Effective Math Instruction

DAY 1

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Introduce yourself.

Describe your role as an educator.

Describe the mathematics you support.

MATH

Schedule for Today

8:00-8:25	- Introductions - Trajectories in mathematics
8:25-9:15	- Critical math content - Evidence-based practices
9:15-9:25	BREAK
9:25-10:25	- Evidence-based practice: Systematic instruction
10:25-11:30	- Evidence-based practice: Fluency with facts - Evidence-based practice: Multiple representations
11:30-1:00	LUNCH
1:00-2:15	- Evidence-based practice: Fluency with computation - Multiple representations: Whole numbers
2:15-2:25	BREAK
2:25-4:00	- Multiple representations: Rational numbers - Multiple representations: Pre-algebra - Multiple representations: Geometry

Schedule for Tomorrow

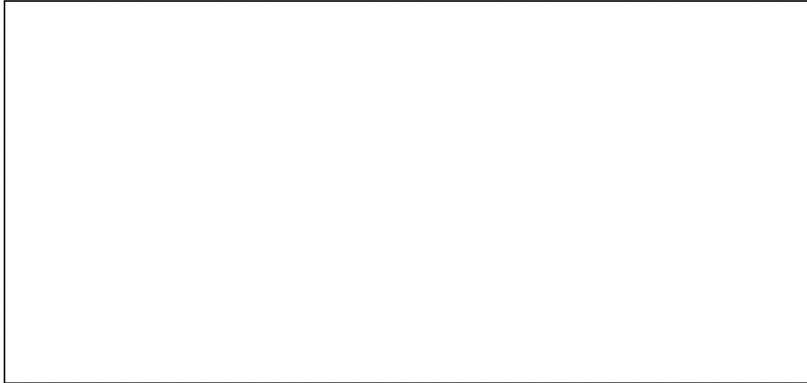| Time	Activity
8:00-9:30	- Language of mathematics
9:30-9:40	BREAK
9:40-10:15	- Evidence-based practice: Word-problem solving
10:15-10:30	- Word-problem solving: Ineffective strategies
10:30-11:00	- Word-problem solving: Attack strategies
11:00-11:30	- Word-problem solving: Total problems
11:30-1:00	LUNCH
1:00-2:15	- Word-problem solving: Difference problems - Word-problem solving: Change problems - Word-problem solving: Equal groups problems - Word-problem solving: Comparison problems
2:15-2:25	BREAK
2:25-4:00	- Word-problem solving: Ratios/Proportions problems - Word-problem solving: Multi-step problems - Wrap-up

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Trajectories in Mathematics

Effective Mathematics Practices
DAY 1

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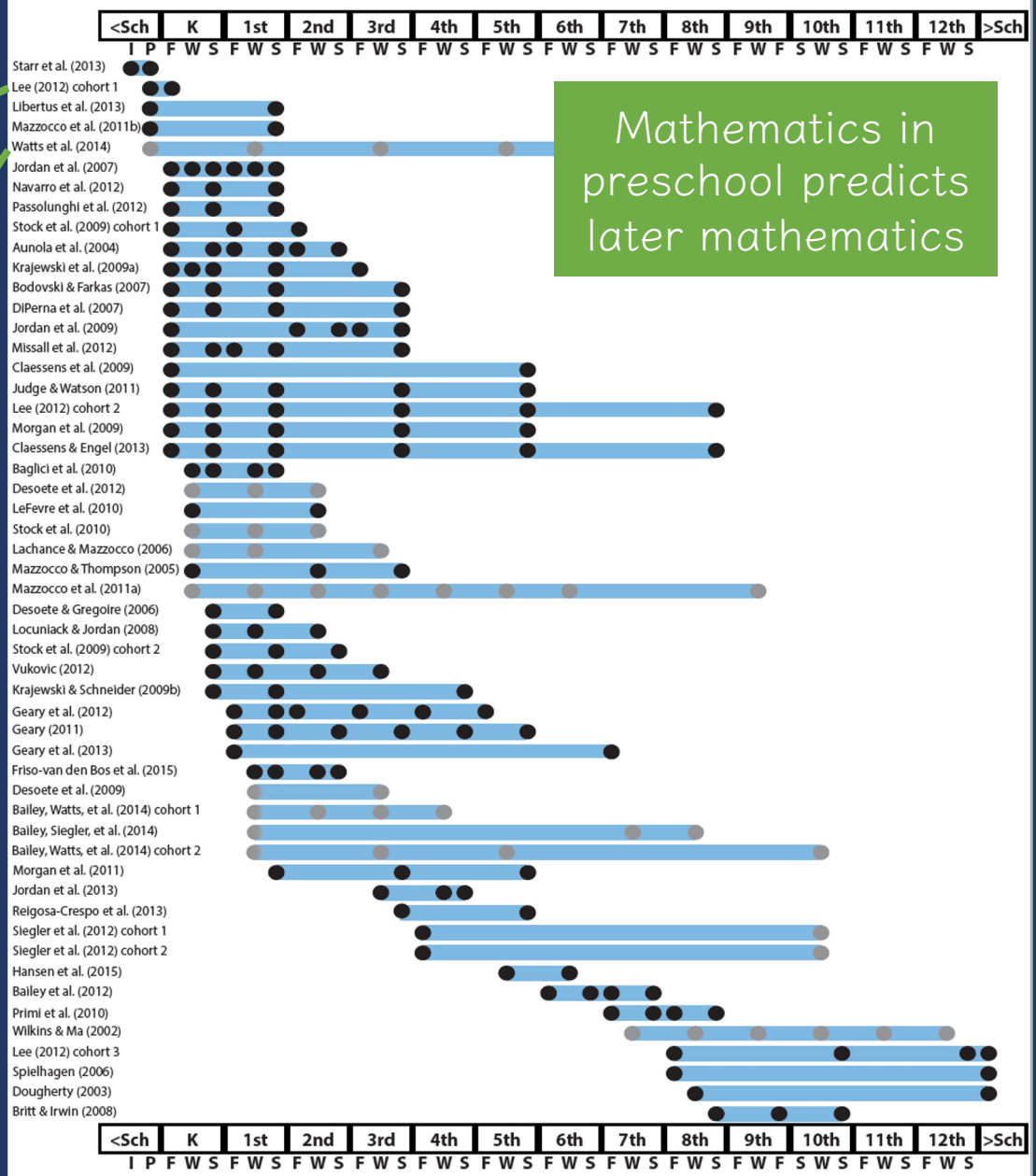
Mathematical Trajectories

Critical Content

Broad math in preK predicted K broad math

Broad math in preK predicted grade 10 broad math

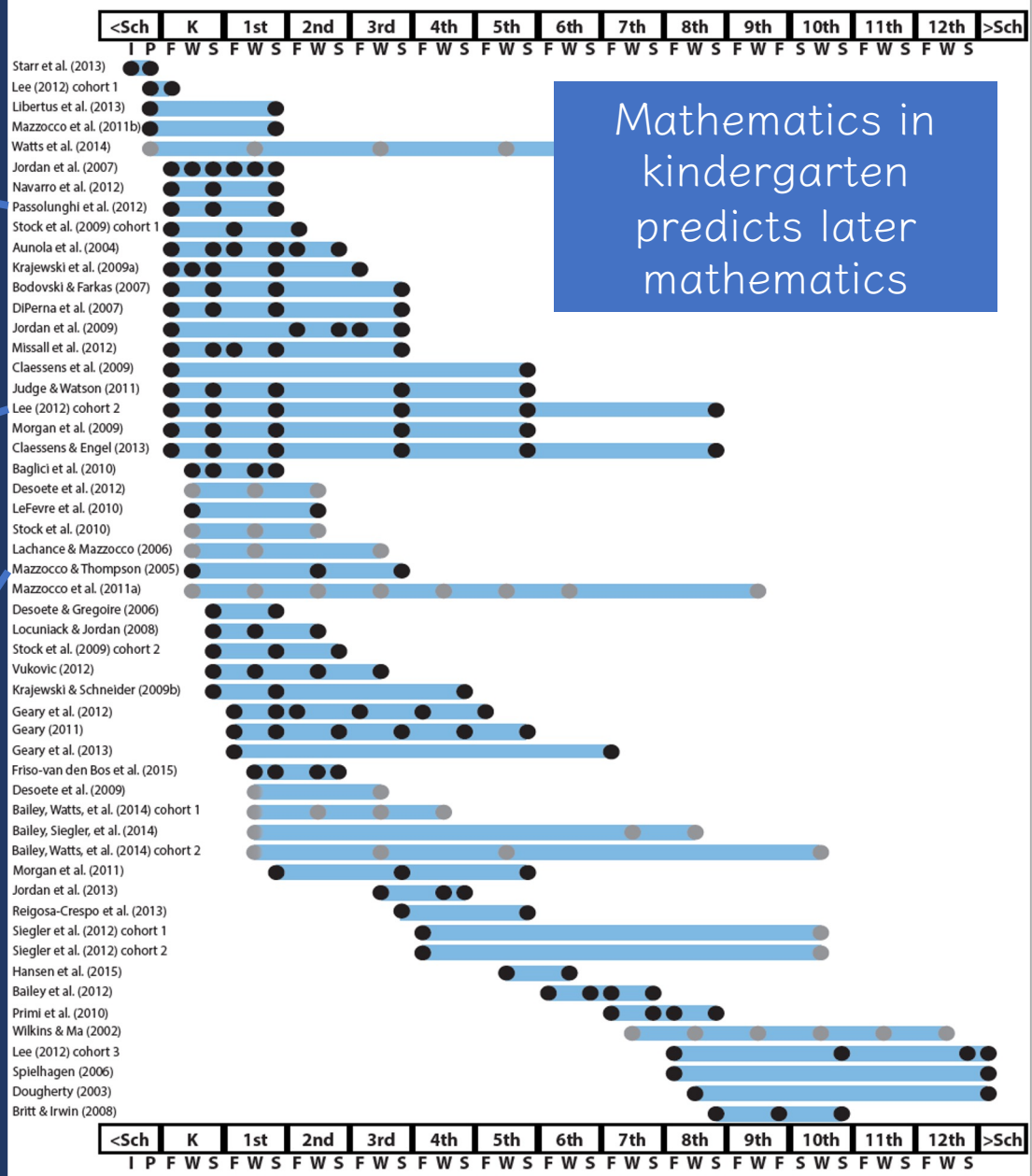Mathematics in preschool predicts later mathematics

Counting in K predicted grade 1 broad math

Broad math in K predicted grade 8 broad math

K math accurately predicted math performance below 10th percentile in grades 2 and 3 with 84% correct classification

