# The Development of Mathematics-Vocabulary Measures for Elementary Students 

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## Thanks

NATIONAL ACADEMY EDUCÁTION


## Language of Mathematics

2,450
thousands


## Language of Mathematics

2,45

## Mathematics Vocabulary



## Mathematics Vocabulary



Identify, compare, and analyze attributes of two- and threedimensional shapes and devel vocabulary to describe the attributes

Describe location and movement using com mon language an geomet ic vocabulary

Children interpret the physical world with geometric ideas (e.g., shape, orientation, s and describe it with correspondir vocabulary. They identify, name, and describe a va

## Mathematics Vocabulary

## COMMON CORE <br> STATE STANDARDS for

Mathematics

COMMON CORE

4. Analyze and compare two- and three-dimensionalshonos in different sizes and orientations, using infor nal language to lescribe their similarities, differences, parts (e.g., number-of cidos and vertices/"corners") and other attributes (e.g., having sides of equal length).

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For exarmpie, The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate $A$ received, candidate C received nearly three votes."
2. Partition circles and rectangloo intono, three, or four equal shares, describe the shares usi $1 g$ the words halves, thirds, half of, a third of, etc., and describe the whoteactionalves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

## Mathematics Vocabulary

## Kevin makes muffins.

- It takes 8 minutes to mix the batter.
- The muffins bake for 17 minutes.
- The muffins then cool for 5 minutes.

What is the total amount of time baking, and cooimy the murims?

## 2003

Click-lllof ne shapes that are quadrilaterals.


## Development of Measures

## Grade 1

1. Term appeared in 2 of 3 first-grade textbook glossaries - Go Math! enVisionMATH Everyday Math
2. Term appeared in a textbook glossary and a standard

- Common Core

3. Term was opposite of selected term

- longest

4. Terms included to alleviate ceiling effects

## Grade 1

- 31 terms appeared in kindergarten glossaries
-45 terms appeared in second-grade glossaries


## Mathematics Vocabulary (1st Grade)

## Answer the questions.

1. Circle the name of each coin.

2. Circle the set with the greatest

3. Circle the set with the least.

4. In the box, write the double of 4.

5. Subtract five circles.

Divide into groups of three.



Show skip counting by twos to 20 .

8. Circle all that show equal shares.

9. Draw a set in the box that is equal.


## 15. Circle the name of each shape.


11. Circle all that show sounting back.
$8,7,6,5$
$3,5,7,9,11$
$14,15,13,12$
$21,20,19,18,17$
12. Circle all that show counting on.
$2,9,7,4$
14. Draw a number line.
$\square$
A. addend
B. difference
C. equal sign
D. greater than
E. minuend
F. minus sign
G. plus sign
H. subtrahend
J. subtrahend
K. sum

| $8,7,6,5$ | $3,5,7,9,11$ | $14,15,13,12$ |
| :--- | :--- | :--- | 21,20,19,18,17

19, 18, 17, 16
$9,10,11,12,13$
$3,4,6,5$


A. octagon
B. parallelogram
C. rectangle
D. rhombus
E. trapezoid

A. decagon
B. heptagon
C. hexagon
D. octagon
E. pentagon

Draw an oval.

Draw a closed figure.

17. Circle all that show an addition sentence

$$
9-2=7 \quad 8=6+2 \quad 10-3=7 \quad 4+6=10 \quad 7+3+1=11
$$

18. Circle all that show a subtraction sentence.
$2=8-6$
$4=3+$
$6+9=15$
$12-3=9$
$9=9$
19. Draw circles to make ten
A. hundreds
B. millions
C. ones
D. tens
E. thousands
20. Circle the line that is shortest.
21. Draw a ball inside the box. Draw a star outside the box.

