

# The Five Essentials for Math Intervention for Students Struggling in Math





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Introduce yourself.

Describe your role as an educator or caregiver.

Describe the mathematics you support.



Share fun things from tonight and tag @sarahpowellphd!





# MATH INTERVENTION

For students  
experiencing  
math  
difficulty

With a  
school-  
identified  
disability

Persistent  
math  
difficulty

Tier 2

Tier 3

Secondary

Targeted

Intensive

Special Education





# Why is mathematics intervention necessary?





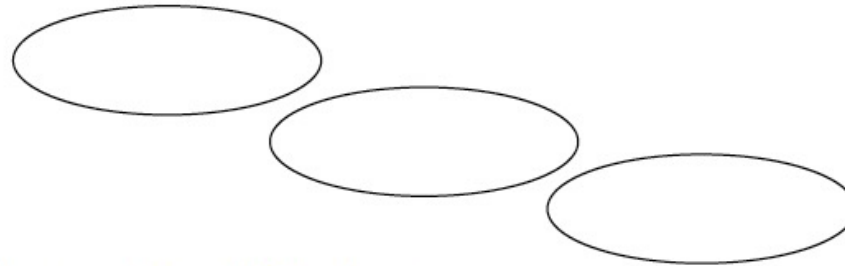
# The Five Essentials for Math Intervention for Students Struggling in Math

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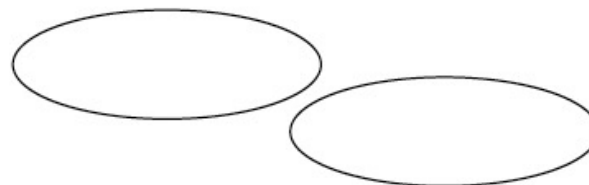
Why is math intervention necessary?

Instructional Platform

Instructional Delivery



Instructional Strategies

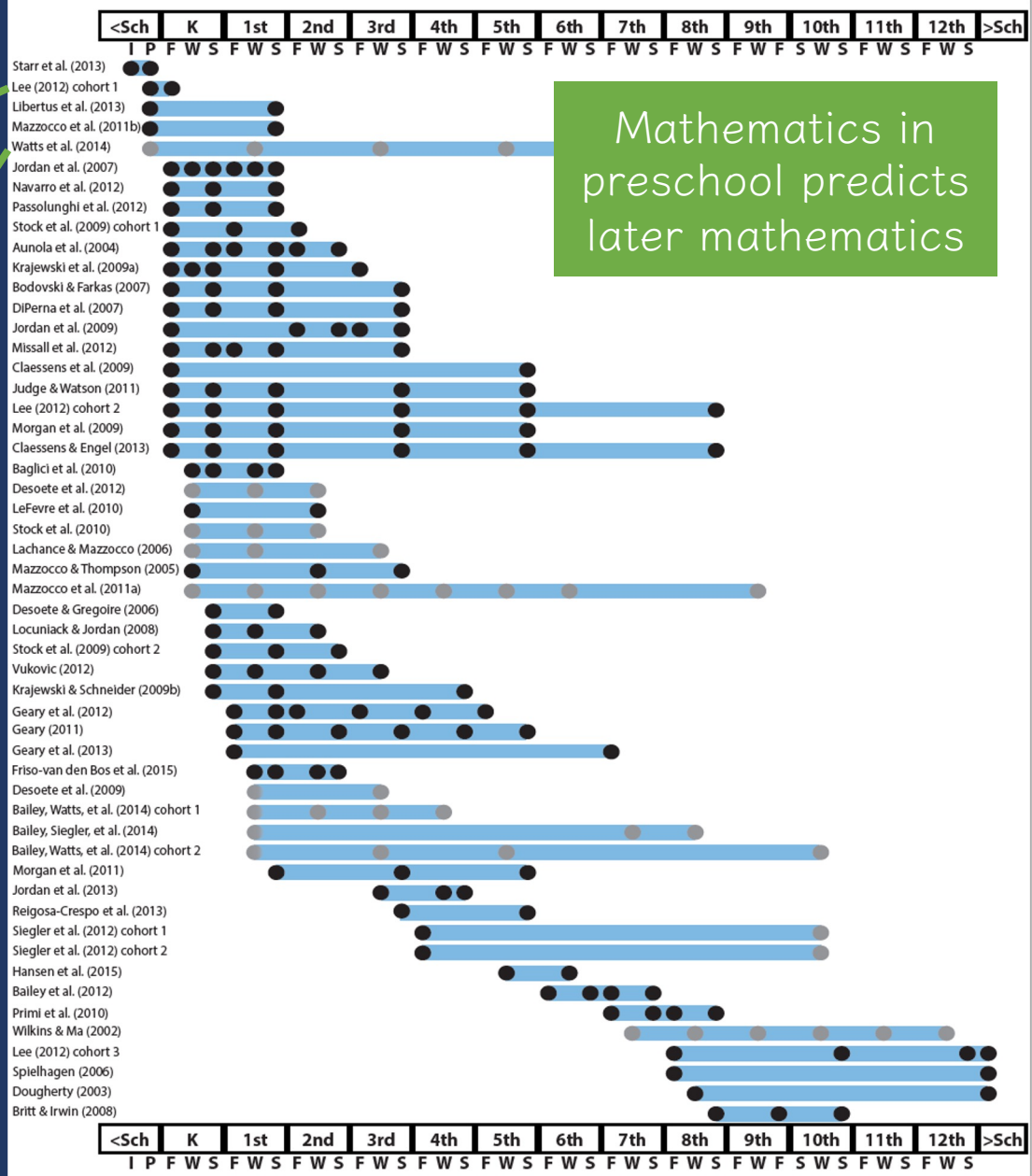




Broad math in preK  
predicted K broad  
math

Broad math in preK  
predicted grade 10  
broad math

Mathematics in  
preschool predicts  
later mathematics







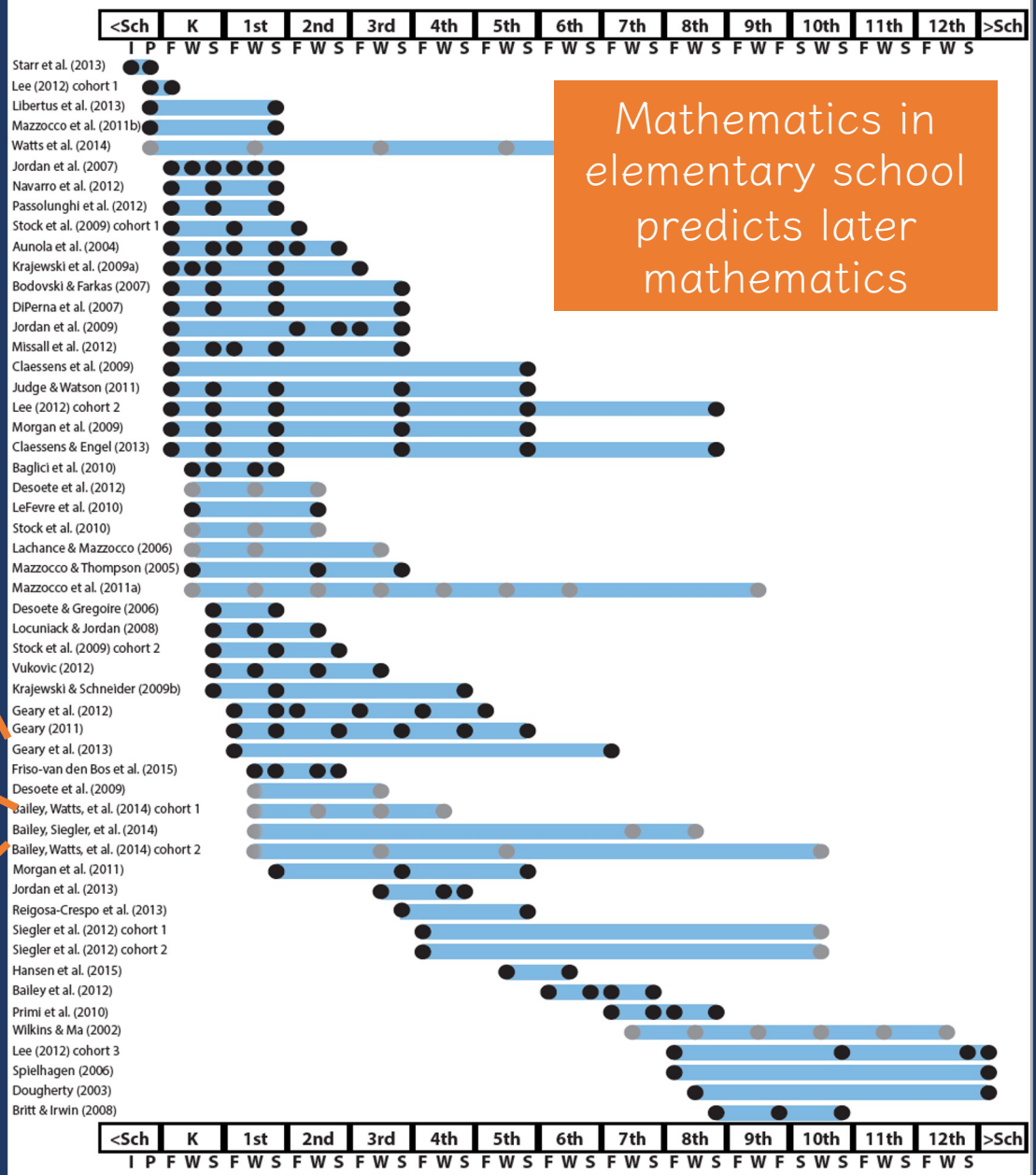


Addition influenced arithmetic with increasing importance from grades 1 to 5

Grade 1 arithmetic predicted arithmetic at grades 2, 3, and 4

Grade 1 broad math predicted broad math at grades 3, 5, and 10

Mathematics in elementary school predicts later mathematics







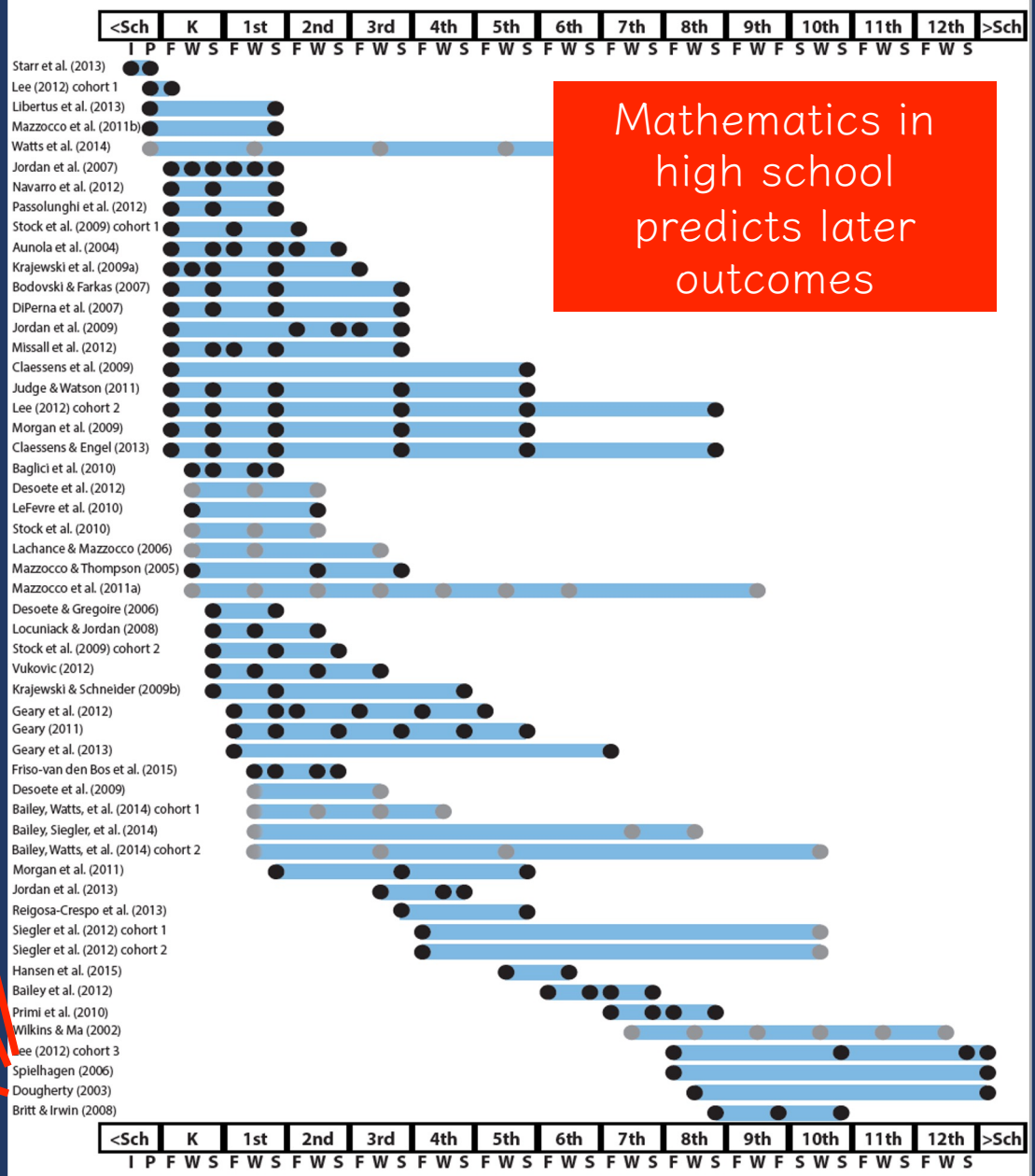


Broad math in grade 8 predicted completion of 4-year college degree

Students who took algebra in grades 8 took more advanced math courses and enrolled in 4-year colleges more often than students who took algebra in grade 9

Numeracy measured in adolescence impacted hourly earnings 7 to 15 years later

Mathematics in high school predicts later outcomes





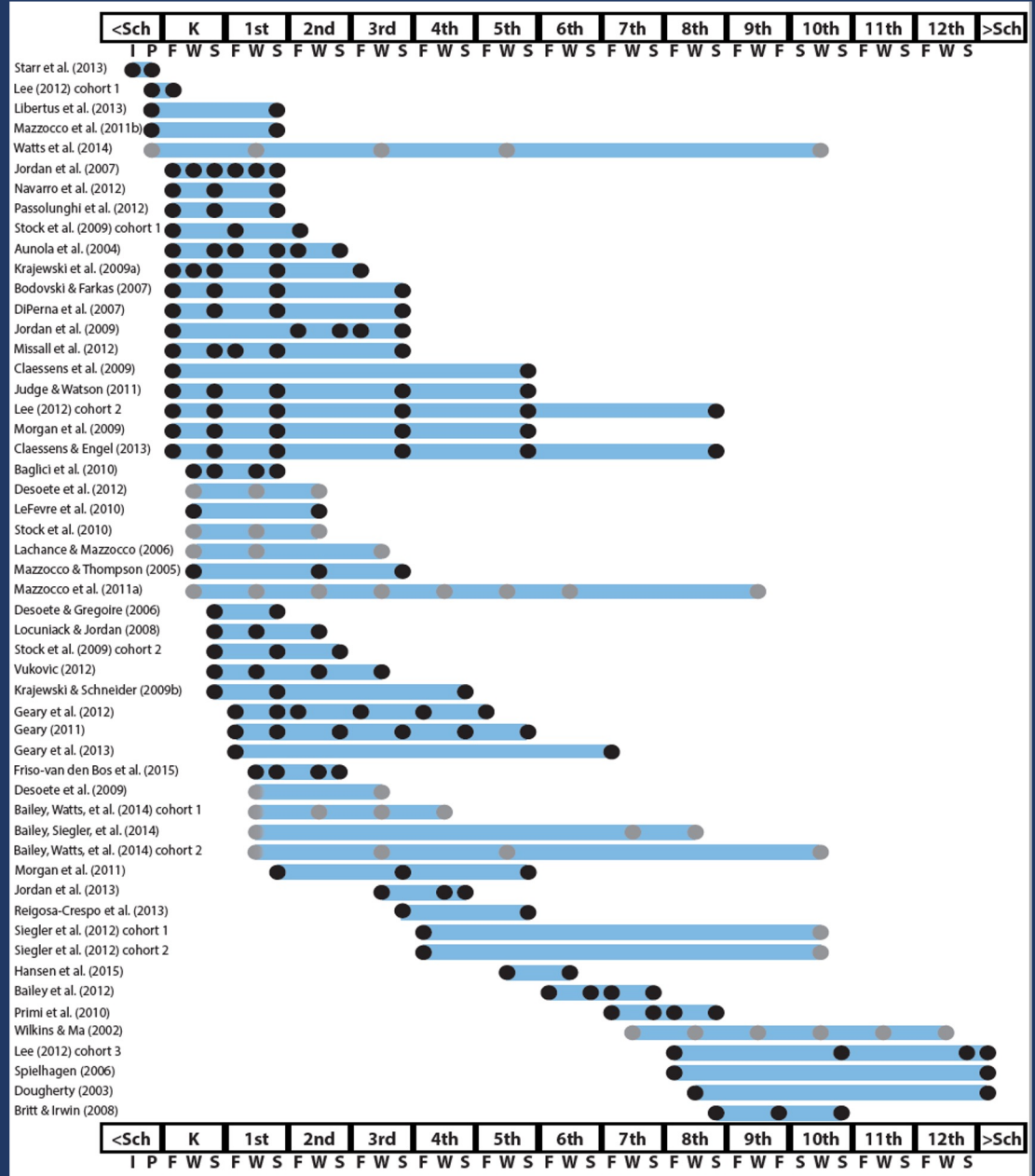
Mathematics in  
preschool predicts  
later mathematics

Mathematics in  
kindergarten  
predicts later  
mathematics

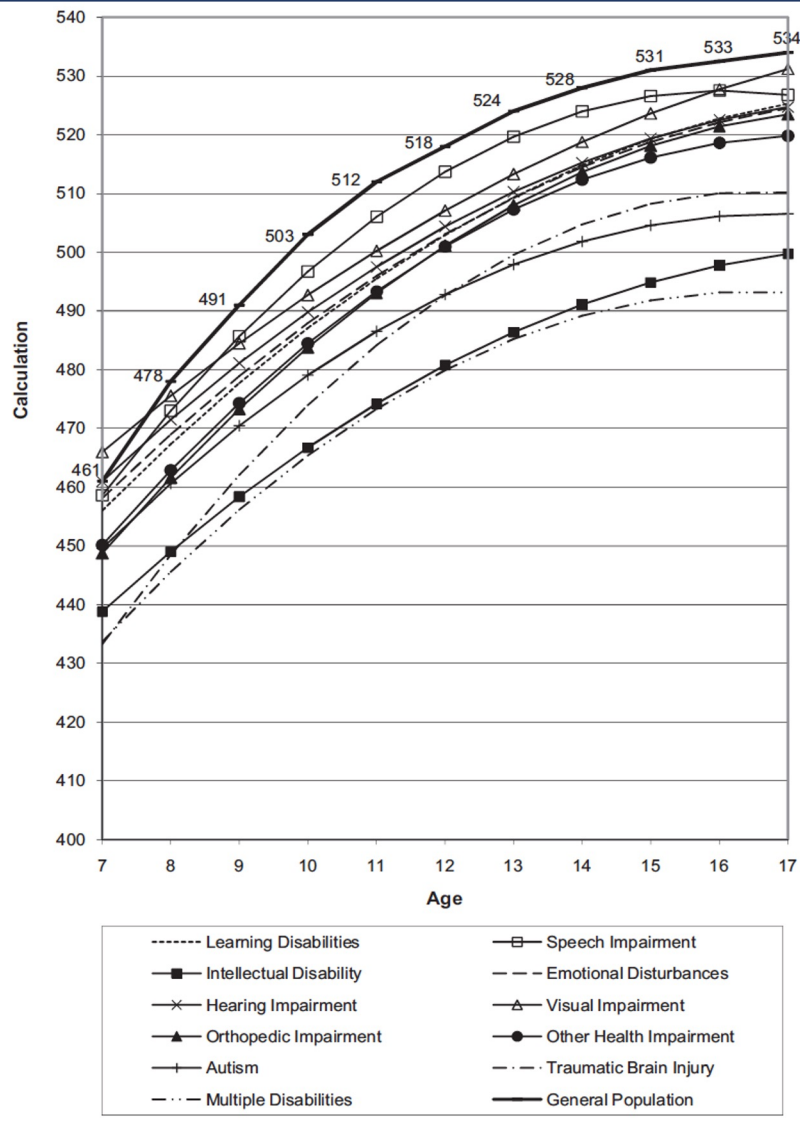
Mathematics in  
elementary school  
predicts later  
mathematics

Mathematics in  
middle school  
predicts later  
mathematics

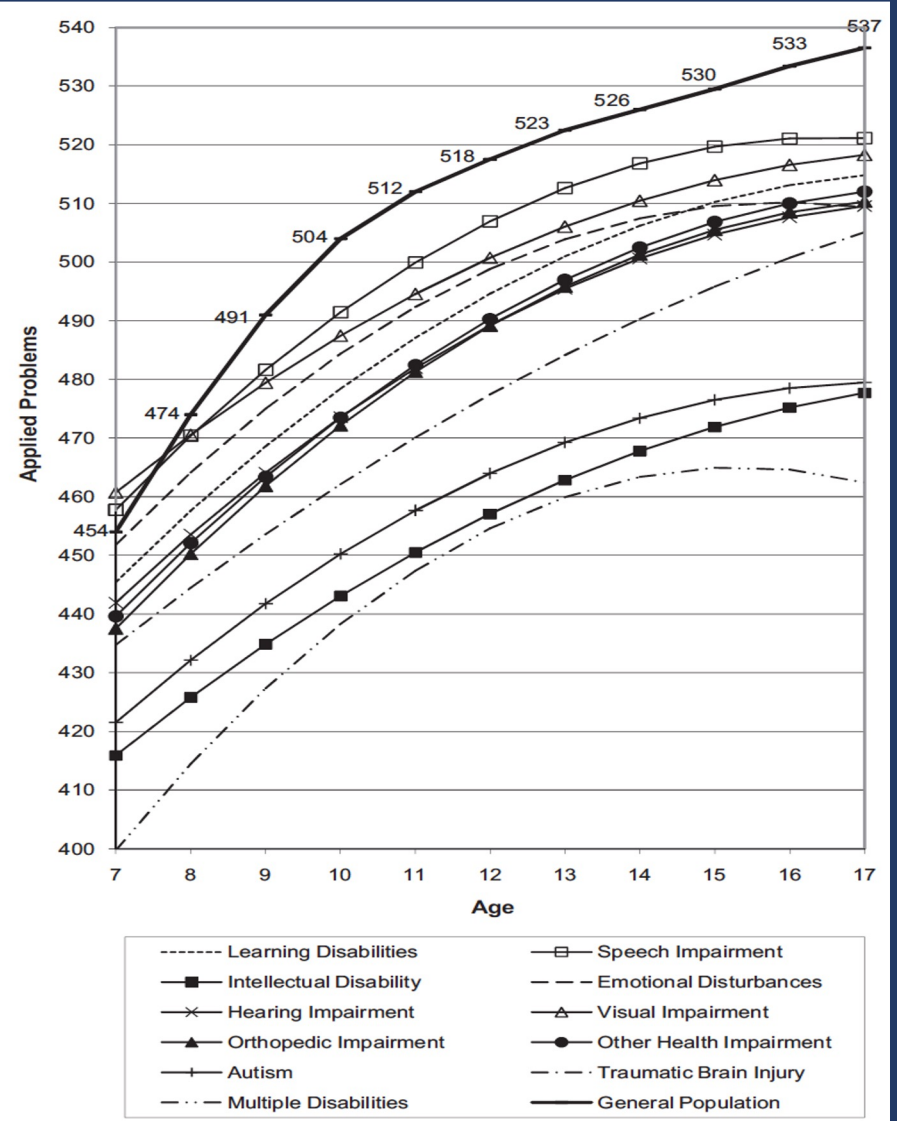
Mathematics in  
high school  
predicts later  
outcomes