Mathematics in kindergarten predicts later mathematics

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

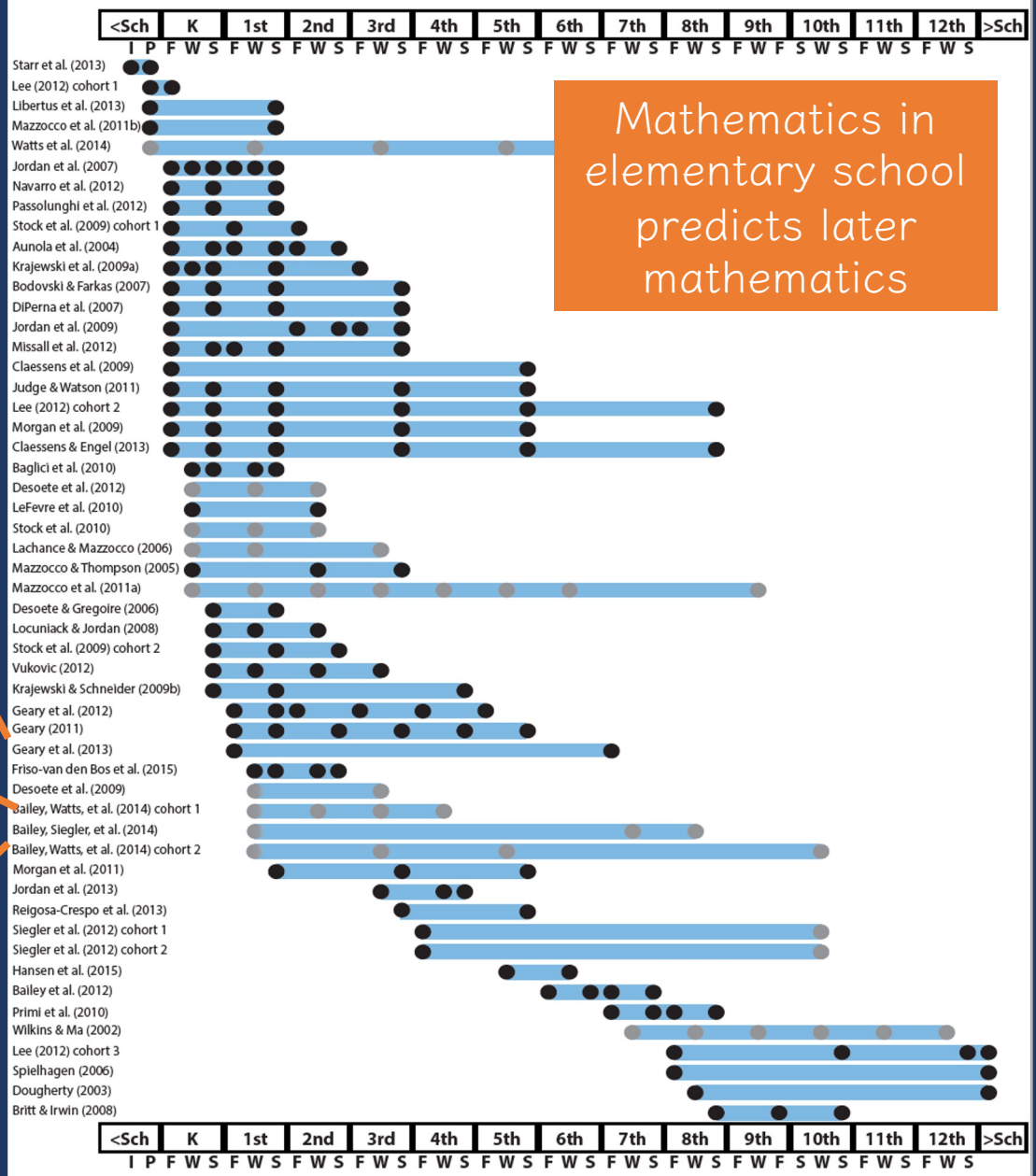Mathematics in elementary school predicts later mathematics

Addition influenced arithmetic with increasing importance from grades 1 to 5

Grade 1 arithmetic predicted arithmetic at grades 2, 3, and 4

Grade 1 broad math predicted broad math at grades 3, 5, and 10

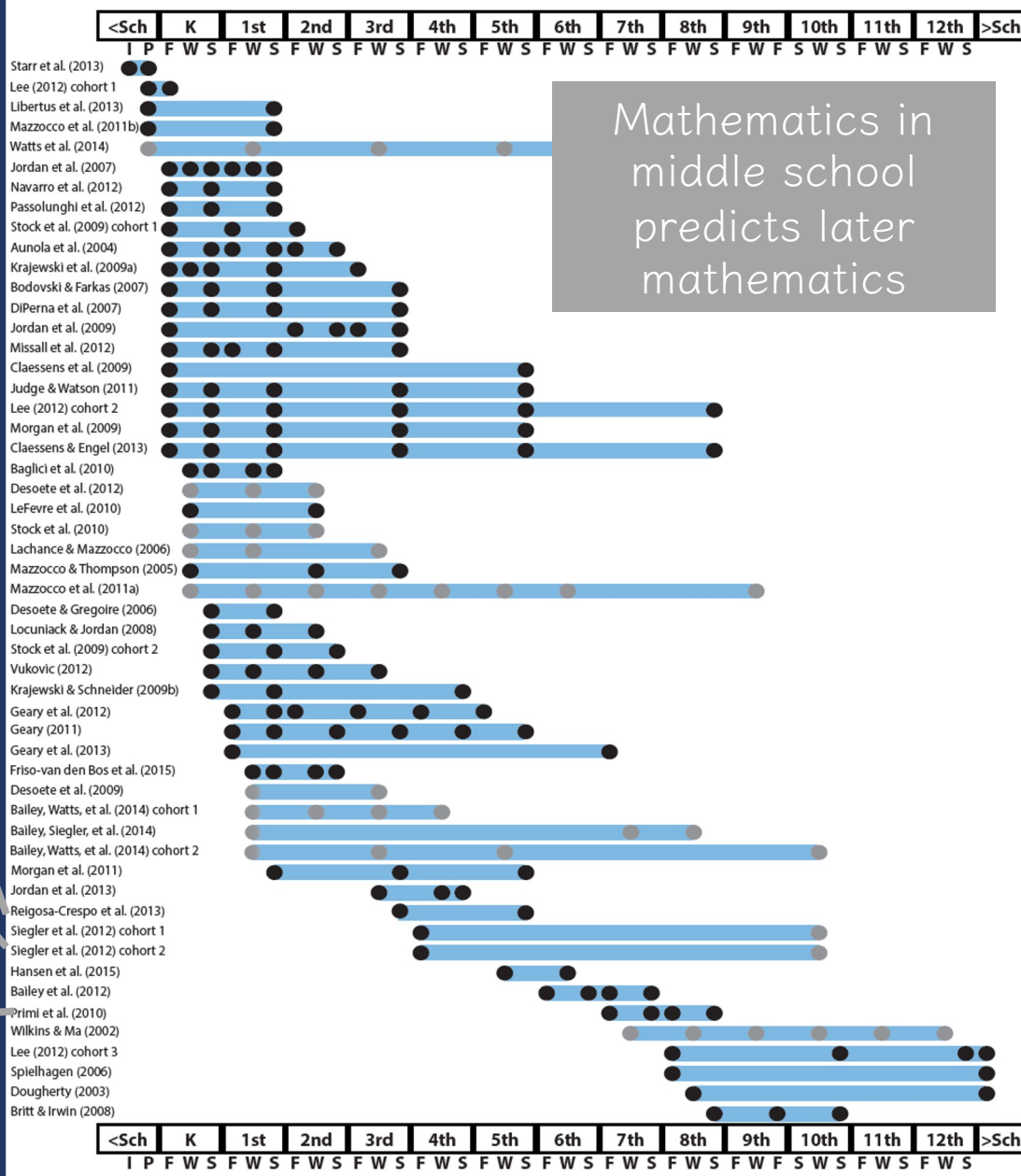Mathematics in middle school predicts later mathematics

Counting and comparison in grades 2 or 4 predicted broad math 1 year later

Fractions at 10-12 years old predicted broad math 5 years later

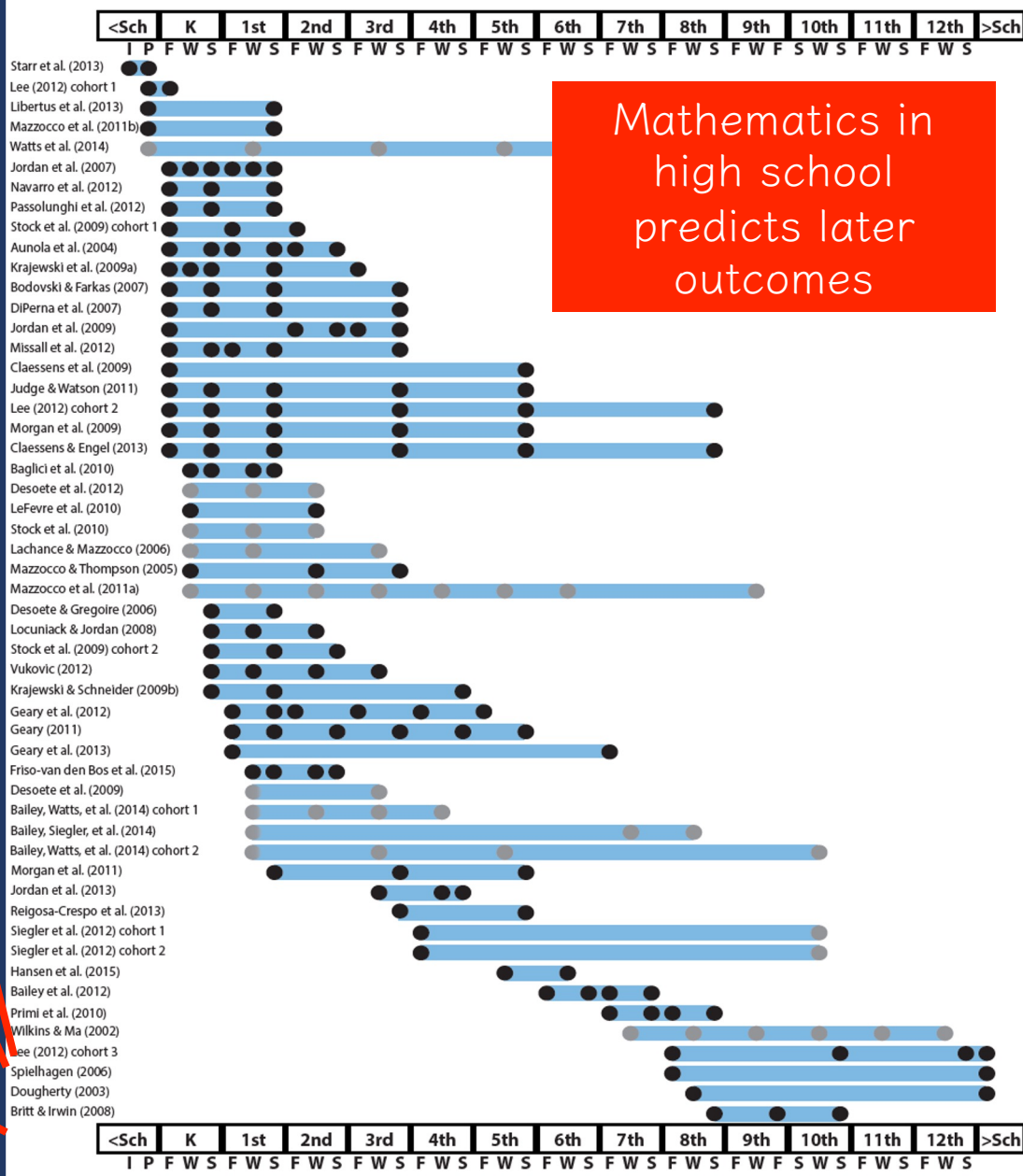Broad math in grade 7 predicted broad math in grade 8

Broad math in grade 8 predicted completion of 4-year college degree

Students who took algebra in grades 8 took more advanced math courses and enrolled in 4-year colleges more often than students who took algebra in grade 9

