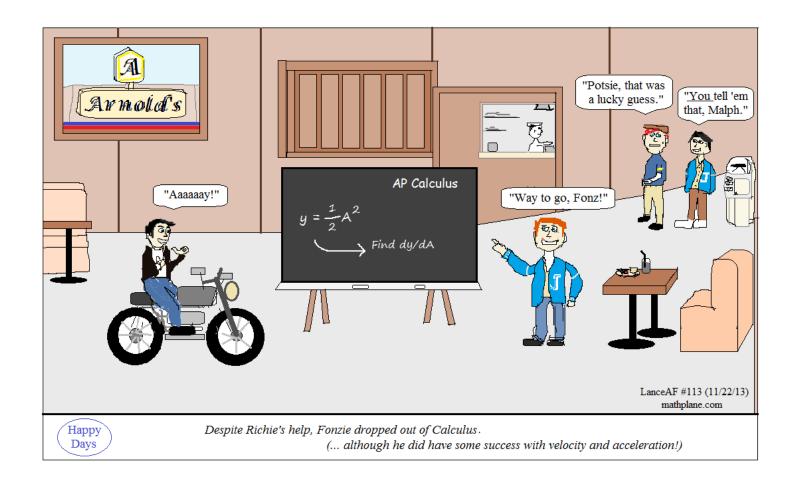
Calculus AB:

Multiple Choice Questions

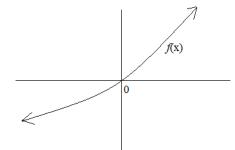
$$f(c) = \frac{1}{b-a} \int_{a}^{b} f(x) dx$$

Topics include differentiation, integrals, mean value theorem, graphs, extrema, differential equations, inverses, logarithms, and more.



Multiple Choice Questions -→

- a) 1
- b) -3/2
- c) 0
- d) undefined
- e) 1/2
- 2) Which is the smallest value?



- a) f(0)
- b) f'(0)
- c) f"(0)
- d) f(3)
- e) f'(3)

3)
$$\int_{0}^{8} x^{1/3} =$$

- a) 2
- b) 6
- c) 8
- d) 12
- e) none of the above

4)
$$h(x) = 5\cos^2(1 - x)$$
 $h'(\frac{1}{2}) =$

$$h'(\frac{1}{2}) =$$

- a) 0
- b) 17
- c) 5
- d) 10
- e) -10

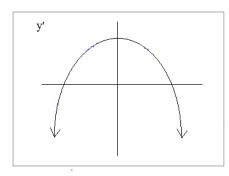
$$5) f(x) = |x^3|$$

What is $\lim_{x \to -1} f'(x)$?

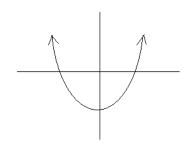
- a) -3
- b) 0
- c) 1
- d) 3
- e) Does not exist

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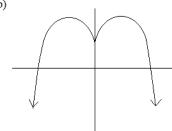
6) Below is a graph of y'. Which graph is possibly y?

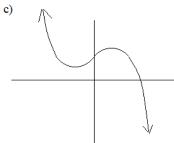


a)

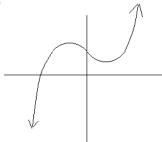


b)

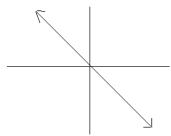




d)



e)



7) Find the instant rate of change with respect to r of

$$V = \frac{1}{6} \text{Tr} r^2 (5 - r)$$

b)
$$\frac{2511}{3}$$

c)
$$\frac{-2511}{6}$$

8) The velocity of a particle is modeled by $v(t) = t^2 - 3t - 10$ When is the particle speeding up?