## Computation



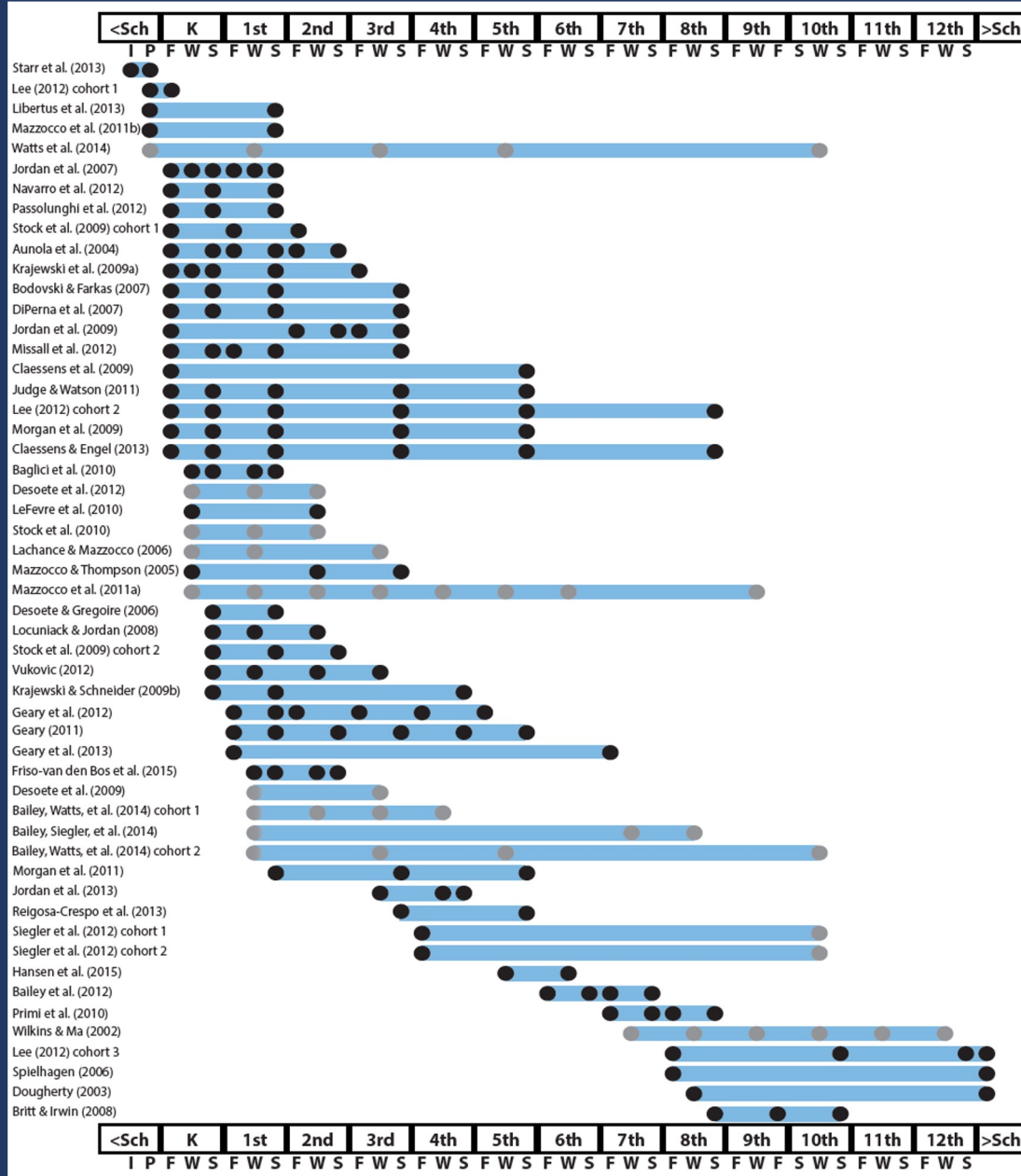
## Problem Solving







With your  
student(s), how  
do you see  
earlier math  
impacting later  
math?





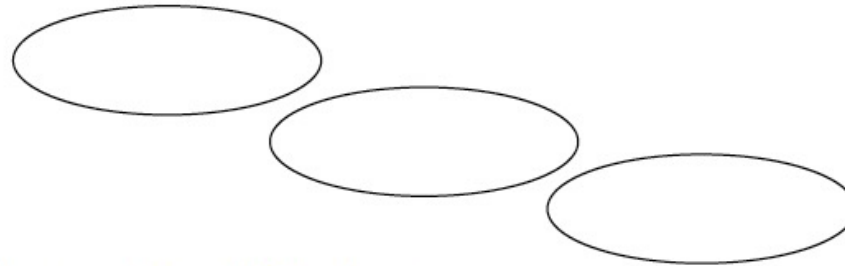
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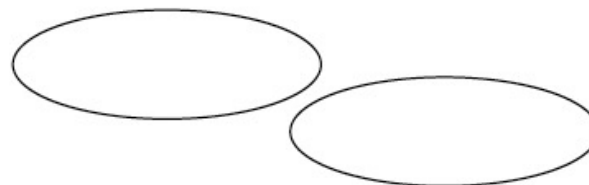
Why is math intervention necessary?

Instructional Platform

Instructional Delivery



Instructional Strategies







# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving  
instruction





## Explicit Instruction

MODELING

PRACTICE

SUPPORTS





## MODELING

Step-by-step  
explanation

Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback





Modeling is  
a dialogue  
between the  
teacher and  
students.

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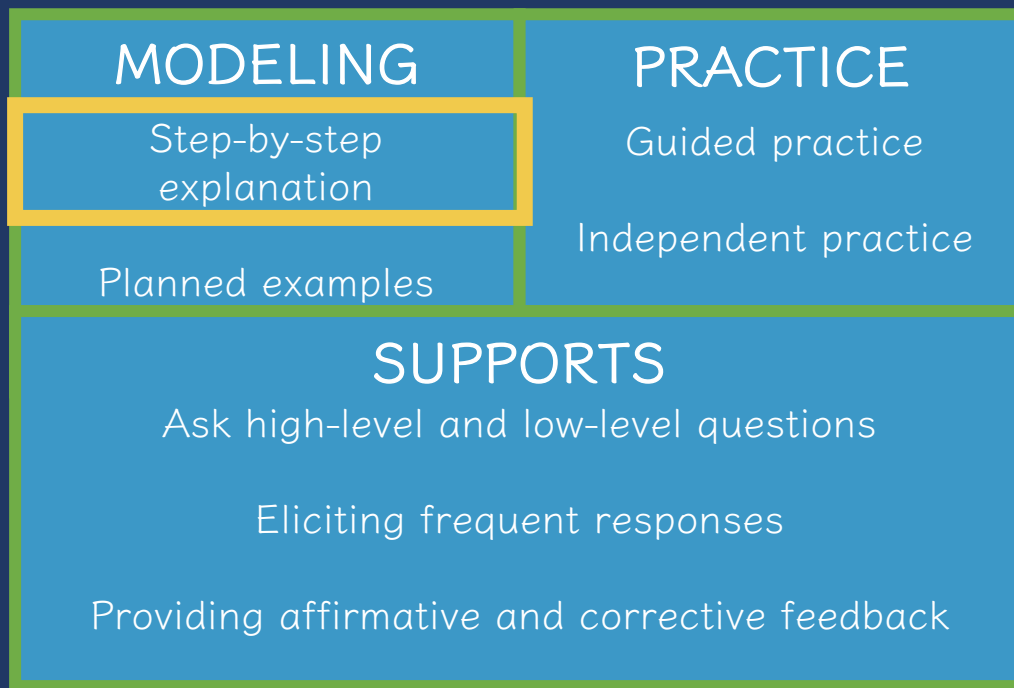
Providing affirmative and corrective feedback





Modeling includes a step-by-step explanation of how to do a mathematical problem.

A teacher may do 1 modeled problem or several.







“Today, we are learning about addition. This is important because sometimes have different amounts – like money – and you want to know how much money you have altogether.”







“Let’s solve this problem. What’s the problem?”

“26 plus 79.”



“To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?”

“Add.”



“How did you know we want to add?”

“There’s a plus sign.”







“The plus sign tells us we want to add. To add, let’s use the partial sums strategy. What strategy?”

“Partial sums.”



“With the partial sums strategy, we start adding in the greatest place value. What’s the greatest place value in this problem?”

“The tens.”



“So, let’s add the tens. What’s 20 plus 70?”

“90.”







"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

"90." 

"90 is the partial sum when you add the tens. What does 90 represent?"

"It's the partial sum of adding 20 plus 70."



"Now, let's add the ones. What should we add?"

"6 plus 9." 







“6 plus 9 equals what?”

“15.” 

“Let’s write 15 below the 90.  
Where do we write the 15?”

“Below the 90.” 

“15 is the partial sum when you add the ones.  
Now, let’s add the partial sums together. What will we add?”

“90 plus 15.” 







“What’s 90 plus 15?”

“How did you add those numbers?”

“So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?”

“105.”



“I added 90 plus 10 then added 5 more.”



“We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums.”





Modeling  
needs to  
include  
planned  
examples.

These  
examples  
should be  
sequenced  
so easier

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What math  
should be  
modeled?

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Step-by-step  
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Ask high-level and low-level questions

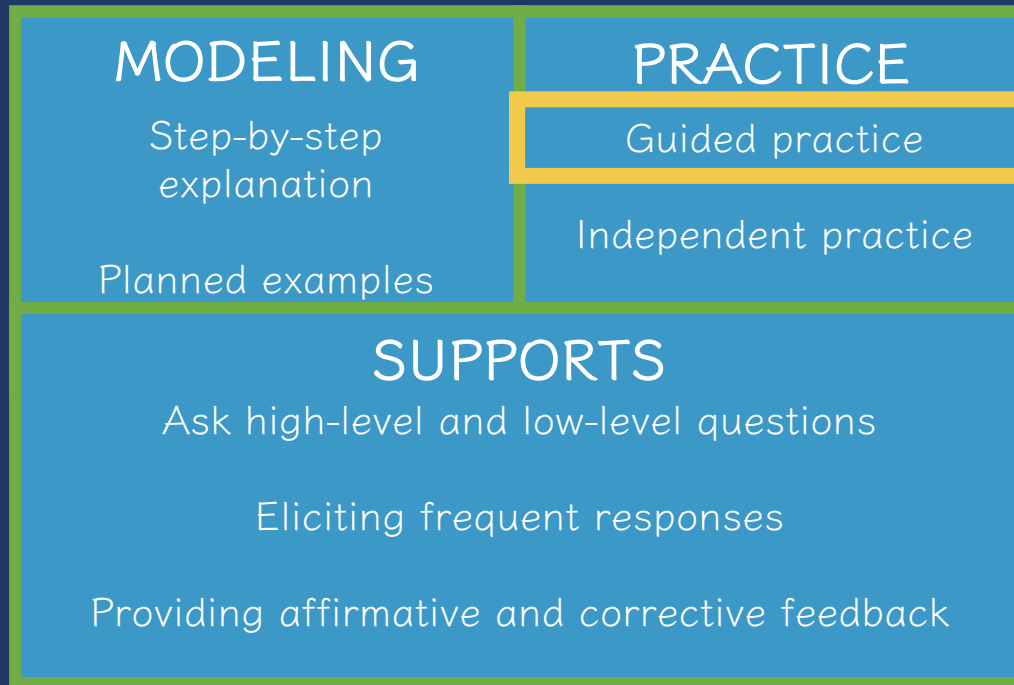
Eliciting frequent responses

Providing affirmative and corrective feedback

Practice continues as a dialogue between the teacher and students.







Guided practice is practice in which the teacher and students practice problems together.



“Let’s work on a problem together.”





## MODELING

Step-by-step  
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Planned examples

## PRACTICE

Guided practice

Independent practice

## SUPPORTS

Ask high-level and low-level questions

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Providing affirmative and corrective feedback

Independent practice is practice in which the students practice independently with teacher support.



“Now, you’ll practice a problem on your own. Use your attack strategy!”





## MODELING

Step-by-step  
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Planned examples

## PRACTICE

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Independent practice

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Ask high-level and low-level questions

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How do you  
engage your  
students in  
guided  
practice?





## MODELING

Step-by-step  
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Eliciting frequent responses

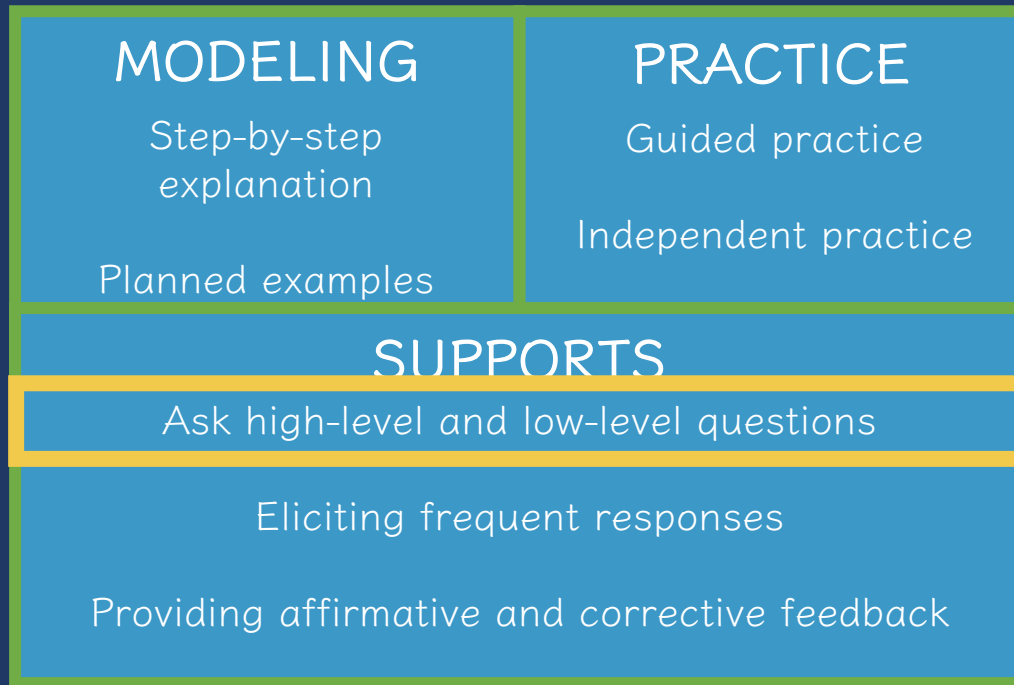
Providing affirmative and corrective feedback

These **Supports** should be used in  
both **Modeling** and **Practice**.



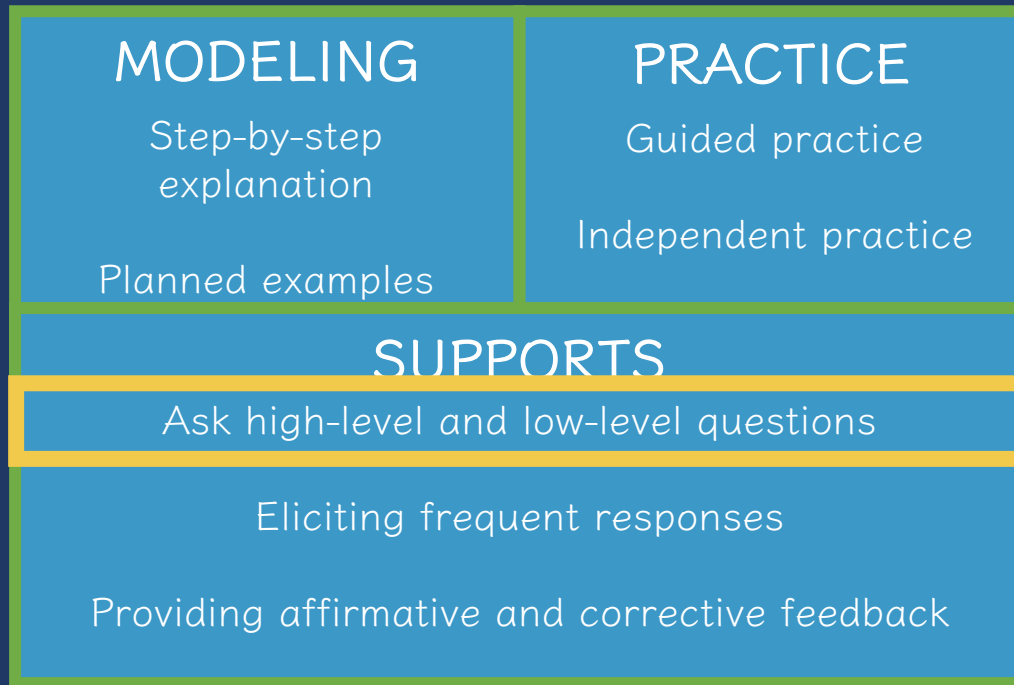


During  
**Modeling and  
Practice**, it is  
essential to  
engage  
students and  
check for  
understanding.





During  
**Modeling** and  
**Practice**, it is  
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understanding.



Ask a  
combination of  
high-level and  
low-level  
questions.



“What is 7 times 9?”

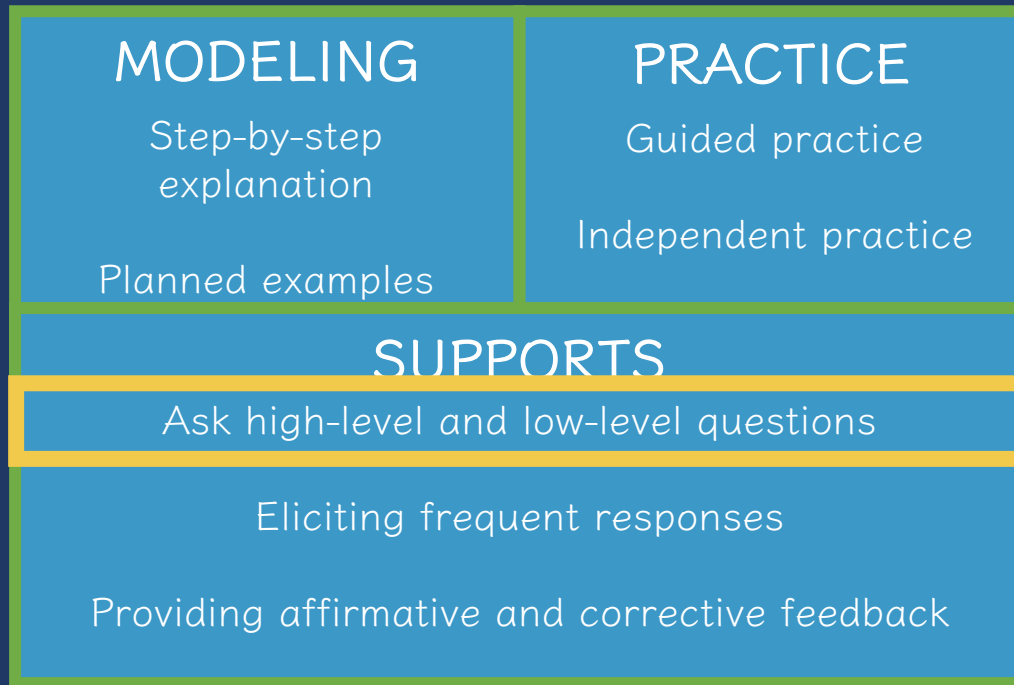


“63.”





During  
**Modeling and  
Practice**, it is  
essential to  
engage  
students and  
check for  
understanding.



Ask a  
combination of  
high-level and  
low-level  
questions.



“Why do you use zero  
pairs?”

“Because a positive 1  
and a negative 1 equal  
0. I use the zero pair  
to help me subtract.”





During **Modeling and Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

## MODELING

Step-by-step  
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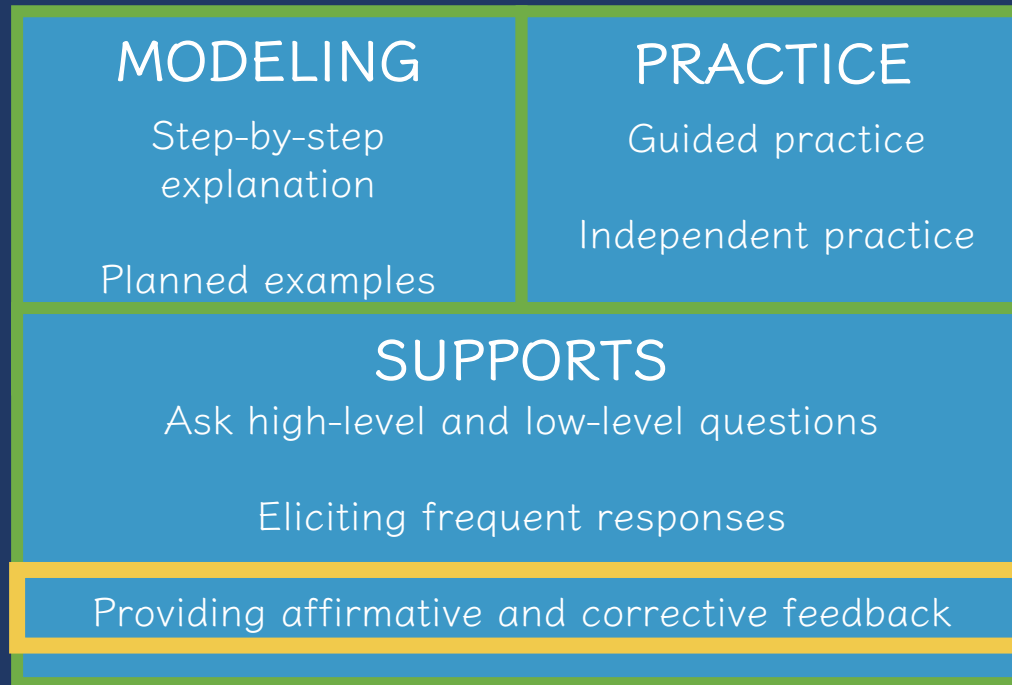


- Oral
- Written
- With manipulatives
- With drawings
- With gestures





During  
**Modeling and  
Practice**,  
students should  
receive  
immediate  
feedback on  
their responses.

