Edexcel GCE

Core Mathematics C4

Silver Level S5

(Mark Scheme)

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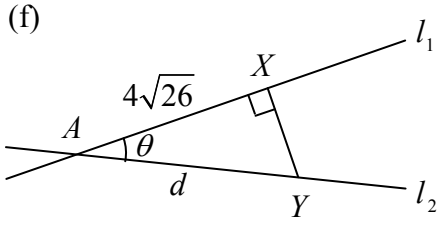
Question Number	Scheme	Marks
1. (a)	$\left\{\cancel{\frac{dy}{dx}} \times \cancel{\frac{dx}{dy}}\right\}$ $\underline{2+6y\frac{dy}{dx}}+\underline{\underline{\left(6xy+3x^2\frac{dy}{dx}\right)}}=\underline{\underline{8x}}$	M1 A1 B1
	$\left\{\frac{dy}{dx}=\frac{8x-2-6xy}{6y+3x^2}\right\}$ *not necessarily required.*	
	At $P(-1,1)$, $m(\mathbf{T})=\frac{dy}{dx}=\frac{8(-1)-2-6(-1)(1)}{6(1)+3(-1)^2}=-\frac{4}{9}$	dM1 A1 **cso** **[5]**
(b)	So, $m(\mathbf{N})=\frac{-1}{-\frac{4}{9}}\left\{=\frac{9}{4}\right\}$	M1
	$\mathbf{N}$: $y-1=\frac{9}{4}(x+1)$	M1
	$\mathbf{N}$: $9x-4y+13=0$	A1 **[3]** **(8 marks)**

Question Number	Scheme	Marks
2.	(a) $y\left(\frac{\pi}{6}\right)\approx 1.2247,\ y\left(\frac{\pi}{4}\right)=1.1180$ accept awrt 4 d.p.	B1 B1 **(2)**
	(b)(i) $I\approx\left(\frac{\pi}{12}\right)(1.3229+2\times 1.2247+1)$ B1 for $\frac{\pi}{12}$	B1 M1
	≈ 1.249 cao	A1
	(ii) $I\approx\left(\frac{\pi}{24}\right)(1.3229+2\times(1.2973+1.2247+1.1180)+1)$ B1 for $\frac{\pi}{24}$	B1 M1
	≈ 1.257 cao	A1 **(6)** **[8]**

Question Number	Scheme	Marks
3. (a)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 2\sqrt{3}\cos 2t$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}t} = -8\cos t \sin t$	M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-8\cos t \sin t}{2\sqrt{3}\cos 2t}$	M1
	$= -\dfrac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\dfrac{2}{3}\right)$	A1 **(5)**
(b)	When $t = \dfrac{\pi}{3}$ $\quad x = \dfrac{3}{2},\ y = 1$ can be implied	B1
	$m = -\dfrac{2}{3}\sqrt{3}\tan\left(\dfrac{2\pi}{3}\right) \quad (=2)$	M1
	$y - 1 = 2\left(x - \dfrac{3}{2}\right)$	M1
	$y = 2x - 2$	A1 **(4)**
(c)	$x = \sqrt{3}\sin 2t = \sqrt{3} \times 2\sin t \cos t$	M1
	$x^2 = 12\sin^2 t \cos^2 t = 12\left(1 - \cos^2 t\right)\cos^2 t$	
	$x^2 = 12\left(1 - \dfrac{y}{4}\right)\dfrac{y}{4}$ or equivalent	M1 A1 **(3)**
		[12]

Question Number	Scheme	Marks
Q4	(a) A: $(-6, 4, -1)$ Accept vector forms	B1 (1)
	(b) $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}.\begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} = 12+4+3 = \sqrt{4^2+(-1)^2+3^2}\sqrt{3^2+(-4)^2+1^2}\cos\theta$	M1 A1
	$\cos\theta = \frac{19}{26}$ awrt 0.73	A1 (3)
	(c) X: $(10, 0, 11)$ Accept vector forms	B1 (1)
	(d) $\overrightarrow{AX} = \begin{pmatrix} 10 \\ 0 \\ 11 \end{pmatrix} - \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}$ Either order	M1
	$= \begin{pmatrix} 16 \\ -4 \\ 12 \end{pmatrix}$ cao	A1 (2)
	(e) $\left\lvert\overrightarrow{AX}\right\rvert = \sqrt{16^2+(-4)^2+12^2}$	M1
	$= \sqrt{416} = \sqrt{16\times 26} = 4\sqrt{26}$ ✱ Do not penalise if consistent incorrect signs in (d)	A1 (2)
	(f)  Use of correct right angled triangle	M1
	$\frac{\left\lvert\overrightarrow{AX}\right\rvert}{d} = \cos\theta$	M1
	$d = \frac{4\sqrt{26}}{\frac{19}{26}} \approx 27.9$ awrt 27.9	A1 (3)
		[12]