a)
$$0 < t < 3/2$$

b)
$$t > 3/2$$

c)
$$t > 5$$

d)
$$-2 \le t \le 3/2$$
 and $t \ge 5$

mathplane.com

10)
$$\int_{\frac{2\pi}{2}}^{x} 2\cos t \, dt =$$

b)
$$\frac{\sin x}{2}$$

d)
$$2(1 - \sin x)$$

11) Determine the equation of the line tangent to the curve

$$x^2 - 3xy = 7$$
 @ (-1,2)

a)
$$8x + 3y = -2$$

b)
$$8x - 3y = -14$$

c)
$$4x - 5y = -14$$

d)
$$4x + 5y = 6$$

e)
$$3x - 2y = -7$$

-1	7	١
1	–	,

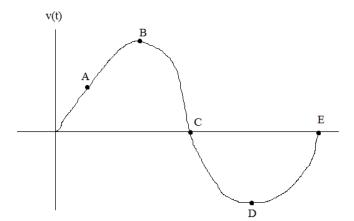
f(3)	g(3)	f'(3)	g'(3)
-1	2	4	-2

$$h(x) = \frac{f(x)}{g(x)} \qquad h'(3) =$$

b)
$$\frac{3}{2}$$

a) A b) B c) C d) D

e) E



- 14)

 - c) -3(ln7)
 - d) (ln7)³
 - e) Does not exist
- 15) What is the absolute minimum on the interval [-2, 1] for

$$f(x) = 6x^3 + 6x^2 - 6x + 14$$
?

- a) -6
- b) -1
- c) 1/3
- d) 2
- e) 12 8/9
- 16) $f(x) = 2x + e^X$

$$g(x) = f^{-1}(x)$$
 for all x

- Since f(0) = 1, what is g'(1)?
- a) -3
- b) -1/3
- c) 0
- d) 1/3
- e) 3

17)
$$\int_{0}^{4} g(x) dx = -10 \qquad \int_{0}^{10} g(x) dx = 4 \qquad g(x) \text{ is an EVEN function}$$

$$\int_{-10}^{-4} g(x) dx =$$

- a) -14
- b) -6
- c) 0
- d) 6
- e) 14

18) If
$$\int_{0}^{7} f(x) dx = 20$$
 what is $\int_{0}^{7} 2f(x) + 4 dx$?

- a) 18
- b) 42
- c) 44
- d) 48
- e) 68

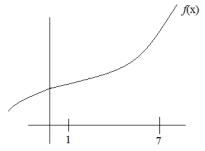
19)
$$f(x) = 3x^2 + 2x + 4$$

What is the average value between x = 1 and x = 4?

- a) 17
- b) 23
- c) 26
- d) 30
- e) 34.5
- 20) Which will give the highest value?



- b) Trapezoid Rule with 6 intervals
- c) Left Riemann Sum using 3 subintervals between 1 and 7
- d) Left Riemann Sum using 6 subintervals between 1 and 7
- e) Right Riemann Sum using 6 subintervals between 1 and 7



Find the value "c" guaranteed by the "integral mean value theorem" (i.e. where the value f(c) equals the <u>average value</u> on the interval [0, 2])

- a) 1
- b) 1.15
- c) 1.75
- d) 2
- e) 3
- 22) For the function $h(x) = x^3 2$

on the interval [-1, 3]

- I. determine the AROC (average rate of change)
 - a) -7
 - b) -1
 - c) 1
 - d) 7
 - e) 28
- II. find the value "c" to verify the Mean Value Theorem
 - a) 1
 - b) 1.53
 - c) 2.57
 - d) 3
 - e) 6
- 23) Find the value of k so g(x) is continuous:

$$g(x) = \begin{cases} k+x & \text{if} \quad x < 10 \\ xk & \text{if} \quad x \ge 10 \end{cases}$$

- b) 1
- c) 10/9
- d) 9
- e) 10
- 24) For the equation $x = \sin t + 3$ find $\frac{dy}{dx}$
 - a) tant
 - b) -tant
 - c) cott
 - d) -cott
 - e) $\frac{-\sin t(\sin t + 3) \cos^2 t}{\left(\sin t + 3\right)^2}$