Numeracy measured in adolescence impacted hourly earnings 7 to 15 years later

Mathematics in high school predicts later outcomes

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/
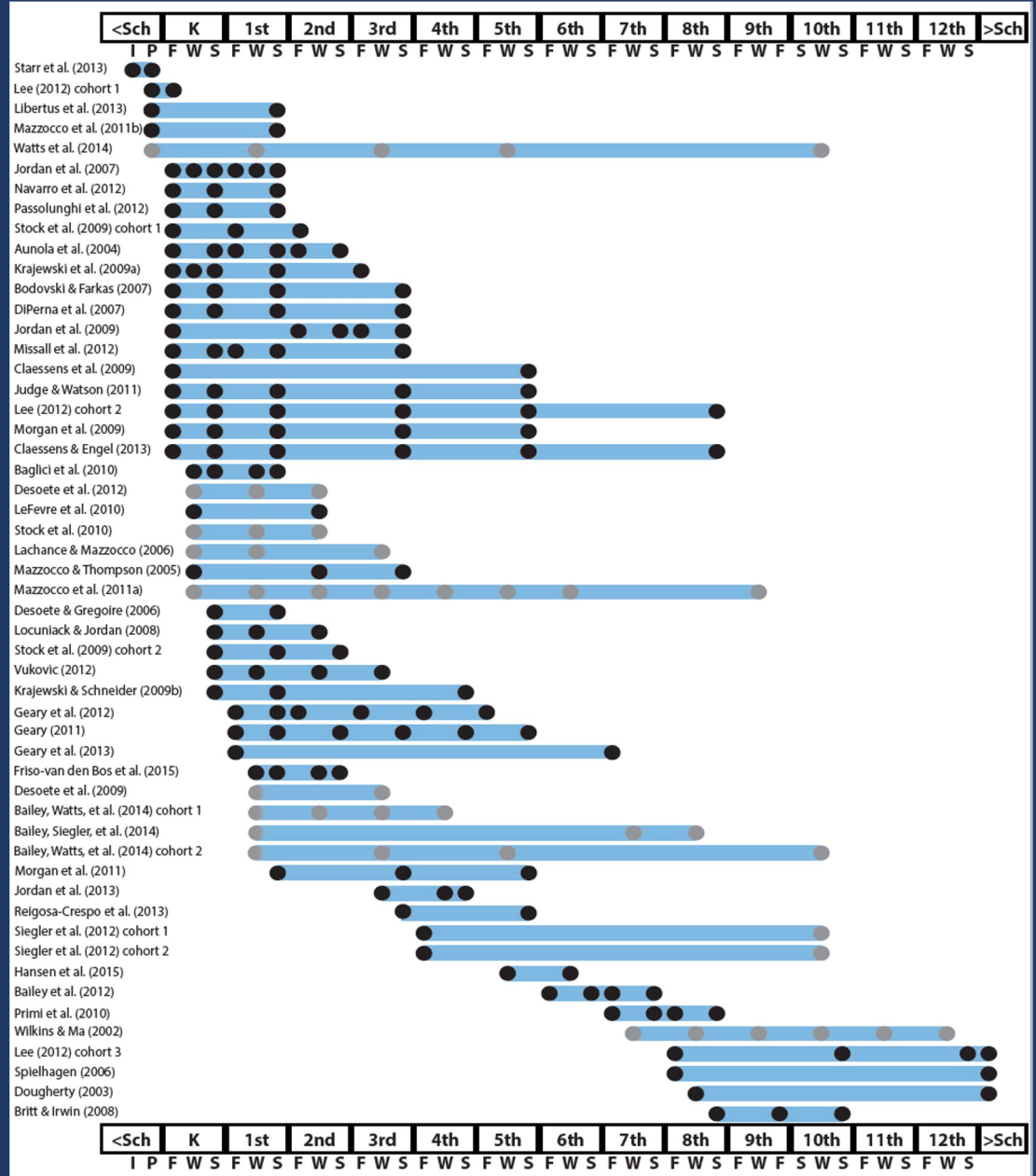
MA+H

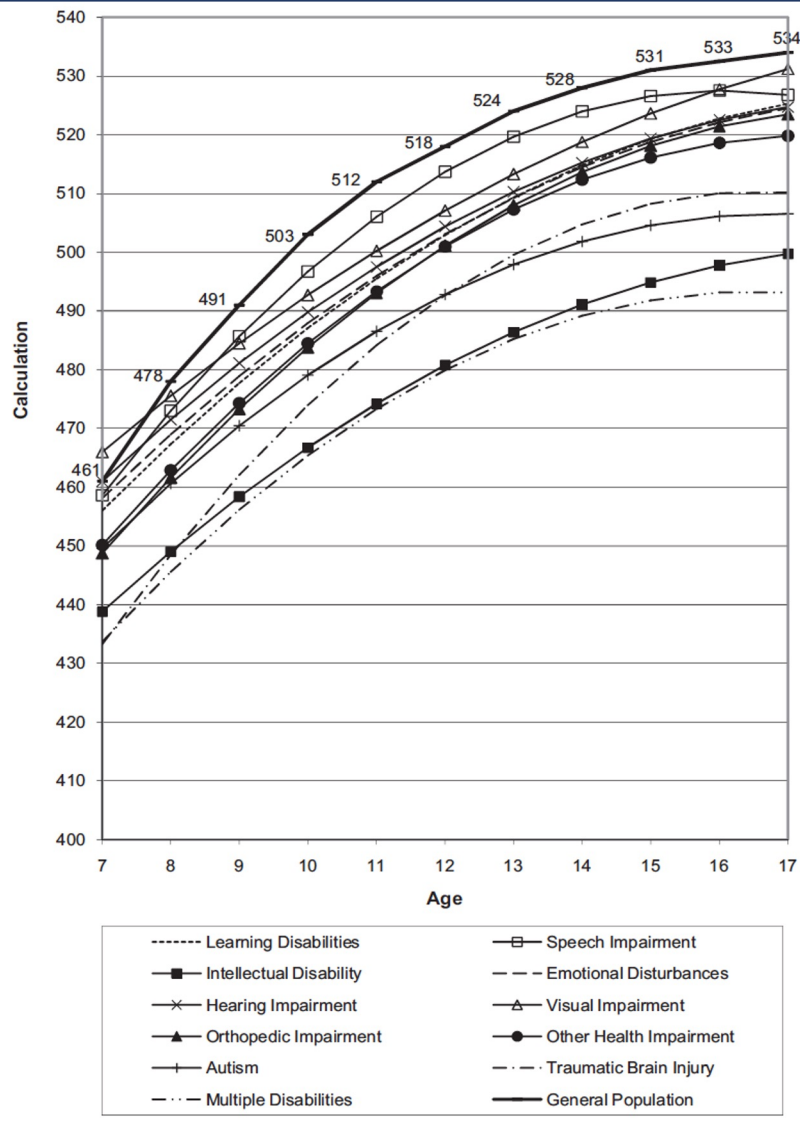Mathematics in preschool predicts later mathematics

Mathematics in kindergarten predicts later mathematics

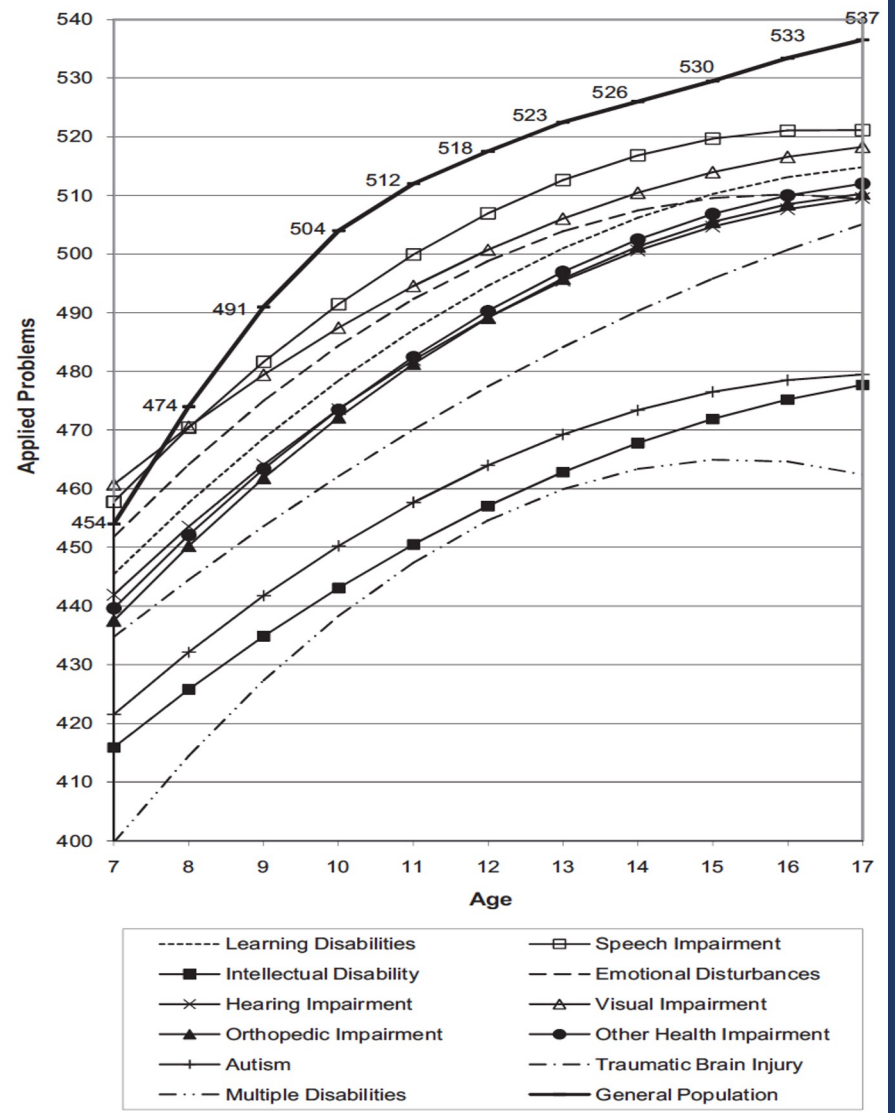Mathematics in elementary school predicts later mathematics

Mathematics in middle school predicts later mathematics

Mathematics in high school predicts later outcomes

| | <Sch | K | 1st | 2nd | 3rd | 4th | 5th | 6th | 7th | 8th | 9th | 10th | 11th | 12th | >Sch |

Starr et al. (2013)
Lee (2012) cohort 1
Libertus et al. (2013)
Mazzocco et al. (2011b)
Watts et al. (2014)
Jordan et al. (2007)
Navarro et al. (2012)
Passolunghi et al. (2012)
Stock et al. (2009) cohort 1
Aunola et al. (2004)
Krajewski et al. (2009a)
Bodovski & Farkas (2007)
DiPerna et al. (2007)
Jordan et al. (2009)
Missall et al. (2012)
Claessens et al. (2009)
Judge & Watson (2011)
Lee (2012) cohort 2
Morgan et al. (2009)
Claessens & Engel (2013)
Baglici et al. (2010)
Desoete et al. (2012)
LeFevre et al. (2010)
Stock et al. (2010)
Lachance & Mazzocco (2006)
Mazzocco & Thompson (2005)
Mazzocco et al. (2011a)
Desoete & Gregoire (2006)
Locuniack & Jordan (2008)
Stock et al. (2009) cohort 2
Vukovic (2012)
Krajewski & Schneider (2009b)
Geary et al. (2012)
Geary (2011)
Geary et al. (2013)
Friso-van den Bos et al. (2015)
Desoete et al. (2009)
Bailey, Watts, et al. (2014) cohort 1
Bailey, Siegler, et al. (2014)
Bailey, Watts, et al. (2014) cohort 2
Morgan et al. (2011)
Jordan et al. (2013)
Reigosa-Crespo et al. (2013)
Siegler et al. (2012) cohort 1
Siegler et al. (2012) cohort 2
Hansen et al. (2015)
Bailey et al. (2012)
Primi et al. (2010)
Wilkins & Ma (2002)
Lee (2012) cohort 3
Spielhagen (2006)
Dougherty (2003)
Britt & Irwin (2008)

MATH

Computation

Problem Solving

Wei, Lenz, & Blackorby (2013)

Provide examples of how you see *earlier* math skills relating to later math skills.

