Points RDM A and REDM CORS from the previous exercises, and a GPS vector from RDM A to REDM with components ΔX = 1321.384 ift , ΔY = 1486.881 ift, ΔZ = 2030.747 ift.

Find: The horizontal "ground" distance between these points using the two methods in this exercise.

Computations:

Method 1. From Exercise 2.4, ellipsoid distance (geodesic) is s = 2842.220 ift

From Exercises 1.3 and 3.1, R_G = 20,923,703 ift at midpoint between RDM A and REDM CORS ARP (which is the same as the average R_G for the two points)

From the ellipsoid heights in Exercise 3.1, the average ellipsoid height is

$$\bar{h} = (3003.015 + 3019.245) / 2 = 3011.130 \text{ ift}$$

So ground distance is

$$D_{grnd} = 2842.220 \times \left(1 + \frac{3011.130}{20{,}923{,}703}\right) = 2842.220 \times 1.000\ 143\ 910 = \underline{\mathbf{2842.629\ ift}}$$

Method 2. Using the given GPS vector components and Δh = 3019.245 – 3003.015 = 16.230 ft gives a horizontal ground distance of

$$D_{grnd} = \sqrt{(1321.384)^2 + (1486.881)^2 + (2030.747)^2 - (16.230)^2} = \underline{\mathbf{2842.629\ ift}}$$

which is the same as that computed for Method 1. For such a short distance, curvature is completely negligible. This can be verified using Equation 3.6, which gives a curvature correction factor of C_C = 1.000 000 0008, or 0.0008 ppm. As stated previously, the curvature correction factor is less than 1 ppm for distances of less than about 20 miles.

Grid versus geodetic bearings

Illustrates misclosure problem with geodetic azimuths, and shows how to convert grid azimuths to geodetic azimuths.

Equation 3.10 Relationship between grid and forward geodetic azimuth from point *A* to *B*

$$\boxed{\alpha_{AB} = t_{AB} + \gamma_A - (t - T)_{AB}}$$

where α_{AB} and t_{AB} = geodetic and grid azimuths from point *A* to *B*, respectively

γ_A = map projection convergence angle at point *A*

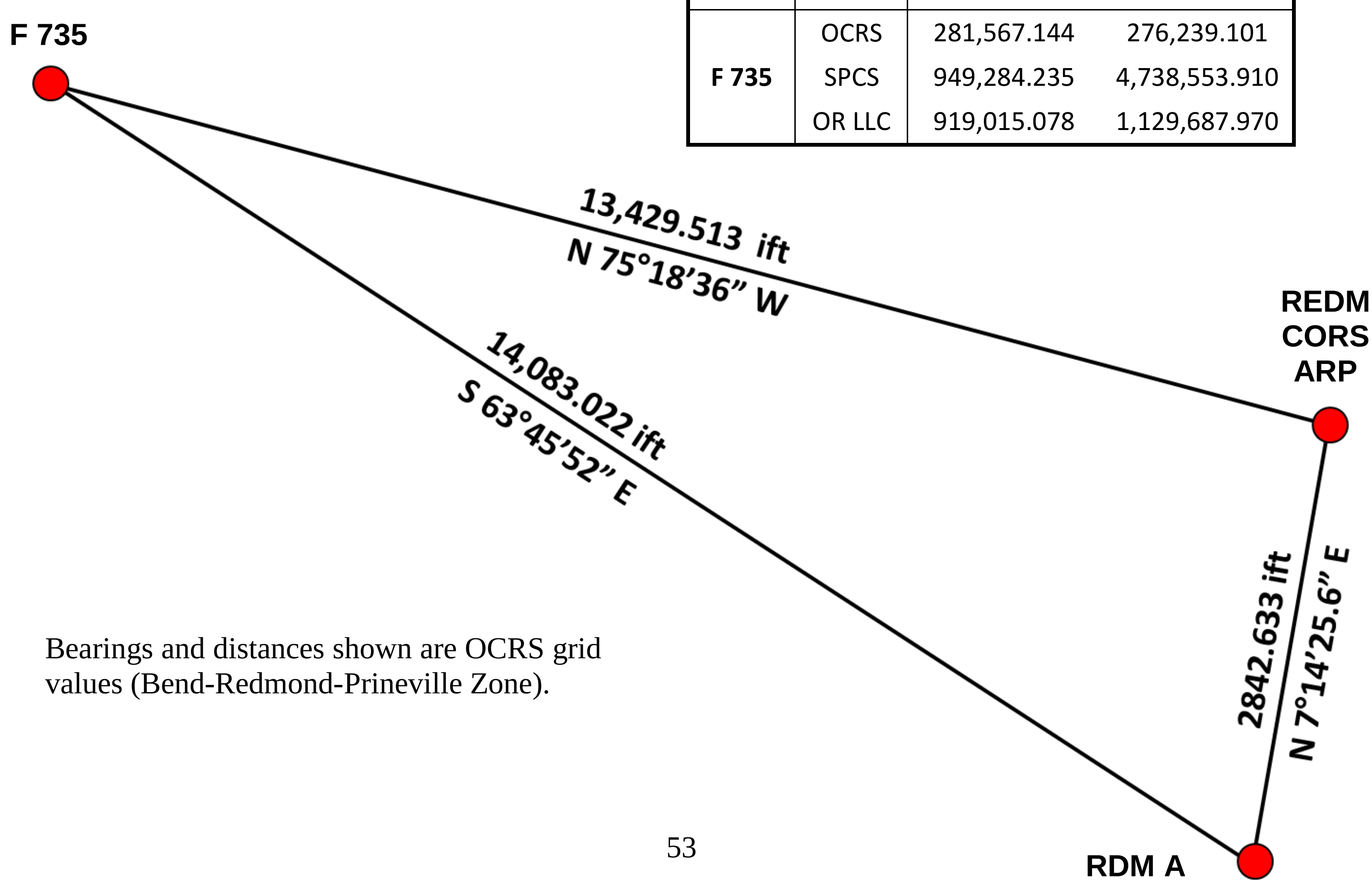
T_{AB} = projected geodetic azimuth from point *A* to *B*

$(t-T)_{AB}$ = arc-to-chord ("second term") correction from *A* to *B* (often negligible)

Example using OCRS, State Plane (SPCS), and Oregon Statewide Lambert (OR LLC) coordinates

Consider closed polygon below formed by points RDM A-, REDM CORS ARP, and F 735. Label the figure with distances, grid azimuths, and geodetic forward and back azimuths.

Grid coords		Northing (ift)	Easting (ift)
RDM A	OCRS	275,341.592	288,871.362
	SPCS	942,945.923	4,751,128.893
	OR LLC	912,676.730	1,142,257.986
REDM CORS ARP	OCRS	278,161.558	289,229.629
	SPCS	945,762.356	4,751,512.467
	OR LLC	915,492.225	1,142,641.877
F 735	OCRS	281,567.144	276,239.101
	SPCS	949,284.235	4,738,553.910
	OR LLC	919,015.078	1,129,687.970