X	1	2	3	4
f(x)	3	1	2	4
f'(x)	-7	-5	-4	- 6
g(x)	4	3	1	2
g'(x)	1/3	1/9	7/9	2/9

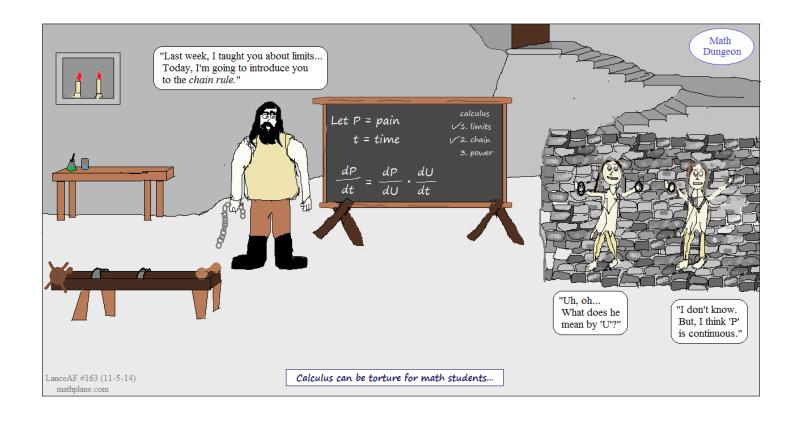
- a) 2/9
- b) -4
- c) 7/9
- d) -5/3
- e) -28/9
- 26) Find the equation of the line tangent to

the graph $y = x^3 - 3x^2 + 5$ at the point of inflection.

- a) y = 6x + 6
- b) y = x + 2
- c) y = -3x + 6
- d) y = -3x + 4
- e) y = x + 8
- 27) If the differential equation $\frac{dy}{dx} = \frac{-x}{4y}$ has a solution containing point (6, 1),

then when x = 2,

- a) y = 1
- b) y = 2
- c) y = 3
- d) y = 4
- e) y = 5



SOLUTIONS-→

Use implicit differentiation to find the instantaneous rate of change

$$3x^2 + 2y\frac{dy}{dx} = 0$$

If
$$x = 1$$
, then $(1)^3 + y^2 = 1$

$$2y\frac{dy}{dx} = -3x^2$$

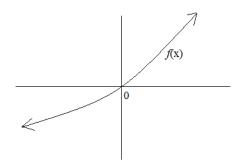
$$\frac{dy}{dx} = \frac{-3x^2}{2x}$$

$$\frac{dy}{dx} = \frac{-3x^2}{2y}$$
 substitute (1, 0) to find slope:
$$\frac{dy}{dx} = \frac{-3}{0}$$

SOLUTIONS

$$\frac{dy}{dx} = \frac{-3}{0}$$

2) Which is the smallest value?



Since the function is increasing,

b)
$$f'(0)$$

$$f'(x) \ge 0$$

c)
$$f''(0)$$

and, since the function is concave

d)
$$f(3)$$

e)
$$f'(3)$$

Therefore,
$$f(0) = 0$$
 is the smallest...

3)
$$\int_{0}^{8} x^{1/3} =$$

- a) 2
- b) 6
- c) 8
- d) 12
- e) none of the above

$$\frac{x^{4/3}}{4/3} \begin{vmatrix} 8 \\ 0 \end{vmatrix} = \frac{3 x^{4/3}}{4} \begin{vmatrix} 8 \\ 0 \end{vmatrix} = \frac{3(16)}{4} - 0 = \boxed{12}$$

4)
$$h(x) = 5\cos^2(1 - x)$$
 $h'(\frac{1}{2}) =$

$$h'(\frac{1}{2}) =$$

- a) 0
- b) 17
- c) 5
- d) 10
- e) -10

$$h'(\mathbf{x}) = 10\cos^{1}(\mathbf{x} - \mathbf{x}) \cdot \sin(\mathbf{x} - \mathbf{x}) \cdot (-1)$$

$$= -10\cos((\upgamma - x)\sin(\upgamma) - x)$$

$$h'(\frac{\pi}{2}) = -10\cos(\frac{\pi}{2})\sin(\frac{\pi}{2})$$

5)
$$f(x) = |x^3|$$
 What is $\lim_{x \to -1} f'(x)$

- a) -3
- b) 0
- c) 1
- d) 3
- e) Does not exist

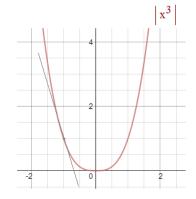
The derivative is the rate of change (slope) of the function.

$$f'(x) = -3x^2$$
 for $x < 0$

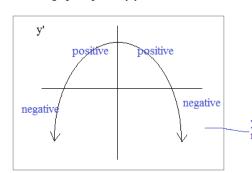
$$=3x^2$$
 for $x > 0$

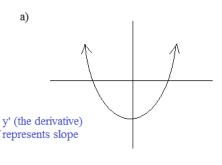
and, the derivative is continuous

$$f'(-1) = -3(-1)^2 = -3$$

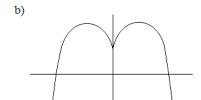


6) Below is a graph of y'. Which graph is possibly y?