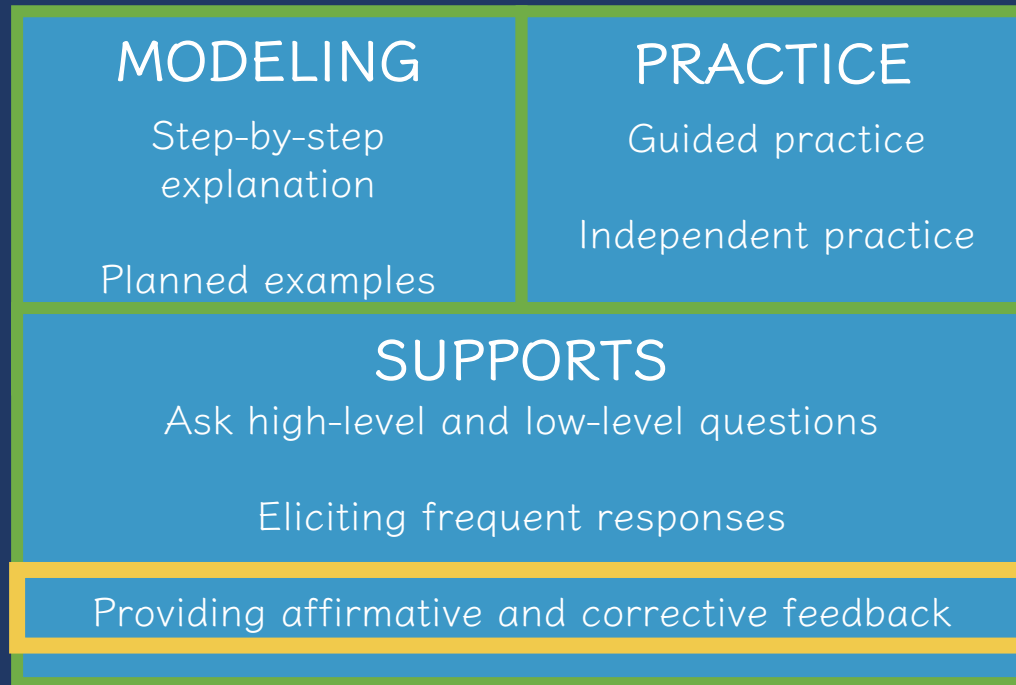


Students  
should receive  
affirmative  
and (when  
necessary)  
corrective  
feedback.





During  
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Practice**,  
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immediate  
feedback on  
their responses.

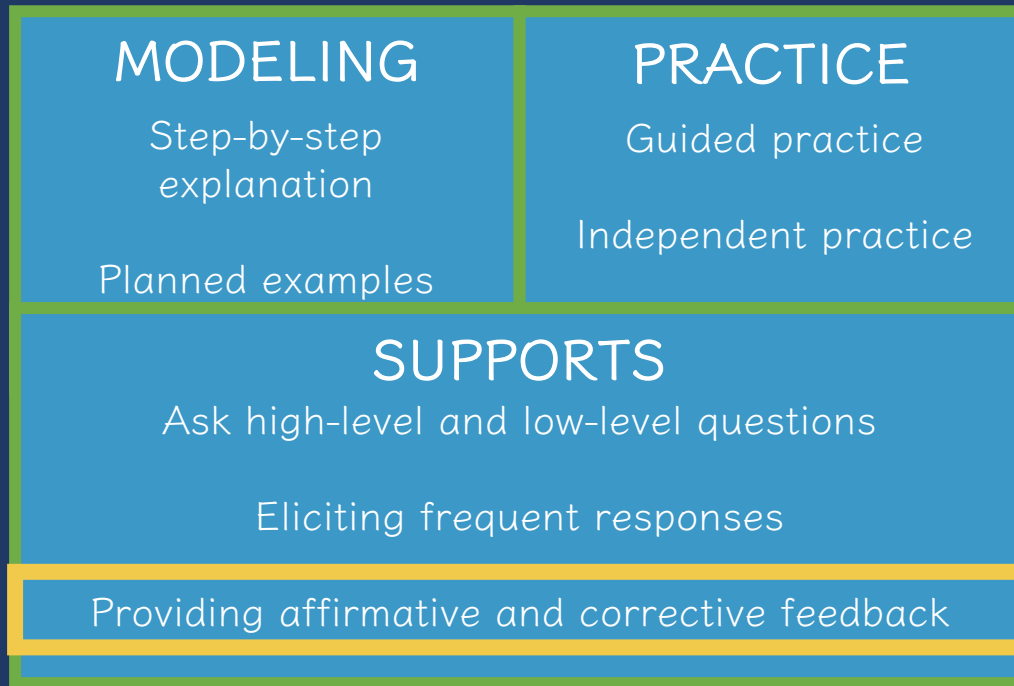


“Nice work using your  
word problem attack  
strategy.”





During  
**Modeling and  
Practice**,  
students should  
receive  
immediate  
feedback on  
their responses.



“Let’s look at that again. Tell me how you added in the hundreds column.”





## MODELING

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Which of these supports do you use most often?







# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

## INSTRUCTIONAL STRATEGIES



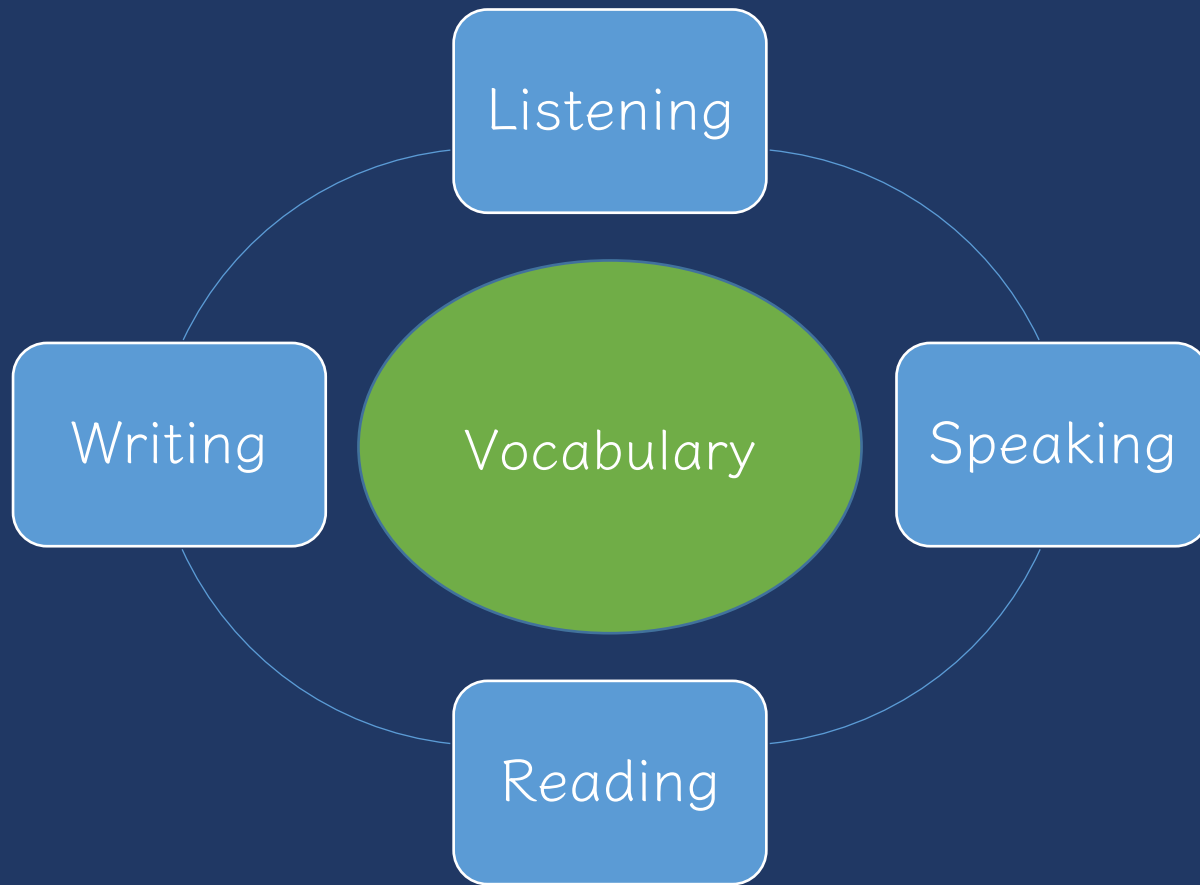


## Mathematical Language

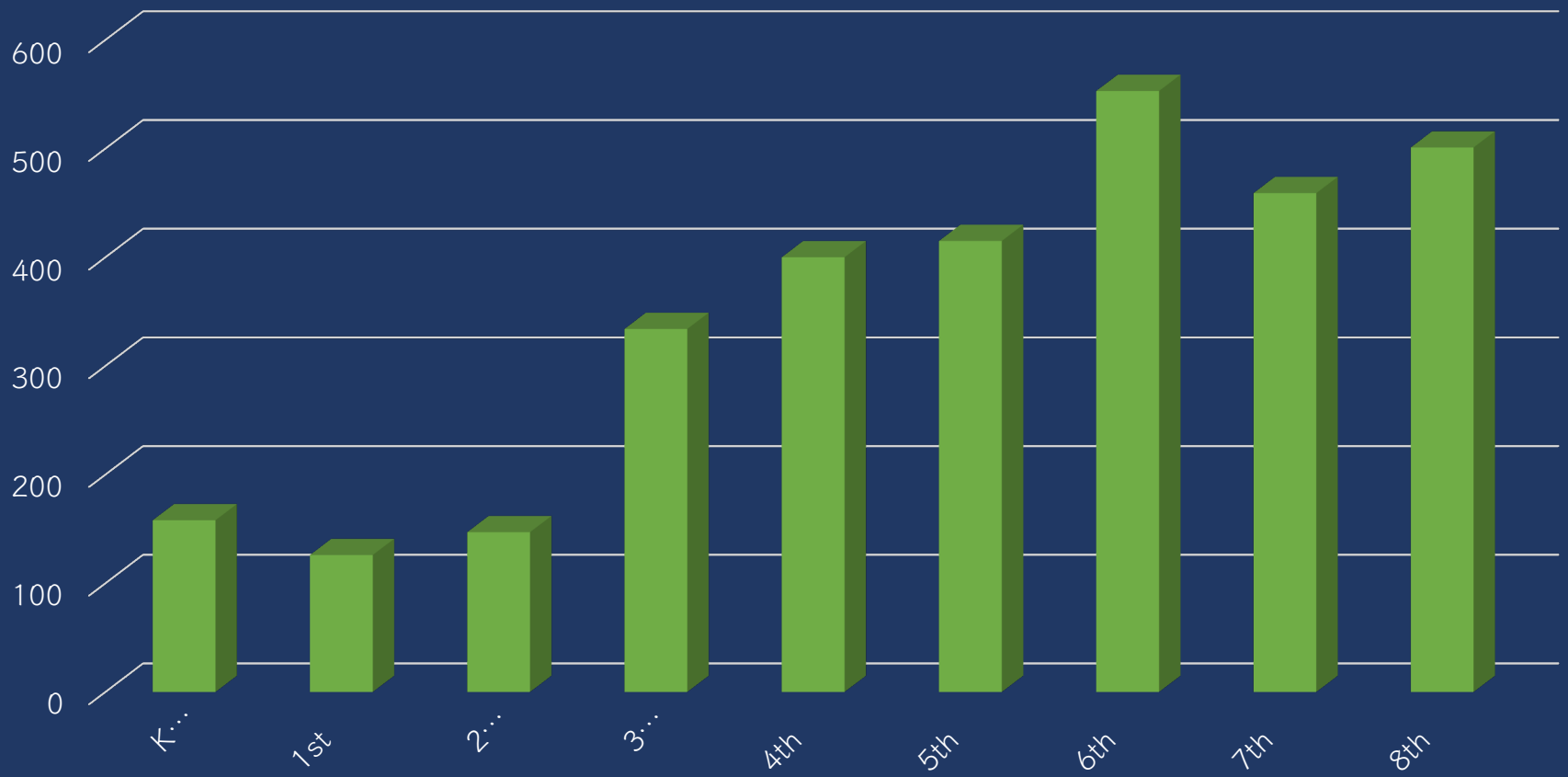
Instead of that...	Say this...











Powell, Bos, & Lin (2019)





1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)





1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)

difference

even

Rubenstein & Thompson (2010)





1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2010)





- 1. Some math terms are shared with English but have different meanings
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- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning



Rubenstein & Thompson (2010)





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- 3. Some math terms are only used in math
- 4. Some math terms have more than one meaning
- 5. Some math terms are similar to other content-area terms with different meanings

variable vs.  
variably  
cloudy

divide vs.  
Continental  
Divide

Rubenstein & Thompson (2010)





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- 5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

eight vs. ate

sum vs.  
some

rows vs.  
rose

base vs.  
bass

Rubenstein & Thompson (2010)





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- 5. Some math terms are similar to other content-area terms with different meanings
- 6. Some math terms are homographs
- 7. Some math terms are related but have distinct meanings

factor vs.  
multiple

hundreds vs.  
hundredths

numerators  
vs.  
denominator

Rubenstein & Thompson (2010)





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- 7. Some math terms are related but have distinct meanings
- 8. An English math term may translate into another language with different meanings

mesa vs.  
tabla

Rubenstein & Thompson (2001)





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- 9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2010)





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- 9. English spelling and usage may have irregularities
- 10. Some math concepts are verbalized in more than one way

one-fourth  
vs. one  
quarter

skip count  
vs. multiples

Rubenstein & Thompson (2010)





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- 11. Informal terms may be used for formal math terms

vertex vs.  
corner

rhombus vs.  
diamond

Rubenstein & Thompson (2010)





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Which of these  
cause difficulty  
for your  
student(s)?

Rubenstein & Thompson (2001)





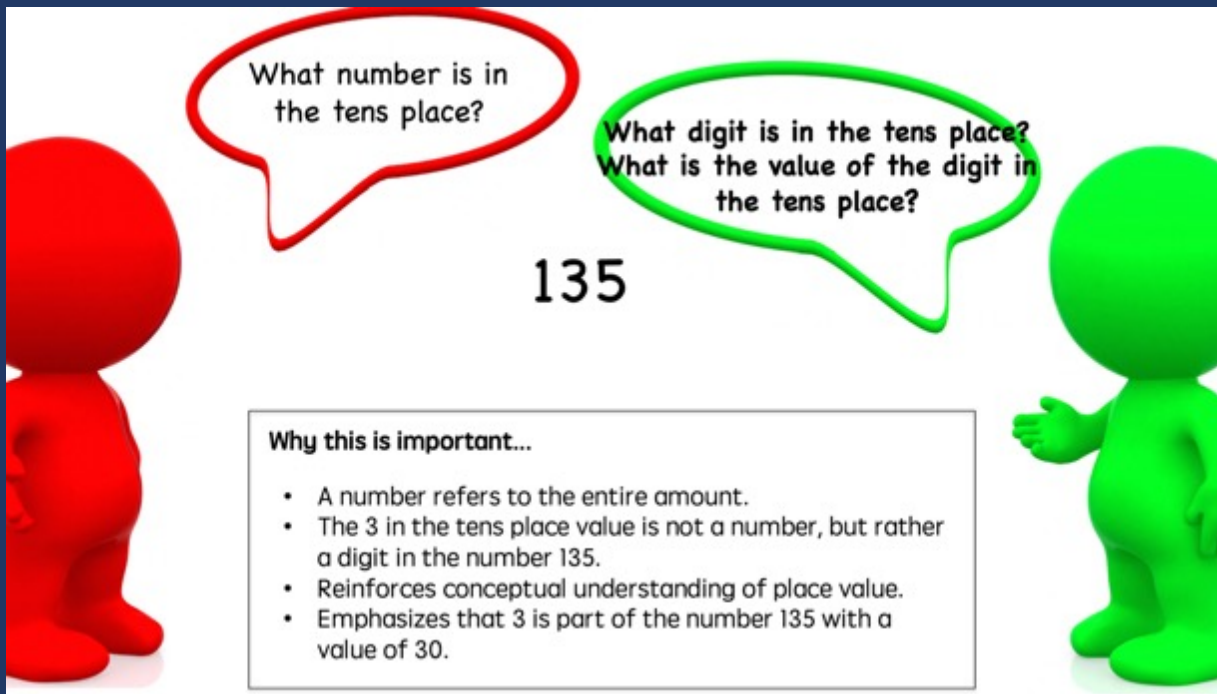
Use formal math language

Use terms precisely





# Use formal math language



What number is in the tens place?

135

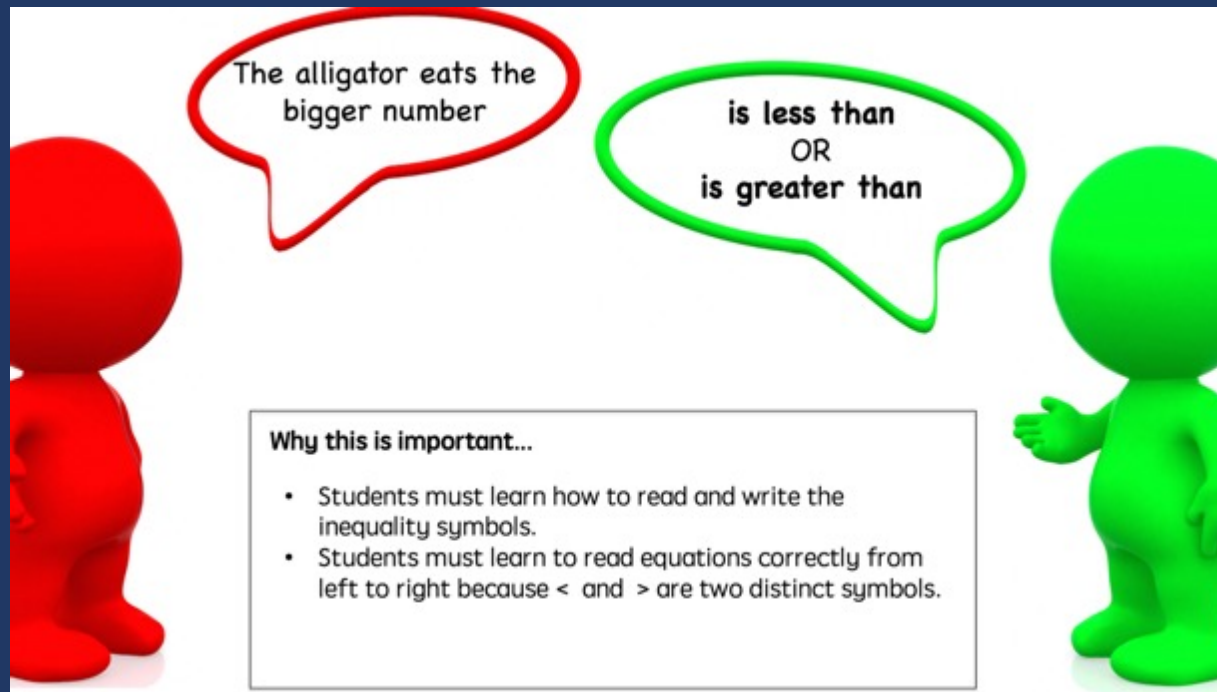
What digit is in the tens place?  
What is the value of the digit in the tens place?

**Why this is important...**

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.



# Use formal math language





# Use formal math language

carry OR borrow

regroup OR trade OR exchange

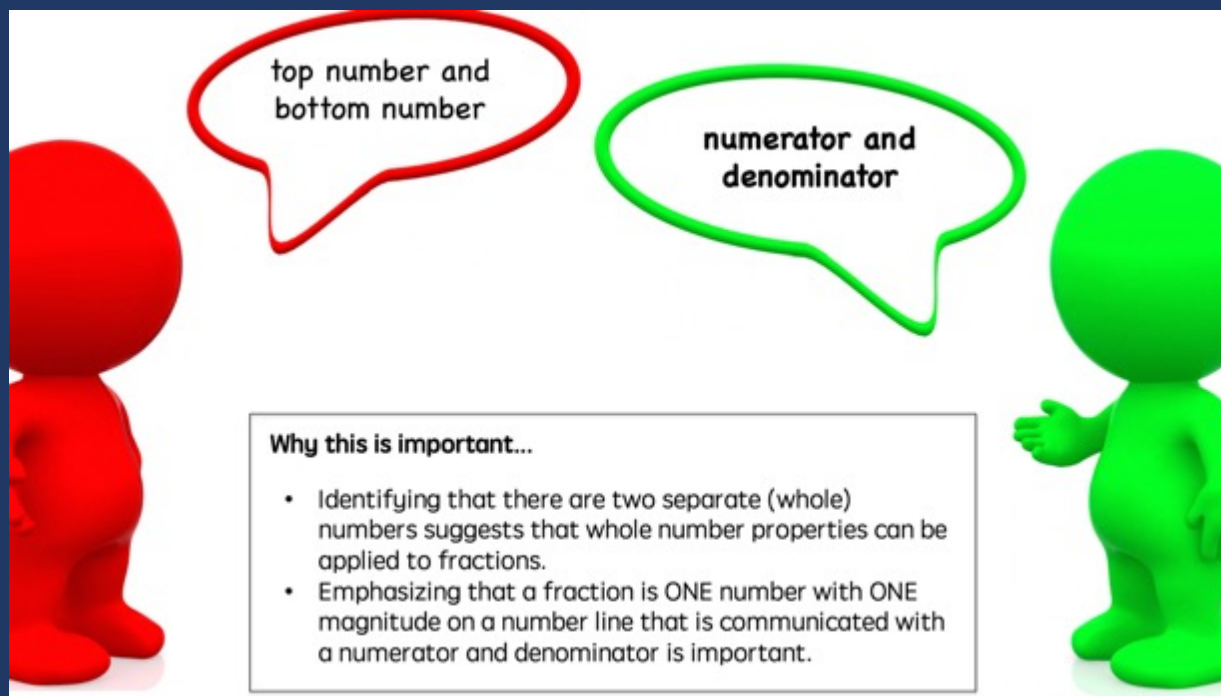
$$\begin{array}{r} 167 \\ + 294 \\ \hline \end{array}$$

**Why this is important...**

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding of regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.