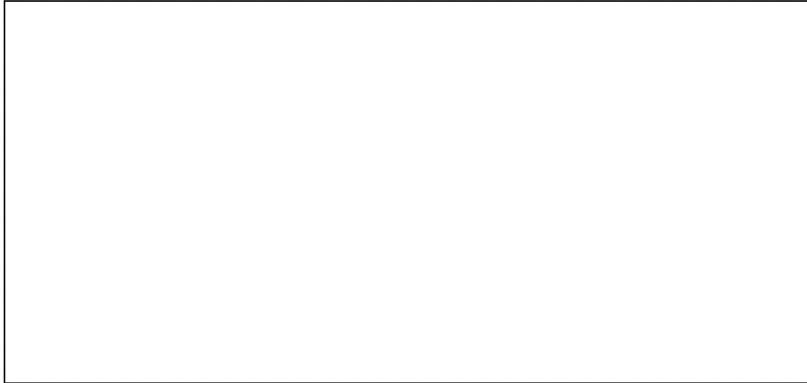# Critical Mathematics Content

Effective Mathematics Practices
DAY 1
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Mathematical Trajectories

Critical Content

continuum of mathematics learning

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division…

Fluently multiply multi-digit whole numbers using the standard algorithm.

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Fluently add and subtract within 5.

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.

Where student IS

Where student NEEDS TO BE

Fluently add and subtract within 5.

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division…

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fluently multiply multi-digit whole numbers using the standard algorithm.

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right…

Compose and decompose numbers from 11 to 19 into ten ones and some further ones…

Use place value understanding to round whole numbers to the nearest 10 or 100.

Where student IS

Where student NEEDS TO BE

Compose and decompose numbers from 11 to 19 into ten ones and some further ones…

Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

Use place value understanding to round whole numbers to the nearest 10 or 100.

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right…

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

M✕A+H

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions…

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations…

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20…

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve real-world and mathematical problems leading to two linear equations in two variables.

Use multiplication and division within 100 to solve word problems…

Solve addition and subtraction word problems, and add and subtract within 10…

Use addition and subtraction within 100 to solve one- and two-step word problems…

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators…

MATH

Solve addition and subtraction word problems, and add and subtract within 10···

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20···

Use addition and subtraction within 100 to solve one- and two-step word problems···

Use multiplication and division within 100 to solve word problems···

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations···

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators···

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions···

Solve real-world and mathematical problems involving the four operations with rational numbers.

Solve real-world and mathematical problems leading to two linear equations in two variables.

MATH

Where student IS

Where student NEEDS TO BE

Use addition and subtraction within 100 to solve one- and two-step word problems…

Use multiplication and division within 100 to solve word problems…

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations…

MATH

Where student IS

Where student NEEDS TO BE

Explain addition and subtraction strategies work, place and properties operations

Understand that the two digits of a two-digit number represent amounts of tens and ones.

addition and action 100 to one-two-step ord ems…

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or relationships.

Apply properties of operations as strategies to multiply and divide…

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division…

Use plication division 100 to e word lems…

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

stand three of a digit ber sent ts of eds, and s.

Find wh numb quotients remaind with up four-d dividends one-di diviso using strateg based place va the prop of opera and/or relation betwe multiplic and divi

Fluently multiply multi-digit whole numbers using the standard algorithm.

multi-word ems with ole rs and whole-ber ers e four ions…

Kindergarten	Grade 1	Grade 2	Grade 3
Comparing and Ordering Numbers			
(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:
(G) compare sets of objects up to at least 20 in each set using comparative language. (H) use comparative language to describe two numbers up to 20 presented as written numerals.	(E) use place value to compare whole numbers up to 120 using comparative language.	(D) use place value to compare and order whole numbers up to 1,200 using comparative language, numbers, and symbols (>, <, or =).	(D) compare and order whole numbers up to 100,000 and represent comparisons using the symbols >, <, or =.
	(F) order whole numbers up to 120 using place value and open number lines.		
	(G) represent the comparison of two numbers to 100 using the symbols >, <, or =.		

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence	Represent and solve problems involving addition and subtraction	Represent and solve problems involving addition and subtraction	Represent & solve problems involving multiplication and division	Use the four operations with whole numbers to solve problems	Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Add and subtract within 20	Understand properties of multiplication and the relationship between multiplication and division	Generalize place value understanding for multi-digit whole numbers	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of numbers to the system of rational numbers	Analyze proportional relationships and use them to solve real-world and mathematical problems	Understand the connections between proportional relationships, lines, and linear equations**
Compare numbers	Add and subtract within 20	Understand place value	Multiply & divide within 100	Use place value understanding and properties of operations to perform multidigit arithmetic	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems	Use properties of operations to generate equivalent expressions	Analyze and solve linear equations and pairs of simultaneous linear equations
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Work with addition and subtraction equations	Use place value understanding and properties of operations to add and subtract	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understandings of arithmetic to algebraic expressions	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Define, evaluate, and compare functions
Work with numbers 11-19 to gain foundations for place value	Extend the counting sequence	Measure and estimate lengths in standard units	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Reason about and solve one-variable equations and inequalities		Use functions to model relationships between quantities
	Understand place value	Relate addition and subtraction to length	Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects	Understand decimal notation for fractions, and compare decimal fractions	Graph points in the coordinate plane to solve real-world and mathematical problems*	Represent and analyze quantitative relationships between dependent and independent variables		
	Use place value understanding and properties of operations to add and subtract		Geometric measurement: understand concepts of area and relate area to multiplication and to addition					
	Measure lengths indirectly and by iterating length units							

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.
** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

https://achievethecore.org/category/774/mathematics-focus-by-grade-level

MATH

Table A.3. Grades 6–8 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*

Legend: ● Grade 6 ● Grade 7 ● Grade 8 ● Outside grades 6–8 ○ Not identified at any level, pre-K–8

Curriculum Focal Points and Connections

Grade 6 Curriculum Focal Points

Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals

Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.

Number and Operations: Connecting ratio and rate to multiplication and division

Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.

Algebra: Writing, interpreting, and using mathematical expressions and equations

Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that

Expectations of the Content Standards

Number and Operations, Grades 6–8

● ● ● Work flexibly with fractions, decimals, and percents to solve problems

● ● Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line

● Develop meaning for percents greater than 100 and less than 1

● ● Understand and use ratios and proportions to represent quantitative relationships

● ● Develop an understanding of large numbers [identified in Grades 4 and 5 Curriculum Focal Points] and recognize and appropriately use exponential, scientific, and calculator notation

● Use factors, multiples, prime factorization, and relatively prime numbers to solve problems

● Develop meaning for integers and represent and compare quantities with them

● ● ● Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers

● ● ● Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals

● ● ● Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems

● ● ● Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

https://www.nctm.org/curriculumfocalpoints/

What is the critical math content for your students?

Evidence-Based Practices

Evidence-Based Practices

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

MA+H

evidence-based practice

A practice that has shown **consistent and positive** results

MA+H

evidence-based practice

evidence-based intervention

An intervention (i.e., packaged program) that has shown **consistent and positive** results

M A T H

evidence-based practice

evidence-based intervention

evidence-based strategy

A method or strategy that has shown **consistent and positive** results

MATH

evidence-based practice

evidence-based intervention

evidence-based strategy

promising practice

A method or strategy that has shown a **positive** result

M∀+H

evidence-based practice

evidence-based intervention

evidence-based strategy

promising practice

no or negative evidence

MA+H

evidence-based practice

Assessment data to show results

Improvement from before intervention

Improvement compared to no treatment students

Replication

Multiple researchers

Multiple students

Multiple times

Setting and students similar to your own

evidence-based intervention

evidence-based strategy

promising practice

M∀T⊢

How do you locate and identify evidence-based practices?

Which evidence-based practices do you plan to use?

Instructional Platform

Instructional Platform

Instructional Delivery

Instructional Strategies

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

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Explicit Instruction

Explicit Instruction

MODELING

PRACTICE

SUPPORTS

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

INSTRUCTIONAL STRATEGIES

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MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice
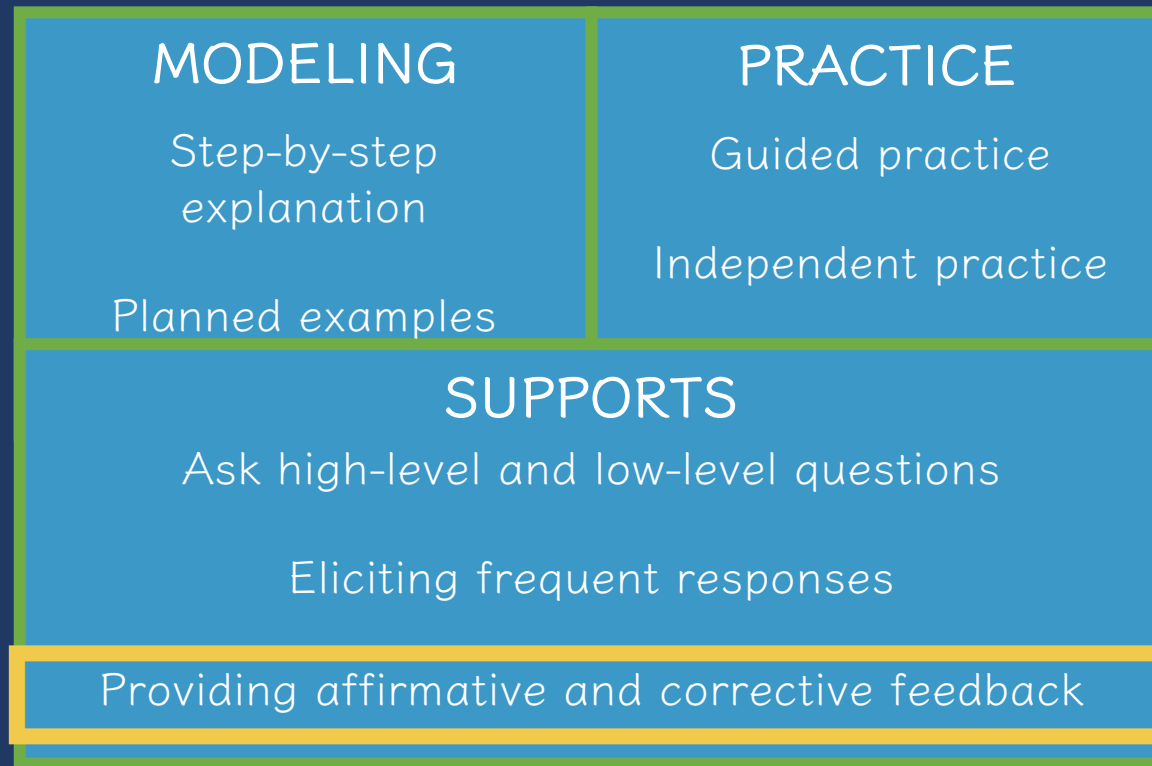
SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MA+H

Modeling is a dialogue between the teacher and students.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MATH

Modeling includes a step-by-step explanation of how to do a math problem.

A teacher may do 1 modeled problem or several.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MA+H

26
+ 79

"Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether."

M∧+H

26
+ 79
___

"Let's solve this problem. What's the problem?

