The Development of **Mathematics-Vocabulary Measures for Elementary** Students

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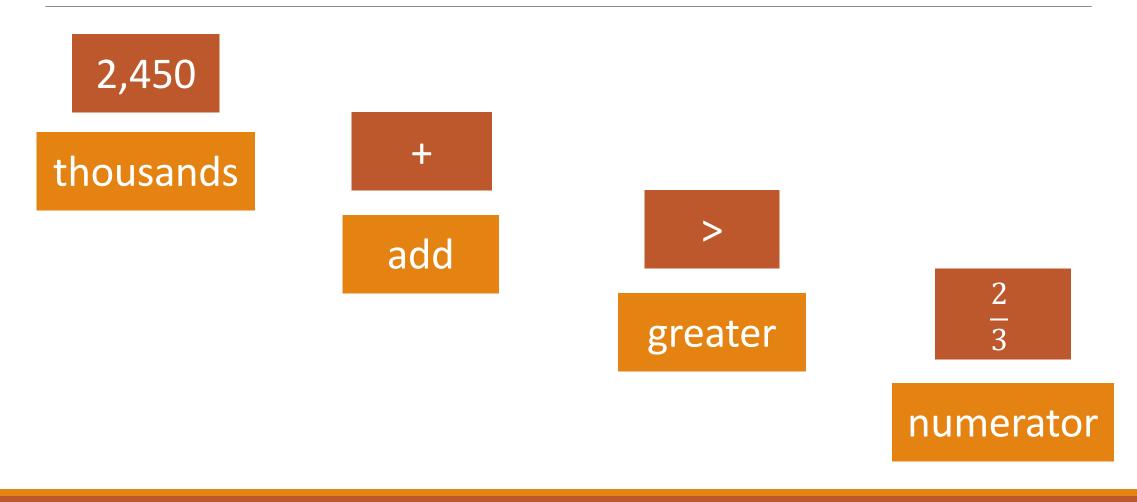
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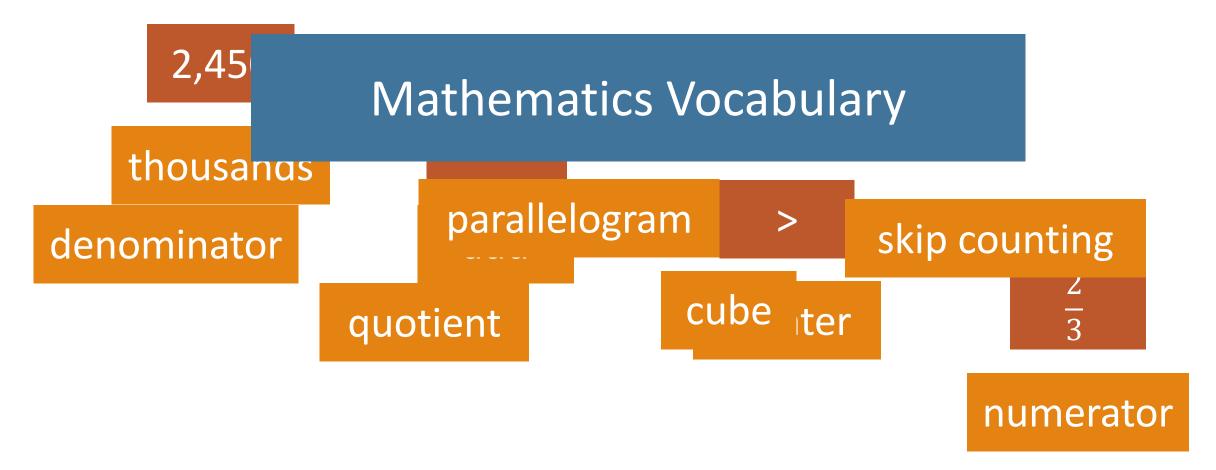


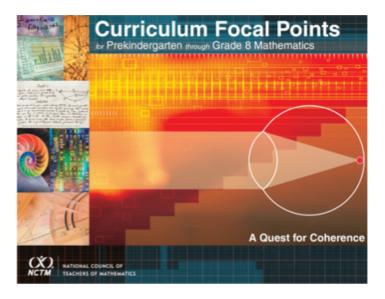
The Meadows Center

Language of Mathematics



Language of Mathematics

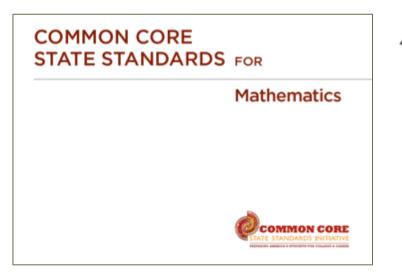




Identify, compare, and analyze attributes of two- and threedimensional shapes and develop vocabulary to describe the attributes