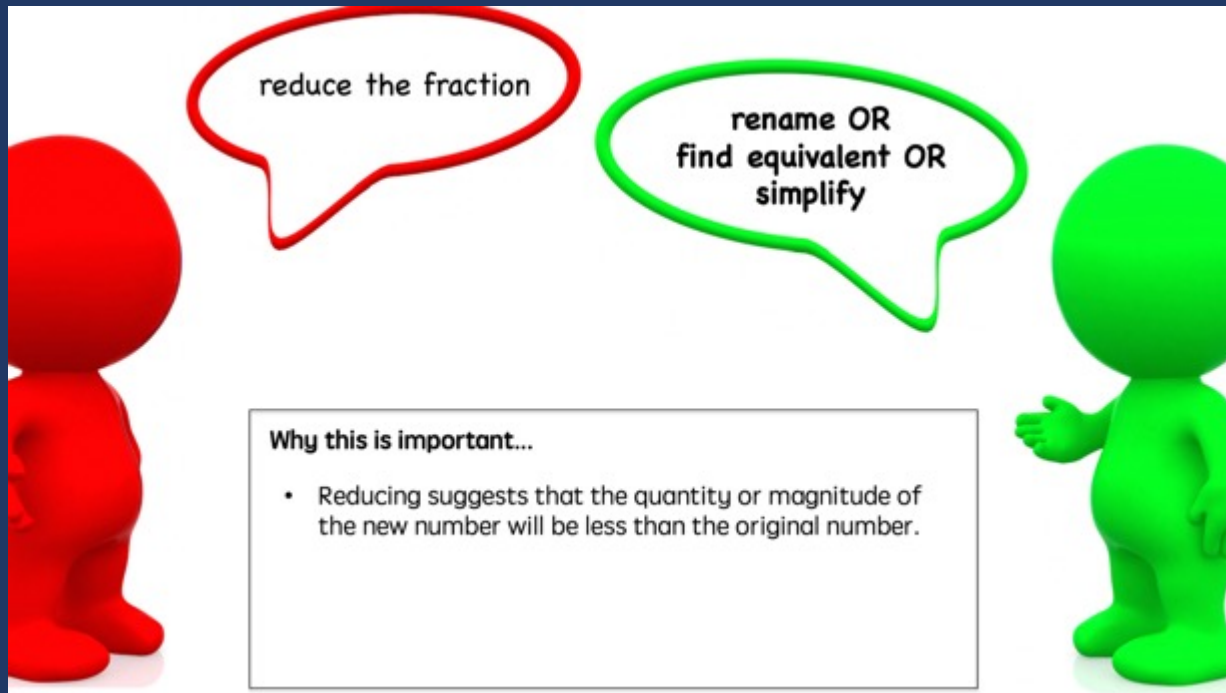


# Use formal math language



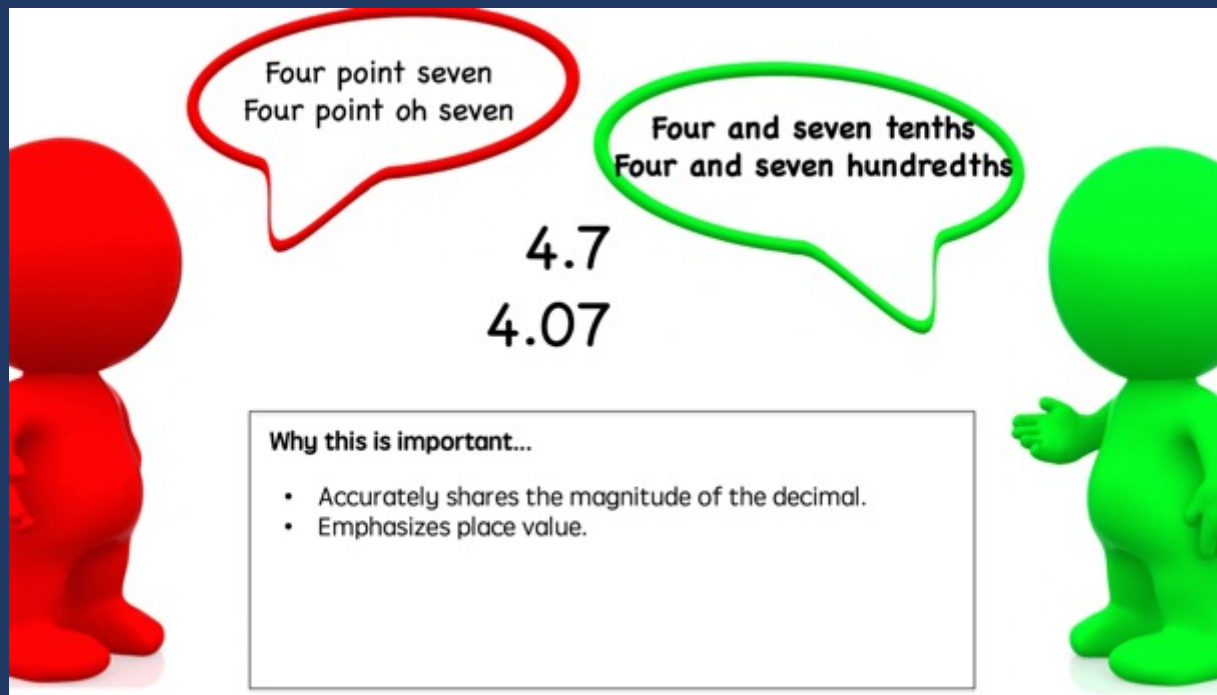


# Use formal math language



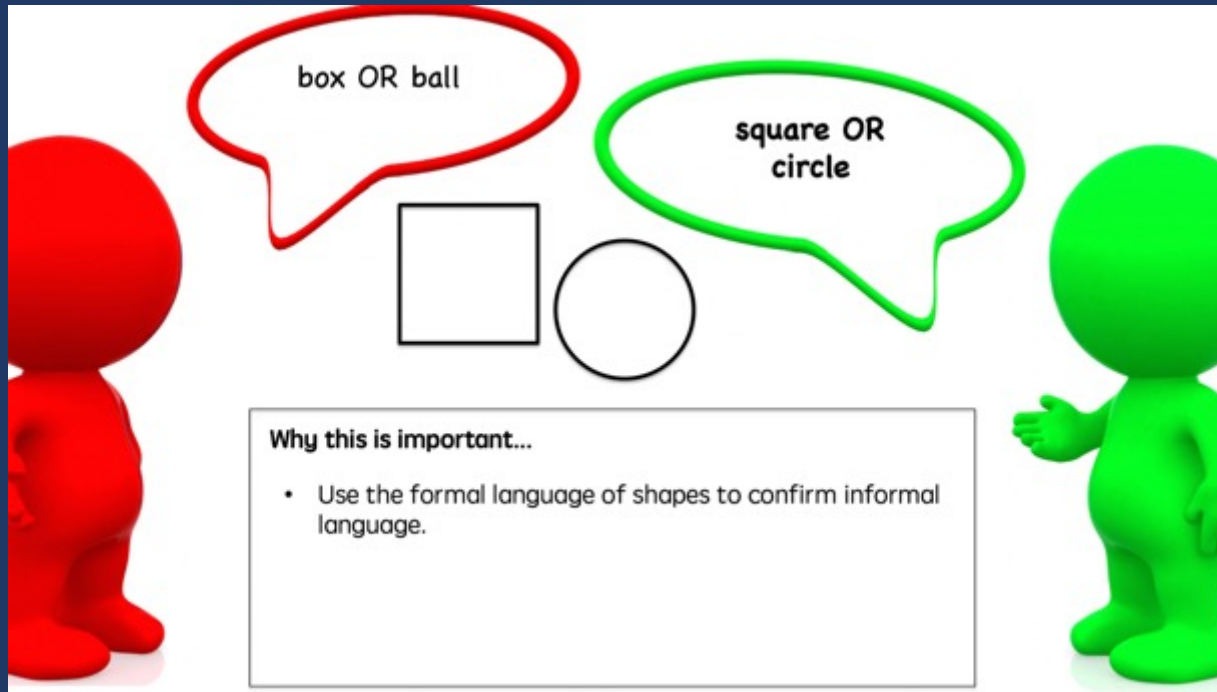


# Use formal math language



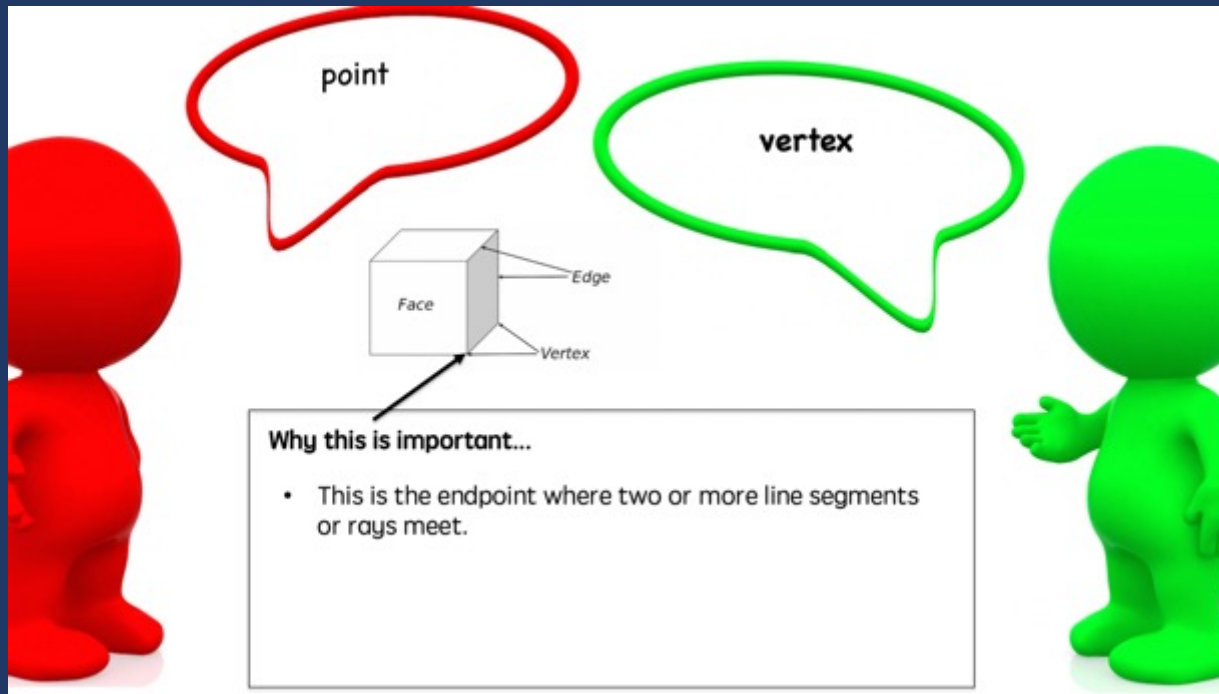


# Use formal math language



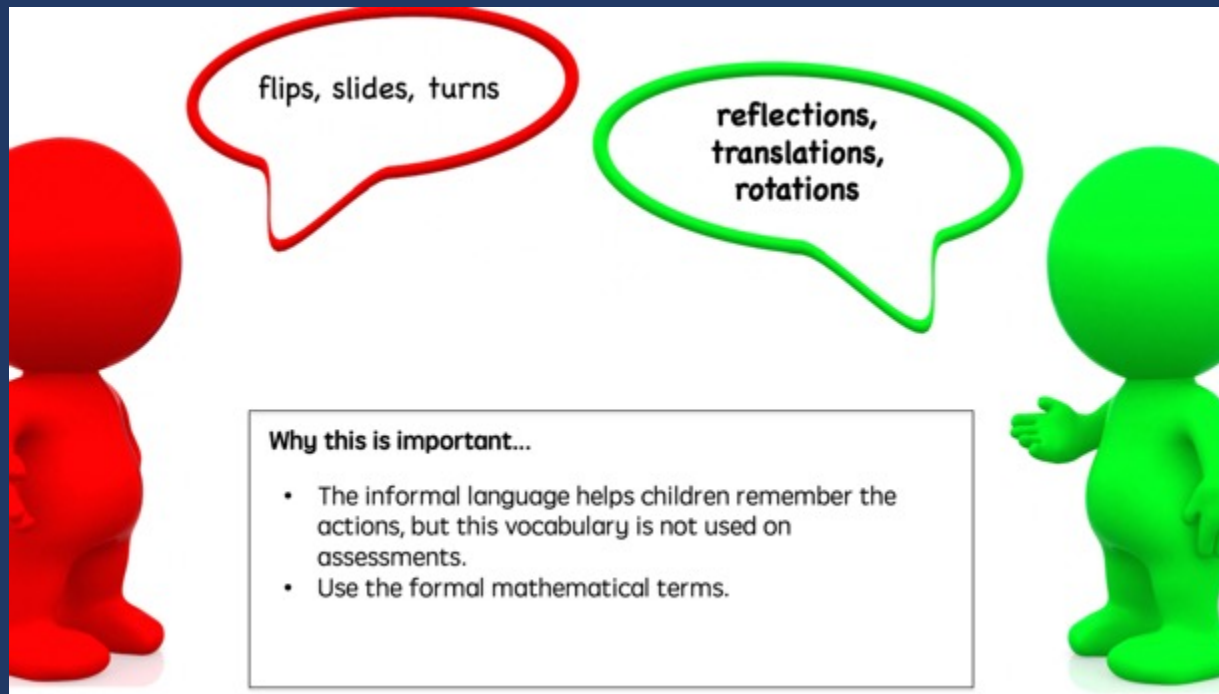


# Use formal math language



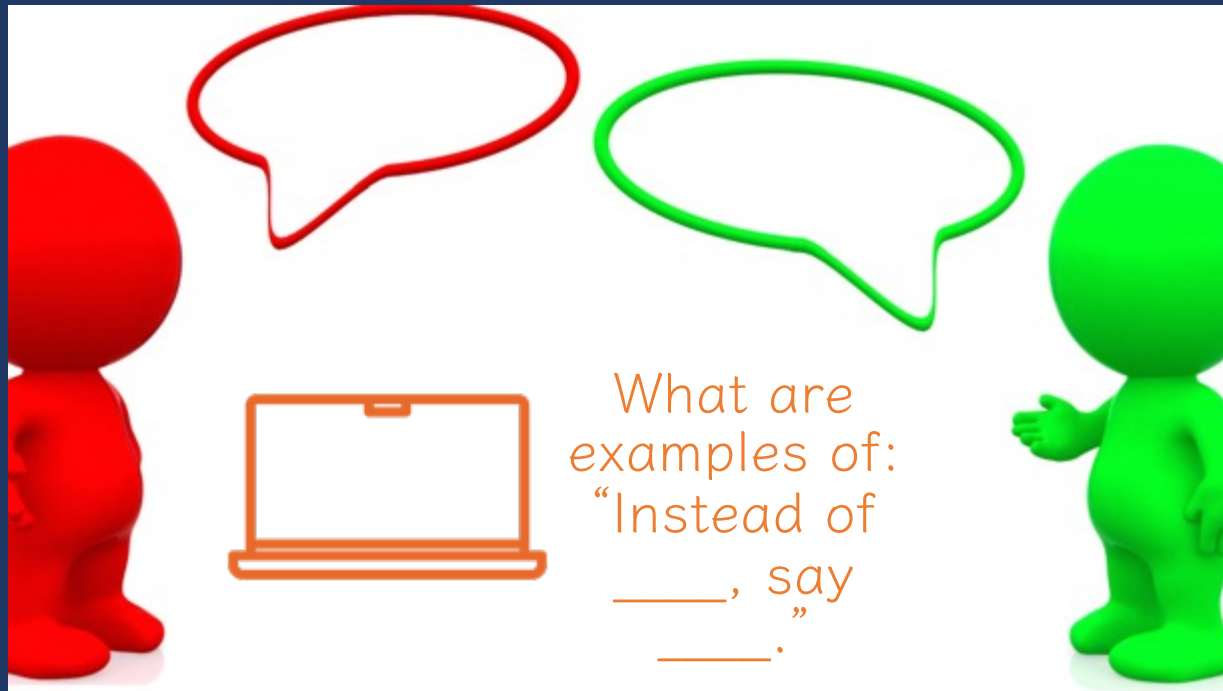


# Use formal math language





# Use formal math language





Use formal math language

Use terms precisely





# Use terms precisely

**Factor**

$$1 \times 8 = 8$$
$$2 \times 4 = 8$$

factor factor

**Multiple**

$$8 \times 1 = 8$$
$$8 \times 2 = 16$$

multiples of 8

E

**Improper fraction**

$$\frac{8}{5}$$

**Mixed number**

$$1\frac{3}{5}$$

**Proportion**

$$\frac{2}{5} = \frac{8}{20}$$

**Ratio**

$$4:3$$

**Proper fraction**

$$\frac{2}{9}$$

**Unit fraction**

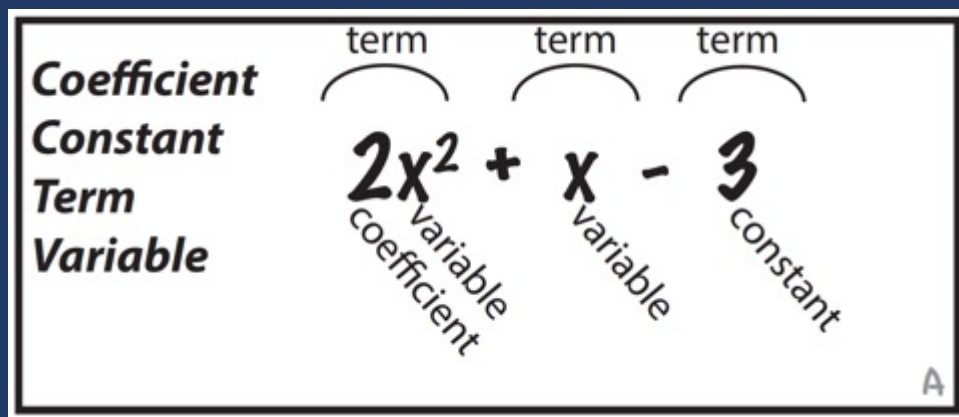
$$\frac{1}{6}$$

D





## Use terms precisely



**Equation**  $9x - 4 = 7x$

**Expression**  $9x - 4$

**Formula**  $a^2 + b^2 = c^2$

**Function**  $f(x)$

**Inequality**  $9x - 4 > 6x$

C





Use terms precisely

### **Quadrilaterals**

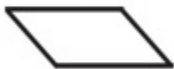
**Kite**



**Rhombus**



**Parallelogram**



**Square**



**Rectangle**

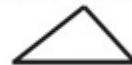


**Trapezoid**



A

**Acute triangle**



**Obtuse triangle**



**Right triangle**



**Equilateral triangle**



**Isosceles triangle**



**Scalene triangle**



C

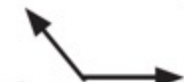


# Use terms precisely

**Acute angle**



**Obtuse angle**



**Right angle**



**Straight angle**

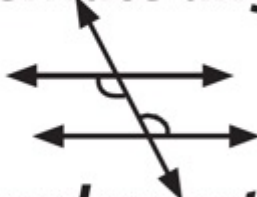


B

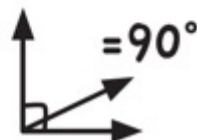
**Adjacent angles**



**Alternate angles**



**Complementary angles**



**Corresponding angles**

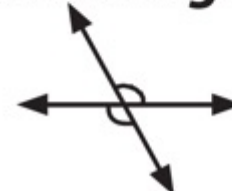


**Supplementary angles**

$= 180^\circ$



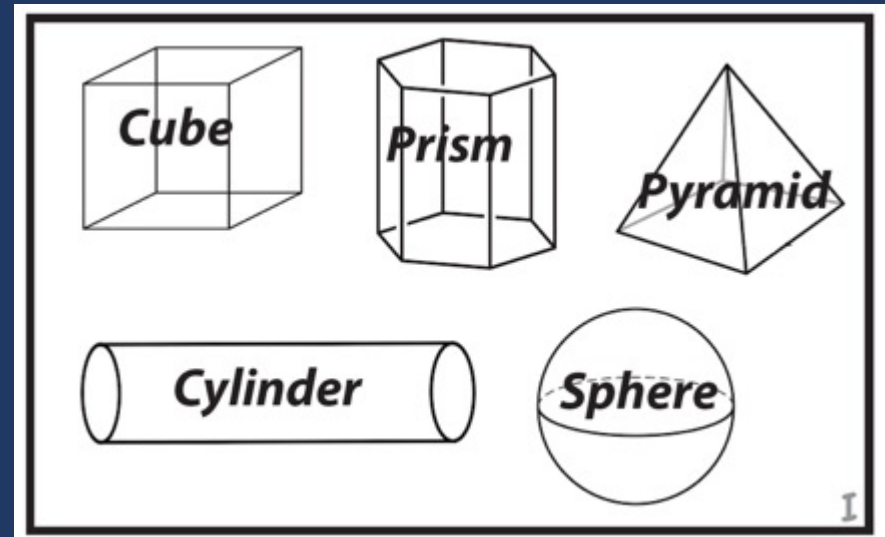
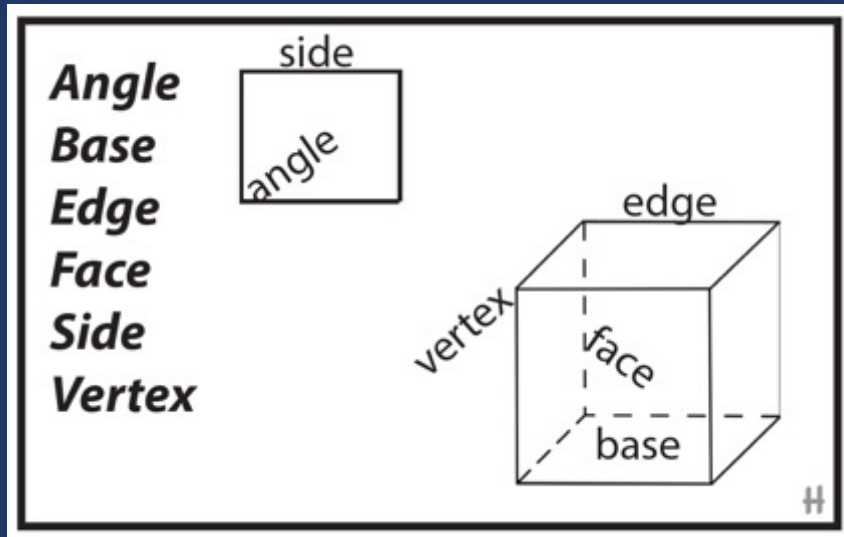
**Vertical angles**



D

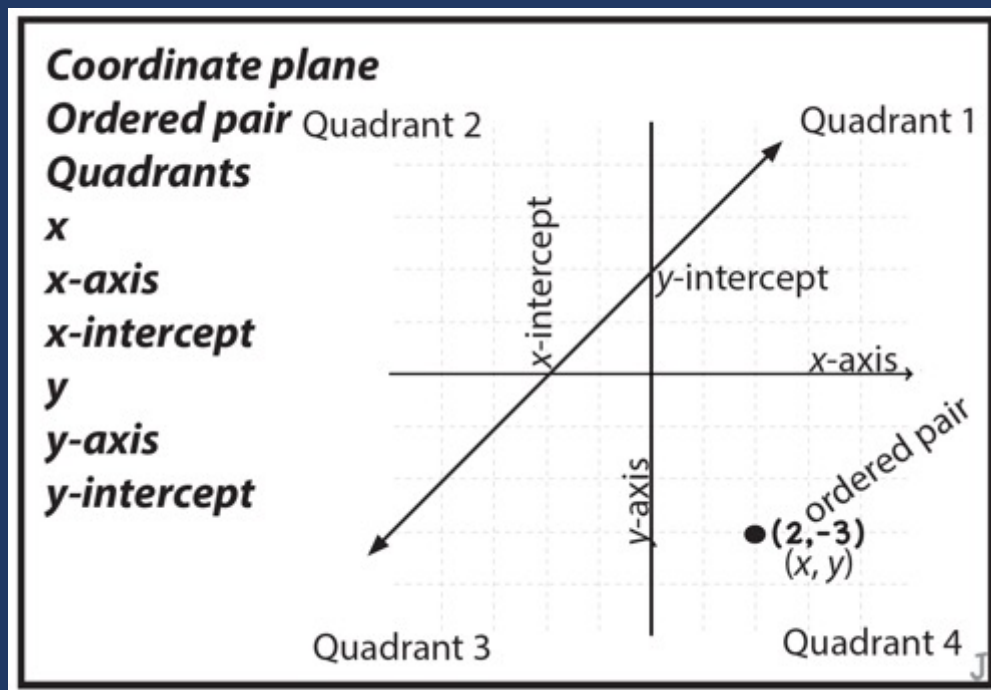


# Use terms precisely





# Use terms precisely





# Use terms precisely



What are terms  
that your  
students do  
not use  
precisely?