"26 plus 79."

"To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?"

"Add."

"How did you know we want to add?"

"There's a plus sign."

$$
\begin{array}{r}
26 \\
+\ 79 \\
\hline
\end{array}
$$

"The plus sign tells us we want to add. To add, let's use the partial sums strategy. What strategy?"

"Partial sums."

"With the partial sums strategy, we start adding in the greatest place value. What's the greatest place value in this problem?"

"The tens."

"So, let's add the tens. What's 20 plus 70?"

"90."

MATH

26
+ 79
———

"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

"90."

"90 is the partial sum when you add the tens. What does 90 represent?"

"It's the partial sum of adding 20 plus 70."

"Now, let's add the ones. What should we add?"

"6 plus 9."

MATH

$$\begin{array}{r} 26 \\ +\ 79 \\ \hline \end{array}$$

"6 plus 9 equals what?"

"15."

"Let's write 15 below the 90. Where do we write the 15?"

"Below the 90."

"15 is the partial sum when you add the ones. Now, let's add the partial sums together. What will we add?"

"90 plus 15."

MATH

26
+ 79

"What's 90 plus 15?"

"How did you add those numbers?"

"So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?"

"105."

"I added 90 plus 10 then added 5 more."

"We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums."

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Modeling needs to include planned examples.

These examples should be sequenced so easier skills lead to more difficult skills.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MATH

MODELING
Step-by-step explanation

Planned examples

PRACTICE
Guided practice

Independent practice

SUPPORTS
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MA+H

Explicit Instruction

Problem	Step-by-Step Explanation

Select a math problem.

Work with a partner to outline a step-by-step explanation.

MATH

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Practice continues as a dialogue between the teacher and students.

MATH

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Guided practice is practice in which the teacher and students practice problems together.

"Let's work on a problem together."

MATH

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Independent practice is practice in which the students practice independently with teacher support.

"Now, you'll practice a problem on your own. Use your attack strategy!"

MATH

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Explicit Instruction

Problem	Practice Opportunities

High-Level Questions

Low-Level Questions

Affirmative Feedback

Corrective Feedback

Describe how you would engage students in practice.

MA+H

MODELING
Step-by-step explanation

Planned examples

PRACTICE
Guided practice

Independent practice

SUPPORTS
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, it is essential to engage students and check for understanding.

MA+H

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"What is 7 times 9?"

"63."

MA+H

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Why do you use zero pairs?"

"Because a positive 1 and a negative 1 equal 0. I use the zero pair to help me subtract."

M A+H

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

MATH

MODELING
Step-by-step explanation

Planned examples

PRACTICE
Guided practice

Independent practice

SUPPORTS
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

- Oral
- Written
- With manipulatives
- With drawings
- With gestures

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, students should receive immediate feedback on their responses.

MA+H

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Nice work using your word problem attack strategy."

MATH

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Let's look at that again. Tell me how you added in the hundreds column."

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Explicit Instruction

Problem

Practice Opportunities

High-Level Questions

Low-Level Questions

Affirmative Feedback

Corrective Feedback

Provide several of your questions.

Provide examples of your feedback.

MATH

MODELING
Step-by-step explanation

Planned examples

PRACTICE
Guided practice

Independent practice

SUPPORTS
Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Explicit Instruction

Problem	Step-by-Step Explanation

Explicit Instruction

Problem	Practice Opportunities

High-Level Questions

Low-Level Questions

Affirmative Feedback

Corrective Feedback

Find a partner.

Share your explicit instruction.

MA+H

Building Fluency

Fluency

Addition

Subtraction

Multiplication

Division

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

MA+H

Building Fluency

Fluency is doing mathematics easily and accurately.

Fluency in mathematics makes mathematics easier.

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both conceptual learning and procedural learning.

MATH

Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Knowing multiples

Identifying shapes

Knowing formulas

MATH

Multiple Representations

Multiple Representations

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

M A + H

Multiple Representations

Abstract

Concrete

Pictorial

Three-dimensional objects

MATH

Abstract

Concrete

Pictorial

Two-dimensional images

MATH

Abstract

Concrete

Pictorial

Two-dimensional images

Modeling Fractions with Cuisenaire Rods

View Hint

Clear

View Help

Trash Can

MA+H

Abstract

Concrete

Pictorial

Two-dimensional images

Virtual Manipulatives

Number & Operations

Place Value

Fractions & Decimals

Integers & Algebra

Geometry

Time & Money

Data & Probability

Extras

Help students see and learn math using different tools!

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FLOWCODE
PRIVACY.FLOWCODE.COM

Fractions & Decimals

fraction strips

fraction strips

fraction strips

Cuisenaire rods

fraction circles

geoboard

geoboard

geoboard

pattern blocks

two-color counters

decimal strips

place value disks

percentage strips

MATH

Numerals and symbols and words

$$2 + 8 = 10 \qquad\qquad 34 = 3 \text{ tens and } 4 \text{ ones}$$

$$x - 6 = 8$$

$$\begin{array}{r} 4{,}179 \\ +569 \\ \hline \end{array}$$

Explore 3 virtual manipulatives.

Share with a partner.

MATH

Building Fluency with Mathematics Facts

100 addition facts

Single-digit addends sum to a single- or double-digit number

$$
\begin{array}{r}
5 \\
+\ 4 \\
\hline
9
\end{array}
$$

5   (addend)
+ 4   (addend)
9   (sum)

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

MATH

Parts put together into a total

Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?

An amount that **increases** or decreases

Premila had $4. Then they earned $5 for cleaning their room. How much money does Premila have now?

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$3 + 9 = \_\_$

Addition

If you have brown eyes:

What's a Total story to show addition?

If you don't have brown eyes:

What's a Change/Join story to show addition?

100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$
\begin{array}{r}
16 \\
-\ \ 8 \\
\hline
8
\end{array}
$$

(minuend)
(subtrahend)
(difference)

An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

Difference

Greater and lesser amounts compared for a difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

9 – 5 = __

If you would chose beaches:

What's a Change/Separate story to show addition?

If you would chose mountains:

What's a Difference story to show addition?

MATH

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

$$
\begin{array}{r}
2 \\
\times\ 3 \\
\hline
6
\end{array}
$$

2   (factor)

× 3   (factor)

6   (product)

Show the groups, show the amount for each group, count product

$$3 \times 2 = 6$$

Show a set, then multiply the set

$$3 \times 2 = 6$$

Equal Groups

Groups multiplied by number in each group for a product

Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?

Set multiplied by a number of times for a product

Vivienne picked 12 apples. Jessica picked 2 times as many apples as Vivienne. How many apples did Jessica pick?

$2 \times 5 = \underline{}$

If you wear glasses:

What's an Equal Groups story to show addition?

If you don't wear glasses:

What's a Comparison story to show addition?

MATH

90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

$$8 \div 4 = 2$$

(dividend) (divisor) (quotient)

Equal Groups

Groups multiplied by number in each group for a product

Stefanie has 12 apples. She wants to share them equally among her 2 friends. How many apples will each friend receive?

Nicole has 12 apples. She put them into bags with 6 apples each. How many bags did Nicole use?

$$12 \div 4 = \_\_$$

If you watch *Stranger Things*:

What's a Partitive story to show addition?

If you watch *Ted Lasso*:

What's a Quotative story to show addition?

MATH

Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends

- Subtraction: single-digit subtrahend

- Multiplication: single-digit factors

- Division: single-digit divisor

$$\begin{array}{r} 5 \\ + \ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ - \ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times \ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 56 \\ \div \ 8 \\ \hline \end{array}$$

MATH

Cover, Copy, Compare

$$9 \times 6 = 54$$

$$8 \times 6 = 48$$

$$7 \times 8 = 56$$

$$6 \times 5 = 3$$

$$9 \times 9 = 81$$

$$6 \times 7 = 42$$

$$8 \times 8 = 64$$

Taped Problems

6×5	8×6	7×9	6×8
9×8	8×5	7×8	6×6
7×7	6×9	5×9	8×4
9×4	6×9	9×5	8×7
6×7	8×8	4×8	5×7

File Folder

$6 + 3 =$	
$1 + 7 =$	8
$6 + 4 =$	1
$7 + 3 =$	10
$2 + 7 =$	9
$5 + 6 =$	11
$4 + 7 =$	11
$7 + 8 =$	15
$6 + 7 =$	13
$7 + 9 =$	16
$7 + 6 =$	13
$8 + 7 =$	15
$7 + 0 =$	7
$9 + 6 =$	15
$6 + 0 =$	6
$6 + 8 =$	14

DAILY and BRIEF

MA+H

Addition | Subtraction

Multiplication | Division

What are five ways you help students build fact fluency?

Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation

$$
\begin{array}{r}
15 \\
+\ 28 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
23 \\
\times\ 9 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
1009 \\
-\ 724 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
7250 \\
\div\ 15 \\
\hline
\end{array}
$$

M A + H

Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ +\ \ 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div\ \ \ 0.14 \\ \hline \end{array}$$

M⋈A+H

Addition	Subtraction
Multiplication	Division

Build fluency with integer computation

$$\begin{array}{r} 1.4 \\ +\ \ \underline{\text{-}3.9} \end{array}$$

$$\begin{array}{r} 6 \\ \times\ \underline{\text{-}12} \end{array}$$

$$-14 - (-7) =$$

$$-135 \div 2 =$$

M×A+H

Building Fluency with Computation

MA+H

Addition and Subtraction Computation

227 + 185 =	232 − 164 =

Partial Sums

A.
```
    74
  + 18
    80
  + 12
    92
```

B.
```
     725
  +  365
   1,000
      80
  +   10
   1,090
```

```
    227
  +  185
```

MA+H

Opposite Change

A.
$$74 \xrightarrow{-4} 70$$
$$+18 \xrightarrow{+4} +22$$
$$\overline{} \ 92$$

B.
$$725 \xrightarrow{+5} 730$$
$$+365 \xrightarrow{-5} +360$$
$$\overline{} \ 1{,}090$$

227
+ 185

MA+H

Addition and Subtraction Computation

227 + 185 =	232 − 164 =

Partial Differences

A.
$$
\begin{array}{r}
62 \\
-\ 17 \\
\hline
+50 \\
-\ 5 \\
\hline
45
\end{array}
$$

B.
$$
\begin{array}{r}
305 \\
-\ 96 \\
\hline
+300 \\
-90 \\
-1 \\
\hline
209
\end{array}
$$

$$
\begin{array}{r}
232 \\
-\ 164 \\
\hline
\end{array}
$$

MATH

Same Change

A.

$$62 \xrightarrow{+3} 65$$
$$-\ 17 \xrightarrow{+3} -20$$
$$\overline{45}$$

B.

$$305 \xrightarrow{+4} 309$$
$$-\ 96 \xrightarrow{+4} -100$$
$$\overline{209}$$

$$232$$
$$-\ \underline{164}$$

M A + H

Add Up

A.

$$62 - 17$$

17
20   3
60   40
62 + 2

    45

B.

$$305 - 96$$

96
100   4
300 200
305 + 5

   209

$$232 - 164$$

M A + H

Multiplication and Division Computation

13 x 7 =

135 ÷ 5 =

13 x 47 =

804 ÷ 12 =

Partial Products

13
× 47

Area (Array)

A.

$$\begin{array}{r} 24 \\ \times\ 43 \end{array}$$

	20	4	
	800	160	40
	60	12	3

$$\begin{array}{r} 1\ 1 \\ 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1,032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \end{array}$$

	100	30	2	
	5000	1500	100	50
	300	90	6	3

$$\begin{array}{r} 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6,996 \end{array}$$