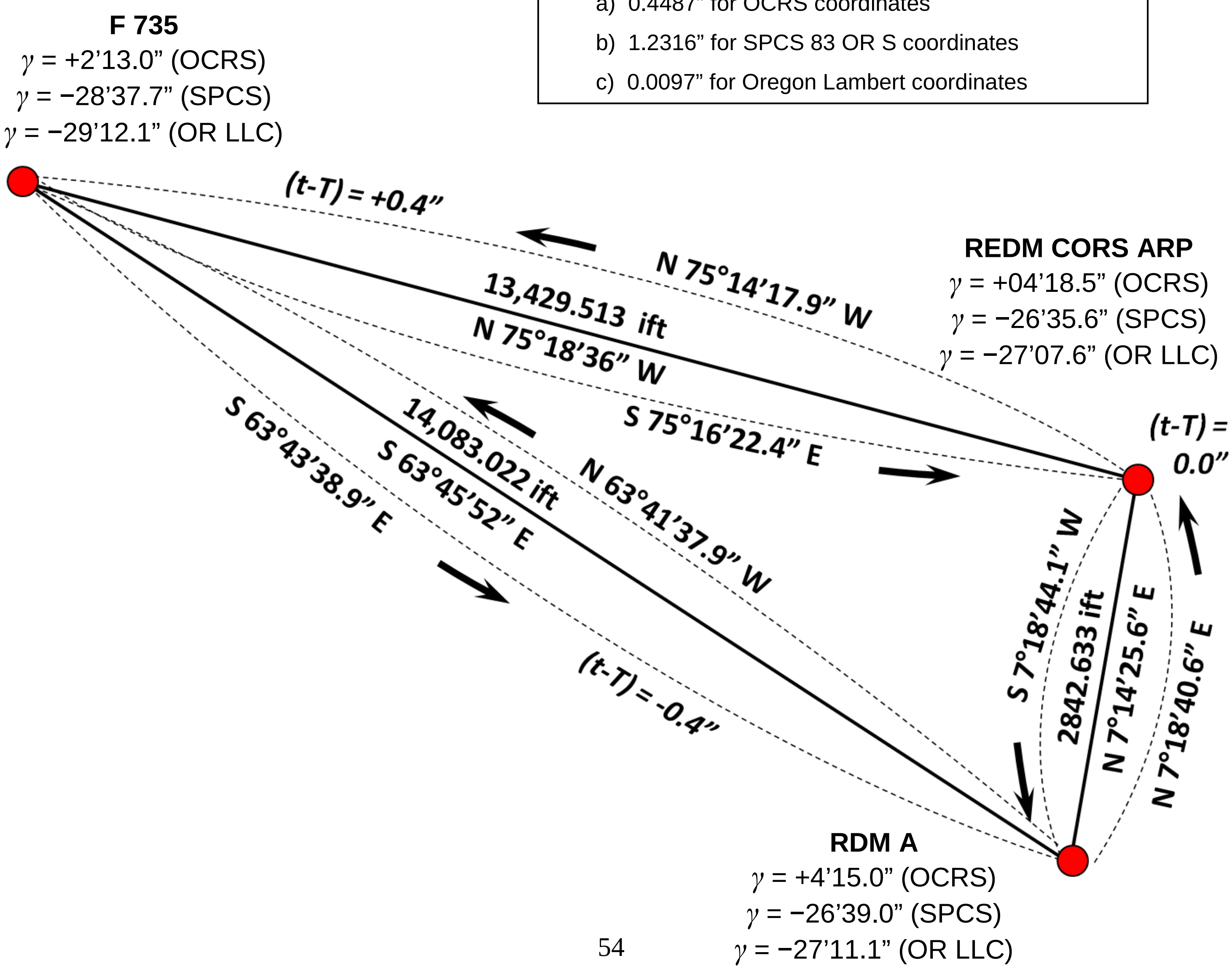
Bearings and distances shown are OCRS grid values (Bend-Redmond-Prineville Zone).

Example solution: Computed using OCRS (Bend-Redmond-Prineville) coordinates

MISCLOSURES (computed using OCRS coordinates)	
Grid bearings and grid distances (misclosure due to rounding)	0.002 ft
Grid bearings and "ground" distances	0.013 ft
Forward geodetic bearings and grid distances	8.485 ft
Forward geodetic bearings and "ground" distances	8.487 ft
Back geodetic bearings and grid distances	7.873 ft
Back geodetic bearings and "ground" distances	7.869 ft
Mean forward & back geodetic bearings and grid distances	0.887 ft
Mean forward & back geodetic bearings and "ground" distances	0.877 ft

Notes

1) Misclosures the same for all grid coordinates systems
2) Arc-to-chord (t-T) corrections in figure are for OCRS
3) Maximum magnitude of arc-to-chord corrections:
    a) 0.4487" for OCRS coordinates
    b) 1.2316" for SPCS 83 OR S coordinates
    c) 0.0097" for Oregon Lambert coordinates



Section 4

VERTICAL DATUMS AND HEIGHT SYSTEMS

How high is it? How deep is it? Where will water go?

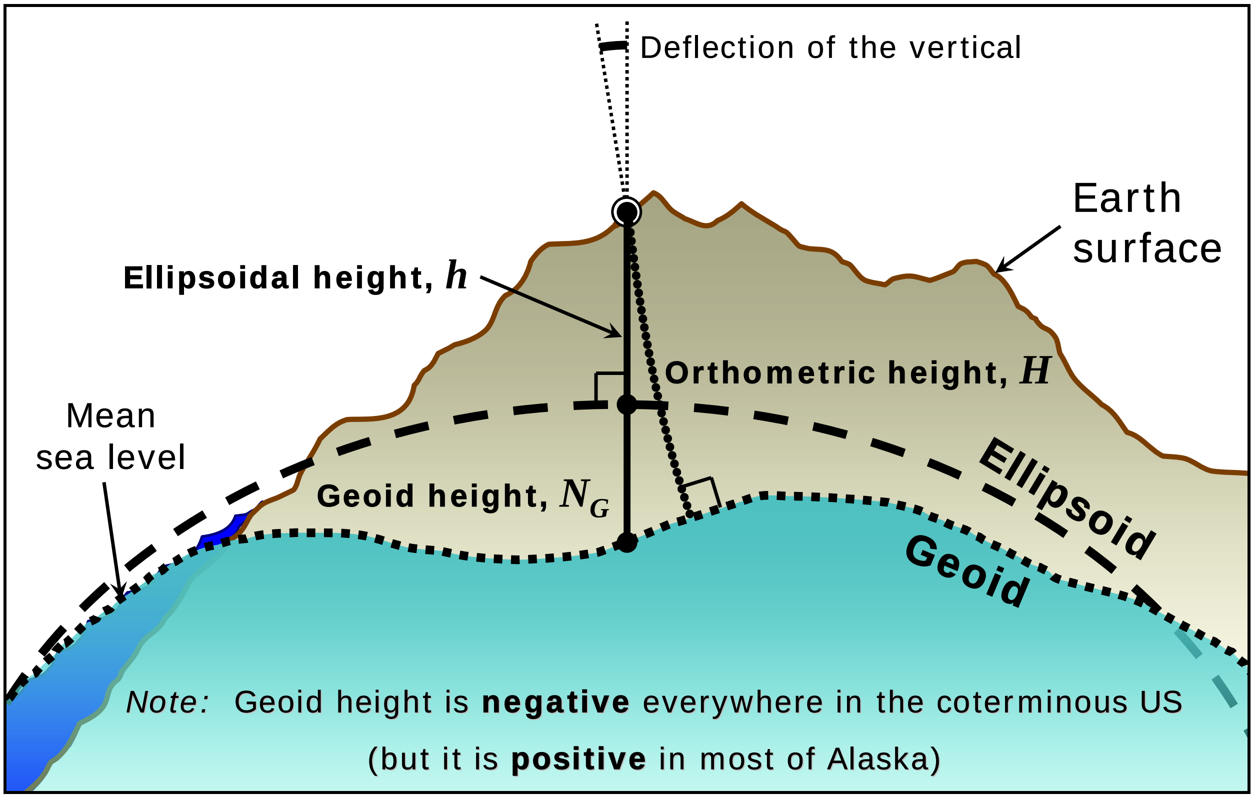
Examples of height determination errors for Oregon

Table 4.1 Examples of various positioning error sources and their magnitudes for Oregon due to vertical datum and height system problems (abbreviations and technical terms are defined in the Glossary).