Use formal math language

Use terms precisely





Word	Lightbulb Word
Definition	Picture

Dunston & Tyminski (2013)





## Integer Definitions

### Zero Pairs

A positive and negative cancel one another;

### Positive

A number that is greater than zero.



### Absolute Value

The distance of a number from zero on a number line; shown as  $||$

### Negative

A number that is less than zero. Identified by a minus sign.

**Numerator:** how many parts of the whole

- Ex.  $\frac{4}{10}$

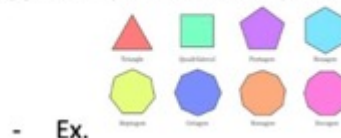
**Odd number:** a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9....

**Percent:** a specific number in comparison to 100

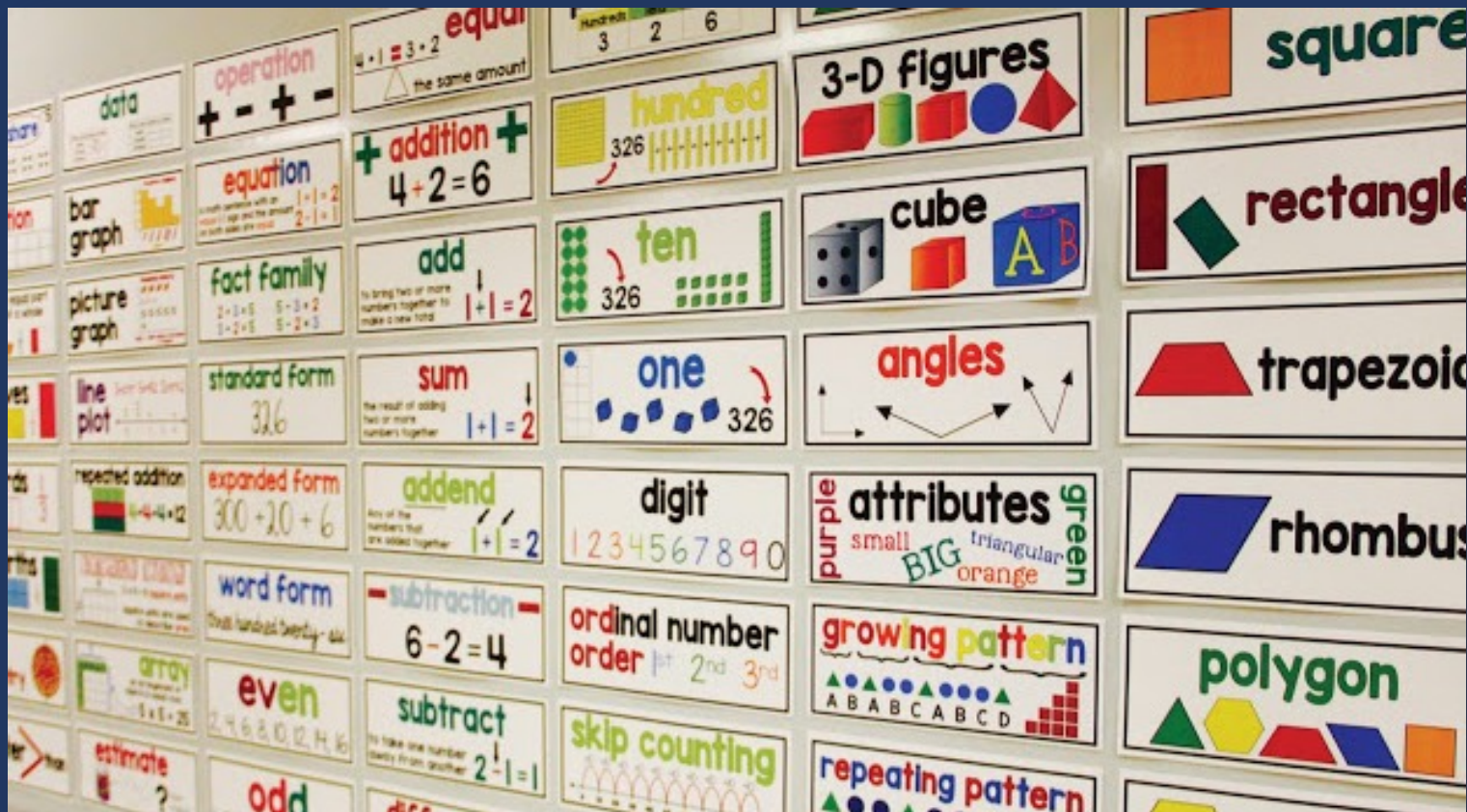
- 74%

**Polygon:** any enclosed shape that is made up of 3 or more straight lines



- Ex.





<https://jillianstarrteaching.com/math-word-walls/>







Dear Feisty Fifth Graders,

Today we have multiple opportunities to do exciting projects! For example, we are going to be doing a science experiment to see how the tilt of a ramp relates to how far a matchbox car will roll. There are several factors we will be looking at in this experiment. I look forward to hearing multiple ideas on how to set up this experiment.

One other thing that factors into our day is that we have an assembly before lunch. We will get to hear music from the high school play. I think we will hear multiple songs.

Sincerely, Ms. Livers

Here is a problem to start your day... in my letter I have used two words that are important math words for today's lesson. Can you find them and tell what they mean in this letter and what they mean when talking about numbers? (Answer this in your math notebook)

Bay-Williams & Livers (2009)





Rating	Word	Definition	Synonym(s)	Example	Sample Problem
2	expression	a mathematical phrase combining operations, numbers and/or variables.	phrase algebraic expression	$6$ $6n$ $6+n$ <div style="display: inline-block; vertical-align: middle;">             no equal sign           </div>	Lucia earns \$8 per hour for babysitting and gets a \$5 tip. Write an <u>expression</u> to represent the amount she would earn if she worked for $x$ hours.
2	variable	a quantity that can change or take many values. (refers to the letter or symbol representing the quantity)	unknown	$x$ $D$ $y$ $T$	The <u>variable</u> $x$ represents the number of hours Charlie works in a week. Write an expression to represent his earnings if he earns \$9 per hour.
1	product	the result when two or more numbers are multiplied	total answer	$3 \times 2 = \textcircled{6}$ ↑ product	The <u>product</u> of 6 and a number is 24. What is the number?
3	quotient	the result of a division (refers to the number of times the divisor divides the dividend)	answer	$18 \div 2 = \textcircled{9}$ $2 \overline{)18}$ <div style="display: inline-block; vertical-align: middle;"> <math>\textcircled{9}</math> ← quotient           </div>	Estimate the <u>quotient</u> when 365 is divided by 12.

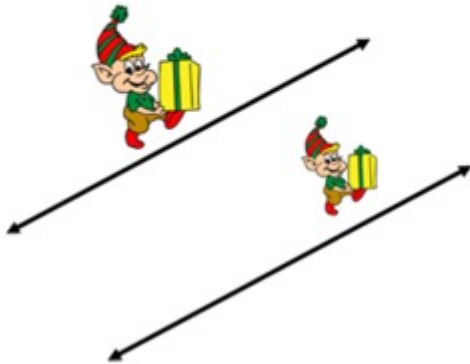
Marin (2018)





### Parallel Lines (Pair of Elves)

Lines that are the same distance apart and will never intersect



The Pair of Elves are the same distance apart and will never intersect.

The Pair of Elves are on Parallel Lines

### Ray (Run Away)

A line that has a starting point but no endpoint



Start here!! Run away and never stop running Ray.

Riccomini et al. (2015)





## Math Word Search #6

### Number Words 51 to 60

Use the word bank to find the number words in the grid below. Words appear horizontally and vertically.



fifty-one  
fifty-two  
fifty-three  
fifty-four

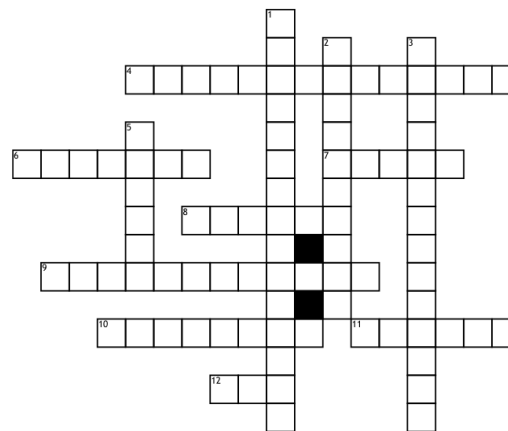
fifty-five  
fifty-six  
fifty-seven

fifty-eight  
fifty-nine  
sixty

© <https://www.puzzlebookninja.com>

Name: \_\_\_\_\_ Date: \_\_\_\_\_

## Circles Vocabulary Practice



### Across

4. What is an angle whose vertex is on the circle?
6. What is a line that intersects the circle at 1 place?
7. What is a segment whose endpoints are on the circle?
8. What is the point in the middle of the circle?

9. What is an angle whose vertex is the center of the circle?
10. What is a chord that goes through the center of the circle?
11. What is a segment whose endpoints are the center and a point on the circle?
12. What is an unbroken part of a circle?


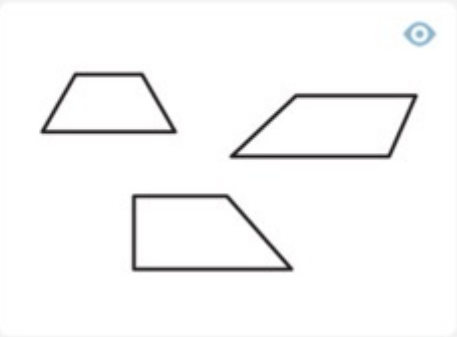
### Down

1. What is the name of the point where a tangent intersects the circle?
2. What is an arc whose endpoints are the endpoints of the diameter?
3. What is an arc that is encased on either side by two different segments?
5. What is a line that intersects the circle at 2 places?







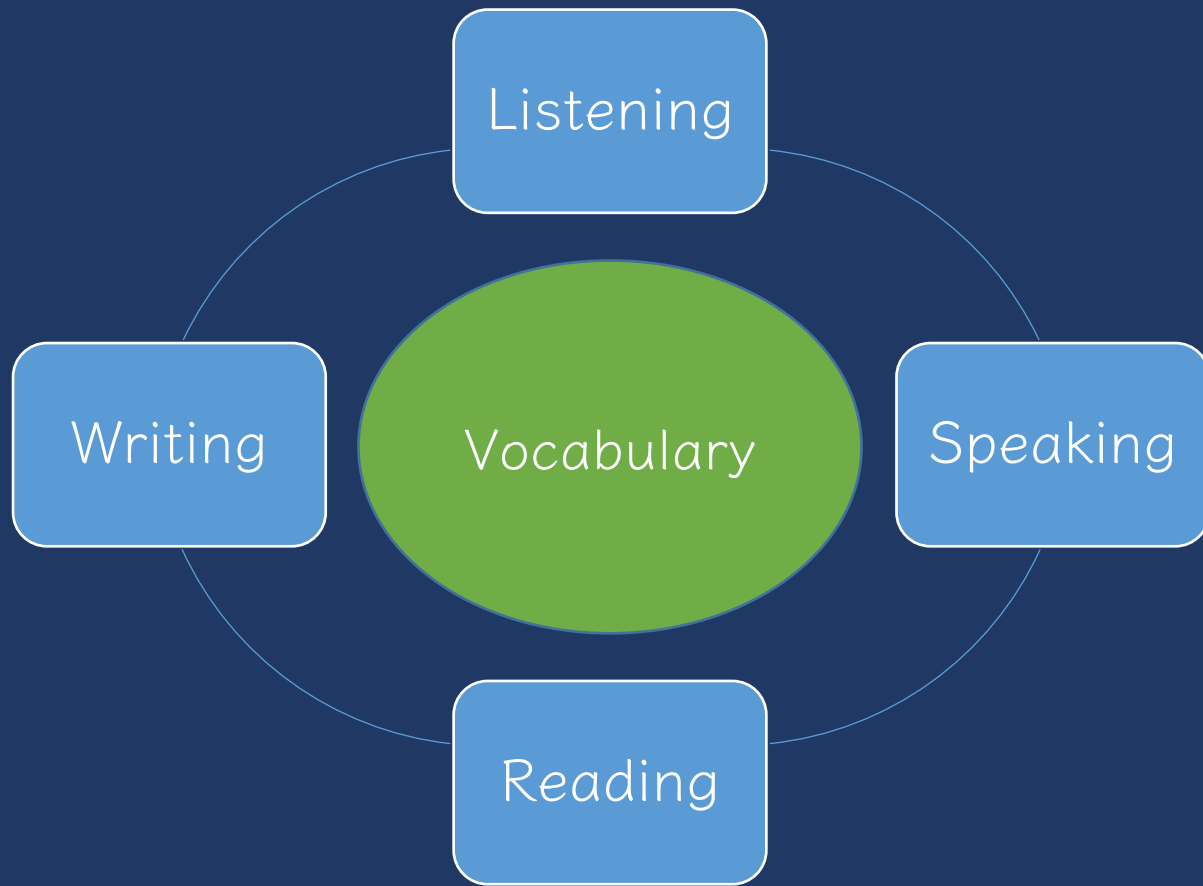
una figura bidimensional (plana) con 4 lados, exactamente 1 par de los cuales son paralelos

Math Lingo

Math Learning Center







What are ways you support the math vocabulary of your student(s)?





# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

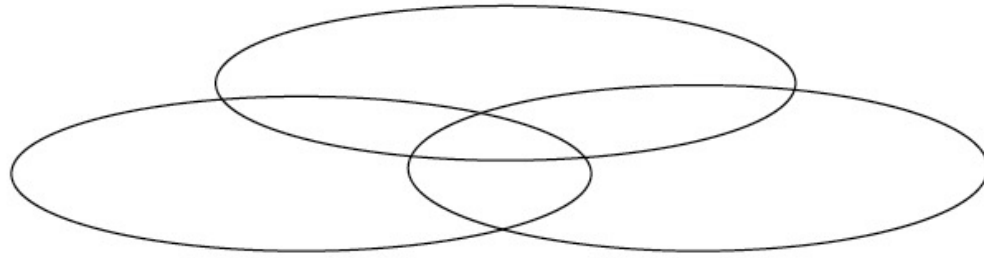
Multiple  
representations

## INSTRUCTIONAL STRATEGIES

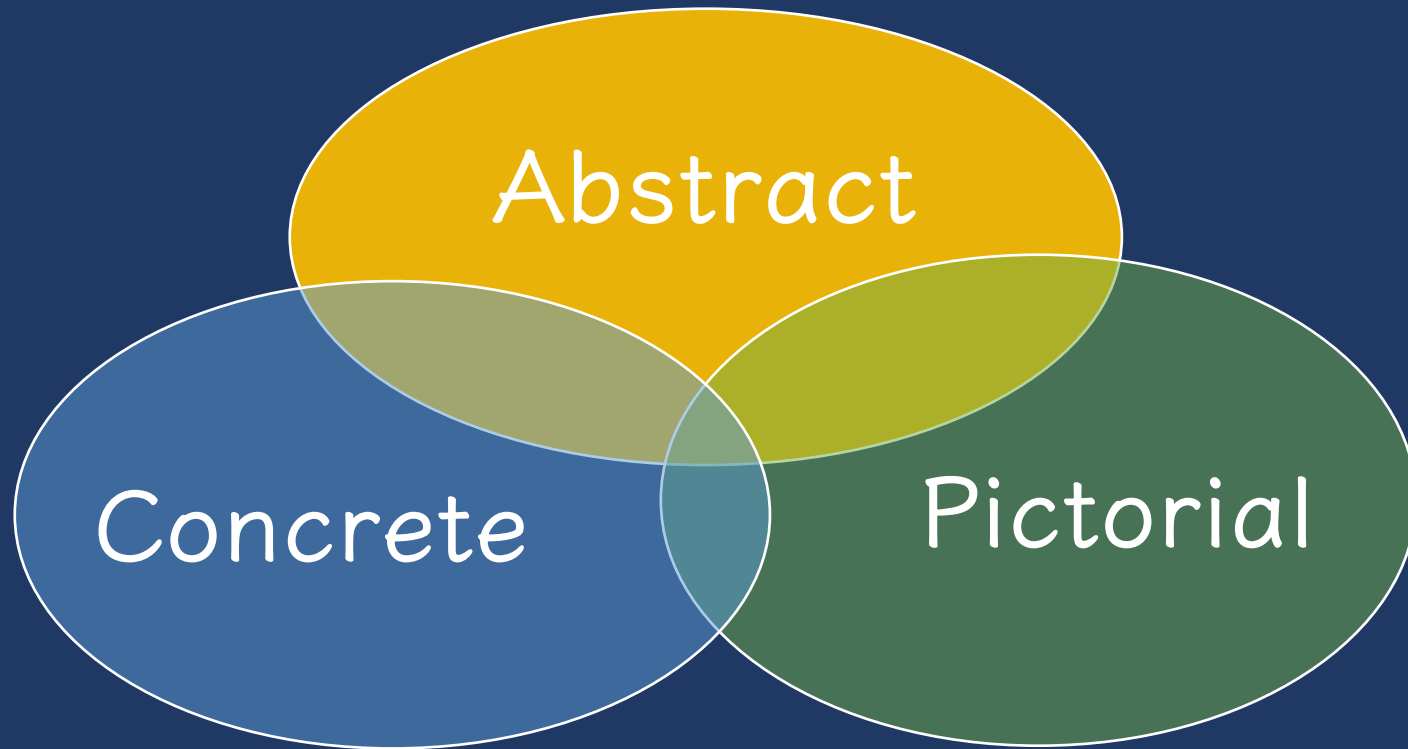




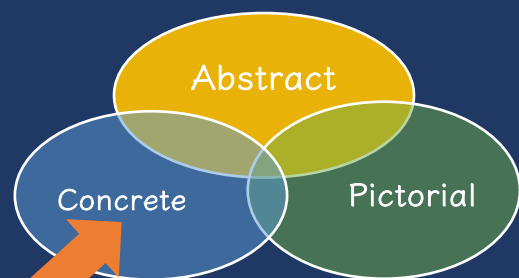
## Multiple Representations



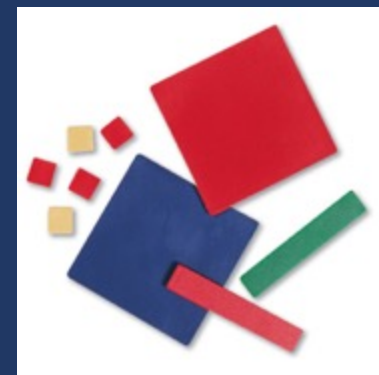
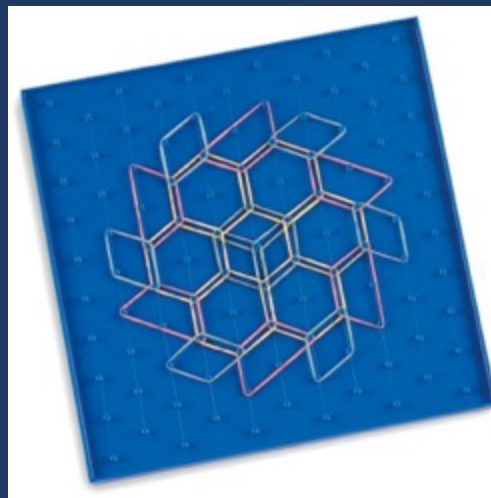




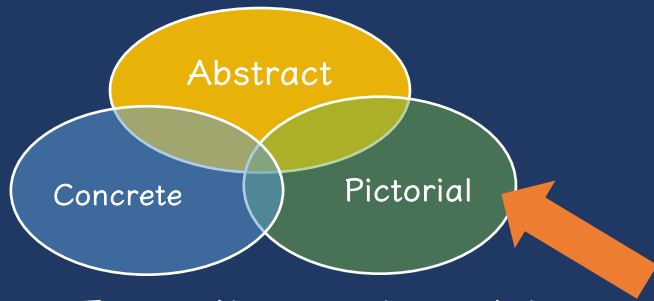




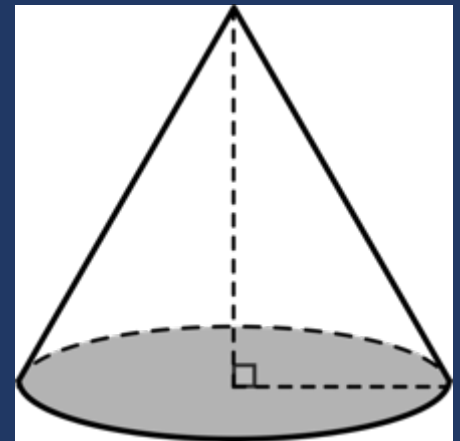
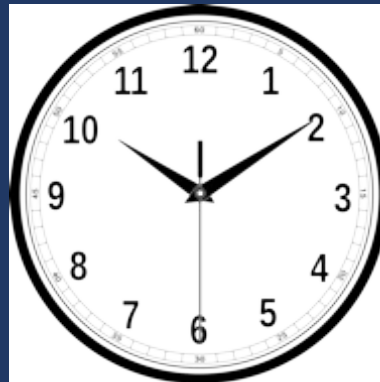
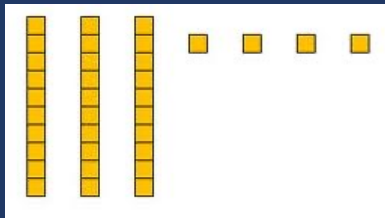
Three-dimensional objects