5.	(*a*)	Lines meet where: $\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$	
		$\mathbf{i}: -9+2\lambda=3+3\mu \quad (1)$	
		Any two of $\mathbf{j}: \lambda = 1-\mu \quad (2)$	M1
		$\mathbf{k}: 10-\lambda = 17+5\mu \quad (3)$	
		(1) – 2**(2)** gives: $-9=1+5\mu \Rightarrow \mu=-2$	M1
		(2) gives: $\lambda = 1--2=3$	A1
		$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$	M1
		Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r}=\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$	A1
		Either check **k:** $\lambda = 3: \text{LHS} = 10-\lambda = 10-3=7$ $\mu=-2: \text{RHS} = 17+5\mu = 17-10=7$	B1 (6)
	(*b*)	$\mathbf{d}_1 = 2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{d}_2 = 3\mathbf{i}-\mathbf{j}+5\mathbf{k}$	
		As $\mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix} = \underline{(2\times3)+(1\times-1)+(-1\times5)} = 0$	M1 A1 (2)
		Then l_1 is perpendicular to l_2.	
	(*c*)	Equating **i** ; $-9+2\lambda=5 \Rightarrow \lambda=7$	
		$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+7\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}5\\7\\3\end{pmatrix}$ ($=\overrightarrow{OA}$. Hence the point A lies on l_1.)	B1 (1)
	(*d*)	Let $\overrightarrow{OX} = -3\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ be point of intersection	
		$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}} = \begin{pmatrix}-8\\-4\\4\end{pmatrix}$	M1 ft
		$\overrightarrow{OB} = \overrightarrow{OA}+\overrightarrow{AB} = \overrightarrow{OA}+2\overrightarrow{AX}$	
		$\overrightarrow{OB} = \begin{pmatrix}5\\7\\3\end{pmatrix}+2\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	M1 ft
		Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i}-\mathbf{j}+11\mathbf{k}}$	A1
			(12 marks)

Question Number	Scheme		Marks
6. (a)	$\left.\begin{matrix} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x & \Rightarrow & v = \frac{1}{2}\sin 2x \end{matrix}\right\}$		
		(see note below)	
	$\text{Int} = \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x.1 \, dx$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= \frac{1}{2}x\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + c$	$\sin 2x \to -\frac{1}{2}\cos 2x$ or $\sin kx \to -\frac{1}{k}\cos kx$ with $k \neq 1$, $k > 0$	dM1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	Correct expression with $+c$	A1
			[4]
(b)	$\int x\cos^2 x \, dx = \int x\left(\frac{\cos 2x + 1}{2}\right) dx$	Substitutes **correctly** for $\cos^2 x$ in the given integral	M1
	$= \frac{1}{2}\int x\cos 2x \, dx + \frac{1}{2}\int x \, dx$		
	$= \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x \, dx$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2 \ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
			7 marks

7.	(*a*)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
		$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
	(*b*)	$x = 8\cos t, \quad y = 4\sin 2t$	
		$\frac{\mathrm{d}x}{\mathrm{d}t} = -8\sin t, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = 8\cos 2t$	M1 A1
		At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	M1
		$\left\{ = \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$	
		Hence m(**N**) = $-\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	M1
		N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	M1
		N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ (*)	A1 cso (6)
	(*c*)	$A = \int_0^4 y\,\mathrm{d}x = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,\mathrm{d}t$	M1 A1
		$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,\mathrm{d}t = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,\mathrm{d}t$	M1
		$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,\mathrm{d}t$	
		$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,\mathrm{d}t$ (*)	A1 (4)
	(*d*)	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	M1 A1
		$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	M1
		$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	A1 (4)
			(16 marks)

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	8		85	6.76	7.85	7.44	6.93	6.36	5.46	4.68	1.82
2	8		82	6.53	7.82	7.40	6.87	6.17	5.25	4.15	2.35
3	12		62	7.45	11.14	9.47	7.67	5.80	4.01	2.40	1.00
4	12		59	7.02		9.02	6.07	4.47	3.15	2.00	0.77
5	12		57	6.81		9.07	7.04	5.25	3.60	2.26	0.94
6	7		54	3.76		5.57	3.77	2.40	1.30	0.65	0.25
7	16		48	7.73		11.16	7.39	5.28	3.41	2.15	1.08
	75		**61**	**46.06**		**59.13**	**45.74**	**35.73**	**26.18**	**18.29**	**8.21**