Positioning error examples for Oregon	Error magnitudes
Using NGVD 29 when NAVD 88 required	3.0 to 5.2 feet (vertical)
Using ellipsoid heights for elevations	Varies from 50 feet to 93 feet (vertical)
Neglecting geoid slope when transferring elevations with GPS	Up to ~0.8 foot vertical per mile horizontal
Using geoid model GEOID09 when GEOID12B is required to derive elevations from ellipsoid heights	Varies from −0.6 foot to +0.8 foot (vertical)
Using leveling without applying NAVD 88 orthometric correction (or using NAVD 88 geopotential numbers)	Can exceed 0.2 foot vertical per mile horizontal
Generating GPS-derived elevations using a best-fit inclined planar correction surface based on ties to inappropriate or inconsistent vertical control (via a vertical "calibration" or "localization")	Varies, but can cause very large systematic vertical errors (can exceed several feet)

Ellipsoid, orthometric, and geoid heights

The relationship between ellipsoidal, orthometric, and geoid heights is shown in the figure below. Note that everywhere in the coterminous US, the geoid height is negative (i.e., the geoid is below the ellipsoid). But in most of Alaska, the geoid height is positive.



Equation 4.1 Relationship between ellipsoidal, orthometric, and geoid heights

$$\boxed{h = H + N_G}$$

where h, H, and N_G are the ellipsoidal, orthometric, and geoid heights, respectively.

Strictly speaking, the relationship in Equation 4.1 is approximate due to deflection of the vertical. However, it is accurate at the sub-millimeter level, and so can be considered exact for all practical purposes.

NGS hybrid geoid model GEOID12B is the first to have accuracies estimated spatially. Because GEOID12B is fit to NAVD 88 benchmarks that have NAD 83(2011) ellipsoid heights, the accuracy is better near benchmarks and becomes worse as distance from benchmarks.

Table 4.2 Estimated accuracy of GEOID12B with respect to NAVD 88 at 95% confidence.

Rules of Thumb

Area	Range	Mean
Oregon	4.1 – 7.5 cm (0.13 – 0.25 ft)	6.1 cm (0.20 ft)
CONUS	3.9 – 8.8 cm (0.13 – 0.29 ft)	5.8 cm (0.19 ft)

Example computation

<u>Given</u>: An NGS Datasheet for conventional NGS control station MARSH (PB0630):

```
PB0630 ***********************************************************************
PB0630  DESIGNATION -   MARSH
PB0630  PID         -   PB0630
PB0630  STATE/COUNTY-   OR/KLAMATH
PB0630  COUNTRY     -   US
PB0630  USGS QUAD   -   CHEMULT (1967)
PB0630
PB0630                         *CURRENT SURVEY CONTROL
PB0630  ______________________________________________________________________
PB0630* NAD 83(1991) POSITION- 43 07 45.66845(N) 121 48 18.75991(W)   ADJUSTED
PB0630* NAVD 88 ORTHO HEIGHT -   1413.441  (meters)    4637.26  (feet) ADJUSTED
PB0630  ______________________________________________________________________
PB0630  LAPLACE CORR     -          0.77  (seconds)                   DEFLEC12B
PB0630  GEOID HEIGHT     -        -21.65  (meters)                    GEOID12B
PB0630  DYNAMIC HEIGHT   -       1412.660 (meters)     4634.70  (feet) COMP
PB0630  MODELED GRAVITY -    980,018.2    (mgal)                      NAVD 88
PB0630
PB0630  HORZ ORDER       -  FIRST
PB0630  VERT ORDER       -  FIRST     CLASS II
```

<u>Find</u>: The ellipsoid height of MARSH in international and US survey feet.

<u>Computations</u>:

Sometimes the only horizontal control station available for a GPS survey was determined using conventional methods. These do not have an ellipsoid height, but there it can be computed if it has an accurate NAVD 88 orthometric height. From the Datasheet we have:

$h = \quad H \quad + \quad N_G$

$h =$ ____________ + __________ = ______________ m = ____________ ift = ____________ sft

<u>Solution</u>:

```
PB0630 ***********************************************************************
PB0630  DESIGNATION -   MARSH
PB0630  PID         -   PB0630
PB0630  STATE/COUNTY-   OR/KLAMATH
PB0630  COUNTRY     -   US
PB0630  USGS QUAD   -   CHEMULT (1967)
PB0630
PB0630                         *CURRENT SURVEY CONTROL
PB0630  ______________________________________________________________________
PB0630* NAD 83(1991) POSITION- 43 07 45.66845(N) 121 48 18.75991(W)   ADJUSTED
PB0630* NAVD 88 ORTHO HEIGHT -   1413.441  (meters)    4637.26  (feet) ADJUSTED  = H
PB0630  ______________________________________________________________________
PB0630  LAPLACE CORR     -          0.77  (seconds)                   DEFLEC12B
PB0630  GEOID HEIGHT     -        -21.65  (meters)                    GEOID12B   = N_G
PB0630  DYNAMIC HEIGHT   -       1412.660 (meters)     4634.70  (feet) COMP
```

h = 1413.441 m + (−21.65 m) **=** <u>1391.79 m</u> = <u>4566.24 ift</u> = <u>4566.23 sft</u>

But, based on the GEOID12B variance model, the accuracy at this location is 5.2 cm (0.17 ft) at 95% confidence, so the ellipsoid height can be reported as:

***h* = <u>4566.2 ft (±0.2 ft at 95% confidence)</u>** *in both international and US survey feet at accuracy of the computation*

Dynamic heights and geopotential numbers

In addition to orthometric heights, H ("elevations"), NGS Datasheets also give *dynamic heights*, H^D. A dynamic "height" is actually not a height in the geometric sense of a distance above a reference surface. Rather, it is a *geopotential number*, C, that has been divided (scaled) by a constant value of gravity, which gives H^D units of length. Both C and H^D represent the gravity potential energy at a point, and changes in H^D are the only "height" differences that give true change in hydraulic head. That is, unconfined water will not flow from one point to another if the water surface at both points has the same H^D, even though the points will generally ***not*** have the same "elevation", H (i.e., $\Delta H^D \neq \Delta H$, although the difference is often small).

Equation 4.2 Relationship between dynamic height and geopotential number

$$\boxed{H^D = \frac{C}{\gamma_0}} \qquad \boxed{H^D = \frac{C}{9.806199}} \text{ [meters]} \qquad \boxed{H^D = \frac{C}{32.172569}} \text{ [ift]}$$

where C = geopotential number (units of m^2/s^2 or ft^2/s^2)

γ_0 = 9.806199 m/s^2 = normal gravity on the GRS 80 ellipsoid at 45° latitude (given on NGS Datasheets as 980.6199 gals, where 1 m/s^2 = 100 gals)

Both the dynamic and orthometric heights shown on NGS Datasheets were originally computed from the same set of adjusted geopotential numbers. The relationship between these two types of heights is given below.

Equation 4.3 Relationship between NAVD 88 dynamic and Helmert orthometric heights

$$\boxed{H^D = \frac{H}{\gamma_0}\bar{g} = \frac{H}{\gamma_0}\left(g + \frac{H}{K}\right) = \frac{H}{\gamma_0}\left(g + \frac{H}{2{,}358{,}000}\right)}$$

(modified from Zilkoski et al., 1992)

where $\bar{g}$ = Helmert mean gravity on the plumbline

g = "Observed" (modeled) NAVD 88 surface gravity (given on NGS Datasheets in milligals, where 1 m/s^2 = 100,000 mgals)

K = 2,358,000 s^2 = 1 / (4.24 × 10^{-7} s^{-2}) is a constant factor for computing Helmert NAVD 88 mean gravity (assumes constant topographic density of 2670 kg/m^3)

Equations 4.4 and 4.5 show that orthometric heights can also be computed from geopotential numbers, as $H = C / \bar{g}$.

Example computation

Given: The NGS Datasheet for NGS station MARSH (in Exercise 4.1, and on the next page):

Find: The geopotential number of MARSH from both dynamic and orthometric height (in ift).