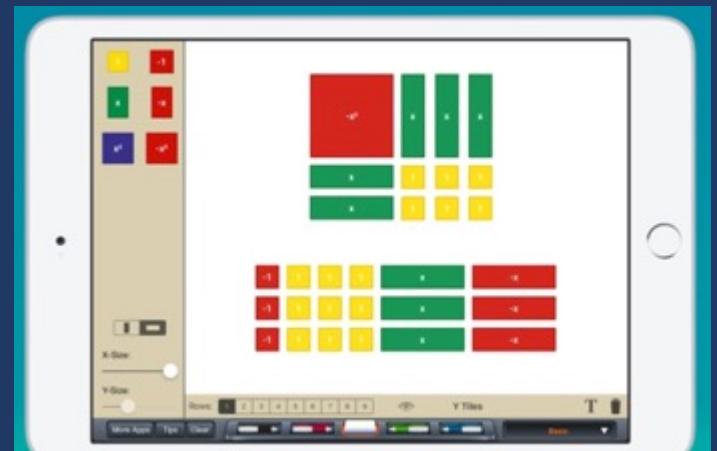
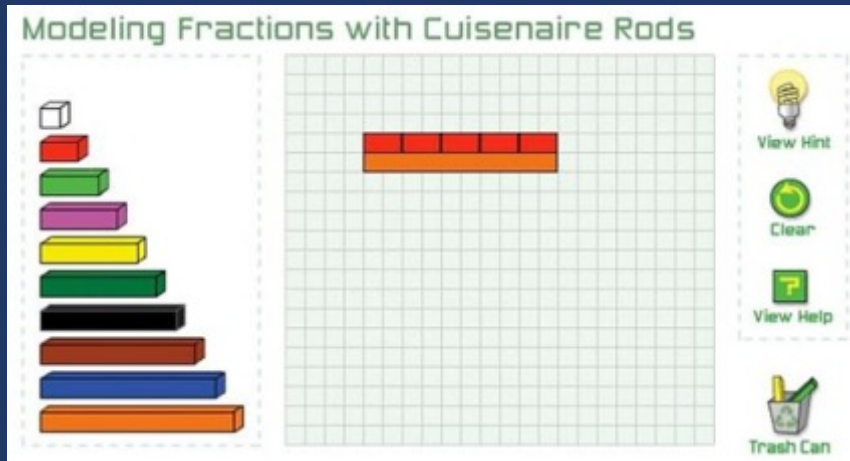
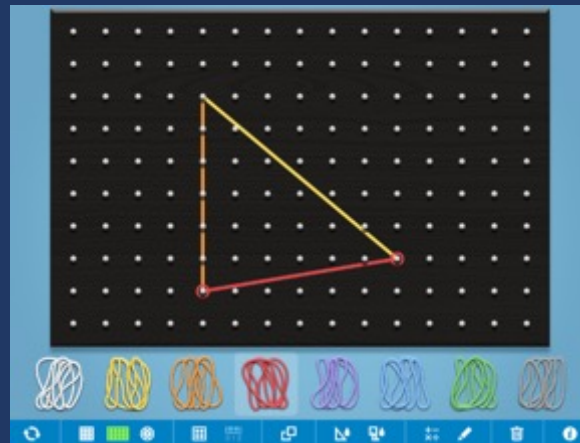
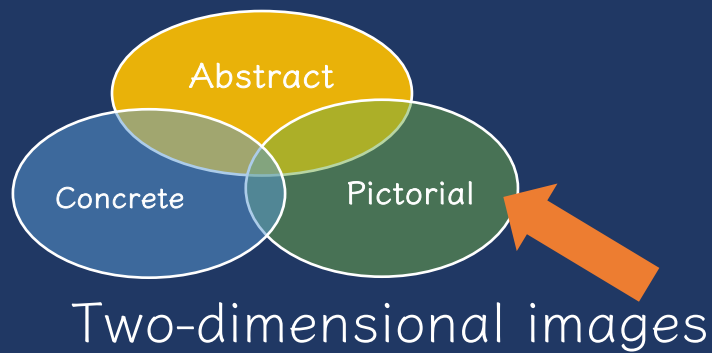




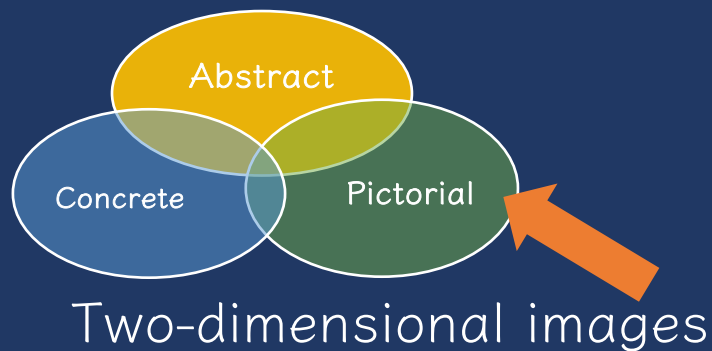
Two-dimensional images











# Virtual Manipulatives

Fall 2020  
EDC 370E

Sarah R. Powell, Ph.D.  
srpowell@austin.utexas.edu  
www.sarahpowellphd.com  
@sarahpowellphd

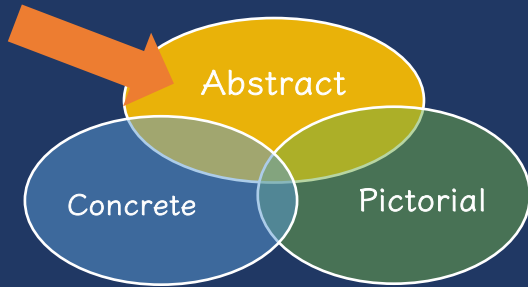
- Number & Operations
- Place Value
- Fractions & Decimals
- Integers & Algebra
- Geometry
- Time & Money
- Data & Probability
- Extras



Fractions & Decimals				







Numerals and symbols and words

$$2 + 8 = 10$$

$$x - 6 = 8$$

34 = 3 tens and 4 ones

$$\begin{array}{r} 4,179 \\ + \quad 569 \\ \hline \end{array}$$







*If you are left handed:*

What's one of your favorite hands-on manipulatives?

*If you are right handed:*

What's one of your favorite virtual manipulatives?







# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

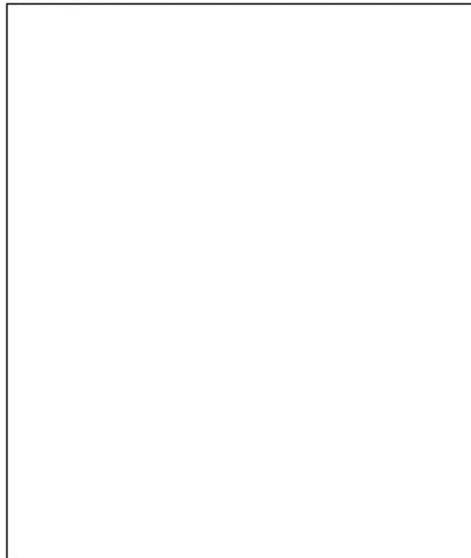
Fluency building



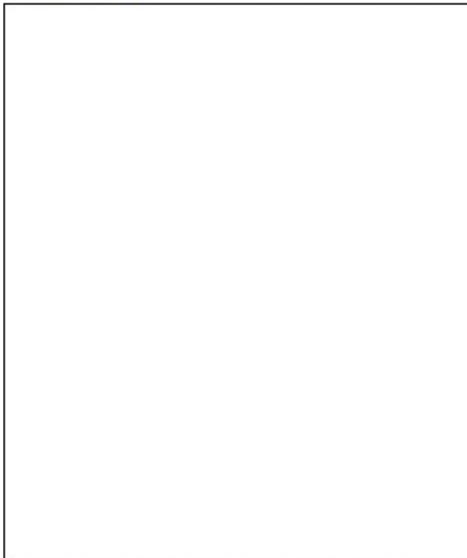


## Fluency

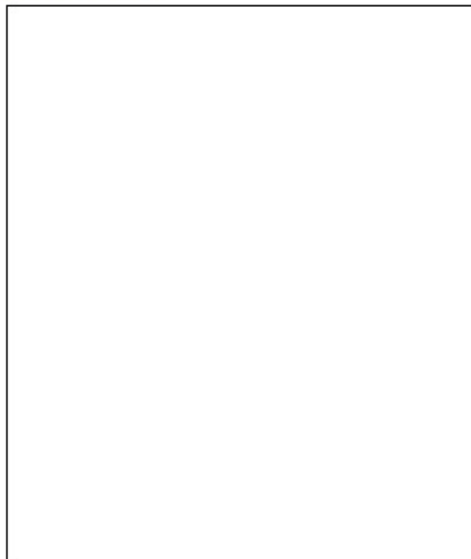
Addition



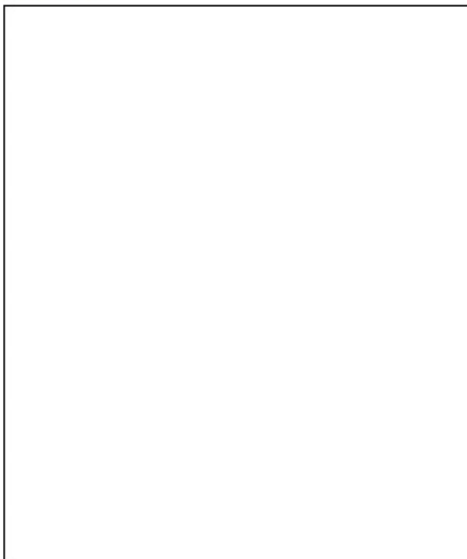
Subtraction



Multiplication



Division





Addition	Subtraction
Multiplication	Division

Fluency is  
doing  
mathematics  
easily and  
accurately.

Fluency  
makes  
mathematics  
easier.

Fluency  
provides less  
stress on  
working  
memory.

Fluency helps  
students build  
confidence  
with  
mathematics.





Addition	Subtraction
Multiplication	Division

It is essential to emphasize  
both conceptual and  
procedural learning.





# Total (Part-Part-Whole, Combine)

Addition

Subtraction

Multiplication

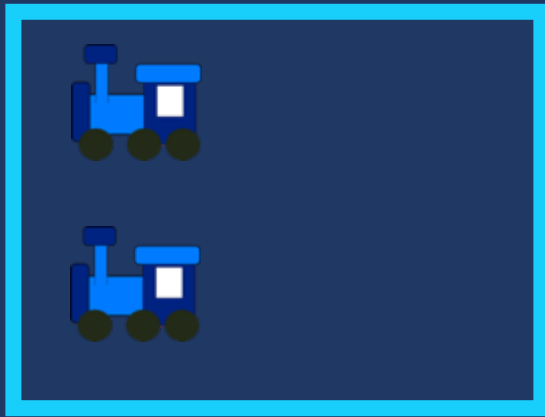
Division



$$2 + 3 = 5$$



# Join (Change Increase)



$$2 + 3 = 5$$

Addition	Subtraction
Multiplication	Division





# Total (Part-Part-Whole, Combine)

Karly saw 4 cardinals and 5 blue jays.  
How many birds did Karly see?

Addition	Subtraction
Multiplication	Division

# Join (Change Increase)

Pia had \$4. Then they earned \$5 for  
cleaning their room. How much money  
does Pia have now?







$$3 + 9 = \underline{\quad}$$

If you have brown eyes:

What's a Total story to show addition?

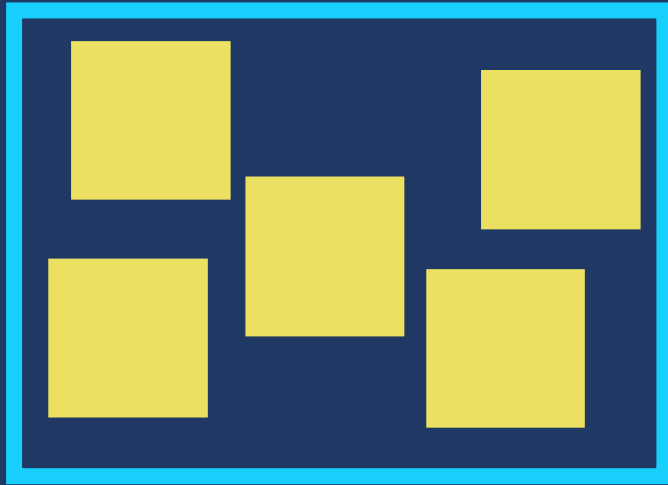
If you don't have brown eyes:

What's a Change/Join story to show addition?





# Separate (Change Decrease)



$$5 - 3 = 2$$

Addition	Subtraction
Multiplication	Division



# Difference (Compare)



$$5 - 3 = 2$$

Addition	Subtraction
Multiplication	Division



# Separate (Change Decrease)

Brady had 9 cookies. Then they ate 2 of the cookies. How many cookies does Brady have now?

Addition	Subtraction
Multiplication	Division

# Difference (Compare)

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)







$$9 - 5 = \underline{\quad}$$

If you were born in Canada:

What's a Change/Separate story to show subtraction?

If you weren't born in Canada:

What's a Difference story to show subtraction?





# Equal Groups

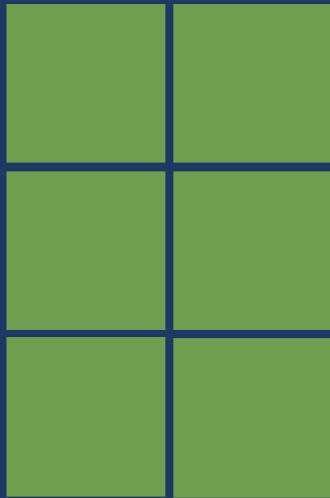


Addition	Subtraction
Multiplication	Division

$$3 \times 2 = 6$$



# Equal Groups (Array)



$$3 \times 2 = 6$$

Addition	Subtraction
Multiplication	Division





# Comparison

Addition	Subtraction
Multiplication	Division



$$3 \times 2 = 6$$



# Equal Groups

Diego has 2 boxes of crayons. There are 8 crayons in each box. How many crayons does Diego have altogether?

Addition	Subtraction
Multiplication	Division

# Comparison

Vivienne picked 2 apples. Jessica picked 8 times as many apples as Vivienne. How many apples did Jessica pick?







$$2 \times 5 = \underline{\quad}$$

If you aren't wearing glasses:

What's an Equal Groups story to show multiplication?

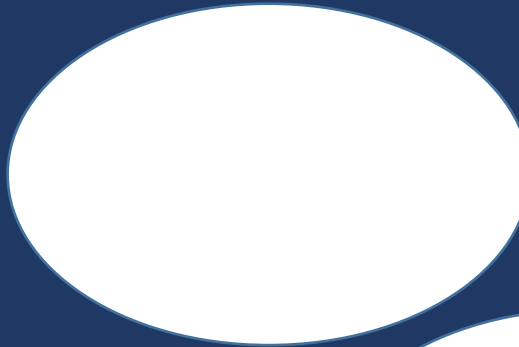
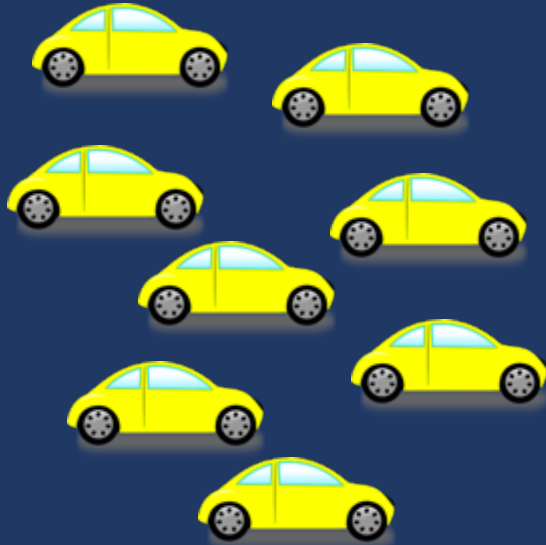
If you are wearing glasses:

What's a Comparison story to show multiplication?





# Partitive Division



Addition	Subtraction
Multiplication	Division

$$8 \div 2 = 4$$



# Quotative Division



$$8 \div 2 = 4$$

Addition	Subtraction
Multiplication	Division



# Partitive

Stefanie has 12 apples. She wants to share them equally among her 2 friends. How many apples will each friend receive?

# Quotative

Nicole has 12 apples. She put them into bags containing 2 apples each. How many bags did Nicole use?

Addition	Subtraction
Multiplication	Division







$$12 \div 4 = \underline{\quad}$$

If you'd watch a comedy show:

What's a **Partitive** story to show division?

If you'd watch a drama how:

What's a **Quotative** story to show division?





Addition

Subtraction

Multiplication

Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ - 4 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ \times 7 \\ \hline \end{array} \quad \begin{array}{r} 56 \\ \div 8 \\ \hline \end{array}$$





Addition

Subtraction

Multiplication

Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ - 724 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div 15 \\ \hline \end{array}$$





Addition

Subtraction

Multiplication

Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ + 3.9 \\ \hline \end{array} \quad \begin{array}{r} 7.892 \\ \div 0.14 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4} \quad \frac{9}{4} - \frac{3}{8}$$





Addition

Subtraction

Multiplication

Division

Build fluency with integer computation

$$-135 \div 2 = \begin{array}{r} 6 \\ \times -12 \\ \hline \end{array}$$

$$-14 - (-7) = \begin{array}{r} 1.4 \\ + -3.9 \\ \hline \end{array}$$





Addition

Subtraction

Multiplication

Division



What type of fluency do  
your students need to  
develop?

How will you practice  
that?







# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving  
instruction





### Problem-Solving Difficulties

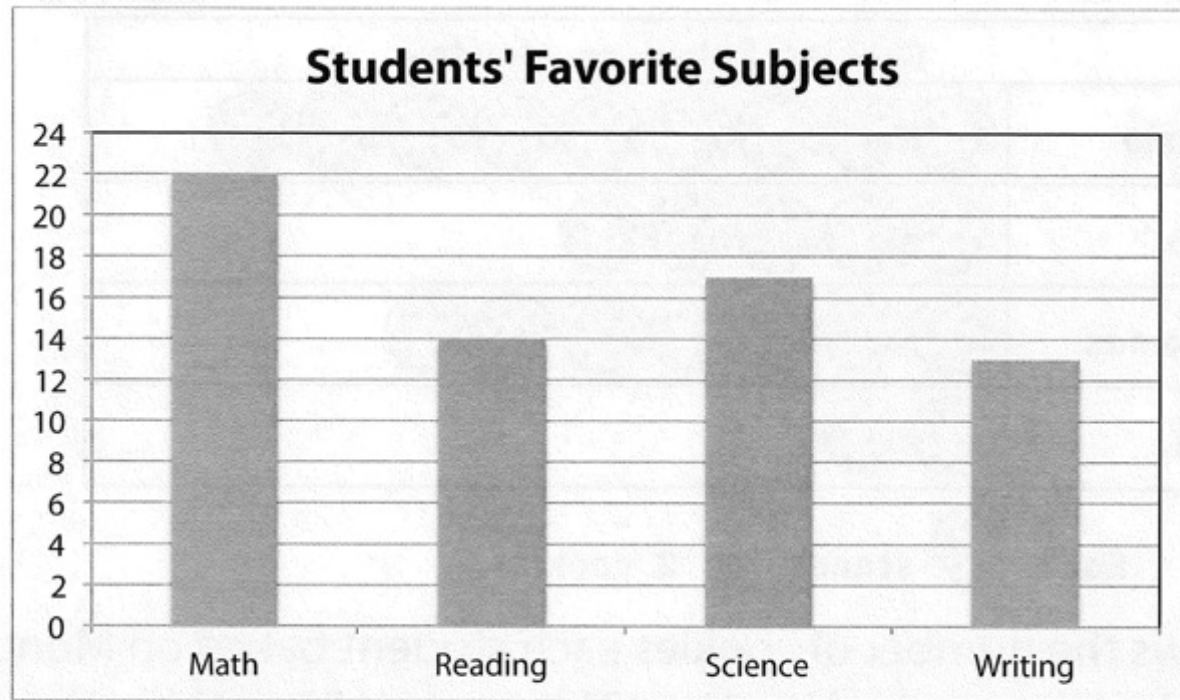

### Attack Strategy

UPS✓
UNDERSTAND <small>Read and explain.</small>
PLAN <small>How will you solve the problem?</small>
SOLVE <small>Set up and do the math!</small>
✓CHECK <small>Does your answer make sense?</small>





J.

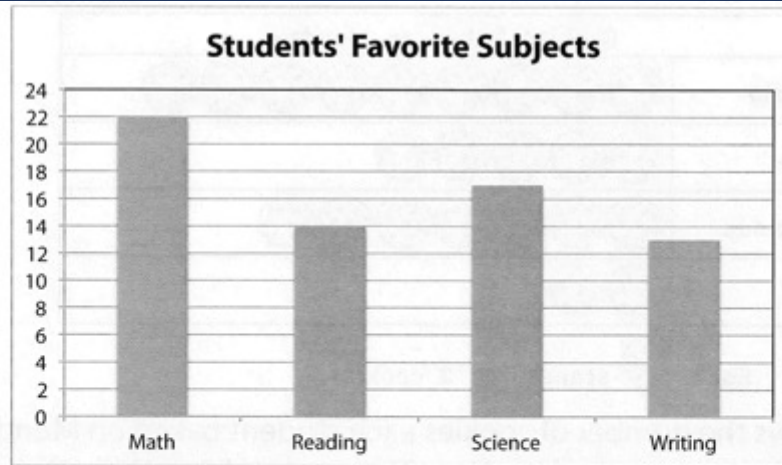


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?