Describe location and movement using common language and geometric vocabulary

Children interpret the physical world with geometric ideas (e.g., shape, orientation, sp and describe it with corresponding vocabulary. They identify, name, and describe a va



- Analyze and compare two- and three-dimensional chapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
 - 1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
- 3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Kevin makes muffins.				
 It takes 8 minutes to mix the batter. 				
 The muffins bake for 17 minutes. 				
The muffins then cool for 5 minutes.				
What is the total amount of time, in minutes, Kevi baking, and cooling the muturs?	n spends mixing, 2003 Click all of the shap quadrilaterals.	bes that are		
			>	

Development of Measures

Grade 1

40

19

1

- 1. Term appeared in 2 of 3 first-grade textbook glossaries • Go Math! enVisionMATH Everyday Math
- Term appeared in a textbook glossary and a standard
 Common Core
- 3. Term was opposite of selected term • *longest*



4. Terms included to alleviate ceiling effects

Grade 1

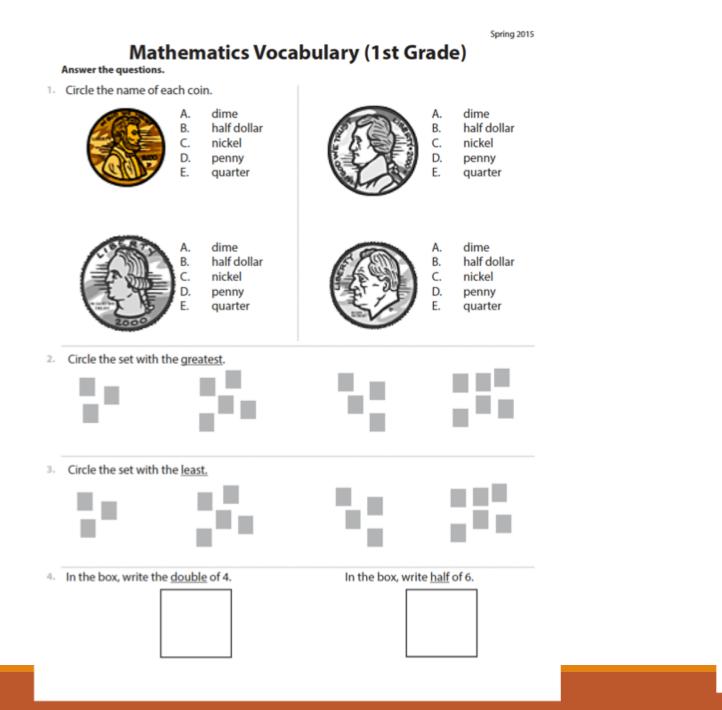
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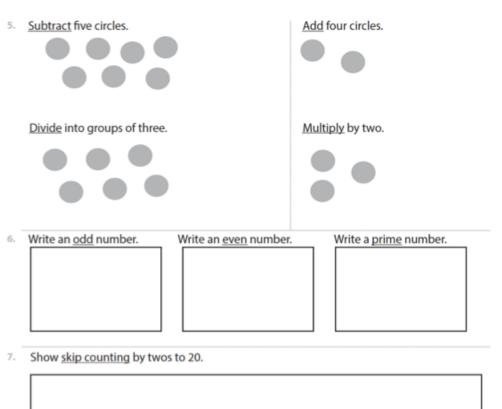
19

1

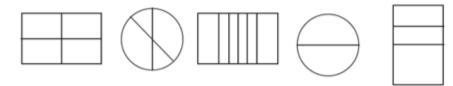
Δ

31 terms appeared in kindergarten glossaries
45 terms appeared in second-grade glossaries