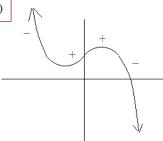




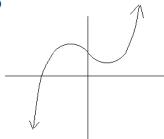
SOLUTIONS



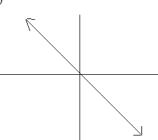
c)



d)



e)



7) Find the instant rate of change with respect to r of

$$V = \frac{1}{6} \text{Tr} r^2 (5 - r)$$

b)
$$\frac{251}{3}$$

c)
$$\frac{-2511}{6}$$

$$V = \frac{1}{6} \uparrow \uparrow \uparrow (5r^2 - r^3)$$

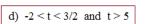
$$\frac{dv}{dr} = \frac{1}{6} \left[10r - 3r^2 \right]$$

Then, at
$$r = 5$$
: $\frac{1}{6} \left[10(5) - 3(5)^2 \right] = \frac{1}{6} (-25)$

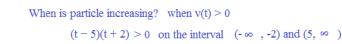
$$=$$
 $\frac{-25}{6}$ $\uparrow \uparrow \uparrow$

8) The velocity of a particle is modeled by $v(t) = t^2 - 3t - 10$ When is the particle speeding up?

- a) 0 < t < 3/2
- b) t > 3/2
- c) t > 5



e)
$$3/2 < t < 5$$



OR, when the direction is decreasing and decelerating.

When is particle accelerating? when
$$a(t) > 0$$
 concave up

A particle is speeding up when the direction is increasing and accelerating

$$a(t) = 2t - 3 \qquad \text{on the interval} \quad (3/2, \infty)$$

**particle is speeding up (in a positive direction) when
$$t > 5$$

When is particle decreasing? when $v(t) \le 0$ (-2, 5)

When is particle decelerating? when $a(t) \le 0$ on the interval $(-\infty, 3/2)$ concave down

**particle is speeding up (in a negative direction) when -2 < t < 3/2

2/9 a)

b) 2/3

c) -52/3

d) -52/9

> e) 0

To find f(x), take the antiderivative of f'(x):

$$\int x^{2} (x^{3} + 1)^{\frac{1}{2}} dx$$

$$\frac{1}{3} \int 3 x^2 (x^3 + 1)^{\frac{1}{2}} dx$$

$$\frac{1}{3} \frac{(x^3+1)^{3/2}}{3/2} = \frac{2}{9} (x^3+1)^{3/2} + C$$

To find C, use a point on f(x):

(2, 0):
$$\frac{2}{9} (2^3 + 1)^{3/2} + C = 0$$

$$f(x) = \frac{2}{9} (x^3 + 1)^{3/2} + (-6)$$

Therefore,
$$f(0) = \frac{2}{9}(1) - 6 = \boxed{\frac{-52}{9}}$$

10)
$$\int_{\frac{2\pi}{2}}^{x} 2\cos t \, dt =$$

- 2sinx
- b) sinx
- c) $2\sin x 1$
- d) $2(1 \sin x)$
- e) 2sinx 2

$$\int_{\frac{1}{2}}^{x} 2\cos t \, dt = 2\sin t \, \left| \begin{array}{c} x \\ \frac{1}{2} \end{array} \right|$$

$$= 2\sin(x) - 2\sin(\frac{\pi}{2}) = 2\sin(x) - 2$$

11) Determine the equation of the line tangent to the curve

$$x^2 - 3xy = 7$$
 @ (-1,2)

To find equation of line, we need a point and the slope...