J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

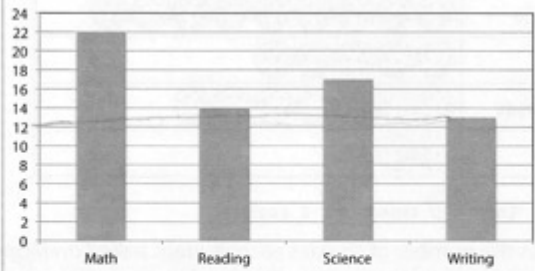
60

|||||

|||||



Students' Favorite Subjects

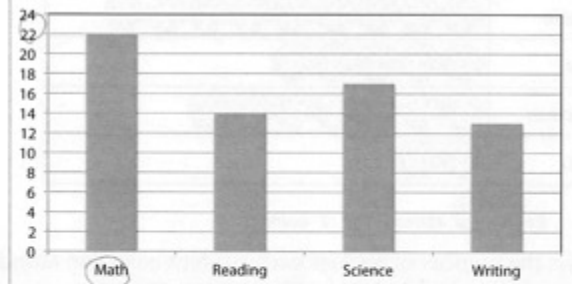


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

22 math

12 writing

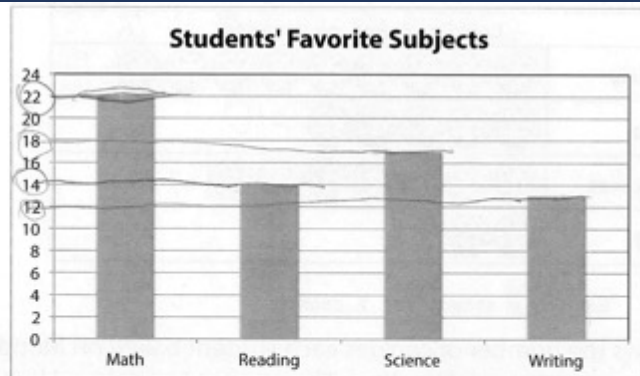
Students' Favorite Subjects



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

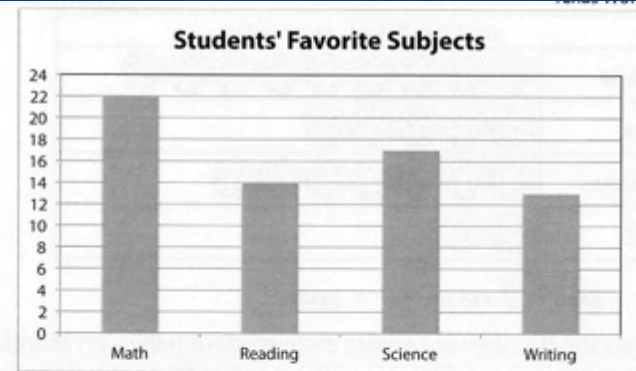
22 math





The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

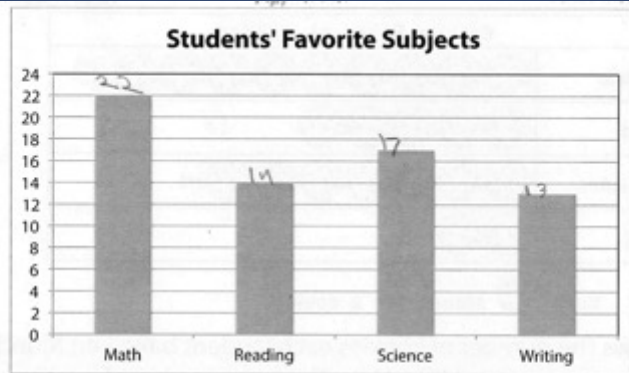
22 Math  
14 Writing



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

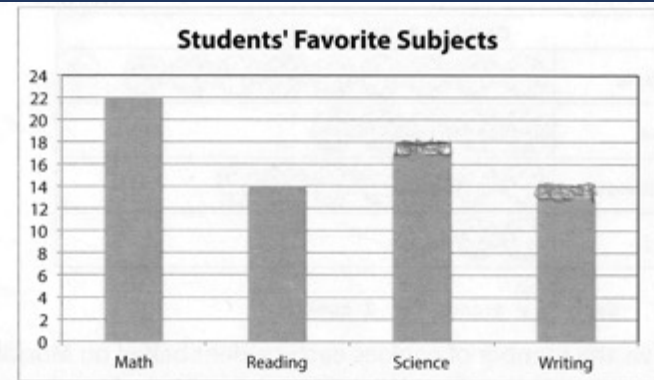
the students chooses they  
love more math d'cos es  
to 22 math





The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

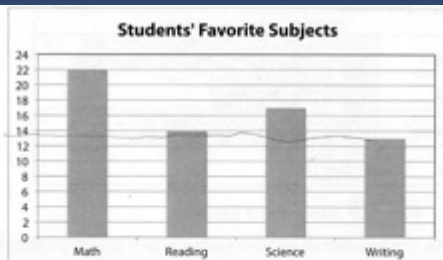
$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

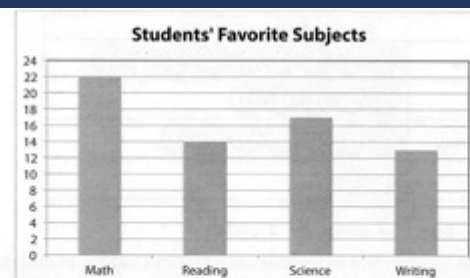
$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$





The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

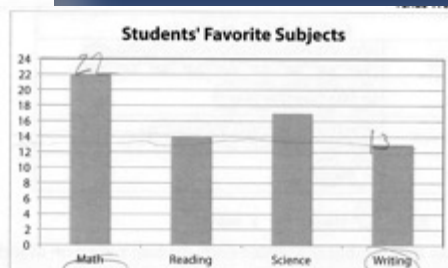
$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ + 12 \\ \hline 34 \end{array}$$

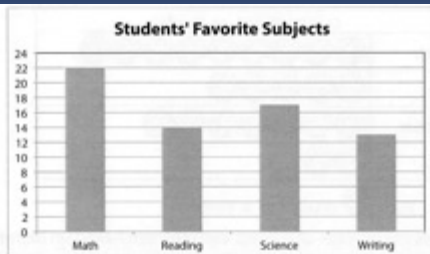
34



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

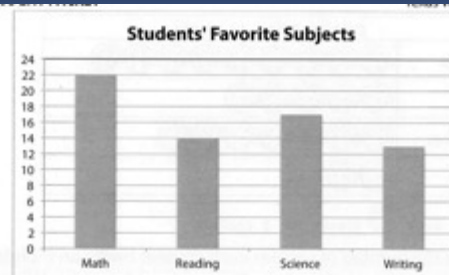
$$\begin{array}{r} 22 \\ + 12 \\ \hline 34 \end{array}$$





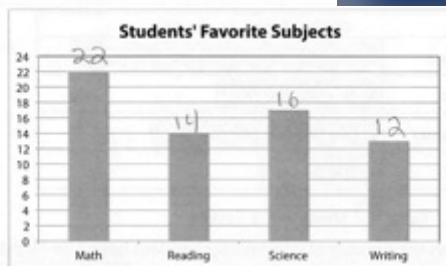
The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 12 \\ \hline 10 \end{array}$$



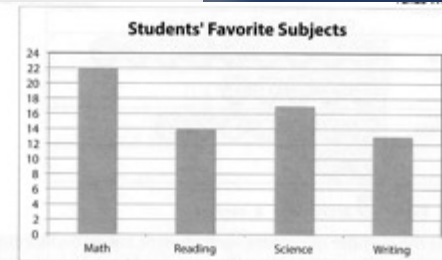
The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

they chose 8% more than reading



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 12 \\ \hline 10 \end{array}$$



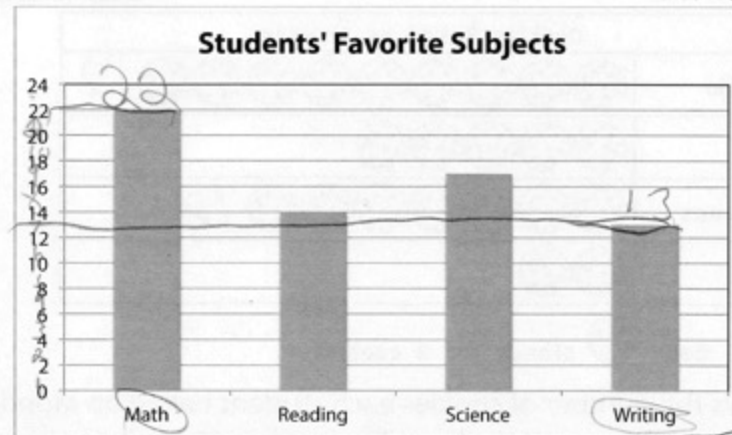
The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 12 \\ \hline 10 \end{array}$$





J.

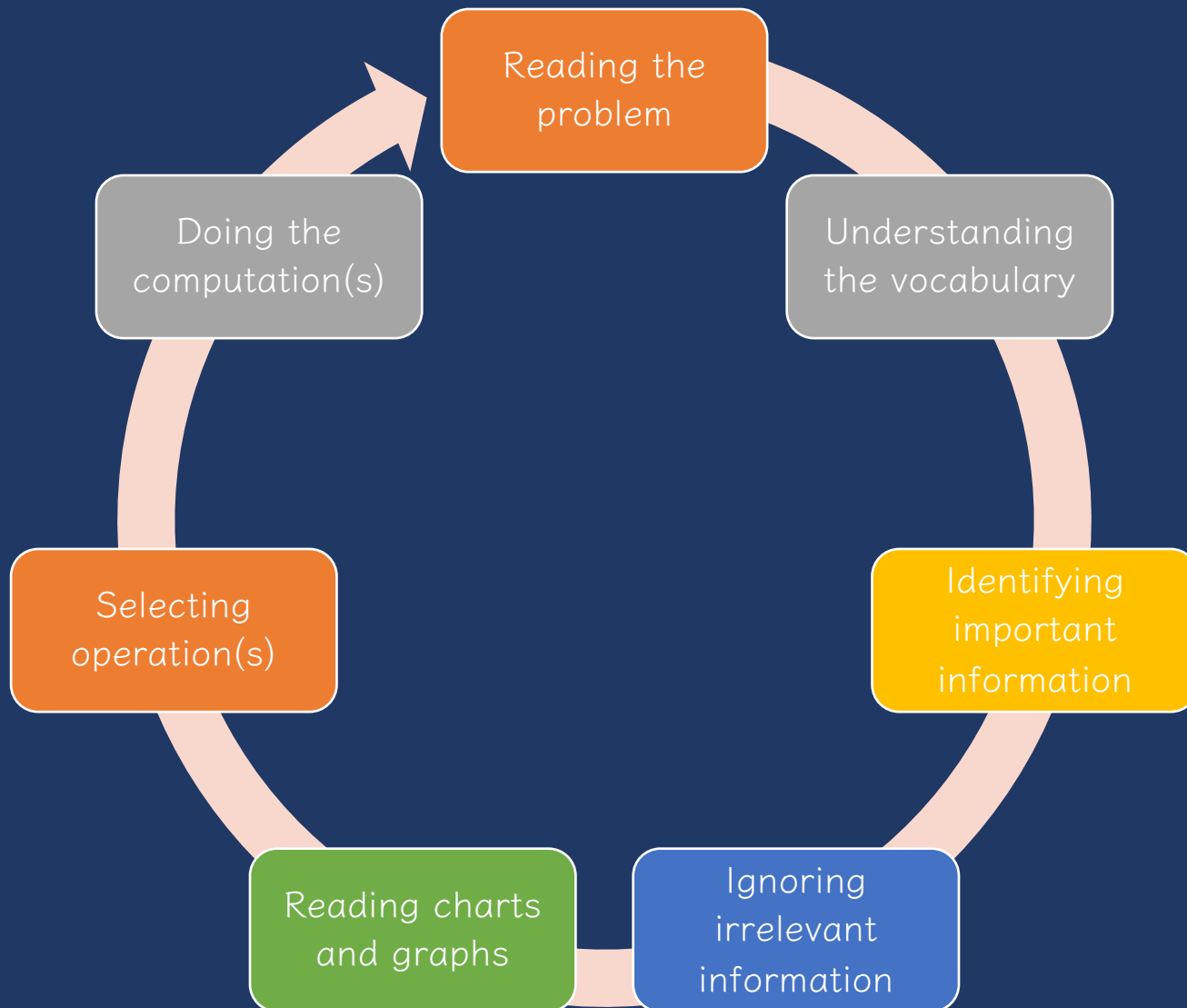


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 13 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$







~~1. Keywords tied to operations~~





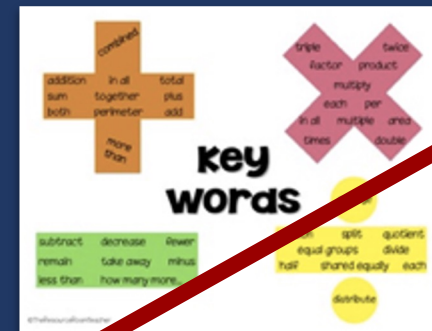
Addition	Subtraction
• Sum • Total	• Fewer • Less than
• Plus • In all	• Exceed • Remain
• And • Join	• Are not • Minus
• Altogether	• Difference
• Perimeter	• How many more
• Together	• Take away
	• Left over
When I say... They mean...	
• Times • Each	• Half • Separate
• Twice • Per	• Split • Quotient
• Area • Product	• Divisor • Cut up
• In all • Multiple	• Dividend • Same
• Equal groups	• Divided by
• Multiplied by	• Cut up
Multiplication	Division

Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?



Key Words Used in Math Word Problems	
<b>Addition Words</b> <ul style="list-style-type: none"> <li>• add</li> <li>• all together or altogether</li> <li>• and</li> <li>• both</li> <li>• combined</li> <li>• how many in all</li> <li>• how much</li> <li>• in all</li> <li>• increased by</li> <li>• plus</li> <li>• sum</li> <li>• together</li> <li>• total</li> </ul>	<b>Subtraction Words</b> <ul style="list-style-type: none"> <li>• change</li> <li>• decreased by</li> <li>• difference</li> <li>• fewer or fewer than</li> <li>• how many are left (or have left)</li> <li>• how many did not have</li> <li>• how many (or much) more</li> <li>• how much longer (shorter, taller, heavier, etc.)</li> <li>• less or less than</li> <li>• lost</li> <li>• minus</li> <li>• need to</li> <li>• reduce</li> <li>• remain</li> <li>• subtract</li> <li>• take away</li> </ul>
<b>Multiplication Words</b> <ul style="list-style-type: none"> <li>• by (dimension)</li> <li>• double</li> <li>• each group</li> <li>• every</li> <li>• factor of</li> <li>• increased by</li> <li>• multiplied by</li> <li>• of</li> <li>• product</li> <li>• times</li> <li>• triple</li> </ul>	<b>Division Words</b> <ul style="list-style-type: none"> <li>• each</li> <li>• each group has</li> <li>• equal shares (parts)</li> <li>• half (or other number)</li> <li>• how many in each</li> <li>• parts</li> <li>• per</li> <li>• percent</li> <li>• quotient of</li> <li>• ratio of</li> <li>• separated</li> <li>• share something equally</li> </ul>



Addition	Subtraction
<ul style="list-style-type: none"> <li>• Sum</li> <li>• Plus</li> <li>• And</li> <li>• Altogether</li> <li>• Perimeter</li> <li>• Together</li> </ul>	<ul style="list-style-type: none"> <li>• Fewer</li> <li>• Less than</li> <li>• Exceed</li> <li>• Remain</li> <li>• Are not</li> <li>• Minus</li> <li>• Difference</li> <li>• How many more</li> <li>• Take away</li> <li>• Left over</li> </ul>
When they say... They mean...	
<ul style="list-style-type: none"> <li>• Times</li> <li>• Each</li> <li>• Twice</li> <li>• Per</li> <li>• Area</li> <li>• Product</li> <li>• In all</li> <li>• Multiple</li> <li>• Equal groups</li> <li>• Multiplied by</li> </ul>	<ul style="list-style-type: none"> <li>• Half</li> <li>• Separate</li> <li>• Split</li> <li>• Quotient</li> <li>• Divisor</li> <li>• Cut up</li> <li>• Dividend</li> <li>• Same</li> <li>• Divided by</li> <li>• Cut up</li> </ul>
Multiplication	Division





**Word-Problem Words Poster Set**

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**Addition**

- to
- join
- both
- increase
- combined
- plus
- in all
- add
- all together
- sum

**Division**

- share equally
- parts
- goes into
- quotient
- each
- divided by
- half (or other fraction)
- split equally

**Subtraction**

- how many more
- remain
- minus
- decrease
- left
- subtract
- less than
- difference
- take away

**Multiplication**

- multiplied by
- product
- times
- total
- by
- equal groups
- twice







*Description of Single-Step Word Problems (n = 132)*

Schema	Occurrence of schema		Any keyword		Schema-specific keywords <sup>a</sup>		Multiple keywords <sup>a</sup>		Keyword(s) led to correct solution <sup>a</sup>	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

<sup>a</sup>When a problem featured a keyword.





### Description of Multi-Step Word Problems ( $n = 84$ )

Schema	Occurrence of schema <sup>a</sup>		Any keyword		Keyword(s) led to correct solution <sup>b</sup>	
	$n$	%	$n$	%	$n$	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

<sup>a</sup>Sum across schemas does not equal 100 because each word problem featured more than one schema.

<sup>b</sup>When a problem featured a keyword.





Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by  $24^{\circ}\text{C}$  per minute for 3 minutes. What was the overall change of the temperature of the substance?





# Important notes about keywords

Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords  
("What does *more than* tell you about?")



But, *do not* tie a keyword to a specific operation!





## 2. Presenting problems by operation






Name: \_\_\_\_\_ Date: \_\_\_\_\_

## Addition Word Problems

Solve the word problems. Show your work.



- Noah had 12 books. He got 6 more books. How many books did Noah have in all?
- Bonnie found 8 rocks on her sidewalk and 7 rocks in her backyard. How many rocks did Bonnie find in all?
- Edward had 5 toy cars. He got 8 more toy cars. How many toy cars did Edward have in all?
- Mariela collected 11 feathers. Then she found 3 more feathers. How many feathers did Mariela have in all?
- LaMonte made 14 cookies. Then he made 5 more cookies. How many cookies did LaMonte have in all?

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4-Digit: S1

## Subtraction Word Problems

- In a botanical garden, there are 5,626 varieties of native and exotic plants. If 2,290 of the plants are exotic, what is the number of native plants?  
\_\_\_\_\_
- If a restaurant uses 7,984 of the 9,151 eggs they had purchased during the month, how many eggs were left unused?  
\_\_\_\_\_
- There are 9,376 people watching a soccer game. If 9,174 of them are adults, how many children are present at the game?  
\_\_\_\_\_
- Matthew scored 3,741 points in a video game while Bryan scored 2,512. How many points more did Matthew score?  
\_\_\_\_\_
- A food-processing company used 4,835 bags of flour in the first week. During the second week, the number increased to 8,572. How many more bags of flour did they use in the second week?  
\_\_\_\_\_

A clockmaker sold 8,948 clocks in 2013. In 2014, he sold 9,407. How many more clocks were sold in 2014?  
\_\_\_\_\_

Teaching Resources @ [www.tutoringhour.com](http://www.tutoringhour.com)

## LONG DIVISION WORD PROBLEMS

- Zookeeper Al wants to give each monkey at the zoo an equal number of bananas. There are 37 monkeys in the zoo and 547 bananas. How many bananas does each monkey get? And how many are left over for him to eat himself?
- Betty has 427 oranges and needs to pack them up equally in 23 boxes. How many oranges go in each box and how much does she have left over?
- Miss King has 1376 pages of scrap paper. She wants to make them into scrap paper packets for her 32 students. How many pages will each packet have? How many extra pages will she have left over?
- Mr. Chong has 1,440 pages of scrap paper. He instead wants to make packets of 40 pages each but forgets to check if that will be enough for his 37 students. Will there be enough packets per student? If not how much more scrap paper does he need?

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Have an attack strategy  
Teach word-problem schemas





# Have an attack strategy

## RIDE

**R**ead the problem.

**I**dentify the relevant information.

**D**etermine the operation and unit for the answer.

**E**nter the correct numbers and calculate, then check the answer.

## RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.





# Have an attack strategy

## STAR

**S**top and read the problem carefully.

**T**hink about your plan and the strategy you will use.

**A**ct. Follow your plan and solve the problem.

**R**everview your answer.

## RICE

**R**ead and record the problem.

**I**llustrate your thinking.

**C**ompute.

**E**xplain your thinking.





# Have an attack strategy

## SUPER

Slowly read the story problem twice.  
Underline the question and circle the numbers you need.  
Picture it. Draw the scenario to show what is happening.  
Explain the problem with a number sentence.  
Rewrite the answer in a sentence.

## SHINES

Slowly and carefully read the problem.  
Highlight or underline key information.  
Identify the question by drawing a circle around it.  
Now solve the problem. Show your work.  
Examine your work for precision, accuracy, and clarity.  
Share your answer by writing a sentence.





# Have an attack strategy

## SOLVE

**S**tudy the problem.

**O**rganize the facts.

**L**ine up the plan.

**V**erify the plan with computation.

**E**xamine the answer.

## R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

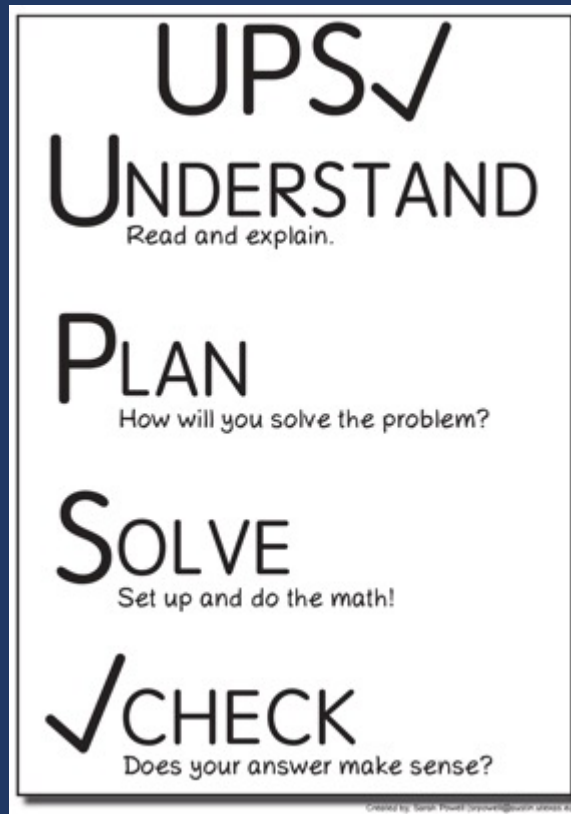
Evaluate steps.

Solve and check.



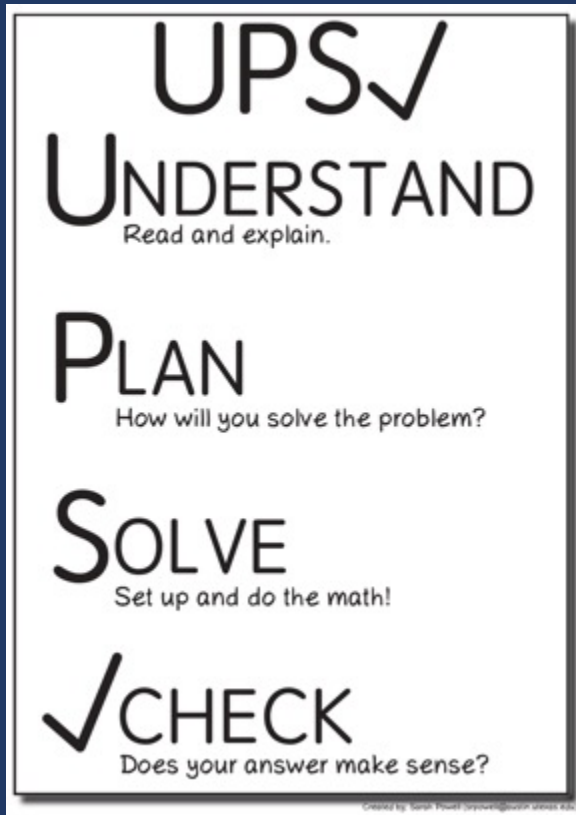


# Have an attack strategy





# Have an attack strategy



What's your favorite attack strategy? Why?