8. Circle all that show equal shares.



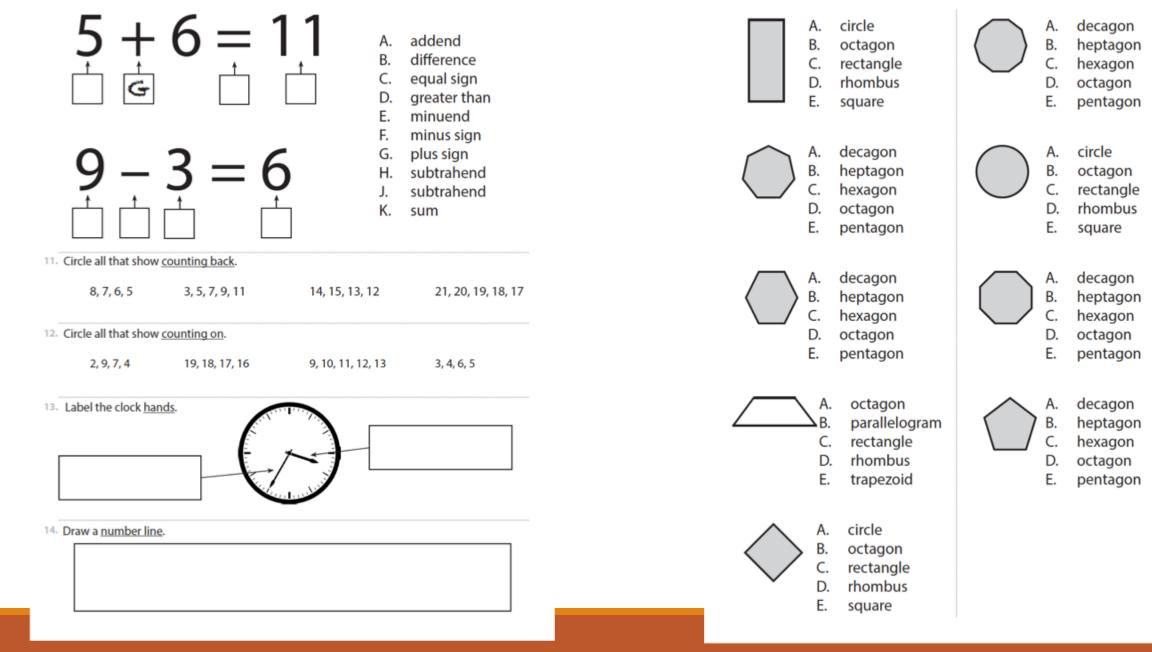
9. Draw a set in the box that is equal.





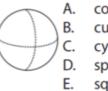
10. Match each part to a word by writing a letter in the box.

15. Circle the name of each shape.

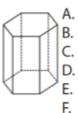


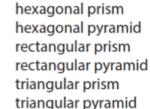
16. Draw a triangle. Draw a square.	20. Label the <u>place value</u> of each digit. A. hundreds B. millions C. ones
Draw an <u>oval</u> . Draw a <u>line</u> .	D. tens E. thousands 21. Draw a ball <u>inside</u> the box. Draw a star <u>outside</u> the box.
Draw a closed figure. Draw an open figure.	
9-2=7 8=6+2 10-3=7 4+6=10 7+3+1=	= 11 22. Circle the line that is <u>longest</u> .
18. Circle all that show a subtraction sentence. $2 = 8 - 6$ $4 = 3 + 1$ $6 + 9 = 15$ $12 - 3 = 9$ $9 = 9$	
19. Draw circles to <u>make ten</u> .	23. Circle the line that is <u>shortest</u> .

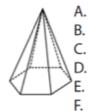
24. Circle the name of each shape.











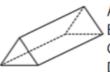
hexagonal pyramid rectangular prism rectangular pyramid triangular prism triangular pyramid

hexagonal prism

E.

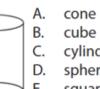


cube cylinder rectangular pyramid square triangular prism



- hexagonal prism Α. hexagonal pyramid B. rectangular prism C.
- D. rectangular pyramid
- triangular prism E.
- triangular pyramid F.

- hexagonal prism Α. Β. hexagonal pyramid rectangular prism C. rectangular pyramid D. triangular prism E.
- F. triangular pyramid
- Α. cone
- B. cube C. cylinder
- sphere D.
- E. square
- cube Α. B. C.
 - cylinder
 - rectangular pyramid
 - D. square
 - E. triangular prism