Computations:

Using the published NAVD 88 dynamic height:

$$C = \gamma_0 \times H^D$$

$$C = \_\_\_\_\_\_\_\_ \times \frac{\_\_\_\_\_\_\_\_\ \text{m}}{0.3048\ \text{m/ift}} = \_\_\_\_\_\_\_\_\ \text{ift}^2/\text{s}^2$$

Using the published NAVD 88 Helmert orthometric height:

$$C = \left(g + \frac{H}{K} \right) \times H$$

$$C = \left(\_\_\_\_\_\_\_\_ + \frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_} \right) \times \_\_\_\_\_\_\_\_ = \frac{\_\_\_\_\_\_\_\_\ \text{m}^2/\text{s}^2}{(0.3048)^2\ \text{m}^2/\text{ift}^2} =$$

$$= \_\_\_\_\_\_\_\_\ \text{ift}^2/\text{s}^2$$

```
PB0630 ***********************************************************************
PB0630  DESIGNATION -  MARSH
PB0630  PID         -  PB0630
PB0630  STATE/COUNTY-  OR/KLAMATH
PB0630  COUNTRY     -  US
PB0630  USGS QUAD   -  CHEMULT (1967)
PB0630
PB0630                         *CURRENT SURVEY CONTROL
PB0630  ______________________________________________________________________
PB0630* NAD 83(1991) POSITION- 43 07 45.66845(N) 121 48 18.75991(W)   ADJUSTED
PB0630* NAVD 88 ORTHO HEIGHT -  1413.441  (meters)    4637.26  (feet) ADJUSTED   = H
PB0630  ______________________________________________________________________
PB0630  LAPLACE CORR    -          0.77  (seconds)                    DEFLEC12B
PB0630  GEOID HEIGHT    -        -21.65  (meters)                     GEOID12B
PB0630  DYNAMIC HEIGHT  -      1412.660 (meters)     4634.70  (feet) COMP      = H^D
PB0630  MODELED GRAVITY -    980,018.2   (mgal)                       NAVD 88   = g
PB0630
PB0630  HORZ ORDER      -  FIRST
PB0630  VERT ORDER      -  FIRST     CLASS II
  :
PB0630.The dynamic height is computed by dividing the NAVD 88
PB0630.geopotential number by the normal gravity value computed on the
PB0630.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
PB0630.degrees latitude (g = 980.6199 gals.).                                  = γ0
PB0630
PB0630.The modeled gravity was interpolated from observed gravity values.
```

Solution:

Using the published NAVD 88 dynamic height:

$$C = 32.172569\ \text{ift/s}^2 \times \frac{1412.660\ \text{m}}{0.3048\ \text{m/ift}} = \underline{\mathbf{149{,}110.6}}\ \mathbf{ift^2/s^2}$$

Using the published NAVD 88 Helmert orthometric height:

$$C = \left(9.800182\ \text{m/s}^2 + \frac{1413.441\ \text{m}}{2{,}358{,}000\ \text{s}^2} \right) \times 1413.441\ \text{m} = \frac{13{,}852.826\ \text{m}^2/\text{s}^2}{(0.3048)^2\ \text{m}^2/\text{ift}^2} = \underline{\mathbf{149{,}110.6}}\ \mathbf{ift^2/s^2}$$

Computing orthometric and dynamic heights from leveling

Leveling, by itself, does not yield true change in orthometric or dynamic heights. But when leveling is combined with surface gravity, the change in geopotential numbers can be computed. If the geopotential number is known for at least one point in a leveling network, then it can be computed at all points in the network. The geopotential numbers can then be converted to orthometric and dynamic heights using the relationships from the previous section, where orthometric height is $H = C / \bar{g}$, and dynamic height is $H^D = C / \gamma_0$.

Equation 4.4 Determining change in geopotential from leveled height differences

$$\boxed{C_B \approx C_A + \left(\frac{g_A + g_B}{2}\right)\Delta n_{AB}}$$

where g_A and g_B = surface gravity at adjacent stations *A* and *B* (in m/s^2 or ft/s^2)

Δn_{AB} = leveled height difference from station *A* and *B* (in same linear units as gravity)

Alternatively, leveled height differences can be converted to orthometric heights and dynamic heights by adding an orthometric correction (*OC*) or dynamic correction (*DC*) to observed leveled height differences between adjacent stations.

Equation 4.5 The NAVD 88 Helmert orthometric correction for leveled height differences

$$\boxed{OC_{AB} \approx \frac{\left[K\left(g_A - g_B\right) - 2\,\Delta n_{AB}\right]\left[2H_A + \Delta n_{AB}\right]}{2\left(K\,g_B + H_A + \Delta n_{AB}\right)}} \quad \text{(modified from Hwang and Hsiao, 2003)}$$

where all variables are as defined previously, and the orthometric correction is added to the observed leveled height difference, i.e., $H_B \approx H_A + \Delta n_{AB} + OC_{AB}$.

Equation 4.6 The dynamic correction for leveled height differences

$$\boxed{DC_{AB} \approx \left(\frac{g_A + g_B}{2\gamma_0} - 1\right)\Delta n_{AB}} \quad \text{(modified from Hofmann-Wellenhof and Moritz, 2005)}$$

where all variables are as defined previously, and the dynamic correction is added to the observed leveled height difference, i.e., $H_B^D \approx H_A^D + \Delta n_{AB} + DC_{AB}$.

"Approximately equal" symbols were used for equations 4.6 – 4.8 because the surface gravity varies continuously along the leveling route. These equations will be exactly true only when the gravity varies linearly between stations. For best results they should be applied to every turning point on a leveling route. However, in most cases, Equation 4.7 (orthometric corrections) should work well for stations less than about 2 km apart. Equations 4.6 and 4.8 (geopotential numbers and dynamic corrections) are more sensitive to variation in surface gravity, and may not give good results even for stations less than 2 km apart, especially in mountainous areas.

Example computation

Given: A leveled height difference of +50.387 ft measured from NGS stations M 504 (PID FQ0543) to L 504 (PID FQ0544). The following data apply to these stations:

	M 504 (*station A*)	**L 504 (*station B*)**
Orthometric height	6104.396 ift	?
Dynamic height	6095.991 ift	?
Surface gravity	32.125673 ift/s^2	32.125305 ift/s^2

Find: The orthometric and dynamic heights of L 504 (in ift). The stations are 6450 ft apart.

Computations: The stations are (slightly) less than about 2 km apart, so using gravity values only at the stations themselves should be adequate (rather than at every leveling turning point).

Alternative 1: Solve using geopotential numbers.

$$C_B = 32.172569 \times 6095.991 + \left(\frac{32.125673 + 32.125303}{2} \right) \times 50.387$$

$$C_B = 196{,}123.7 + 32.125489 \times 50.387 = \underline{197{,}742.4}\ \text{ift}^2/\text{s}^2$$

Orthometric height: $$H_B = \frac{C_B}{\bar{g}_B} = \frac{197{,}742.4}{32.125305 + \dfrac{6104.396 + 50.387}{2{,}358{,}000}} = \underline{\mathbf{6154.847}}\ \mathbf{ift}$$

Dynamic height: $$H_B^D = \frac{C_B}{\gamma_0} = \frac{197{,}742.4}{32.172569} = \underline{\mathbf{6146.304}}\ \mathbf{ift}$$

Alternative 2: Solve using orthometric and dynamic corrections.

$$OC_{AB} = \frac{\left[\, 2{,}358{,}000 \times (32.125673 - 32.125305) - 2 \times 50.387 \right] \times \left[\, 2 \times 6104.396 + 50.387 \right]}{2 \times (2{,}358{,}000 \times 32.125305 + 6104.396 + 50.387)}$$

$$OC_{AB} = \frac{\left[\, 766.970 \,\right] \times \left[\, 12{,}259.179 \,\right]}{151{,}515{,}248} = \underline{+0.062}\ \text{ft}$$

$$DC_{AB} = \left(\frac{32.125673 + 32.125305}{2 \times 32.172569} - 1 \right) \times 50.387 = (-0.001463) \times 50.387 = \underline{-0.074}\ \text{ft}$$

Orthometric height:

$$H_B = H_A + \Delta n_{AB} + OC_{AB} = 6104.396 + 50.387 + 0.062 = \underline{\mathbf{6154.845}}\ \mathbf{ift}$$

Dynamic height:

$$H_B^D = H_A^D + \Delta n_{AB} + DC_{AB} = 6095.991 + 50.387 + (-0.074) = \underline{\mathbf{6146.304}}\ \mathbf{ift}$$

Check: The NGS Datasheet for station L 504 gives:

H_B = 1875.997 m = 6154.846 ift and H_B^D = 1873.393 m = 6146.302 ift ✓

Section 5

DOCUMENTATION AND ACCURACY REPORTING

Is it in the right place? By how much? How do you know?