13
$\times\ \ 47$

Lattice

13
× 47

Multiplication and Division Computation

13 x 7 =	135 ÷ 5 =
13 x 47 =	804 ÷ 12 =

Partial Quotients

A.
$$
\begin{array}{r}
12 \overline{)158} \\
-120 \quad 10 \\
\hline
38 \\
-36 \quad +3 \\
\hline
2 \quad 13 \text{ R2}
\end{array}
$$

B.
$$
\begin{array}{r}
34 \overline{)970} \\
-680 \quad 20 \\
\hline
290 \\
-170 \quad 5 \\
\hline
120 \\
102 \quad +3 \\
\hline
18 \quad 28 \text{ R18}
\end{array}
$$

804
÷ 12

MA+H

Lattice

804

÷ 12

MA+H

Division as Fractions

A. $12 \overline{)158}$

$$\frac{100}{12} \quad 8\frac{4}{12}$$

$$\frac{50}{12} \quad 4\frac{2}{12}$$

$$\frac{8}{12} + \frac{8}{12} = 12\frac{14}{12} = 13\frac{2}{12}$$

B. $34 \overline{)970}$

$$\frac{900}{34} \quad 26\frac{16}{34}$$

$$\frac{70}{34} + 2\frac{2}{34}$$

$$\frac{0}{34} \qquad = 28\frac{18}{34}$$

804
÷ 12

Multiple Representations: Fractions

Fraction Concepts

Fraction	Length	Area	Set
$\frac{2}{3}$			
$\frac{1}{4}$			
$1\frac{1}{2}$			
$\frac{3}{7}$			

LENGTH   AREA   SET

LENGTH

Fractions are appropriated by length

$\frac{2}{3}$

Fraction tiles/bars

MATH

LENGTH

Fractions are appropriated by length

$\dfrac{2}{3}$

Cuisenaire rods

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Fraction Concepts

Fraction	Length	Area	Set
$\frac{2}{3}$			
$\frac{1}{4}$			
$1\frac{1}{2}$			
$\frac{3}{7}$			

Show fractions with the length model.

MA+H

Shapes divided into equal sections

$$\frac{2}{3}$$

Fraction circles

AREA

Shapes divided into equal sections

$$\frac{2}{3}$$

Geoboards

MATH

AREA

Shapes divided into equal sections

$\frac{2}{3}$

Pattern blocks

Shapes divided into equal sections

$$\frac{2}{3}$$

Legos

MA+H

Fraction Concepts

Fraction	Length	Area	Set
$\dfrac{2}{3}$			
$\dfrac{1}{4}$			
$1\dfrac{1}{2}$			
$\dfrac{3}{7}$			

Show fractions with the area model.

M▧A+H

SET — Individual shapes match the fraction

$\dfrac{2}{3}$

Two-color counters

MATH

Fraction Concepts

Fraction	Length	Area	Set
$\frac{2}{3}$			
$\frac{1}{4}$			
$1\frac{1}{2}$			
$\frac{3}{7}$			

Show fractions with the set model.

M×A+H

Fraction Concepts

Fraction	Length	Area	Set
$\dfrac{2}{3}$			
$\dfrac{1}{4}$			
$1\dfrac{1}{2}$			
$\dfrac{3}{7}$			

Select a grade-level fractions task. Use representations to model the task.

M×A+H

Multiple Representations: Pre-Algebra

Integers

Addition and Subtraction

Problem	Representation
3 + (-5)	
-2 + 6	
3 + (-1)	
-3 − 4	
2 − 5	
-2 − (-6)	
-3 − 2	

Ways to Teach Addition and Subtraction

MAT and CUBES

TWO-COLOR COUNTERS

NUMBER LINE

3 + (-5)

−

+

3 + (-5)

3 + (-5)

Integers
Addition and Subtraction

Problem	Representation
3 + (-5)	
-2 + 6	
3 + (-1)	
-3 – 4	
2 – 5	
-2 – (-6)	
-3 – 2	

Use representations to model the addition problems.

MA+H

-3 – 4

-3 − 4

-3 – 4

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

M A + H

-3 – 4

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

M∆+H

Integers

Addition and Subtraction

Problem	Representation
3 + (-5)	
-2 + 6	
3 + (-1)	
-3 – 4	
2 – 5	
-2 – (-6)	
-3 – 2	

Use representations to model the subtraction problems.
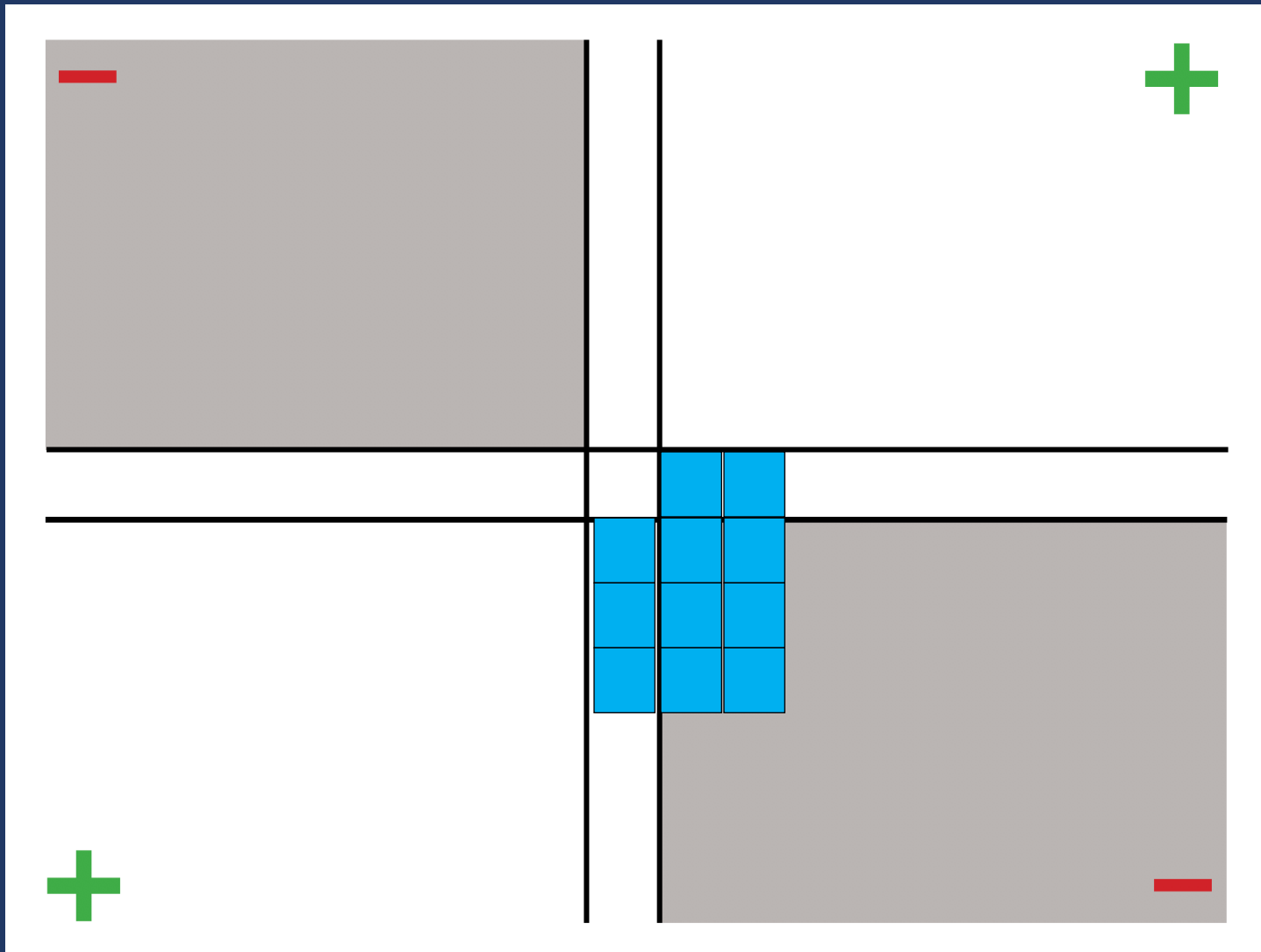
MATH

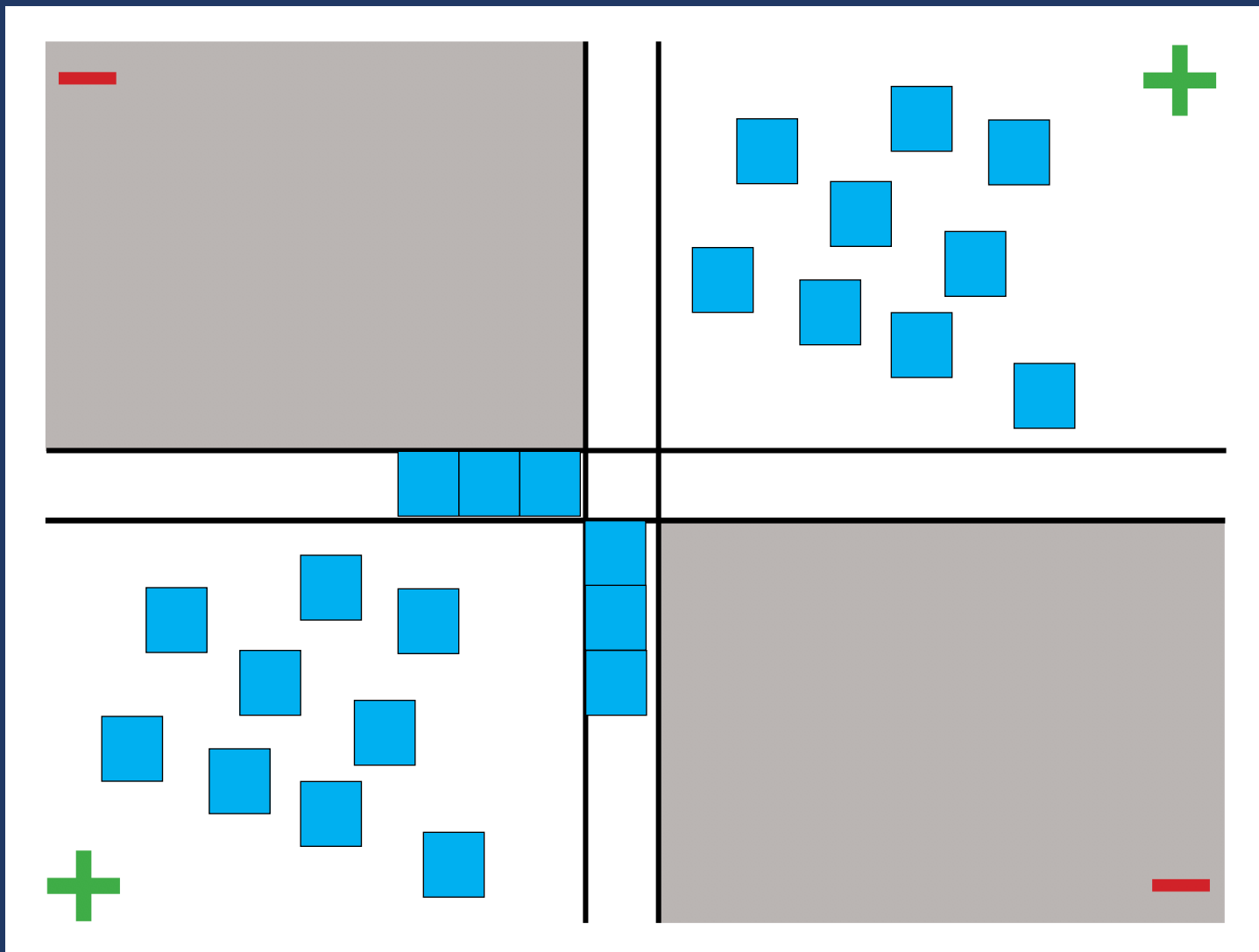# Ways to Teach Multiplication and Division