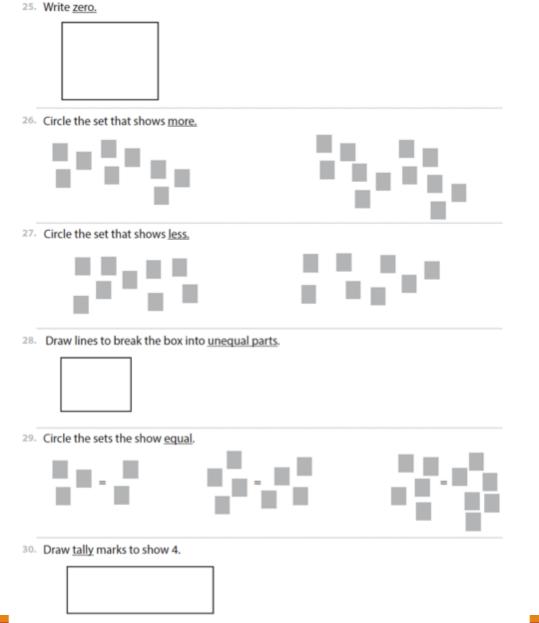


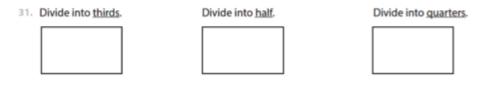
- cylinder



square







32. Separate 2 from the set.



33. Take away 4 from the set.



34. Color the bottom <u>row</u> with your pencil.

35. Color one <u>column</u> with your pencil.

Grades 3 and 5

77

6

12

- 1. Term appeared a textbook glossary at grade 3 and 5 • Go Math! enVisionMATH Everyday Math
- 22 2. Term appeared in a textbook glossary and a standard
 Common Core
 - 3. Term in a glossary but not explicit in a standard
- 16 4. Term explicitly named in standards but not a glossary
 - 5. Term related to previously-selected terms

Grades 3 and 5

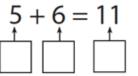
13	33
22	
6	
16	

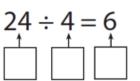
12

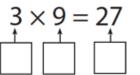
- In the Common Core:
 - 22 introduced in kindergarten
 - •9 in grade 1
 - 18 in grade 2
 - •13 in grade 3
 - 14 in grade 4
 - •4 in grade 5
 - 14 in grade 6 or above

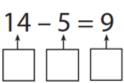
Answer the questions. Try the easy problems first, then go back and try the harder problems.

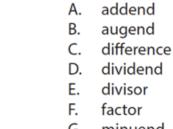
1. Identify each part. Write the letter in the box. You may repeat letters and/or not use all letters.





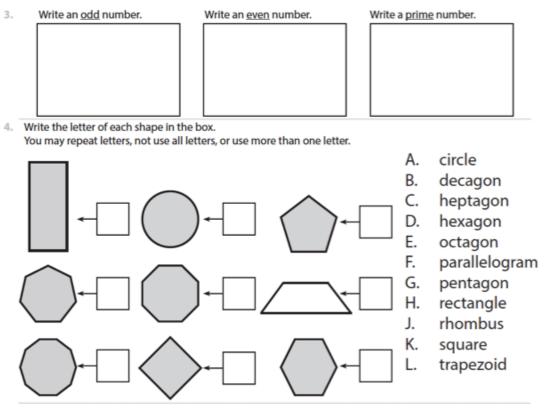






- dividend divisor
- factor
- G. minuend
- multiplicand Η.
- multiplier J.
- K. plus
- product L.
- quotient Μ.
- subtrahend N.
- О. sum

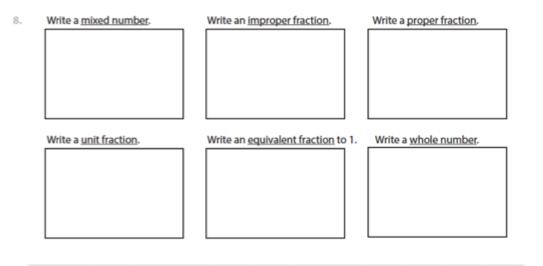
Draw an isoceles triangle. Draw an obtuse triangle. Draw an acute triangle. 2. Draw a scalene triangle. Draw an equilateral triangle. Draw a right triangle.



5. Write a number with a denominator of 9 and a numerator of 4.

6. Draw a number line.

7. Write 537 in expanded form.



Α.

B.

C.

D. E.

F.

G.

line

point

ray

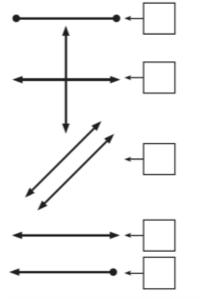
intersecting lines

perpendicular lines

line segment

parallel lines

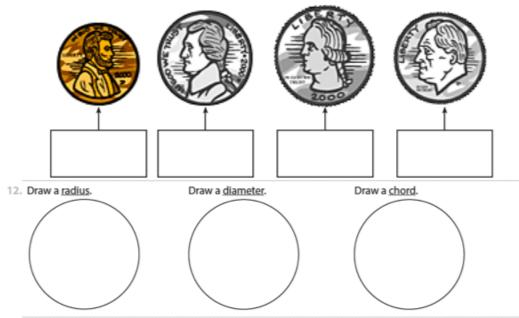
9. Identify each. Write the letter in the box. You may repeat letters and/or not use all letters.