Examples of documentation and accuracy reporting errors

Table 5.1 Examples of various positioning error sources and their magnitudes due to documentation and accuracy reporting problems (abbreviations and technical terms are defined in the Glossary).

Documentation error examples	Problem
Documenting geodetic datum as "WGS-84" when data actually referenced to NAD 83	Perpetuates confusion about "equivalence" of WGS-84 and NAD 83
Listing grid coordinates (such as SPCS) as "NAD 83"	NAD 83 is a geodetic datum, not a grid coordinate system
Documenting geodetic datum as "GRS-80"	GRS-80 is a reference ellipsoid, not a datum
Documenting vertical datum as "Mean Sea Level" (MSL)	There is no MSL datum in the US (name changed to NGVD 29 in 1976)
Using precision as an accuracy estimate with data containing systematic errors (e.g., incorrect reference coordinates)	Accuracy estimate is meaningless
Reporting horizontal error using unscaled standard deviation, rather than at the 95% confidence level (as specified by the FGDC)	Gives error estimates at 39% confidence level
Reporting vertical error using unscaled standard deviation, rather than at the 95% confidence level (as specified by the FGDC)	Gives error estimates at 68% confidence level
Using radial and circular estimates for horizontal error rather than semi-major axis of horizontal error ellipse	Typically makes errors appear less than actual
Using trivial vectors in GPS network adjustments	Varies, but always makes errors appear less than actual
Relying on precision computed by baseline processor for a single GPS vector as an indicator of accuracy	Varies, but precision value usually very optimistic and will not reveal systematic errors

Computing accuracy estimates from standard deviations

Accuracies for GNSS stations are given on the NGS Datasheet as linear values for the horizontal and ellipsoid height components (in centimeters) scaled to the 95% confidence level. The horizontal accuracy of a station is computed from the standard deviations in the north and east components, along with the horizontal correlation coefficient. The height accuracy is computed from the ellipsoid height standard deviation. The standard deviation and horizontal correlation values were computed in the constrained least-squares adjustments performed for determining the published coordinates. A hyperlink on the Datasheet immediately below the published accuracies opens an Accuracy Datasheet that given the standard deviations and horizontal correlations, along with other information.

For passive GNSS control, both "network" and "local" accuracies are given. The network accuracy represents the accuracy of the station with respect to the NSRS. Local accuracy is the accuracy of the station with respect to another station that was processed simultaneously. It represents the error of the adjusted GNSS observations between the two stations and is a property of the station pair, not of one station or the other. The median values of all local accuracies are given on the Datasheet, along with the median distance between local accuracy station pairs. The complete list of all local accuracies and distances are given on the Accuracy Datasheet. Local accuracies are not given for CORS, because the method used for determining CORS coordinates is not amenable to that approach.

The following approach is used for computing accuracies on the NGS Datasheet.

Equation 5.1 Ellipsoid height accuracy on the NGS Datasheet (at the 95% confidence level)

$$\boxed{E_{95}^{h} = 1.9600 \times \sigma_h}$$

where E_{95}^{h} is the ellipsoid height error ("accuracy") at 95% confidence

σ_h is the ellipsoid height standard deviation

1.9600 is the *univariate* (one-dimensional) scalar for a confidence level of 95%. See Table 5.2 below for this and other scalars at various confidence levels.

Equation 5.2 Horizontal accuracy on the NGS Datasheet (at the 95% confidence level)

$$\boxed{E_{95}^{Horz} = a\,(1.960790 + 0.004071\ C + 0.114276\ C^2 + 0.371625\ C^3)} \quad \text{(Leenhouts, 1985)}$$

where E_{95}^{Horz} is the radius of error circle ("horizontal accuracy") at 95% confidence

$C = b / a$

a and b are the error ellipse semi-major and semi-minor axes, computed as follows

Equation 5.3 Horizontal error ellipse axes computed from standard deviations and covariance

$$\boxed{a,\ b = \sqrt{\frac{1}{2}\left[\sigma_N^2 + \sigma_E^2 \pm \sqrt{\left(\sigma_N^2 - \sigma_E^2\right)^2 + 4\sigma_{NE}^2}\right]}}$$

The "±" operator in Equation 5.3 allows computation of both *a* and *b* with this one equation, and *a* is always greater than *b*. The horizontal covariance is computed as follows.

Equation 5.4 Horizontal covariance computed from the horizontal correlation coefficient

$$\boxed{\sigma_{NE} = \rho\, \sigma_N\, \sigma_E}$$

where σ_{NE} is the horizontal covariance

ρ is the horizontal correlation coefficient

The orientation (rotation) of a horizontal error ellipse can be computed from the standard deviations and covariance.

Equation 5.5 Horizontal error ellipse rotation computed from standard deviation and covariance

$$\boxed{\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{NE}}{\sigma_E^2 - \sigma_N^2}\right)}$$

where θ is the rotation angle of the semi-major axis, with respect to the east direction (positive counterclockwise). If $\sigma_N > \sigma_E$, rotation is with respect to the *positive* east axis. If, rotation is $\sigma_N < \sigma_E$,with respect to the *negative* east axis. If $\sigma_N = \sigma_E$, then θ = ±45°, where the sign of the rotation is determined by the sign of σ_{NE}.

Table 5.2 Values used to scale standard errors (accuracies) to various confidence levels. The *univariate scalar* is used for single error components, such as vertical error. The *bivariate scalar* is used for dual (two-dimensional) error components, such as horizontal error, and can be used to scale an error ellipse to a desired confidence level. The *trivariate scalar* is rarely used but is provided here for the sake of completeness. It is for three-dimensional error components and can be used for scaling an error ellipsoid to a desired confidence level. In all cases, these scalars are based on the normal probability distribution of random variables, and the multivariate scalars are for jointly distributed random variables.

Univariate scalars		**Bivariate scalars**		**Trivariate scalars**	
Scalar, c_X^1	**Confidence level,** X	**Scalar,** c_X^2	**Confidence level,** X	**Scalar,** c_X^3	**Confidence level,** X
0.6745	50.00%	1.0000	39.35%	1.0000	19.87%
1.0000	68.27%	1.1774	50.00%	1.5382	50.00%
1.6449	90.00%	2.0000	86.47%	2.0000	73.85%
1.9600	95.00%	2.1460	90.00%	2.5003	90.00%
2.0000	95.45%	2.4477	95.00%	2.7955	95.00%
2.5758	99.00%	3.0000	98.89%	3.0000	97.07%
3.0000	99.73%	3.0349	99.00%	3.3682	99.00%
3.2905	99.90%	3.7169	99.90%	4.0331	99.90%

In contrast to the preceding definition of horizontal accuracy, and for the sake of completeness, a horizontal (circular) accuracy can be computed that consistent with the approach used by the National Standard for Spatial Data Accuracy (NSSDA) as developed by the Federal Geographic Data Committee (1998, Part 3).

Equation 5.6 Horizontal (circular) accuracy computed at the 95% confidence level per NSSDA

$$\boxed{CEP_{95} = 2.4477\,\frac{\sigma_N + \sigma_E}{2}}$$

where CEP_{95} is the estimated Circular Error Probable (horizontal accuracy) at 95% confidence (note that *CEP* is typically computed at the 50% confidence level)

σ_N and σ_E are the north and east standard deviations, respectively

2.4477 is the *bivariate* (two-dimensional) scalar for 95% confidence, per Table 5.2

Again, for the sake of completeness, note that the trivariate scalar can be used to scale the estimated Spherical Error Probable (*SEP*) to a desired confidence level. As with *CEP*, typically *SEP* is computed at 50% confidence.