22. Circle the line that is longest.
$\qquad$
$\qquad$

A. hexagonal prism
B. hexagonal pyramid
C. rectangular prism D. rectangular pyramid
E. triangular prism
F. triangular pyramid

A. hexagonal prism
B. hexagonal pyramid
C. rectangular prism rectangular pyramid
E. triangular prism
F. triangular pyramid

A. cube
B. cylinder
rectangular pyramid square
E. triangular prism

A. cube
B. cylinder
C. rectangular pyramid
D. square
E. triangular prism

A. cone
B. cube
C. cylinder
D. sphere
E. square

A. hexagonal prism
B. hexagonal pyramid
C. rectangular prism
D. rectangular pyramid
E. triangular prism
F. triangular pyramid
23. Write zero.

24. Circle the set that shows more.

25. Circle the set that shows less.

26. Draw lines to break the box into unequal parts.

27. Circle the sets the show equal.

28. Draw tally marks to show 4.
$\square$
$\square$

29. Separate 2 from the set.

30. Take away 4 from the set.

31. Color the bottom row with your pencil

32. Color one column with your pencil.


## Grades 3 and 5

1. Term appeared a textbook glossary at grade 3 and 5 - Go Math! enVisionMATH Everyday Math

22 2. Term appeared in a textbook glossary and a standard

- Common Core

3. Term in a glossary but not explicit in a standard
4. Term explicitly named in standards but not a glossary
5. Term related to previously-selected terms

## Grades 3 and 5

- In the Common Core:
- 22 introduced in kindergarten
- 9 in grade 1
- 18 in grade 2
- 13 in grade 3
- 14 in grade 4
- 4 in grade 5
- 14 in grade 6 or above


## Mathematics Vocabulary

## Answer the questions. Try the easy problems first, then go back and try the harder problems.

. Identify each part. Write the letter in the box. You may repeat letters and/or not use all letters.

A. addend
B. augend
C. difference
D. dividend
$24 \div 4=6$
E. divisor
F. factor
G. minuend
H. multiplicand
J. multiplier
K. plus
L. product
M. quotient
N. subtrahend
O. sum
3. Write an odd number

Write an even number.

4. Write the letter of each shape in the box.

You may repeat letters, not use all letters, or use more than one letter.

5. Write a number with a denominator of 9 and a numerator of 4 .

6. Draw a number line.

Draw an equilateral triangle.
Draw a right triangle.

8. Write a mixed number.

9. Identify each. Write the letter in the box. You may repeat letters and/or not use all letters.
10. Write the reciprocal of $6 / 5$.
A. intersecting lines
B. line
C. line segment
D. parallel lines
E. perpendicular lines
F. point
G. ray
$\square$


Write a proper fraction.

11. Write the name of each coin in the box.
17. Write the name of each shape.

18. Draw a regular polygon.


19. Label the place value of each digit.

## $\square \square$

 2,568,901.347

What is the name for the item between the 1 and 3?

What number is in the tenths place?
What number is the the ones place?

20. Write a positive integer.

Write a negative integer.

21. Label the clock hands.
22. Divide into thirds.

Divide into halves.


Divide into fourths.

23. Label the parts of the figure.


What is this figure? (Circle all options.)
Two-dimensional
Solid figure
Three-dimensional
Open figure
24. Subtract five.

25. Show skip counting by twos to 20 .

26. Draw a line around the perimeter of the shape.

Shade the area of the shape.
27. Draw a right angle. Draw an obtuse angle.

Draw an acute angle.

28. Draw an array for 4 times 2.

29. Circle the two congruent shapes.

30. Complete the conversions.


5,280 feet $=1$ $\square$
1 gallon $=4$
1 pound $=16$ $\square$
1000 meters $=1$ $\square$ 2,000 pounds $=1$ $\square$

32. What does more mean? $\square$
$\square$
$\square$


What is a formula?