10. Write the reciprocal of 6/5.



11. Write the name of each coin in the box.

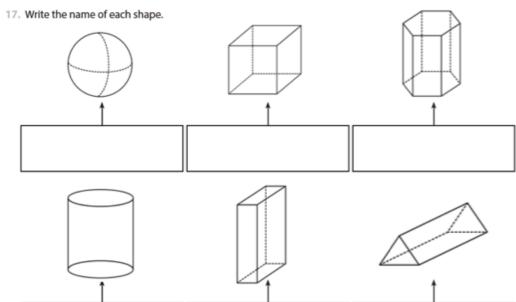


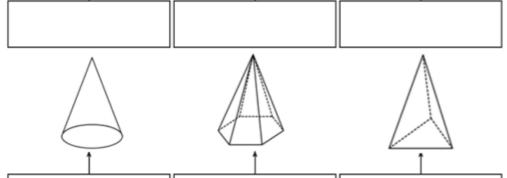
13. Write three-hundred, twenty-five in standard form.

14. Write an expression.

15. Write an equation.

16. Write an inequality.

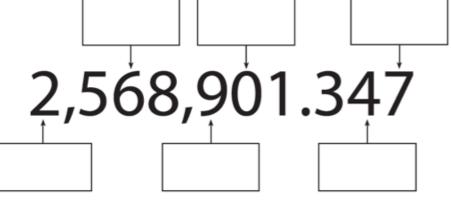




18. Draw a regular polygon.

Draw an irregular polygon.





What do the commas separate?

What is the name for the item between the 1 and 3?

What number is in the tenths place?

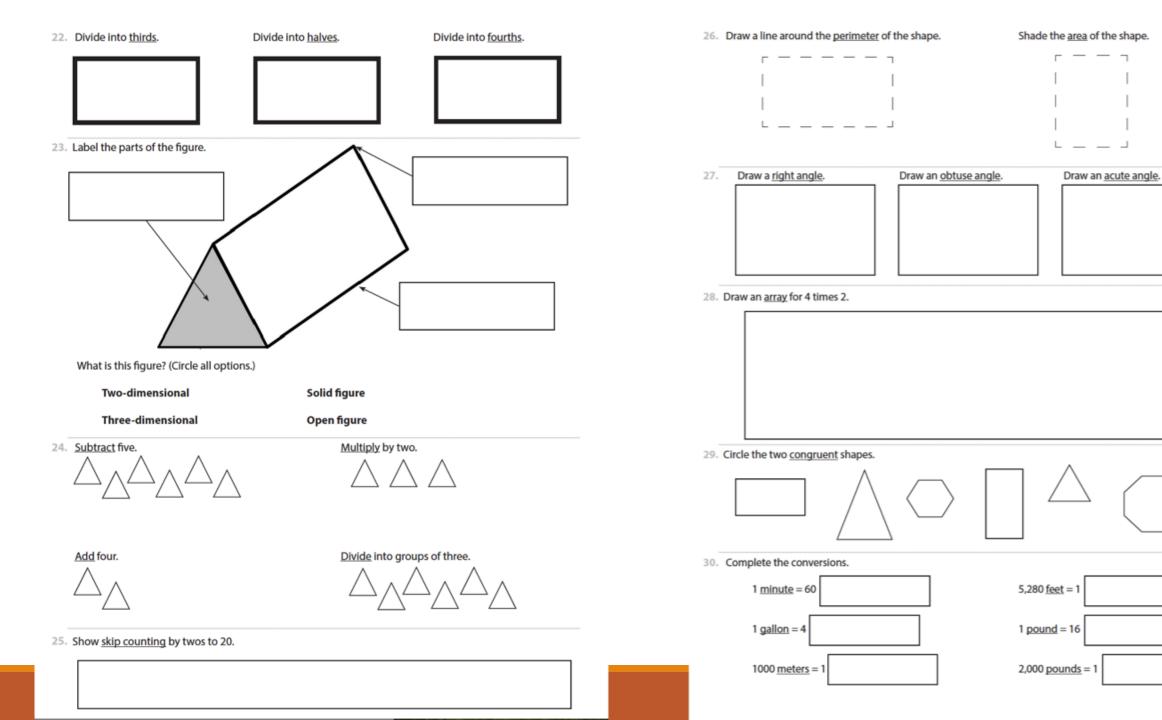
What number is the the ones place?

