Math 1497 – Calc 2

Review of some Calc 1

limits

Falling Bodies

$s(t) = 16t^2$ $\quad t$ sec

$s(t)$ position ft

what is the instantaneous velocity at $t=1$

over $[1,2]$ $\quad V_{ave} = \dfrac{s(2)-s(1)}{2-1} = \dfrac{16(2)^2 - 16(1)^2}{2-1} = 48$

$[1,1.1]$ $\quad V_{ave} = \dfrac{s(1.1)-s(1)}{1.1-1} = \dfrac{16(1.1)^2-16}{.1}$

$= 33.6$

$[1,1.01]$ $\quad V_{ave} = 32.16$ ft/sec

$$\lim_{\Delta t \to 0} \frac{s(t+\Delta t)-s(t)}{\Delta t} = \frac{16(t+\Delta t)^2 - 16t^2}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{\cancel{16t^2} + 32\Delta t \cdot t + 16\Delta t^2 - \cancel{16t^2}}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{\cancel{\Delta t}(32t + 16\Delta t)}{\cancel{\Delta t}} = 32t$$

$V(t) = 32t$

reintroduction of limit – "getting close"

<u>Ex.</u> $f(x) = \dfrac{x^2 - 1}{x - 1}$

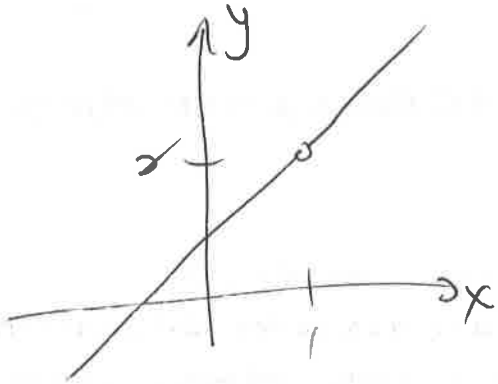
problem at $x = 1$

(1) table

x	.9	.99	1	1.01	1.1
f	1.9	1.99	?	2.01	2.1

table say $\displaystyle\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$

Graph



Graph say $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$

Analytical

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$$

$$\lim_{x\to 1} x+1 = 2$$

All say the same

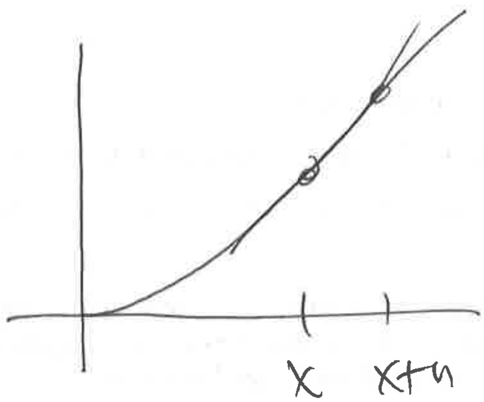
In general

$$\lim_{x\to a} f(x) = L$$

δ ε – won't do

Tangent line

$f(x) = x^2$



slope secant

$$\frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \frac{(2x+h)\cancel{h}}{\cancel{h}}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \text{slope tangent}$$

$$\lim_{h \to 0} 2x + h = 2x$$

at $x = 1$ slope $= 2$ pt $(1, 1^2)$

tangent $y - 1 = 2(x - 1)$

Derivative $y = f(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

notation $y', f', \frac{dy}{dx}, \frac{df}{dx}$

many properties of limits

Also limits at infinity & infinite limits
- talk more tomorrow

Continuity

Derivative Rules given $y = f(x)$ $y = g(x)$
c const

$(f \pm g)' = f' \pm g'$

$(cf)' = cf'$

$(fg)' = f'g + fg'$ prod.

$(f/g)' = \frac{gf' - fg'}{g^2}$ quot

Standard Derivatives - handout

Chain Rule - Composite functions

Ex $y = \sqrt{x^2+1}$

let $u = x^2+1$, so $y = \sqrt{u} = u^{1/2}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$ each one easy

$$= \frac{1}{2} u^{-1/2} \cdot 2x = \frac{x}{\sqrt{u}} = \frac{x}{\sqrt{x^2+1}}$$