## Pilot Studies

## Grade 1

- 104 first-grade students from 6 classrooms

|  | $n$ | $\%$ |
| :--- | ---: | ---: |
| Gender: |  |  |
| Male | 55 | 52.9 |
| Female | 49 | 47.1 |
| Race/ethnicity: | 12 | 11.5 |
| African American | 1 | 1.0 |
| Asian | 37 | 35.6 |
| White | 54 | 51.9 |
| Hispanic | 1 | 1.0 |
| English learners | 7 | 6.7 |
| Retained | 3 | 2.9 |
| Special education |  |  |

## Measures



## Grade 1 Mathematics Vocabulary

- $\alpha=.85$
- Item-by-item analysis for reliability
- Deletion of only five terms would have increased $\alpha$ by . 02
- Opted to keep all terms for analysis


## Grade 1 Mathematics Vocabulary

|  | Raw Score |  |  | Correlations |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Variables | $M$ | $S D$ |  | WD | MF |
| GMRT WD | 27.57 | 8.56 |  | - | - |
| WJ-III MF | 31.17 | 14.29 |  | .526 | - |
| Mathematics Vocabulary | 36.30 | 8.10 |  | .697 | .586 |

## Grade 1 Mathematics Vocabulary

-Range: 15-55

- No significant differences based on gender or retained status
- Significant differences:
- For English learner ( $n=1$ )
- Students with disabilities < students without disabilities
- African American < Hispanic < Caucasian


## Grade 1 Mathematics Vocabulary

| Predictor | $B$ | SE B | $\beta$ | $t$ | $p$ | $R^{2}$ | $\Delta R^{2}$ |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model 1: |  |  |  |  |  |  |  |
| $\quad$ Intercept | 18.088 | 1.940 |  | 9.325 | $<.001$ |  |  |
| GMRT word decoding | .661 | .067 | .697 | 9.826 | $<.001$ | .486 |  |
| Model 2: |  |  |  |  |  |  |  |
| $\quad$ Intercept | 16.779 | 1.844 |  | 9.157 | $<.001$ |  |  |
| GMRT word decoding | .509 | .074 | .538 | 6.874 | $<.001$ |  |  |
| WJ-III math fluency | .172 | .044 | .304 | 3.881 | $<.001$ | .553 | .067 |

## Grade 1 Mathematics Vocabulary

-Accuracy by item introduction:

- Kindergarten
67.1\%
- First grade 48.8\%
- Second grade 29.2\%
-Accuracy by category:
- Technical
42.0\%
- Subtechnical 56.4\%
- General 91.1\%
- Symbolic 54.5\%


## Grades 3 and 5

| Variable | Grade $3(n=65)$ |  | Grade $5(n=128)$ |  | Overall sample ( $N=193$ ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $n$ | (\%) ${ }^{\text {a }}$ | $n$ | (\%) ${ }^{\text {a }}$ | $n$ | (\%) |
| Gender |  |  |  |  |  |  |
| Female | 26 | (40.0) | 61 | (47.7) | 87 | (45.1) |
| Male | 39 | (60.0) | 67 | (52.3) | 106 | (54.9) |
| Race/ethnicity |  |  |  |  |  |  |
| African American | 4 | (6.2) | 15 | (11.7) | 19 | (9.8) |
| Caucasian | 20 | (30.8) | 47 | (36.7) | 67 | (34.7) |
| Hispanic | 41 | (63.1) | 55 | (43.0) | 96 | (49.7) |
| Other | 0 | (0.0) | 11 | (8.6) | 11 | (5.7) |
| School-identified disability | 12 | (18.5) | 8 | (6.3) | 20 | (10.4) |
| English learner | 17 | (26.2) | 24 | (18.8) | 41 | (21.2) |
| Retained | 8 | (12.3) | 5 | (3.9) | 13 | (6.7) |
|  | M | (SD) | M | (SD) | M | (SD) |
| GMRT vocabulary ${ }^{\text {b }}$ (standard score) | 98.94 | (13.98) | 97.65 | (11.25) | 98.09 | (12.22) |
| WRAT Math Computation (standard score) | 113.92 | (12.97) | 107.66 | (14.23) | 109.77 | (14.10) |
| Mathematics vocabulary (raw score) | 35.57 | (14.02) | 57.51 | (20.61) | 50.20 | (21.32) |