The point is (-1, 2), and to find the slope, use implicit differentiation:

a)
$$8x + 3y = -2$$

b)
$$8x - 3y = -14$$

c)
$$4x - 5y = -14$$

d)
$$4x + 5y = 6$$

e)
$$3x - 2y = -7$$

$$2x - 3(y + xy') = 0$$

$$2x = 3y + 3xy'$$

$$2x - 3y = 3xy'$$

$$y' = \frac{2x - 3y}{3x}$$

$$y - 2 = \frac{8}{3}(x + 1)$$

$$y = \frac{8}{3}x + \frac{14}{3}$$

$$3y - 6 = 8x + 8$$

$$8x - 3y = -14$$

@ (-1, 2)
$$y' = \frac{2(-1) - 3(2)}{3(-1)} = \frac{8}{3}$$

$$y-2=\frac{8}{3}(x+1)$$

$$y - 2 = \frac{3}{3}(x + 1)$$

$$y = \frac{8}{3}x + \frac{14}{3}$$

$$8x - 3y = -14$$

f(3)	g(3)	f'(3)	g'(3)
-1	2	4	-2

$$h(x) = \frac{f(x)}{g(x)} \qquad h'(3) =$$

b)
$$\frac{3}{2}$$

c) 3

d) 5

e) -3

Use quotient rule:

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$$

$$h'(3) = \frac{f'(3)g(3) - g'(3)f(3)}{g(3)g(3)}$$

$$= \frac{4(2) - (-2)(-1)}{(2)(2)} = \boxed{\frac{3}{2}}$$

AP Calculus Questions

13) The graph shows the velocity of an object (moving right and left) along the x-axis as a function of time. Which point corresponds to the position farthest to the right?

SOLUTIONS

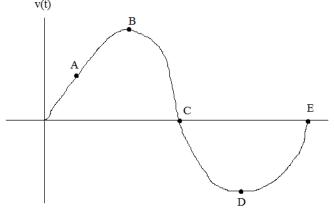
a) A

b) B

c) C

d) D

e) E



The velocity graph shows the rate of change of the object. so, if the graph is above the x-axis, the object is moving in a positive direction (i.e. to the right). So, from t = 0 until point C, the particle is moving to the right... After point C, the particle begins moving in a negative direction (i.e. to the left)...

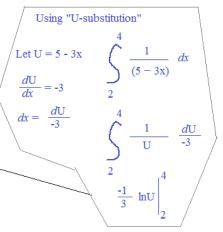


- a) 8
- b) <u>-ln</u>7
- c) $-3(\ln 7)$
- d) (ln7)
- e) Does not exist

$$-\frac{1}{3} \int_{2}^{4} \frac{\text{"derivative of function"}}{\frac{-3}{(5-3x)}} \frac{dx}{\ln(\text{function"})}$$



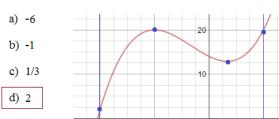
$$-\frac{1}{3}(\ln 7 - \ln 1) = -\frac{1}{3}(\ln 7 - 0)$$



15) What is the absolute minimum on the interval [-2, 1]

$$f(x) = 6x^3 + 6x^2 - 6x + 14$$
?

To find critical values and relative minimums, set derivative equal to zero...



 $f'(x) = 18x^2 + 12x - 6$ $0 = 6(3x^2 + 2x - 1)$ (3x - 1)(x + 1)

$$f''(x) = 36x + 12$$

$$0 = 36x + 12$$

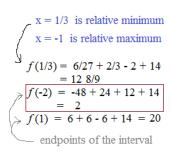
$$x = -1/3 \text{ point of inflection}$$

$$x > -1/3 \text{ (concave up)}$$

x = -1, 1/3

e) 12 8/9
$$f(x) = 2x + e^{X}$$

x < -1/3 (concave down)



 $g(x) = f^{-1}(x)$ for all x

Since f(0) = 1, what is g'(1)?

- a) -3
- b) -1/3
- c) 0
- d) 1/3

e) 3

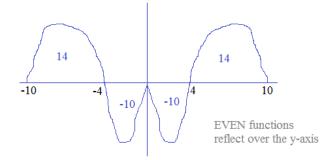
Since inverses are reflections over y = x, it follows that their slopes are reciprocals...

