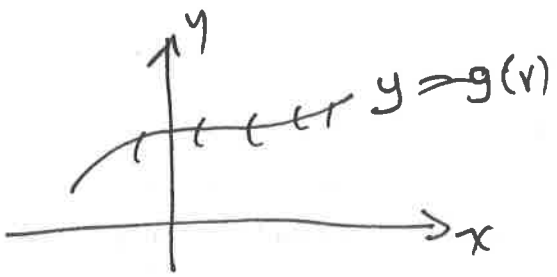
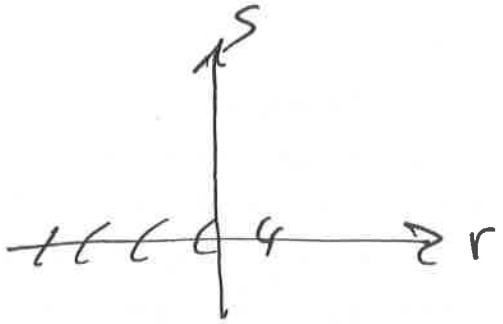
Math 4315 - PDEs

$u(x, g(x)) = f(x)$





$s = 0, \quad x = r, \quad y = g(r) \quad u = f(r)$

Recall from Calc 3

$u = F(x, y) \qquad x = f(r), \quad y = g(r)$

$$\frac{du}{dr} = \frac{\partial u}{\partial x}\frac{dx}{dr} + \frac{\partial u}{\partial y}\frac{dy}{dr} \qquad \rightarrow \quad u = U(r)$$

$$= p \cdot 1 + q \cdot g'(r) = \frac{dU}{dr}$$

So $\quad x = r, \quad y = r \quad u(x, x) = 1$

$$\frac{du(x,x)}{dr} = \frac{d(1)}{dr} = 0$$

$$p + q = 0$$

Ex1 $u_x - u_y^2 = 2x$ $u(x,-x) = x^2$

So $F = p - q^2 - 2x$

$F_x = -2,\ F_y = 0,\ F_u = 0,\ F_p = 1,\ F_q = -2q$

CF $x_s = 1$

$y_s = -2q$

$u_s = p - 2q^2$

$p_s = -(-2+0) = 2$

$q_s = 0$

$s = 0$

$x = r$

$y = -r$

$u = r^2$

$p - q = 2r$

$p - q^2 = 2r$

$\Rightarrow q^2 - q = 0$

$q = 0 \quad p = 2r$

$q = 1 \quad p = 2r+1$

two cases

Case 1 $q = 0,\ p = 2r$

$\because q_s = 0 \quad q = e(r) \quad s = 0 \ q = 0 \Rightarrow q \equiv 0$ always

So $u_x = 2x \Rightarrow u = x^2 + c$

$u(x,-x) = x^2 \Rightarrow c = 0 \Rightarrow u(x,y) = x^2$

Case 2 $\quad q=1,\ p=2r+1$

$s=0$

$x_s=1 \qquad x=r$

$y_s=-2q \qquad y=r$

$u_s=p-2q \qquad u=r^2$

$p_s=2 \qquad p=2r+1$

$q_s=0 \qquad q=1$

(1) $q_s=0 \quad q=e(r) \quad s=0,\ q=1 \Rightarrow e(r)=1$ $\boxed{q=1}$

(2) $x_s=1 \quad x=s+a(r) \quad s=0 \ x=r \Rightarrow a(r)=r$ $\boxed{x=s+r}$

*1 $y_s=-2q \quad y=-2s+b(r) \quad s=0,\ y=-r \Rightarrow b=-r$

$=-2$

$\therefore \boxed{y=-2s-r}$

*2 $p_s=2 \quad p=2s+d(r) \quad s=0 \ p=2r+1 \Rightarrow d(r)=2r+1$

$\Rightarrow \boxed{p=2s+2r+1}$

(3) $u_s=p-2q=2s+2r+1-2=2s+2r-1$

$u=s^2+(2r-1)s+c(r) \quad s=0 \ u=r^2$

$\Rightarrow c(r)=r^2$

$\boxed{u=s^2+(2r-1)s+r^2}$

$=(s^2+2rs+r^2)-s=(s+r)^2-s \Rightarrow \boxed{x^2+x+y=u}$

Ex2 $u_x u_y = u$ $\quad u(x, \frac{1}{x}) = 1$

$F = pq - u$

$F_x = 0 \quad F_y = 0 \quad F_u = -1, \quad F_p = q \quad F_q = p$

CE $\qquad s = 0$

$x_s = q \qquad x = r$

$y_s = p \qquad y = \frac{1}{r}$

$u_s = 2pq = 2u \qquad u = 1$

$p_s = p \qquad p = \frac{1}{r}$

$q_s = q \qquad q = r$

$p - \frac{q}{r^2} = 0$

$pq = 1$

$q = r^2 p$

$\Rightarrow r^2 p^2 = 1 \Rightarrow p = \pm \frac{1}{r}$

Case (i) +ve $\qquad q = \pm r$

(1) $p_s = p \Rightarrow p = a(r) e^s \qquad s = 0 \quad p = \frac{1}{r} \Rightarrow a(r) = \frac{1}{r}$

So $\boxed{p = \frac{1}{r} e^s}$

(2) $q_s = q \Rightarrow q = b(r) e^s \qquad s = 0 \quad q = r \Rightarrow b(r) = r$

So $\boxed{q = r e^s}$

(3) $x_s = q = re^s \Rightarrow x = re^s + c(r)$

$s = 0 \quad x = r \Rightarrow c(r) = 0$

$$\boxed{x = re^s}$$

(4) $y_s = p = \frac{1}{r}e^s \Rightarrow y = \frac{1}{r}e^s + d(r)$

$s = 0, \; y = \frac{1}{r} \Rightarrow d(r) = 0$

$$\boxed{y = \frac{1}{r}e^s}$$

(5) $u_s = 2u \Rightarrow u = e(r)e^{2s}$

$s = 0 \quad u = 1 \Rightarrow e(r) = 1$

$$\boxed{u = e^{2s}}$$

Now $xy = re^s \cdot \frac{1}{r}e^s \Rightarrow$ $\boxed{u = xy}$ Sol^n

$= e^{2s}$

Second Sol^n $u = (\sqrt{xy} - 2)^2$