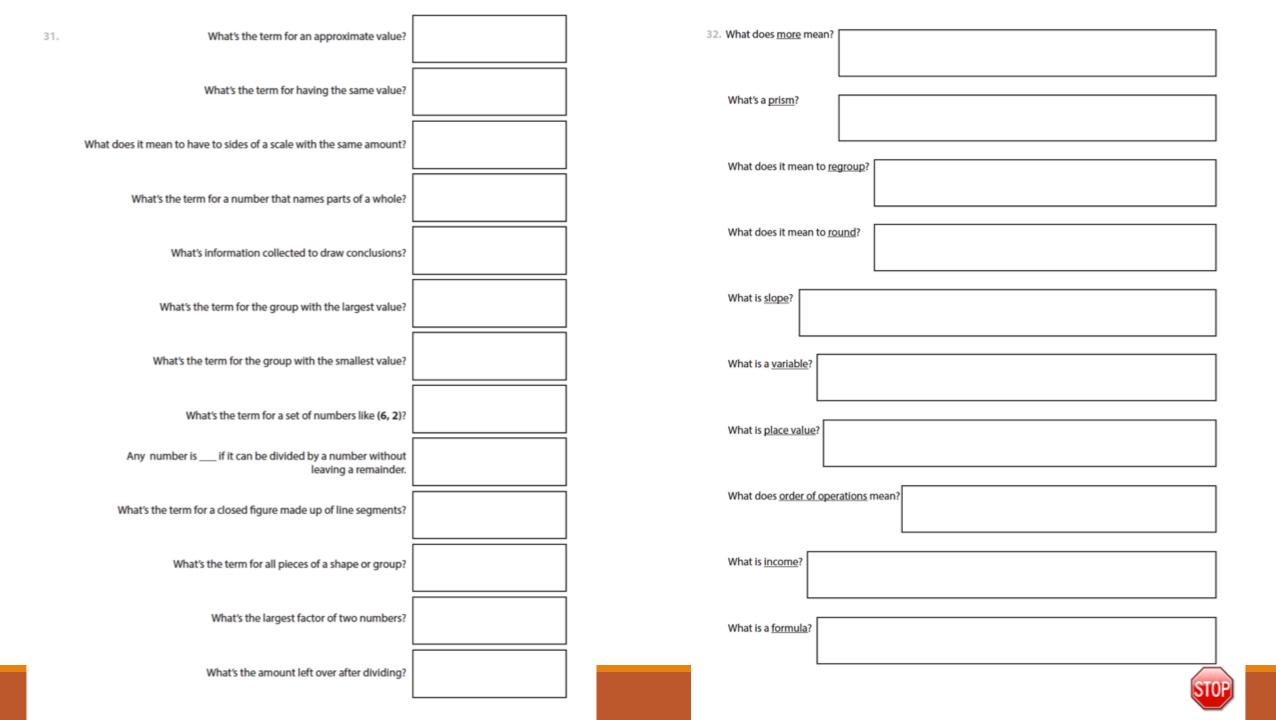
20. Write a positive integer.



Write a negative integer.

21. Label the clock hands.



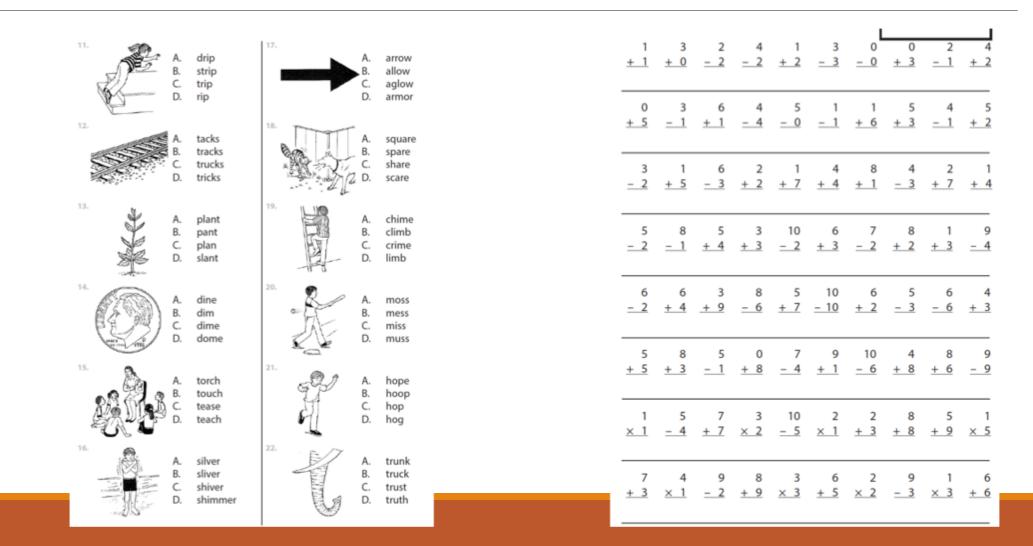


Pilot Studies

Grade 1

 104 first-grade students from 6 classrooms 		n	%
	Gender:		
	Male	55	52.9
	Female	49	47.1
	Race/ethnicity:		
	African American	12	11.5
	Asian	1	1.0
	White	37	35.6
	Hispanic	54	51.9
	English learners	1	1.0
	Retained	7	6.7
	Special education	3	2.9

