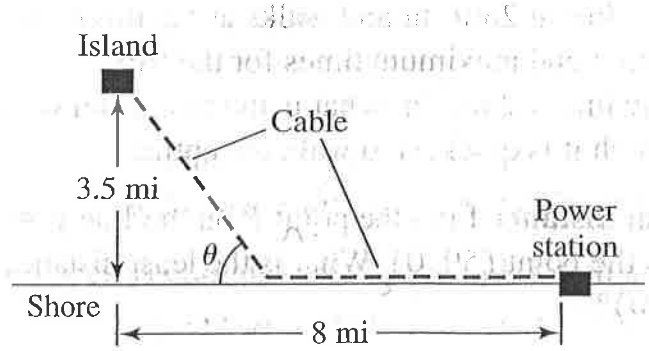
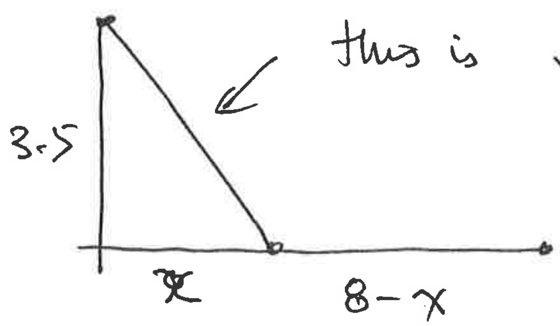
Math 1496 – Calc I

37. Laying cable An island is 3.5 mi from the nearest point on a straight shoreline; that point is 8 mi from a power station (see figure). A utility company plans to lay electrical cable underwater from the island to the shore and then underground along the shore to the power station. Assume that it costs \$2400/mi to lay underwater cable and \$1200/mi to lay underground cable. At what point should the underwater cable meet the shore to minimize the cost of the project?



So first we draw a picture & label



this is $\sqrt{x^2+3.5^2}$

The cost is $C = 2400\sqrt{x^2+3.5^2} + 1200(8-x)$

$$C' = \frac{2400(2x)}{2\sqrt{x^2+3.5^2}} - 1200$$

we set this $= 0$ so

$$\frac{2400x}{\sqrt{x^2+3.5^2}} = 1200$$

or $2x = \sqrt{x^2+3.5^2} \Rightarrow 4x^2 = x^2+3.5^2$

$$\Rightarrow 3x^2 = 3.5^2 \Rightarrow x = \frac{3.5}{\sqrt{3}} = \frac{7\sqrt{3}}{6}$$

Now the second derivative test

$$C'' = 2400\left[\frac{1\sqrt{x^2+3.5^2} - \frac{x\,(2x)}{2\sqrt{x^2+3.5^2}}}{x^2+3.5^2}\right]$$

$$= 2400\,\frac{(x^2+3.5^2-x^2)}{(x^2+3.5)\sqrt{x^2+3.5^2}}$$

$$= \frac{2400\,(3.5)^2}{(x^2+3.5^2)^{3/2}}$$

Now $C''>0$ when $x = \frac{7\sqrt{3}}{6}$ so a min