Equation 5.7 Three-dimensional (spherical) accuracy computed at the 95% confidence level

$$\boxed{SEP_{95} = 2.7955\,\frac{\sigma_N + \sigma_E + \sigma_h}{3}}$$

Example computation

Given: The NGS Datasheet for station J 99 (QE0722):

```
1         National Geodetic Survey,   Retrieval Date = FEBRUARY  1, 2016
 QE0722 ***********************************************************************
 QE0722  DESIGNATION -  J 99
 QE0722  PID         -  QE0722
 QE0722  STATE/COUNTY-  OR/POLK
 QE0722  COUNTRY     -  US
 QE0722  USGS QUAD   -  MONMOUTH (1986)
 QE0722
 QE0722                         *CURRENT SURVEY CONTROL
 QE0722  ______________________________________________________________________
 QE0722* NAD 83(2011) POSITION- 44 48 11.97228(N) 123 13 20.88312(W)   ADJUSTED
 QE0722* NAD 83(2011) ELLIP HT-    53.898 (meters)        (06/27/12)   ADJUSTED
 QE0722* NAD 83(2011) EPOCH   -  2010.00
 QE0722* NAVD 88 ORTHO HEIGHT -    76.442  (meters)     250.79  (feet) ADJUSTED
 QE0722  ______________________________________________________________________
 QE0722  NAD 83(2011) X  - -2,483,622.713 (meters)                     COMP
 QE0722  NAD 83(2011) Y  - -3,792,127.021 (meters)                     COMP
 QE0722  NAD 83(2011) Z  -  4,471,905.090 (meters)                     COMP
 QE0722  LAPLACE CORR    -          -8.54  (seconds)                   DEFLEC12B
 QE0722  GEOID HEIGHT    -         -22.54  (meters)                    GEOID12B
 QE0722  DYNAMIC HEIGHT  -          76.439 (meters)     250.78  (feet) COMP
 QE0722  MODELED GRAVITY -     980,572.9   (mgal)                      NAVD 88
 QE0722
 QE0722  VERT ORDER      -  SECOND    CLASS 0
 QE0722
 QE0722  Network accuracy estimates per FGDC Geospatial Positioning Accuracy
 QE0722  Standards:
 QE0722         FGDC (95% conf, cm)     Standard deviation (cm)     CorrNE
 QE0722            Horiz  Ellip           SD_N   SD_E   SD_h      (unitless)
 QE0722  -------------------------------------------------------------------
 QE0722  NETWORK     0.81   1.45           0.37   0.28   0.74        0.13256717
 QE0722
 QE0722  Click here for local accuracies and other accuracy information.
 QE0722
```

E_{95}^{Horz} E_{95}^{h} σ_N σ_E σ_h ρ

Find: The horizontal and ellipsoid height network accuracy at the 50%, 95%, and 99% confidence levels using standard deviations and horizontal correlation coefficient values on the Accuracy Datasheet for this station. Also compute the circular error probable (*CEP*), spherical error probable (*SEP*), and the horizontal error ellipse axes and rotation angle, at 95% confidence. Give the final results in feet.

Computations: The ellipsoid height network accuracy (error) is one-dimensional, so the univariate scalars from Table 5.2 should be used to scale the ellipsoid height standard deviation from the Accuracy Datasheet to the required confidence levels:

$$E_X^h = c_X^1 \times \sigma_h$$

where c_X^1 is the univariate scalar at the X% confidence level.

E_{50}^{h} = 0.6745 × 0.74 = 0.50 cm = **0.016 ft (at 50% confidence)**

E_{95}^{h} = 1.9600 × 0.74 = 1.45 cm = **0.048 ft (at 95% confidence)**

E_{99}^{h} = 2.5758 × 0.74 = 1.91 cm = **0.063 ft (at 99% confidence)**

Computing the horizontal network accuracy first requires computing the error ellipse semi-major and semi-minor axes from the north and east standard deviations and horizontal correlation coefficient on the Accuracy Datasheet. First the horizontal covariance is computed:

$$\sigma_{NE} = \rho\, \sigma_N\, \sigma_E = +0.13256717 \times 0.37 \text{ cm} \times 0.28 \text{ cm} = \underline{+0.01373396 \text{ cm}^2}.$$

The standard error ellipse axes can now be computed using Equation 5.3. Note that there is a "±" symbol in the equation — *a* is computed for the case where "±" is "+", and *b* is computed for the case where "±" is "–":

$$a, b = \sqrt{\frac{1}{2}\left[\sigma_N^2 + \sigma_E^2 \pm \sqrt{\left(\sigma_N^2 - \sigma_E^2\right)^2 + 4\sigma_{NE}^2}\right]}$$

$$= \sqrt{\frac{1}{2}\left[0.37^2 + 0.28^2 \pm \sqrt{\left(0.37^2 - 0.28^2\right)^2 + 4 \times 0.01373396^2}\right]} = \begin{cases} \underline{a = 0.374 \text{ cm} = 0.012 \text{ ft}} \\ \underline{b = 0.274 \text{ cm} = 0.009 \text{ ft}} \end{cases}$$

Since the *a* and *b* dimensions are for the standard error ellipse, the scalar is 1, which corresponds to a confidence level of 39.35% (as shown in Table 5.2) for the bivariate (2-D) case. The coefficients in Equation 2 for horizontal accuracy are already scaled to 95% confidence. First compute $C = 0.274 / 0.374 = 0.73262$, which gives:

$$E_{95}^{Horz} = 0.374 \times (1.960790 + 0.004071 \times 0.73262 + 0.114276 \times 0.73262^2 + 0.371625 \times 0.73262^3)$$

E_{95}^{Horz} = 0.81 cm = **0.027 ft (at 95% confidence)**

To get the accuracy at different confidence levels, simply multiply the 95% confidence accuracy by the ratio of the bivariate scalar of the desired confidence level to the 95% scalar of 2.4477:

E_{50}^{Horz} = (1.1774 / 2.4477) × 0.81 = 0.39 cm = **0.013 ft (at 50% confidence)**

E_{99}^{Horz} = (3.0349 / 2.4477) × 0.81 = 1.00 cm = **0.033 ft (at 99% confidence)**

The circular error probable and spherical error probable at 95% confidence are given by equations 5.6 and 5.7:

CEP_{95} = 2.4477 × (0.37 + 0.28) / 2 = 0.80 cm = **0.026 ft (at 95% confidence)**

and

SEP_{95} = 2.7955 × (0.37 + 0.28 + 0.74) / 3 = 1.30 cm = **0.042 ft (at 95% confidence)**

Surveying & mapping spatial data requirements & recommendations

These should be explicitly specified in surveying and mapping projects

1. Completely define the coordinate system

a. Linear unit (e.g., international foot, U.S. survey foot, meter)
   i. Use same linear unit for horizontal and vertical coordinates
b. Geodetic datum (recommend North American Datum of 1983)
   i. Should include “datum tag”, e.g., 1986, 1991, 1998, 2007, 2011, as necessary, as well as epoch date for modern high-accuracy positions, e.g., 2010.00
   ii. WGS 84, ITRF/IGS, and NAD 27 are usually **NOT** recommended
c. Vertical datum (e.g., North American Vertical Datum of 1988)
   i. If GPS used for elevations, recommend using a modern geoid model (e.g., GEOID12B)
   ii. Recommend using NAVD 88 rather than NGVD 29 when possible
d. Map projection type and parameters (e.g., Transverse Mercator, Lambert Conformal Conic)
   i. Special attention required for low-distortion grid (a.k.a. “ground”) coordinate systems
      1) Avoid scaling of existing coordinate systems (e.g., “modified” State Plane)

2. Require *direct* referencing of the NSRS (National Spatial Reference System)

a. Ties to published control strongly recommended (e.g., National Geodetic Survey control)
   i. Relevant component of control must have greater accuracy than positioning method used
      1) E.g., network accuracies that meet project needs, 2^{nd} order (or better) for vertical control
b. NGS Continuously Operating Reference Stations (CORS) can be used to reference the NSRS
   i. Free Internet GPS post-processing service: OPUS (Online Positioning User Service)