MAT and CUBES

ALGEBRA TILES/ALGEBLOCKS

(2)(-3)

MATH

$9 \div (-3)$

Solving Equations

With Cups/Plates and Counters

Problem	Representations
x + 2 = 5	
x + 2 = 5	
4 + x = 6	
5 = x − 3	
-2 = x + 3	

With Algebra Tiles

Problem	Representations
x + 2 = 5	
4 + x = 6 **x + 3 = 7**	
y − 4 = 2	
5 = x − 3 **y − 2 = 5**	

cups/plates and counters

algebra tiles

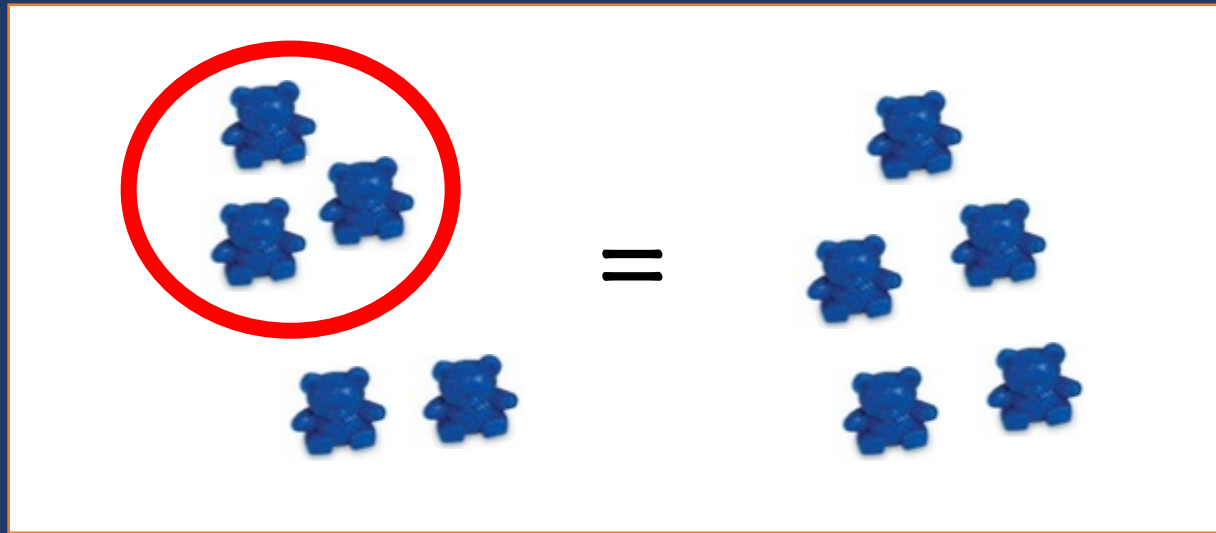# Algeblocks

zero pairs

-1 and 1 equal 0

properties of equality

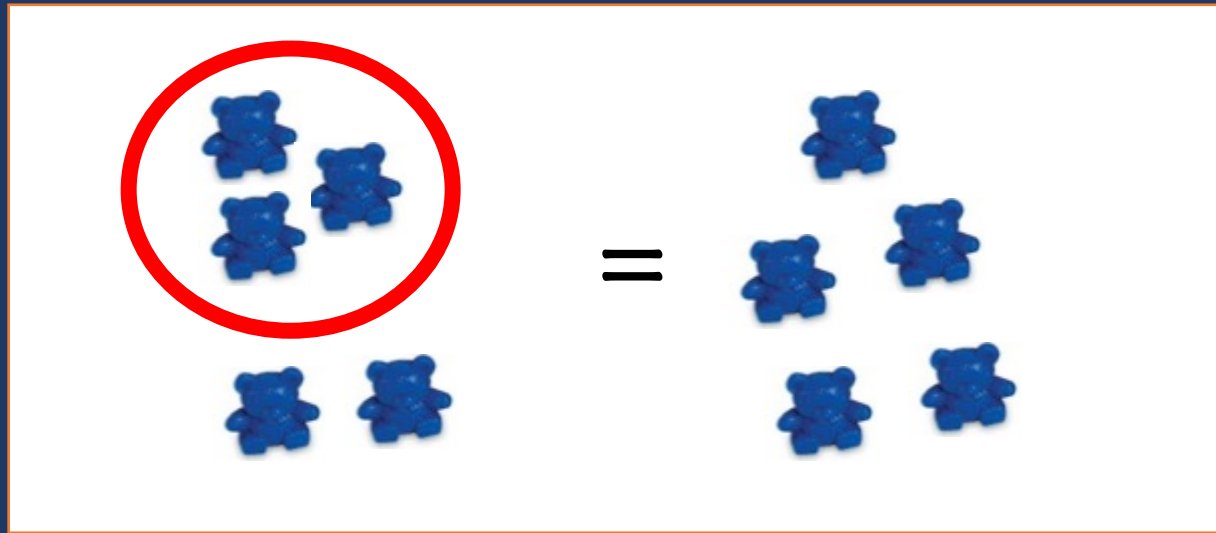If you do something to one side of the equal sign, you do the same thing to the other side.
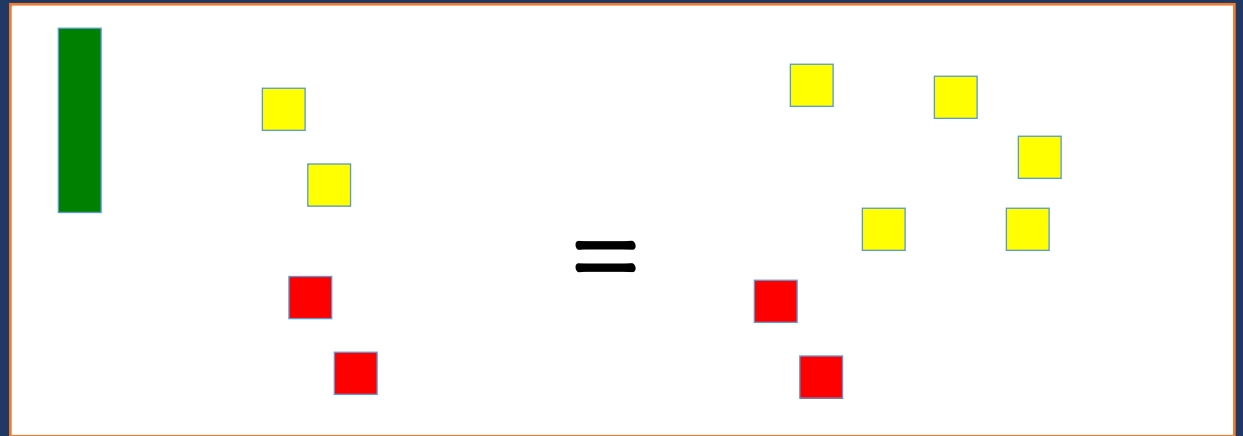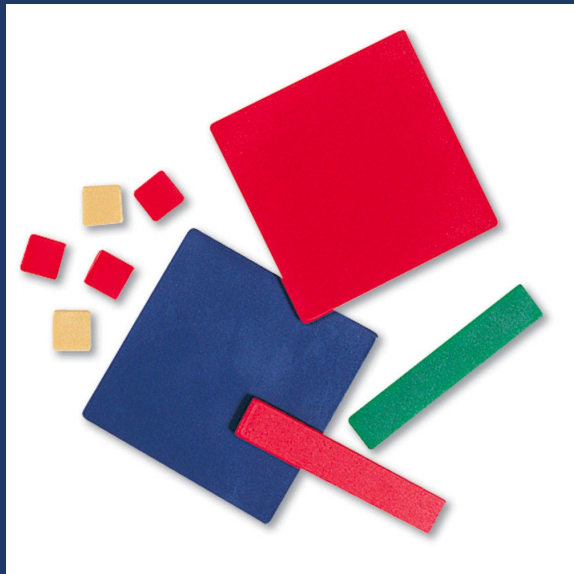
M∆+H

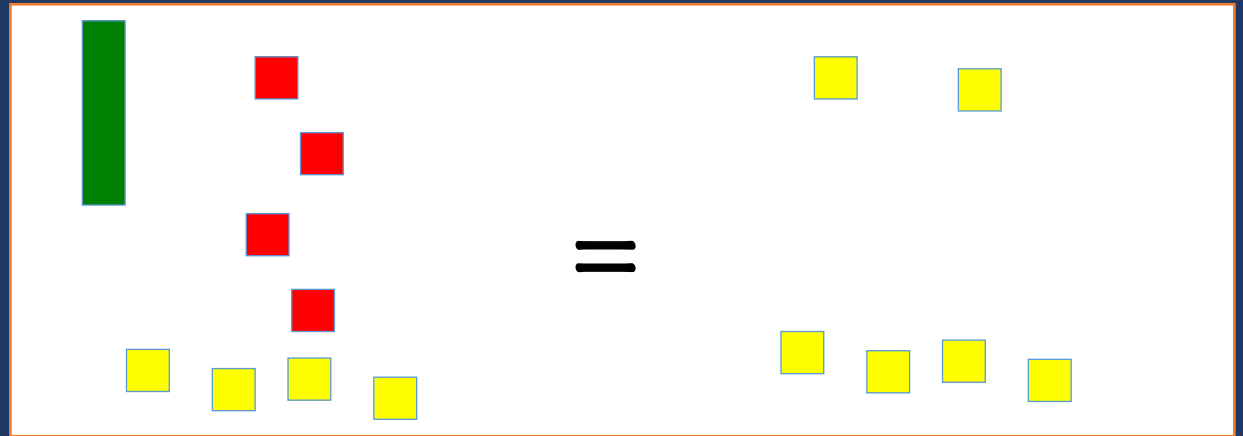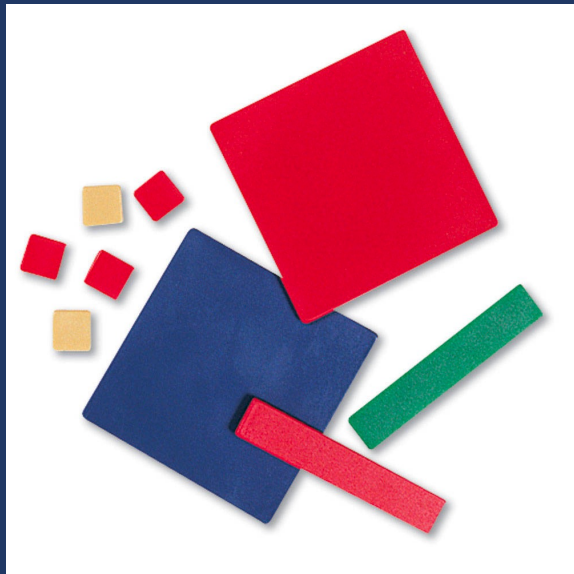cups/plates and counters