 $f'(x) = 2 + e^{X}$ at (0, 1), the slope is $2 + e^{0} = 3$ therefore, the slope of g(x) at (1, 0) should be 1/3

g'(1) = 1/3

17)
$$\int_{0}^{4} g(x) dx = -10 \qquad \int_{0}^{10} g(x) dx = 4 \qquad g(x) \text{ is an EVEN function}$$

$$\int_{-10}^{-4} g(x) dx =$$



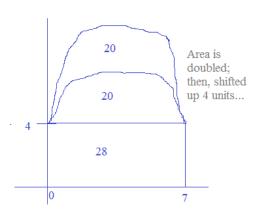
possible sketch

18) If
$$\int_{0}^{7} f(x) dx = 20$$
 what is $\int_{0}^{7} 2f(x) + 4 dx$?



$$2\int_{0}^{7} f(x) dx + \int_{0}^{7} 4 dx$$

$$2 \cdot (20) + 4x \Big|_{0}^{7} = 68$$



19)
$$f(x) = 3x^2 + 2x + 4$$

What is the average value between x = 1 and x = 4?

- a) 17
- b) 23
- c) 26
- d) 30
- e) 34.5

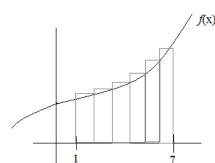
Area under curve between
$$=$$

$$\begin{cases} 4 \\ 3x^2 + 2x + 4 \end{cases} dx$$
 1 and 4

$$x^3 + x^2 + 4x \Big|_{1}^{4} = 96 - 6 = 90$$

Therefore, the average value =
$$\frac{90}{(4-1)}$$
 = 30

- 20) Which will give the highest value?
 - a) $\int_{0}^{1} f(x)$
 - b) Trapezoid Rule with 6 intervals
 - c) Left Riemann Sum using 3 subintervals between 1 and 7
 - d) Left Riemann Sum using 6 subintervals between 1 and 7
 - e) Right Riemann Sum using 6 subintervals between 1 and 7



overestimates the area under the curve...

Find the value "c" guaranteed by the "integral mean value theorem"

(i.e. where the value f(c) equals the <u>average value</u> on the interval [0, 2])

a) 1

- c) 1.75
- d) 2

e) 3

First, find the average value on the interval:

$$\int_{0}^{2} x^{2} + 1 \, dx = \frac{x^{3}}{3} + x \bigg|_{0}^{2} = \frac{8}{3} + 2 - (0/3 + 0) = \frac{14}{3} \quad \text{area under the curve (i.e. total value on interval [0, 2])}$$

average value: $\frac{\frac{14}{3}}{(2-0)} = \frac{7}{3}$ average value

since the function is continuous and closed on the interval, there must be a value "c" such that f(c) = average value

so, where does the function equal $\frac{7}{3}$? $\frac{7}{3} = x^2 + 1$

$$x = \frac{2\sqrt{3}}{3}$$
 approx. 1.15

 $f(c) = \frac{1}{b-a} \int f(x) dx$

We don't include -1.15 (because it is not in the interval)

on the interval [-1, 3]

I. determine the AROC (average rate of change)

22) For the function $h(x) = x^3 - 2$

- a) -7
- b) -1
- c) 1

- Average
- Of
- Change
- II. find the value "c" to verify the Mean Value Theorem
 - a) 1 b) 1.53
- Instantaneous Rate Of

Change

 $h'(x) = 3x^2 - 0$

- c) 2.57
- d) 3
- e) 6

- at point "c" h'(c) = 7

 - c = -1.53 or 1.53

If function is continuous and differentiable...

there exists at least one point c where



- rate of change
- average rate of change between a and b

- 23) Find the value of k so g(x) is continuous:
 - a) 9/10
 - b) 1 c) 10/9
 - d) 9
 - e) 10

- $g(x) = \begin{cases} k+x & \text{if } x < 10 \\ xk & \text{if } x \ge 10 \end{cases}$
- to be continuous, each part of the piecewise function must meet:

find $\frac{dy}{dy}$

- k + x = xk at x = 10:
 - 10 + k = 10k10 = 9kk = 10/9
- (Note: although the function is continuous at x = 10, it is NOT differentiable because the slopes/ instanteous rates of change are different...)