Measures



•*α* = .85

- Item-by-item analysis for reliability
 - Deletion of only five terms would have increased α by .02
 - Opted to keep all terms for analysis

	Raw	Score	Correlations	
Variables	M	SD	WD	MF
GMRT WD	27.57	8.56	_	_
WJ-III MF	31.17	14.29	.526	_
Mathematics Vocabulary	36.30	8.10	.697	.586

- •Range: 15 55
- •No significant differences based on gender or retained status
- •Significant differences:
 - For English learner (*n* = 1)
 - Students with disabilities < students without disabilities
 - African American < Hispanic < Caucasian

Predictor	В	SE B	β	t	P	R^2	ΔR^2
Model 1:							
Intercept	18.088	1.940		9.325	<.001		
GMRT word decoding	.661	.067	.697	9.826	<.001	.486	
Model 2:							
Intercept	16.779	1.844		9.157	<.001		
GMRT word decoding	.509	.074	.538	6.874	<.001		
WJ-III math fluency	.172	.044	.304	3.881	<.001	.553	.067

- Accuracy by item introduction:
 - Kindergarten 67.1%
 - First grade 48.8%
 - Second grade 29.2%

- •Accuracy by category:
 - Technical 42.0%
 - Subtechnical 56.4%
 - General **91.1%**
 - Symbolic **54.5%**

Grades 3 and 5

Variable	Grade 3	(n = 65)	Grade 5 (a	n = 128)	Overall samp	Overall sample $(N = 193)$	
	n	(%) ^a	n	(%) ^a	n	(%)	
Gender							
Female	26	(40.0)	61	(47.7)	87	(45.1)	
Male	39	(60.0)	67	(52.3)	106	(54.9)	
Race/ethnicity							
African American	4	(6.2)	15	(11.7)	19	(9.8)	
Caucasian	20	(30.8)	47	(36.7)	67	(34.7)	
Hispanic	41	(63.1)	55	(43.0)	96	(49.7)	
Other	0	(0.0)	11	(8.6)	11	(5.7)	
School-identified disability	12	(18.5)	8	(6.3)	20	(10.4)	
English learner	17	(26.2)	24	(18.8)	41	(21.2)	
Retained	8	(12.3)	5	(3.9)	13	(6.7)	
	М	(SD)	М	(SD)	M	(SD)	
GMRT vocabulary ^b (standard score)	98.94	(13.98)	97.65	(11.25)	98.09	(12.22)	
WRAT Math Computation (standard score)	113.92	(12.97)	107.66	(14.23)	109.77	(14.10)	
Mathematics vocabulary (raw score)	35.57	(14.02)	57.51	(20.61)	50.20	(21.32)	

Measures

11. They should display it.

A. show

B. play with

- C. go around D. look at
- D. 100Ku
- 12. a sore knuckle
 - A. back of the neck
 - B. shoulder
 - C. tip of a toe
 - D. part of a finger

13. a big jug

- A. container
- B. cork
- C. jumble D. drink

14. She was active.

- A. lazy
- B. noisy
- C. doing things
- D. tired

15. a good excuse

- A. note
- B. reason
- C. movement D. example

16. It might rise.

- A. bloom B. go fast
- C. ride away

17. t	he	good	<u>waxi</u>	ng	
-------	----	------	-------------	----	--

A. waltzing B. polishing

C. rocking

D. answering

18. The others peered at it.

- A. looked closely B. smiled
- C. pecked
- D. made loud noises

19. They might discuss it.

- A. discover
- B. decide on
- C. talk about D. be upset about

20. It may vibrate.

- A. come loose B. break open
- C. inflate D. shake

21. the important mission

- A. homeworkB. thing that is lost
- C. task D. session

22. They can <u>weave</u>. A. make cloth

B. say good-bye C. paint

1. 1 + 1 =	2. <u>- 1</u>	^{3.} Write the missing number: 28, 29,, 31, 32	4. 8-4 =	s. 2 + 7 =
6. 9 <u>+ 3</u>	7. 8 – = 5	8. 32 24 <u>+ 40</u>	9. <u>- 15</u>	10. 3 × 4 =
11. 68 <u>+ 23</u>	12. 6 ÷ 2 =	13. <u>- 17</u>	14. 229 5,048 <u>+ 63</u>	15. 17 <u>× 4</u>
16. 724 <u>- 597</u>	17. Round 357 to the nearest ten. Answer	$\frac{15}{5} =$	$\frac{1}{3} + \frac{1}{3} = $	20. $2\frac{1}{2} + 1\frac{1}{2} = $