## Measures

| 11. They sho <br> A. <br> B. <br> C. <br> D. | Id display it. <br> show <br> play with <br> go around <br> look at |
| :---: | :---: |
| 12. a sore knuckle |  |
| A. <br> B. <br> C. <br> D. | back of the neck shoulder tip of a toe part of a finger |
| 13. a bigjug |  |
| A. <br> B. <br> C. <br> D. | container cork jumble drink |
| 14. She was active. |  |
| A. <br> B. <br> C. <br> D. | lazy <br> noisy <br> doing things <br> tired |
| 15. a good excuse |  |
| A. <br> B. <br> C. <br> D. | note <br> reason <br> movement example |
| 16. It might rise. |  |
| A. <br> B. <br> C. | bloom go fast ride away |



## Grades 3 and 5 Mathematics Vocabulary

- $\alpha=.92$ at Grade 3
- $\alpha=.96$ at Grade 5


## Grades 3 and 5 Mathematics Vocabulary

Measure
Grade 3
Grade 5
GMRT $^{\text {a }}$ WRAT MV GMRT ${ }^{\text {b }}$ WRAT MV

1. General vocabulary (GMRT)
2. Math computation (WRAT)
3. Mathematics vocabulary (MV)

4. Mathematics vocabulary (MV) 0.606 $\square$

## Grades 3 and 5 Mathematics Vocabulary

- Grade 3
- Range = 6-68
- $M=35.57$ (14.02)
-Grade 5
- Range = 5-100
- $M=57.51$ (20.61)
- No significant differences based on gender or retained status


## Grades 3 and 5 Mathematics Vocabulary



## Grades 3 and 5 Mathematics Vocabulary



## Grade 5 Mathematics Vocabulary

| Variable | Typical ( $n=70$ ) |  | MD-only ( $n=16$ ) |  | RD-only ( $n=18$ ) |  | $M D R D(n=10)$ |  | Excluded ( $n=14$ ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $n$ | (\%) | $n$ | (\%) | $n$ | (\%) | $n$ | (\%) | $n$ | (\%) |
| Gender |  |  |  |  |  |  |  |  |  |  |
| Female | 35 | (27.3) | 6 | (4.7) | 8 | (6.3) | 7 | (5.5) | 5 | (3.9) |
| Male | 35 | (27.3) | 10 | (7.8) | 10 | (7.8) | 3 | (2.3) | 9 | (7.0) |
| Race |  |  |  |  |  |  |  |  |  |  |
| African American | 6 | (4.7) | 4 | (3.1) | 1 | (0.8) | 3 | (2.3) | 1 | (0.8) |
| Caucasian | 30 | (23.4) | 4 | (3.1) | 5 | (3.9) | 2 | (1.6) | 6 | (4.7) |
| Hispanic | 28 | (21.9) | 5 | (3.9) | 11 | (8.6) | 5 | (3.9) | 6 | (4.7) |
| Other | 6 | (4.7) | 3 | (2.3) | 1 | (0.8) | 0 | (0.0) | 1 | (0.8) |
| English learner | 12 | (9.4) | 3 | (2.3) | 6 | (4.7) | 1 | (0.8) | 2 | (1.6) |
| Retained | 0 | (0.0) | 1 | (0.8) | 1 | (0.8) | 0 | (0.0) | 3 | (2.3) |
| Special education | 0 | (0.0) | 1 | (0.8) | 2 | (1.6) | 3 | (2.3) | 2 | (1.6) |
|  | M | (SD) | M | (SD) | M | (SD) | M | (SD) | M | (SD) |
| WRAT Math Computation | 40.14 | (3.22) | 32.50 | (1.79) | 38.61 | (2.06) | 31.60 | (2.46) | 36.93 | (2.27) |
| GMRT Vocabulary | 29.46 | (5.33) | 23.19 | (4.17) | 13.50 | (3.05) | 15.10 | (2.13) | 21.64 | (3.10) |
| Mathematics Vocabulary | 68.13 | (15.12) | 46.00 | (19.37) | 46.56 | (16.10) | 21.90 | (10.37) | 57.07 | (13.27) |