- 24) For the equation $x = \sin t + 3$

 - a) tant
 - b) -tant
 - c) cott
 - d) -cott
 - e) $\frac{-\sin t(\sin t + 3) \cos^2 t}{\left(\sin t + 3\right)^2}$
- We can find $\frac{dx}{dt} \longrightarrow \cos t + 0$
 - then, find $\frac{dy}{dt} \longrightarrow -\sin t$
- $\frac{dy}{dx} \leftarrow \frac{\frac{dy}{dt}}{\frac{dx}{dx}} = \frac{-\sin t}{\cos t} = -\tan t$
- These show how x and y change as t changes
- To find $\frac{dy}{dx}$, we combine the fractions (rates of change)

(chain rule)

$$\frac{\frac{dy}{dt}}{\frac{dx}{dx}} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dx}$$

25) Find D_x of g(f(x)) when x = 2

X	1	2	3	4	
f(x)	3	1	2	4	
f'(x)	-7	-5	-4	-6	
g(x)	4	3	1	2	
g'(x)	1/3	1/9	7/9	2/9	

- a) 2/9
- b) -4
- c) 7/9
- d) -5/3
- e) -28/9
- 26) Find the equation of the line tangent to the graph $y = x^3 - 3x^2 + 5$ at the point of inflection.
 - a) y = 6x + 6
 - b) y = x + 2
 - c) y = -3x + 6
 - d) y = -3x + 4
 - e) y = x + 8

SOLUTIONS

Using the chain rule of compositions, Derivative: $g(f(x))' = g'(f(x)) \cdot f'(x)$

$$f(2) = 1$$
 $g'(1) = 1/3$

$$f'(2) = -5$$

For equation of line, we need a point and the slope...

The point will occur at the point of inflection...

$$y' = 3x^2 - 6x$$

$$y'' = 6x - 6$$

set y'' = 0: 6x - 6 = 0 x = 1 (point of inflection)

plug x = 1 into the original equation,

$$y = (1)^3 - 3(1)^2 + 5 = 3$$

Then, the slope at (1, 3):

plug x = 1 into the derivative equation,

$$y' = 3(1)^2 - 6(1)$$
 $y' = -3$

Therefore, slope is -3...

$$y-3 = -3(x-1)$$
 or $y = -3x+6$

27) If the differential equation $\frac{dy}{dx} = \frac{-x}{4y}$ has a solution containing point (6, 1),

then when x = 2,

Separable differential equations...

4y dy = -x dx

cross multiply / separate the variables...

a)
$$y = 1$$

b)
$$y = 2$$

c)
$$y = 3$$

d)
$$y = 4$$

e)
$$y = 5$$

$$\int 4y \, dy = \int x \, dx$$
 integrate

$$2y^2 + C = \frac{-x^2}{2} + C$$
 combine

$$\frac{x^2}{2} + 2y^2 = C \qquad \text{find } C$$

$$\frac{6^2}{2} + 2(1)^2 = C$$
 $C = 20$

$$\frac{x^2}{2} + 2y^2 = 20$$

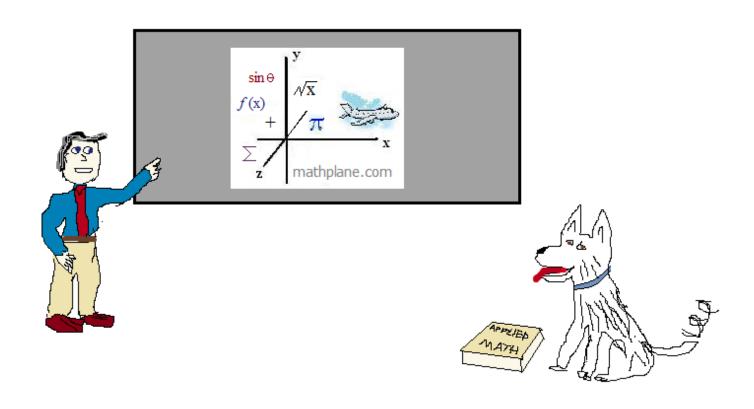
finally, substitute x = 2

$$\frac{2^2}{2} + 2y^2 = 20$$

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers.



Also, at TeachersPayTeachers, Facebook, Google+, and Pinterest.