or $y = f(g(x))$ $y' = f'(g(x)) \cdot g'(x)$

we prefer the "u" way

Implicit Differentiation

Ex $x^2 + 2xy + y^4 = y^2$

$$2x + 2(xy' + y) + 4y^3 y' = 2y'$$

$$(2x + 4y^3 - 2)y' = -2x - 2y$$

$$y' = \frac{-2x-2y}{2x+4y^3-2}$$

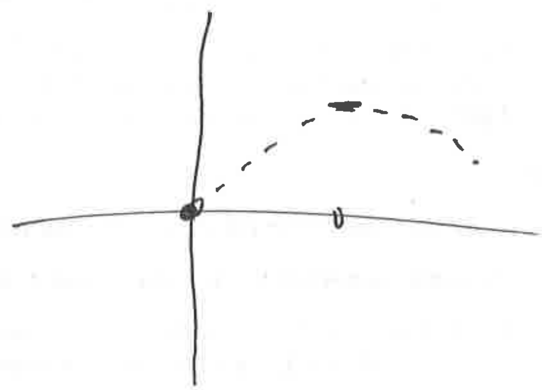
Application

(1) Related Rates

(2) Optimization Problems

using Calc. we can ~~app~~ explore how graphs change.

Ex $f(x) = xe^{-x}$



$f(0) = 0$

Now what?

$f'(x) = 1 \cdot e^{-x} - xe^{-x}$

$= (1-x)e^{-x}$

$f'(x) = 0$ when $x = 1$

$f(1) = e^{-1} > 0$

what happens when we pass $x=1$

x		1	
$1-x$	$+$	0	$-$
e^{-x}	$+$	$+$	$+$
$(1-x)e^{-x}$	$+$	0	$-$
slope	/	—	\

second derative

- concavity concave up $\cup$

concave down $\cap$

$$f'' = -1e^{-x} - (1-x)e^{-x}$$

$$= -(2-x)e^{-x}$$

$$= (x-2)e^{-x}$$

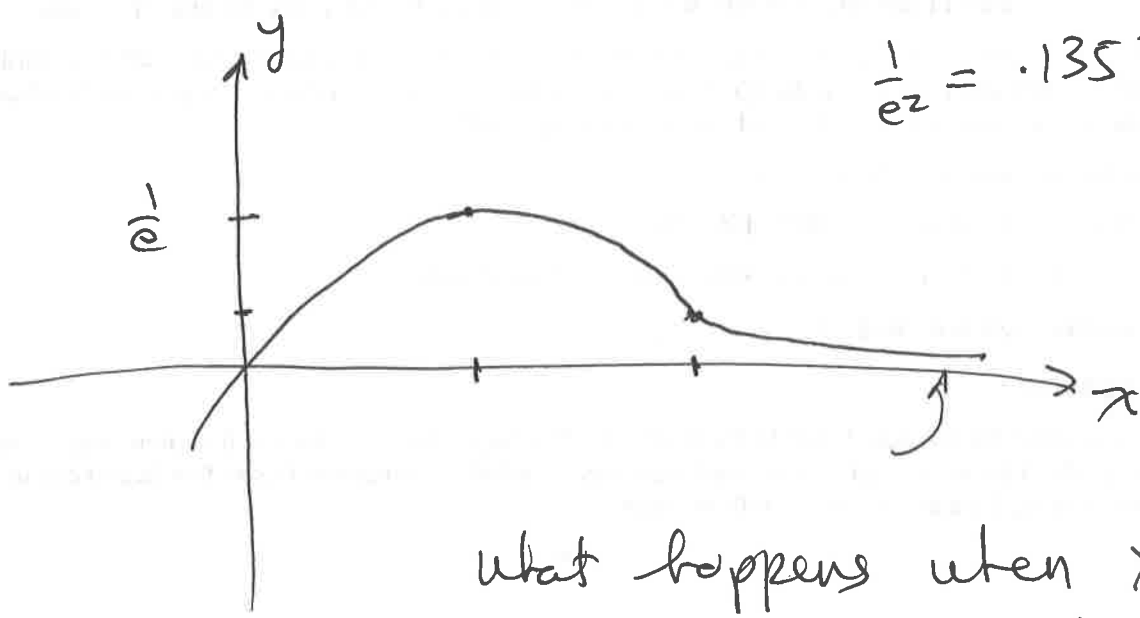
x		2	
$x-2$	$-$	0	$+$
$(x-2)e^{-x}$	$-$	0	$+$
conc	$\cap$	pi	$\cup$

change in concavity

- point of inflection

$\frac{1}{e} = .3679$

$\frac{1}{e^2} = .1353$



what happens when $x \to \infty$

tomorrow!