## Grade 5 Mathematics Vocabulary

Typical $>$ MD $=$ RD $>$ MDRD
MD > MDRD
ES = 1.41
RD > MDRD
Typical > MDRD
ES = 1.67

MD < Typical
ES = 1.37
RD < Typical
ES = 3.12

ES = 1.40

## AN INVESTIGATION OF THE <br> MATHEMATICS-VOCABULARY KNOWLEDGE OF FIRST-GRADE STUDENTS

stract
Compdency with mathematics requires use of numerals nation vocalowary (es ast anderstanding and use of mathmatio woxatuarary (eg, add, more, triangle). Currently, no
mesurse exist in which the primary fuuction is to guge athematics wocabulary understanding. We created a 4 -item mathematics vocatkulary messure for first gnide . pliotod the assesment with 109 first-grade students. We also adminitereds standard dized messures of geeceal word knowlogge and mathematiss fluency to investigte sults indicated a wide variablity in how first grade stuInts intep pret mahematks woabulary termas but stong eliabliliy for the mathematics wocabulary measure.
.menial, sudents solve problems by munipulating numerals and inter. preting symbial, bume based vocabulary component. Educators and students use language to tead ang karn about mathematics, and this oral and written languge is filled with vocabuulary erms spedific to mathematics (e.e, slim, octagon) and vocabulury terms from ever) y language that have mathemakal meaning (e.g, more, quaricrs). Furthermore

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Differences in the Mathematics-Vocabulary Knowledge of Fifth-Grade Students With and Without Learning Difficulties

Suzanne R. Forsyth © and Sarah R. Powell The University of Tecas at Austin







INTRODUCTION
In the United States, federal legisilation requires annual
iountability testing, to measure studen countability testing to measure student progerss and mar
dates that schools use evidence-based practiess to mitigat performance gaps amongs student groups (Every Child Suc prow ement Ach, 2004). Many studensts, however, continue to
 on hish-stakes assessments, even when reveriving eviden
based instruction. One reason for persistent failure on mat ematics assessments may be thar students with learning dificulties, with or withouta diagnosed disability, struggle with
the mathematics language used for classoom instruction, if the mathematiss language used for
textbooks, and on assessments.
 ematics difficulty without reading difficulty (MD-onl)
ratidig dificuly w without mathematics difficulty (RD-onl),


 the elementary grodes. We compared the performance
students with MD-only, RD-only, and MDRD to stude without dificiculty in mathematicsor reading tn this sintrodic tion, we discuss the language of mathenatics sicoountery
in classoom leanning and on hight-stakes assesments. W.
 RD-only, and MDRD encounter when answering question
on such assesment. Finally, we sate the purpose and
rescarch questions of this stuly.


The Language of Mathematics
Most preschool language and vocabulary skills are learned
lrowgho oral interactions, and by the tiny children enter el rrough oral interactions, and by the tine children enter el
mentary school, significant differences in vecabulary ac quisition and languge comprechnsion are apparent (Beck
McKeown, \& Kucan, 2013: Hart \& Risley, 1999). Knowl dge of specific mathematics language (i.e., quantity and spa
 lume, Sims, \& Lonigan, 2011; Toll \& Van Luit, 2014). These atcomes in both mathematics and reading than attention lated behavioral, social, and ealy mating skills (Duncaa As stude
 asthematics skills, including numeration, calculation, ge-
 al., 2010). At the third-grade level. Vukowic and Lesuux
$(2013)$ determined that plonoological skils and general lanuage abilitites weve move stogronly corcelated to collaula-
to and wort-problem skills than working memory and on and word-problem skills than working memory and
vsuospatial procesing. respectively. By late elenentary shool, oral languages skills speso impact fration competenc Chow \& Jacobs, 2016; Secthaler. Fuchss Star, \& Bryant, Mastering mathematical language has been compared
in dificuly to learning a second language (Wakefield difficulty to learning a seoond language (Wakeficla
Doo). Conceptual meaning in mathenatics is constructed
 otation, (b) visual representations, and (c) oral and writcon


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