# Teach word-problem schemas

Total

Equal Groups

Difference

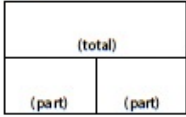

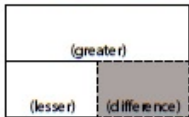
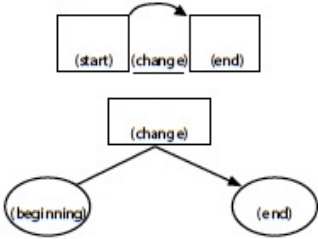
Comparison

Change

Ratios/Proportions





Schema and Definition	Equations and Graphic Organizers	Examples	Variations
<b>Total (Combine; Part-part-whole)</b> Parts combined for a sum	$P1 + P2 = T$ (part + part = total) 	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?  Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?	More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
<b>Difference (Compare)</b> Sets compared for a difference	$B - s = D$ (bigger - smaller = difference)  $G - L = D$ (greater - less = difference) 	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?  Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?  Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
<b>Change (Join; Separate)</b> An amount that increases or decreases	$ST \pm C = E$ (start $\pm$ change = end) 	End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now?  Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting?  Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with?  End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?  Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?	Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?  Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?

Powell & Fuchs (2018).

Material collected from: Griffin & Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch & Greeno, 1985; Van de Walle, Karp, & Bay-Williams, 2013.





Total

Part-part-whole  
Combine

**Parts** put together into a **total**

Daniela saw **3** canoes and **8** kayaks. How many boats did Daniela see?

Daniela saw **11** boats. If **3** of the boats were canoes, how many were kayaks?

Daniela saw **11** boats. **8** of the boats were kayaks, how many were canoes?

Total

Part

Part





Total

Part-part-whole  
Combine

“Are parts put together for a total?”

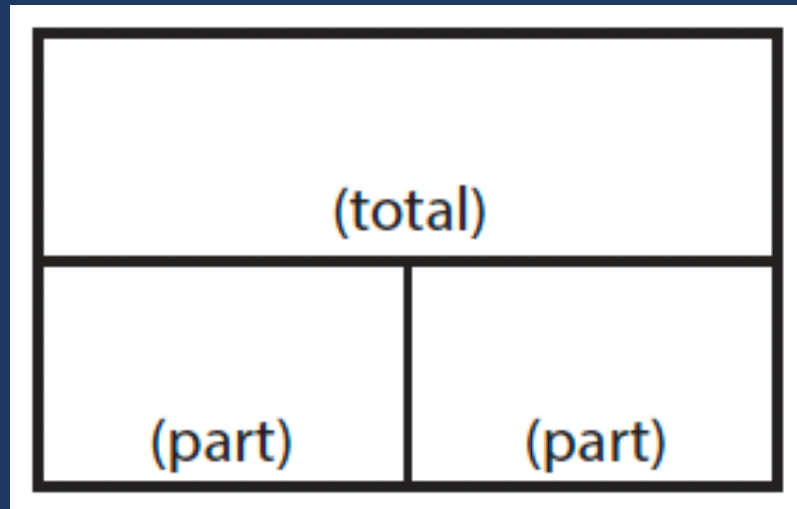




Total

Part-part-whole  
Combine

$$P1 + P2 = T$$





# Difference

Compare

## Greater and lesser amounts compared for a difference

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have?

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does Tracy have?

Difference

Greater  
amount

Lesser  
amount





Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”





# Difference

Compare

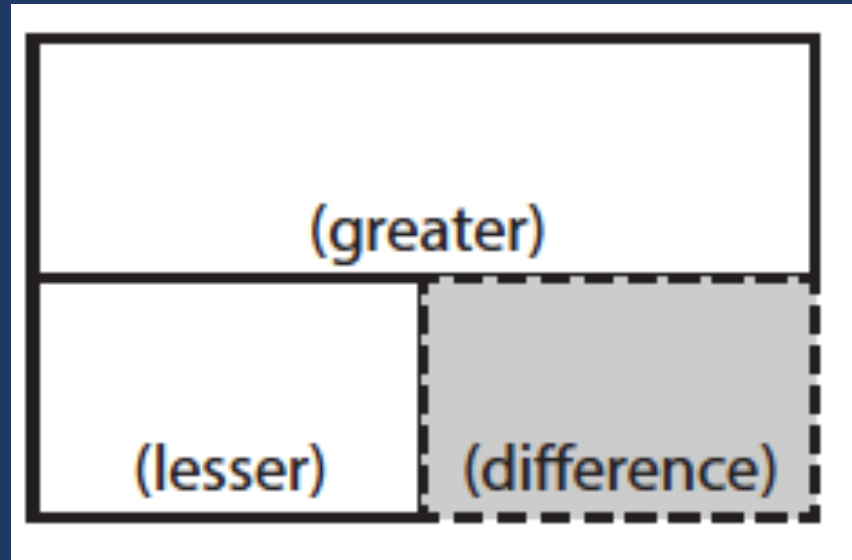
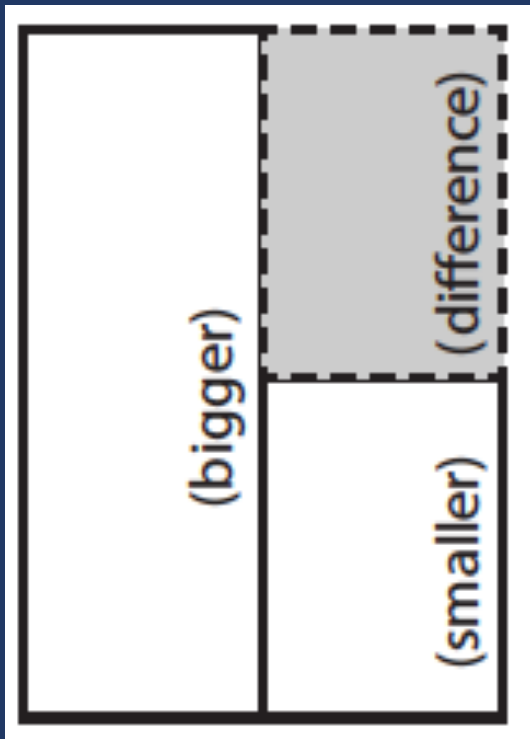
G

—

L

=

D





Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

End  
amount

Change  
amount

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Start  
amount

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?





Change

Separate

An amount that increases or decreases

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

End  
amount

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Change  
amount

Samantha baked some cookies. She ate 3 of the cookies and has 17 cookies left. How many cookies did Samantha bake?

Start  
amount





Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

Change

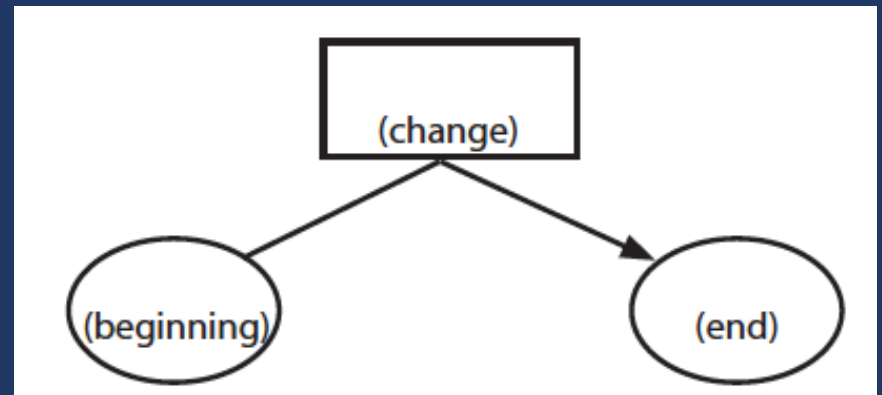
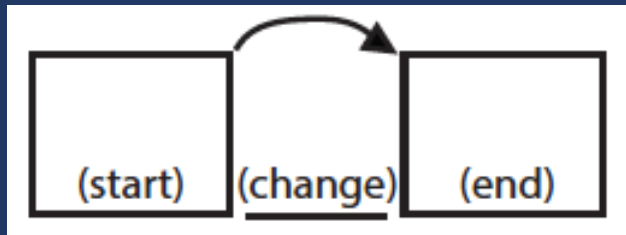
“Does an amount increase or decrease?”



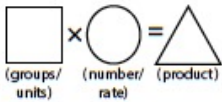
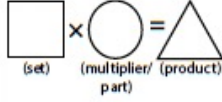
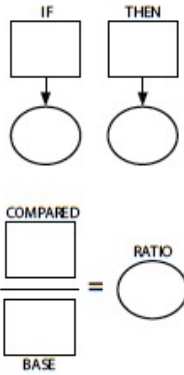


# Change

$$ST + / - C = E$$





Schema and Definition	Graphic Organizers	Examples			Variations
<b>Equal Groups (Vary)</b> A number of equal sets or units		<b>Product unknown:</b> Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	<b>Groups unknown:</b> Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	<b>Number unknown:</b> Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	<b>With rate:</b> Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
<b>Comparison</b> One set as a multiple or part of another set		<b>Product unknown:</b> Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	<b>Set unknown:</b> Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	<b>Times unknown:</b> Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	<b>With fraction:</b> Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
<b>Proportions</b>		<b>Subject unknown:</b> Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	<b>Object unknown:</b> Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	<b>Ratio unknown:</b> Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	<b>With percentage:</b> Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?  <b>With unit rate:</b> Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.





# Equal Groups

Array  
Vary

Groups multiplied by number in each group for a product

Toni has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Toni have altogether?

Groups

Toni has 24 crayons. They want to place them equally into 2 boxes. How many crayons will Toni place in each box?

Number in each group

Toni has 24 crayons. They put them into boxes with 12 crayons each. How many boxes did Toni use?

Product





## Equal Groups

“Are there groups with an equal number in each group?”





# Equal Groups

Array  
Vary

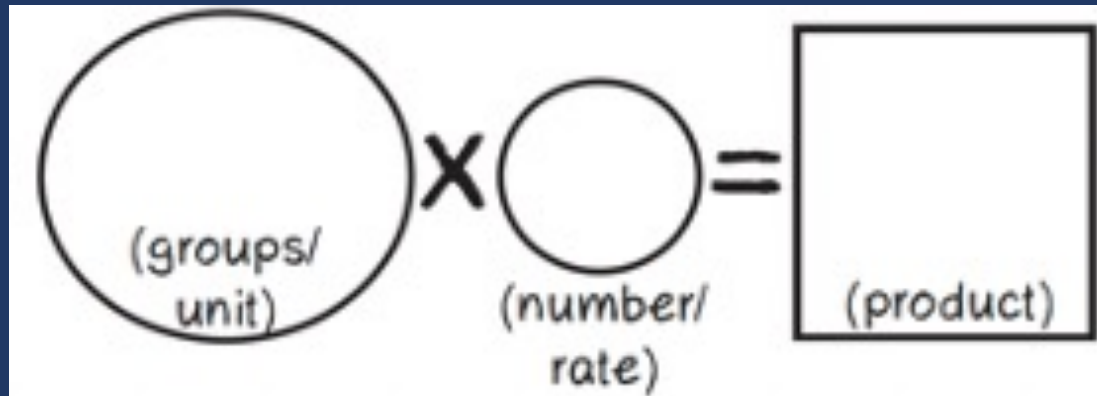
GR

×

N

=

P





## Comparison

**Set** multiplied by a number of **times**  
for a **product**

Brooke ran **6** minutes. Shaleeni ran **4** times longer than Brooke. How many minutes did Shaleeni run?

Set

Number  
of times

Product





## Equal Groups

“Are there groups with an equal number in each group?”

## Comparison

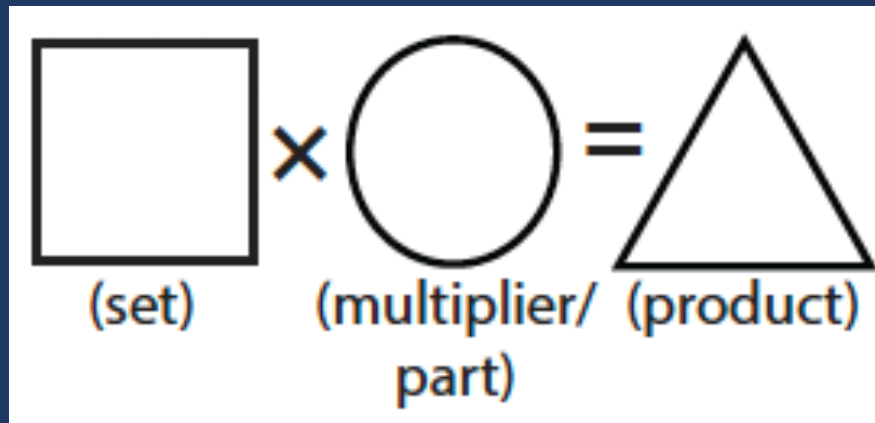
“Is a set compared a number of times?”





# Comparison

$$S \times T = P$$





# Ratios/Proportions

## Description of **relationships** among quantities

Emma typed **56** words in **2** minutes. At this rate, how many words could Emma type in **7** minutes?

Melissa baked cookies and brownies. The ratio of cookies to brownies was **3:5**. If she baked **25** brownies, how many cookies did she bake?





## Equal Groups

“Are there groups with an equal number in each group?”

## Comparison

“Is a set compared a number of times?”

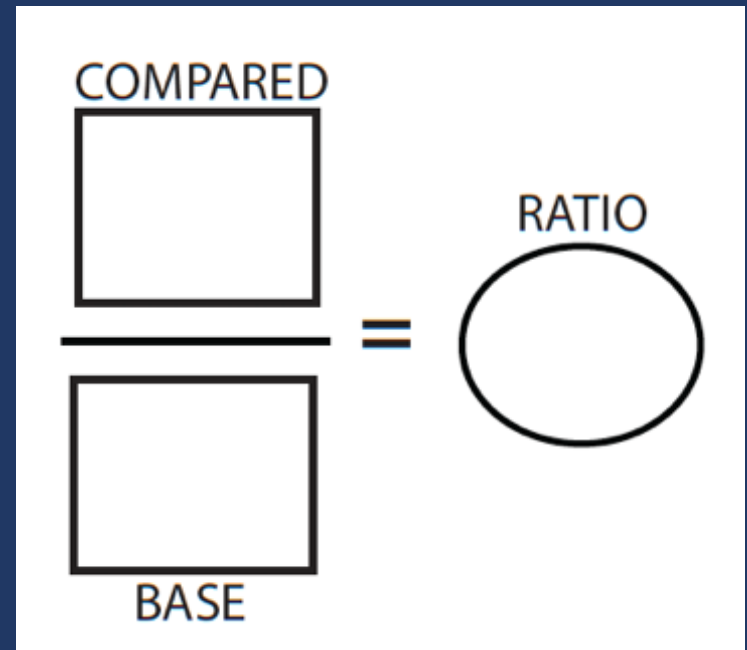
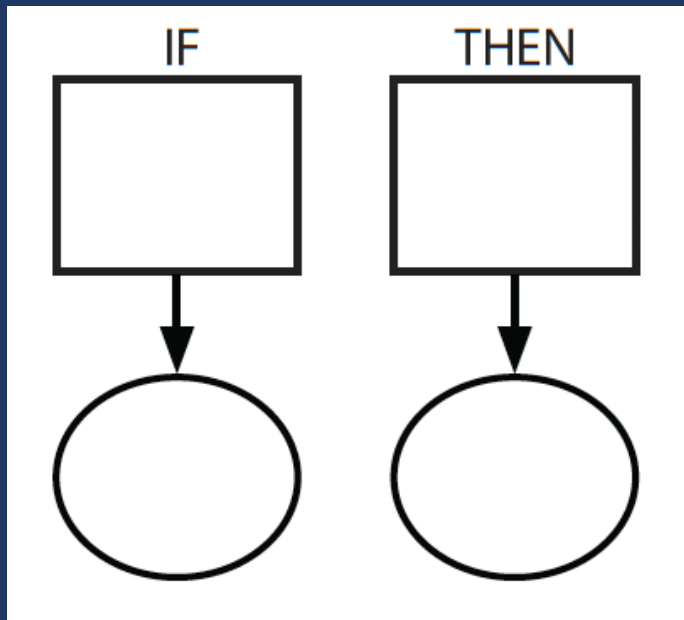
## Ratios/Proportions

“Are there relationships among quantities - if this, then this?”





# Ratios/Proportions





# Teach word-problem schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions







# Pirate Math Equation Quest

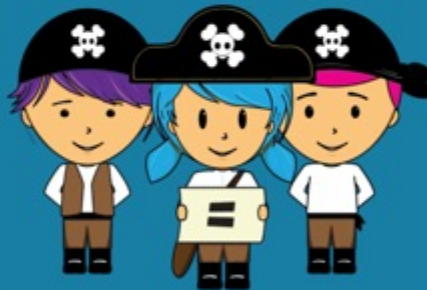
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**Welcome to Pirate Math Equation Quest!**

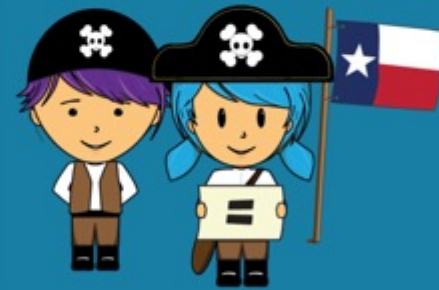
**Individual Word-Problem  
Intervention**



**Small-Group Word-Problem  
Intervention**



**Small-Group Word-Problem  
Intervention for STAAR**



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# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit  
instruction

Precise  
language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving  
instruction





National Center on  
**INTENSIVE INTERVENTION**  
at American Institutes for Research

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Intensive Intervention ▾

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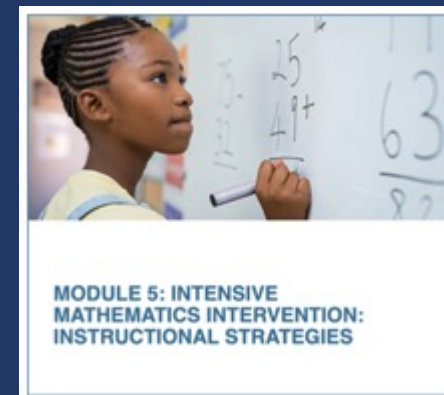
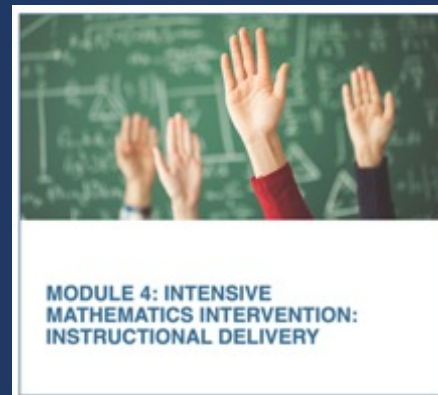
Information For... ▾

## Intensive Intervention in Mathematics Course Content

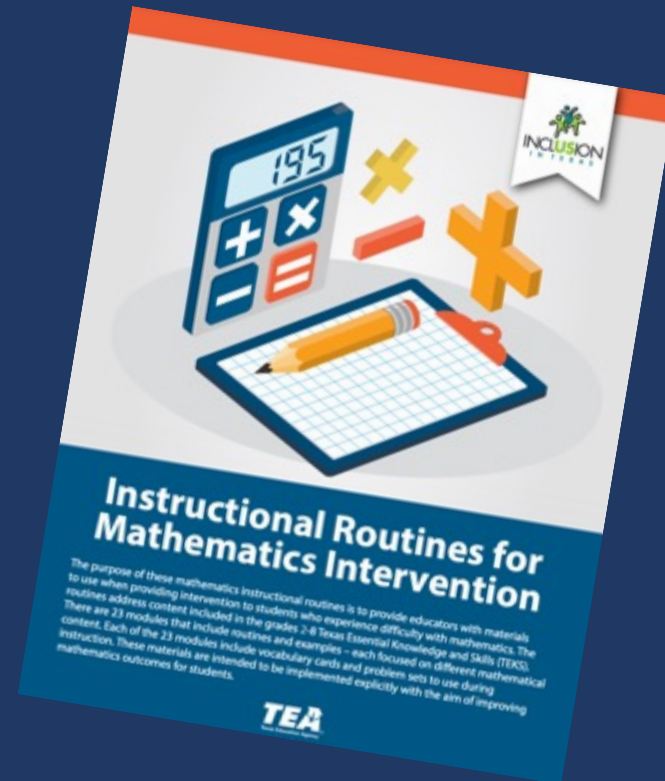
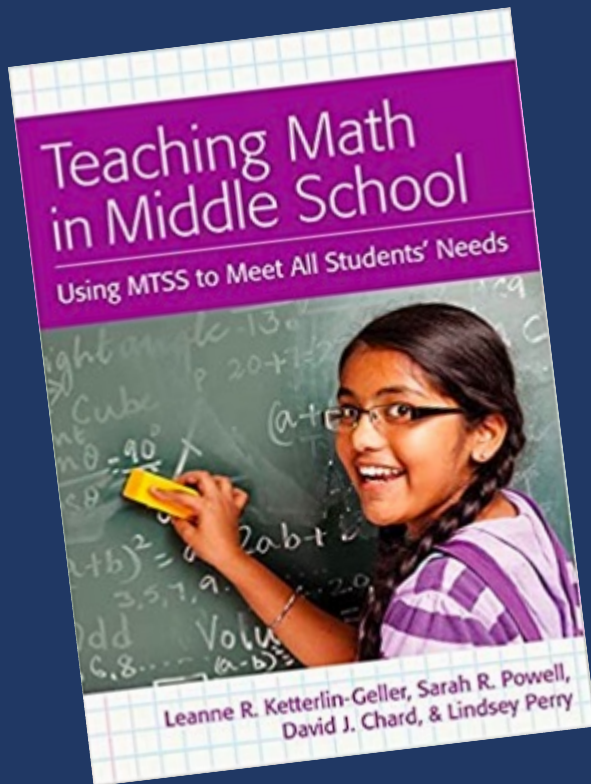
NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.







[https://www.inclusionintexas.org/apps/pages/index.jsp?uREC\\_ID=2155039&type=d&pREC\\_ID=2169859](https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859)





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