3. Specify *accuracy* requirements (*not* precision)

a. Use objective, defensible, and robust methods (published ones are recommended)
   i. Mapping and surveying: National Standard for Spatial Data Accuracy (NSSDA)
      1) Require occupations (“check shots”) of known high-quality control stations
   ii. Surveys performed for establishing control or determining property boundaries:
      1) Appropriately constrained and over-determined least-squares adjusted control network
      2) Beware of “cheating” (e.g., using “trivial” GPS vectors in network adjustment)

4. Documentation is *essential* (metadata!)

a. Require a report detailing methods, procedures, and results for developing final deliverables
   i. This must include any and all post-survey coordinate transformations
      1) E.g., published datum transformations, computed correction surfaces, “rubber sheeting”
b. Documentation should be complete enough that someone else can reproduce the product
c. For GIS data, recommend that accuracy and coordinate system information be included as feature attributes (not just as separate, easy-to-lose and easy-to-ignore metadata files)

Example of surveying and mapping documentation (*metadata*)

Basis of Bearings and Coordinates

Linear unit: International foot (ift)

Geodetic datum: North American Datum of 1983 (2011) epoch 2010.00

Vertical datum: North American Vertical Datum of 1988 (see below)

System: Oregon Coordinate Reference System

Zone: Bend-Redmond-Prineville

Projection: Lambert Conformal Conic (one-parallel)
Standard parallel and latitude of grid origin: 44° 40’ 00” N
Longitude of central meridian: 121° 15’ 00” W
Northing at grid origin: 130,000.000 m (~426,509.18635 ift)
Easting at central meridian: 80,000.000 m (~262,467.19160 ift)
Scale factor on central meridian: 1.000 12 (exact)

All distances and bearings shown hereon are projected (grid) values based on the preceding projection definition. The projection was defined to minimize the difference between projected (grid) distances and horizontal (“ground”) distances at the topographic surface within the design area of this coordinate system.

The basis of bearings is geodetic north. Note that the grid bearings shown hereon (or implied by grid coordinates) do not equal geodetic bearings due to meridian convergence.

Orthometric heights (elevations) were transferred to the site from NGS control station “C 30” (PID QD0823) using GNSS with NGS geoid model “GEOID12B” referenced to the current published 1st order NAVD 88 height of this station (1049.170 m).

The survey was conducted using GNSS referenced to the National Spatial Reference System. A partial list of point coordinates is given below (additional coordinates are available upon request). Accuracy estimates are at the 95% confidence level and are based on an appropriately constrained and weighted least-squares adjustment of redundant observations.

Point #1, NGS control station C 30 (PID QD0823), constrained (off site)		**Estimated accuracy**
Latitude = 44° 06' 53.98076'' N	Northing = 225,363.515 ift	Horiz = ±0.024 ift
Longitude = 121° 17' 27.31006'' W	Easting = 251,718.529 ift	Ellipsoid ht = ±0.076 ift
Ellipsoid height = 3372.940 ift	Elevation = 3442.159 ift	Elevation FIXED

Point #1002, 1/2” rebar with aluminum cap, derived coordinates		**Estimated accuracy**
Latitude = 44° 06' 31.96763'' N	Northing = 223,132.860 ift	Horiz = ±0.034 ift
Longitude = 121° 16' 51.33054'' W	Easting = 254,342.973 ift	Ellipsoid ht = ±0.086 ift
Ellipsoid height = 3395.610 ift	Elevation =3464.760 ift	Elevation = ±0.094 ift

Point #1006, 1/2” rebar with plastic cap, derived coordinates		**Estimated accuracy**
Latitude = 44° 06' 28.79196'' N	Northing = 222,811.061 ift	Horiz = ±0.047 ift
Longitude = 121° 16' 45.17852'' W	Easting = 254,791.795 ift	Ellipsoid ht = ±0.088 ift
Ellipsoid height = 3391.047 ift	Elevation = 3460.184 ift	Elevation = ±0.097 ift

GLOSSARY

Below is a list of the abbreviations and terms used in this workbook. In the interest of brevity, the definitions are highly general and simplified. Please note also that this list gives only a portion of the terms and abbreviations frequently encountered in GPS positioning and geodesy. Terms in *italics* within the definitions are also defined in this glossary. Cited references are listed at the end of the workbook.

Autonomous position. A *GPS* position obtained with a single receiver using only the ranging capability of the *GPS* code (i.e., with no *differential correction*).

Cartesian coordinates. Coordinates based on a system of two or three mutually perpendicular axes. *Map projection* and *ECEF* coordinates are examples two- and three-dimensional Cartesian coordinates, respectively.

Confidence interval or level. A computed probability that the "true" value will fall within a specified region (e.g., 95% confidence level). Applies only to randomly distributed errors.

CORS (Continuously Operating Reference Stations). A nationwide system of permanently mounted *GPS* antennas and receivers that collect *GPS* data continuously. The CORS network is highly accurate and constitutes the geometric foundation of the *NSRS*. CORS data can be used to correct *GPS* survey and mapping results, and the data are freely available over the Internet.

Datum transformation. Mathematical method for converting one *geometric* or *vertical datum* to another (there are several types, and they vary widely in accuracy).

Differential correction. A method for removing much of the error in an autonomous *GPS* position. Typically requires at least two simultaneously operating *GPS* receivers, with one of the two at a location of known geodetic coordinates.

ECEF (Earth-Centered, Earth-Fixed). Refers to a global three-dimensional (X, Y, Z) *Cartesian coordinate* system with its origin at the Earth's center of mass, and "fixed" so that it rotates with the solid Earth. The Z-axis corresponds to the Earth's conventional spin axis, and the X- and Y-axes lie in the equatorial plane. Widely used for geodetic and *GPS* computations.

Ellipsoid height. Straight-line height above and perpendicular to a reference *ellipsoid*. This is the type of height determined by *GPS*, and it does not equal elevation. Can be converted to orthometric heights ("elevations") using a *geoid* model.

Ellipsoid normal. A line perpendicular to the reference *ellipsoid* along which *ellipsoid heights* and *geoid* heights are measured.

Ellipsoid. A simple mathematical model of the Earth, historically corresponding to mean sea level or (the *geoid*) and used as part of a *geometric datum* definition. Constructed by rotating an ellipse about its semi-minor axis. Less frequently referred to by the older and more generic term "spheroid."

FBN (Federal Base Network). Nationwide network of passive *GPS* control stations observed using *GPS* and adjusted by the *NGS*. Nationwide readjustment of the FBN were completed in 2007 and 2012.

FGDC (Federal Geographic Data Committee). Develops and promulgates information on spatial data formats, accuracy, specifications, and standards. Widely referenced by other organizations. Includes the Federal Geodetic Control Subcommittee (FGCS) and the *NSSDA*.

Geodesic. Usually the shortest distance between two points on the surface of an *ellipsoid*. Analogous to the great circle for the shortest distance between two points on a sphere (although a geodesic path around an ellipsoid generally does not return to its beginning point).

Geographic "projection". Not a true *map projection* in the sense that it does not transform geodetic coordinates (latitude and longitude) into linear units. However, it is a projection in the sense that it represents geodetic coordinates on a regular flat grid, such that the difference in angular units (e.g., decimal degrees) is equal in all directions. Because of meridian convergence, this results in an extremely distorted coordinate system, especially at high latitudes, and the distortion varies greatly with direction. Also called a Plate Carrée projection.

Geoid. Surface of constant gravity equipotential (a level surface) that best corresponds to global mean sea level. Often used as a reference surface for *vertical datums*.

Geometric datum. Reference system for computing geodetic coordinates (latitude, longitude, and ellipsoid height or *ECEF* X, Y, and Z) of a point. Typically refers to a particular *ellipsoid* and a set of constants for defining its location and orientation with respect to the physical Earth.

GNSS (Global Navigation Satellite System). A general term for all satellite systems used for navigation, mapping, surveying, and timing. Includes *GPS* and other similar systems such as GLONASS (Russia) and Galileo (Europe).