$x + 2 = 5$

cups/plates and counters

$x + 2 = 5$

algebra tiles

$x + 2 = 5$

algebra tiles
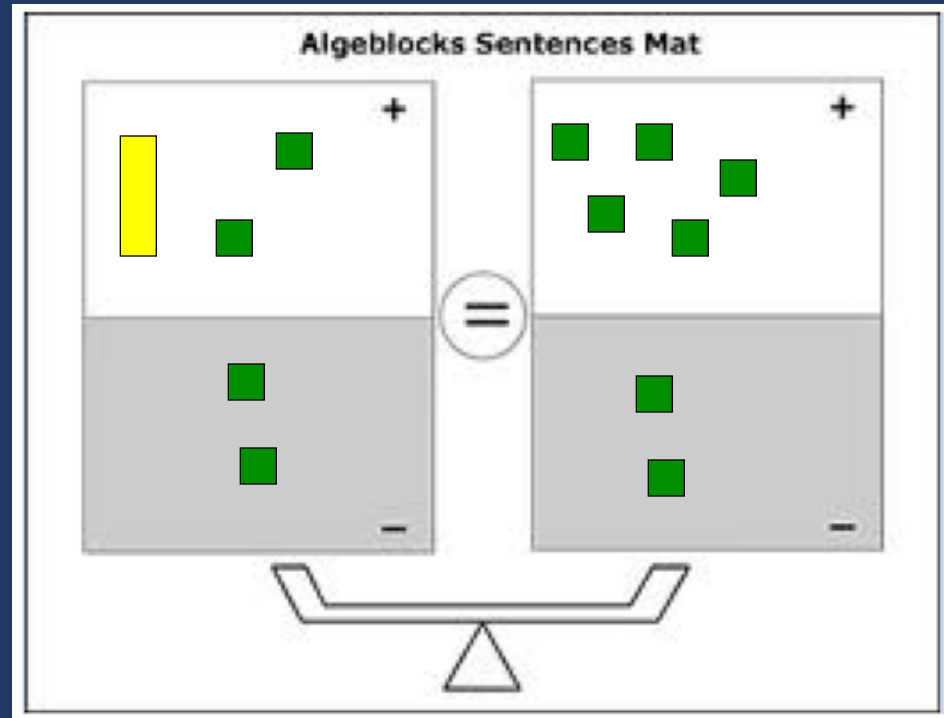
$$y - 4 = 2$$

$$x + 2 = 5$$

Algeblocks

$x + 2 = 5$

Algeblocks

-1 = y − 4

Algeblocks Sentences Mat

Solving Equations

With Cups/Plates and Counters

Problem	Representations
x + 2 = 5	
x + 2 = 5	
4 + x = 6	
5 = x – 3	
-2 = x + 3	

With Algebra Tiles

Problem	Representations
x + 2 = 5	
4 + x = 6 **x + 3 = 7**	
y – 4 = 2	
5 = x – 3 **y – 2 = 5**	

Use representations to solve the problems.

M A + H

Solving Equations
With Algeblocks

Problem	Representations
$x + 2 = 5$	
$4 + x = 6$	
$-2 = x + 3$	
$-1 = y - 4$	
$2x + 2 = 6$	
$x + 4 = 3x$	
$2(x + 3) = x + 4$	
$2x - 4 = 1 + 3x$	
$3y - 5 = -y - 1$	
$1 - x = x + 1$	

Algeblocks

$$2x + 2 = 6$$

Algeblocks Sentences Mat

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Algeblocks

$$2(x + 3) = 4$$

Algeblocks Sentences Mat

M A+H

Solving Equations

With Algeblocks

Problem	Representations
$x + 2 = 5$	
$4 + x = 6$	
$-2 = x + 3$	
$-1 = y - 4$	
$2x + 2 = 6$	
$x + 4 = 3x$	
$2(x + 3) = x + 4$	
$2x - 4 = 1 + 3x$	
$3y - 5 = -y - 1$	
$1 - x = x + 1$	

Use representations to solve the problems.

M A T H

Multiple Representations: Geometry

Geometry

Select a grade-level geometry task. Use representations to model the task.

MA+H

📅 Schedule for Today

8:00-8:25	- Introductions - Trajectories in mathematics
8:25-9:15	- Critical math content - Evidence-based practices
9:15-9:25	BREAK
9:25-10:25	- Evidence-based practice: Systematic instruction
10:25-11:30	- Evidence-based practice: Fluency with facts - Evidence-based practice: Multiple representations
11:30-1:00	LUNCH
1:00-2:15	- Evidence-based practice: Fluency with computation - Multiple representations: Whole numbers
2:15-2:25	BREAK
2:25-4:00	- Multiple representations: Rational numbers - Multiple representations: Pre-algebra - Multiple representations: Geometry

Schedule for Tomorrow

8:00-9:30	- Language of mathematics
9:30-9:40	BREAK
9:40-10:15	- Evidence-based practice: Word-problem solving
10:15-10:30	- Word-problem solving: Ineffective strategies
10:30-11:00	- Word-problem solving: Attack strategies
11:00-11:30	- Word-problem solving: Total problems
11:30-1:00	LUNCH
1:00-2:15	- Word-problem solving: Difference problems - Word-problem solving: Change problems - Word-problem solving: Equal groups problems - Word-problem solving: Comparison problems
2:15-2:25	BREAK
2:25-4:00	- Word-problem solving: Ratios/Proportions problems - Word-problem solving: Multi-step problems - Wrap-up

National Center on
INTENSIVE INTERVENTION
at American Institutes for Research ■

Intensive Intervention ▾   Tools Charts ▾   Implementation Support ▾   Intervention Materials ▾

Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a high-leverage practice in special education , and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center , developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.
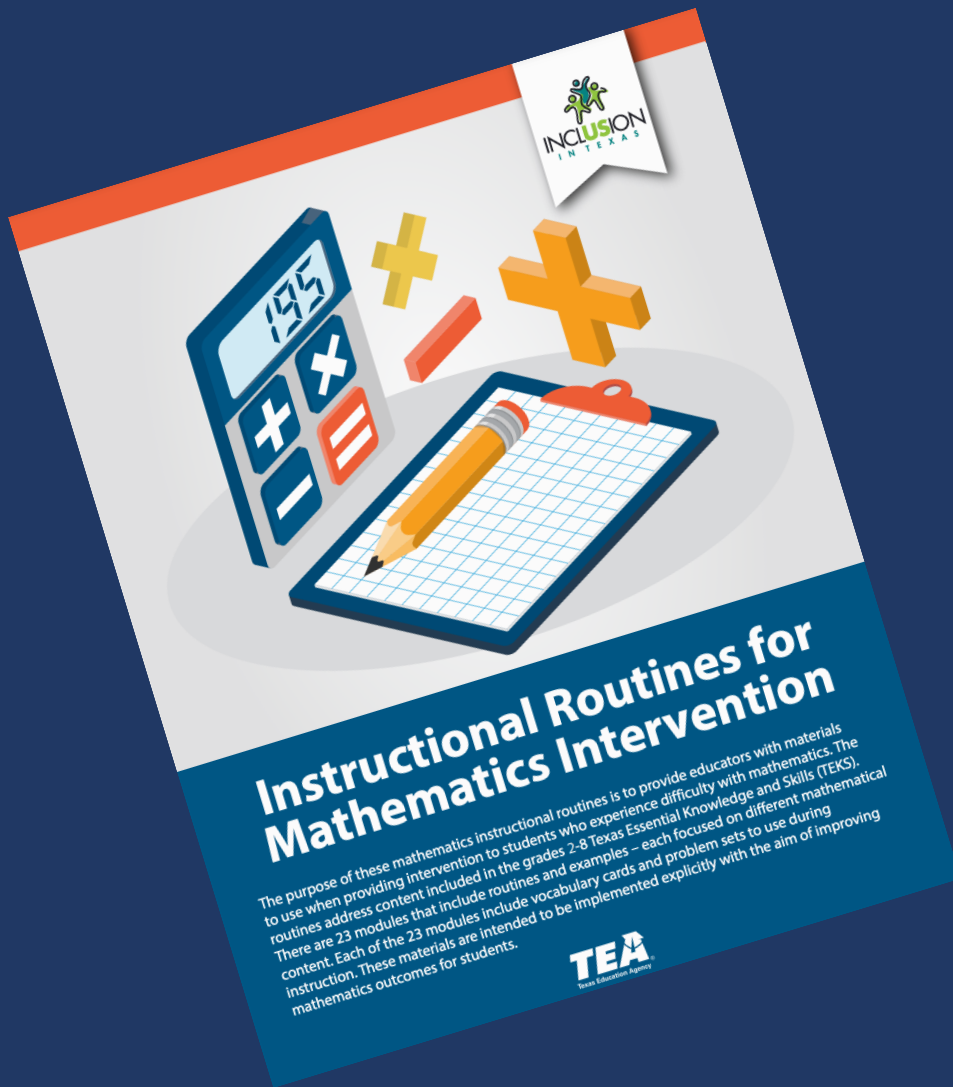
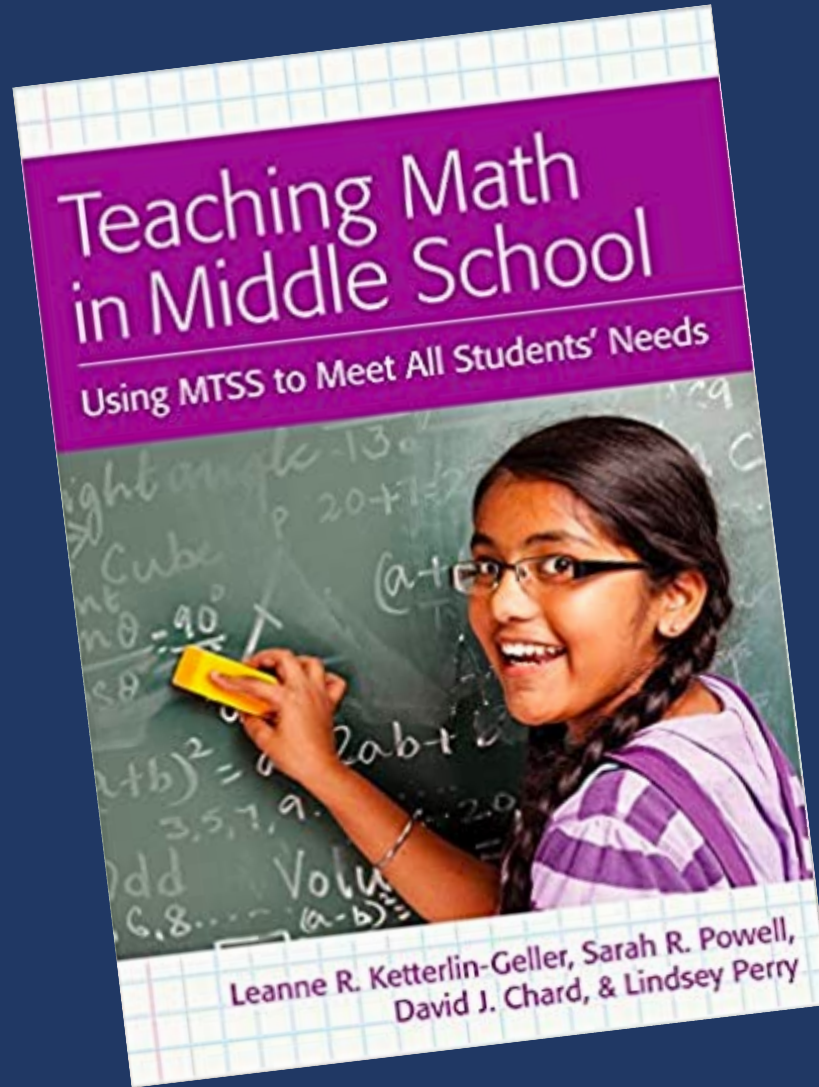**MODULE 4: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL DELIVERY**

MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES

MA+H

Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

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