GPS (Global Positioning System). A constellation of satellites used for navigation, mapping, surveying, and timing. Microwave signals transmitted by the satellites are observed by GPS receivers to determine a three-dimensional position. Accuracy varies greatly depending on the type of receiver and methods used.

Grid distance. The horizontal distance between two points on a flat plane. This is the type of distance obtained from *map projections*.

Ground distance. The horizontal distance between two points as measured on the curved Earth surface. There is no widely accepted definition of a “horizontal ground distance”. In this workbook, it is defined as the *geodesic* (*ellipsoid*) distance scaled to the mean topographic ellipsoid height of the endpoints using the geometric mean radius of curvature at the mean latitude of the endpoints.

GRS-80 (Geodetic Reference System of 1980). The reference *ellipsoid* currently used for many *geometric datums* throughout the world, including *NAD 83* and *ITRF*.

HARN (High Accuracy Reference Network). Network of *GPS* stations adjusted by the *NGS* on a state-by-state basis. Most HARNs was determined in the 1990s. Previously referred to as a High Precision *GPS* (or Geodetic) Network (HPGN).

IERS (International Earth Rotation and Reference System Service). An international organization for defining celestial and terrestrial reference systems (such as *ITRF*) used by the geodetic, geophysics, and astronomic communities. Formally the International Earth Rotation Service.

IGS (International GNSS Service). A voluntary international organization for providing high-quality *GNSS* data, products, and services. Includes periodically determining global reference systems using GNSS techniques corresponding to a specific date (epoch), e.g., IGS08 epoch 2005.00. Each IGS realization is combined with other geodetic techniques to define an *ITRF* realization (e.g., IGS08 was used to determine ITRF2008).

International Foot. Linear unit adopted by the US in 1959, and defined such that one foot equals exactly 0.3048 meter. Shorter than the *US Survey Foot* by 2 *parts per million* (ppm).

ITRF (International Terrestrial Reference Frame). Global geodetic reference system determined using multiple geodetic techniques and not referenced to any particular tectonic plate. A new ITRF is determined periodically and is referenced to a specific date (epoch), e.g., ITRF2008 epoch 2005.00. Each ITRF is a realization of the International Terrestrial Reference System (ITRS). See Soler (2007), and Soler and Snay (2004) and Snay (2012) for information on its relationship to *NAD 83* and *WGS 84*.

Local geodetic horizon. A “northing”, “easting”, and “up” planar coordinate system defined at a point such that the northing-easting plane is perpendicular to the *ellipsoid normal*, north corresponds to true geodetic north, and “up” is in the direction of the *ellipsoid normal* at that point.

Map projection. A functional (one-to-one) mathematical relationship between geodetic coordinates (latitude, longitude) on the curved *ellipsoid* surface, and grid coordinates (northings, eastings) on a planar (flat) map surface. All projections are distorted, in that the relationship between projected coordinates differs from that between their respective geodetic coordinates. See Snyder (1987) for details.

NAD 27 (North American Datum of 1927). *Geometric* ("horizontal") *datum* of the US prior to *NAD 83*, and superseded by *NAD 83* in 1986. This is the datum of *SPCS* 27 and *UTM* 27.

NAD 83 (North American Datum of 1983). Current official *geometric* (historically called "horizontal") *datum* of the US. Replaced *NAD 27* in 1986, which is the year of the initial NAD 83 realization. This is the datum of *SPCS* 83 and *UTM* 83. See Schwarz (1986) and Snay (2012) for details.

NADCON. Mapping-quality d*atum transformation* computer program developed by the NGS for transforming coordinates between *NAD 27* and *NAD 83*, and also between the *NAD 83* 1986 adjustment and the various *HARN* adjustments. See Dewhurst (1990) for details.

NAVD 88 (North American Vertical Datum of 1988). Current official vertical datum of the US. Replaced *NGVD 29* in 1991. See Zilkoski *et al.* (1992) for details.

NDGPS (National Differential GPS). A nation-wide system of "beacons" (permanently mounted *GPS* receivers and radio transmission equipment) that transmits real-time *differential corrections* which can be used by *GPS* receivers equipped with the appropriate radio receivers. Operated and maintained by the US Coast Guard. See US Coast Guard (2005) for details.

NGS (National Geodetic Survey). Federal agency within the Department of Commerce responsible for defining, maintaining, and providing access to the *NSRS* within the US and its territories.

NGVD 29 (National Geodetic Vertical Datum of 1929). Previous *vertical datum* of the US, superseded by *NAVD 88* in 1991. Not referenced to the *geoid* and called "Mean Sea Level" (MSL) datum prior to 1976.

NSRS (National Spatial Reference System). The framework for latitude, longitude, height, scale, gravity, orientation and shoreline throughout the US. Consists of geodetic control point coordinates and sets of models describing relevant geophysical characteristics of the Earth, such as the *geoid* and surface gravity. Defined and maintained by the *NGS* (see Doyle, 1994, for details).

NSSDA (National Standard for Spatial Data Accuracy). *FGDC* methodology for determining the positional accuracy of spatial data (see Federal Geographic Data Committee, 1998).

OPUS (Online Positioning User Service). A free *NGS* service that computes *NSRS* and *ITRF/IGS* coordinates with respect to the *CORS* using raw *GPS* data submitted via the Internet.

Orthometric correction. A correction applied to leveled height differences which reduces systematic errors due to non-parallel gravity equipotential surfaces.

Parts per million (ppm). A method for conveniently expressing small numbers, accomplished by multiplying the number by 1 million (e.g., 0.00001 = 10 ppm). Exactly analogous to percent, which is "parts per hundred."

SPCS (State Plane Coordinate System). A system of standardized *map projections* covering each state with one or more zones such that a specific distortion criterion is met with respect to the *ellipsoid* (usually 1:10,000). Can be referenced to either the *NAD 83* or *NAD 27* datums (SPCS 83 and SPCS 27, respectively). See Stem (1990) for details.

Triangulation. A method for determining positions from angles measured between points (requires at least one distance to provide scale).

Trilateration. A method for determining positions from measured distances only.

Trivial vector. A *GPS* vector (computed line connecting two *GPS* stations) that is not statistically independent from other *GPS* vectors determined at the same time.

US Survey Foot. Linear unit of the US prior to 1959, and defined such that one foot equals exactly 1200 / 3937 meter. Longer than the *International Foot* by 2 *parts per million* (ppm).

UTM (Universal Transverse Mercator). A grid coordinate system based on the Transverse Mercator *map projection* which divides the Earth (minus the polar regions) into 120 zones in order to keep map scale error within 1:2500 with respect to the *ellipsoid*. Can be referenced to either the *NAD 83* or *NAD 27* datums (UTM 83 and UTM 27, respectively). See NGA (2014b and 2014c) for details.

Vertical datum. Reference system for determining "elevations", typically through optical leveling. Modern vertical datums typically use the *geoid* as a reference surface and allow elevation determination using *GPS* when combined with a *geoid* model.

WAAS (Wide Area Augmentation System). A system of geosynchronous satellites and ground *GPS* reference stations developed and managed by the Federal Aviation Administration and used to provide free real-time *differential corrections*. See Federal Aviation Administration (2005) for details.

WGS 84 (World Geodetic System of 1984). Reference *ellipsoid* and *geometric datum* of *GPS*, defined and maintained by the National Geospatial-intelligence Agency (NGA). Recent realizations of WGS 84 are aligned with *ITRF* and can be considered equivalent at the cm level. See NGA (2014a) for details.

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Control survey marks and datasheets: www.ngs.noaa.gov/datasheets
The Geodetic Tool Kit: www.ngs.noaa.gov/TOOLS
Online Positioning User Service (OPUS): www.ngs.noaa.gov/OPUS
Continuously Operating Reference Stations (CORS): www.ngs.noaa.gov/CORS
The Geoid Page: www.ngs.noaa.gov/GEOID
NGS State Geodetic Advisors: www.ngs.noaa.gov/ADVISORS

Pacific Northwest Regional Geodetic Advisor (Oregon, Washington, and Idaho):

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