• α = .92 at Grade 3

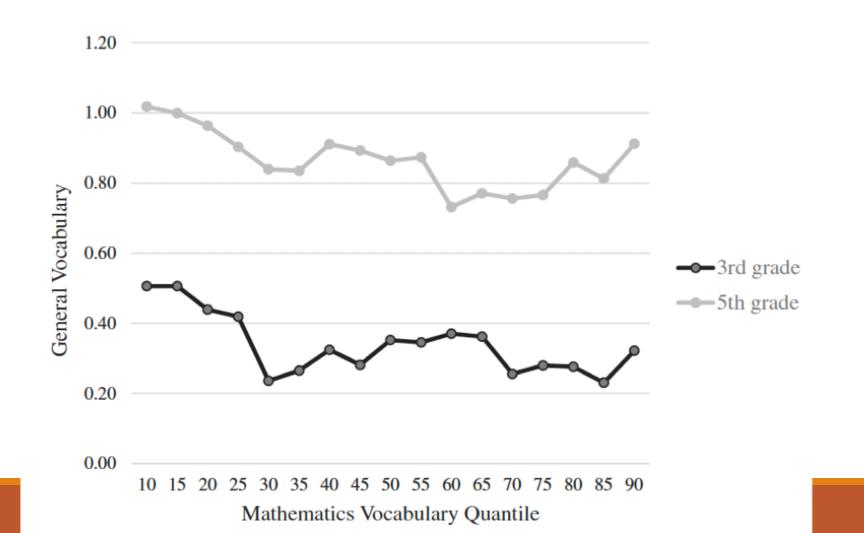
• α = .96 at Grade 5

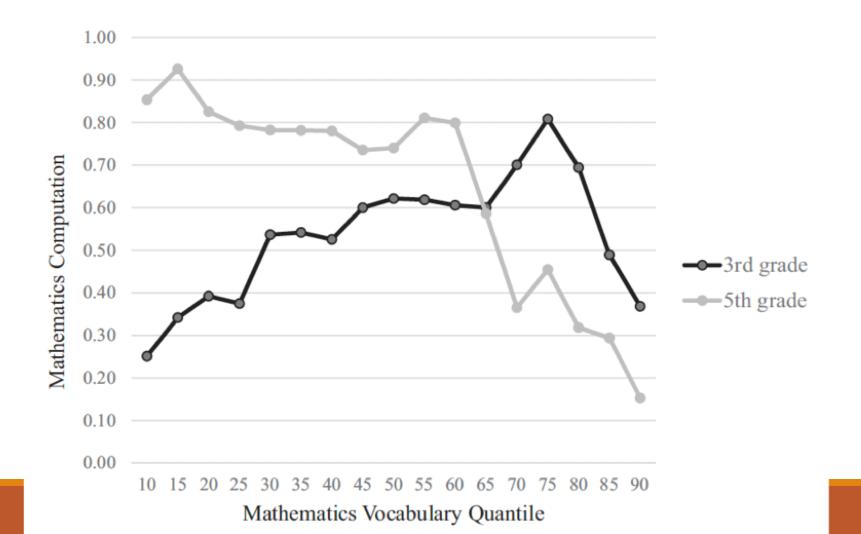
Measure	Grade 3		Grade 5			
	GMRT ^a	WRAT	MV	GMRT ^b	WRAT	MV
1. General vocabulary (GMRT)	_			_		
2. Math computation (WRAT)	0.573*	-		0.372*	-	
3. Mathematics vocabulary (MV)	0.606	0.669	-	0.659	0.626	-

•Grade 3

- Range = 6 68
- *M* = 35.57 (14.02)
- •Grade 5
 - Range = 5 100
 - *M* = 57.51 (20.61)

•No significant differences based on gender or retained status





Variable	Typical $(n = 70)$		MD-only $(n = 16)$		RD-only $(n = 18)$		MDRD (n = 10)		Excluded $(n = 14)$	
	n	(%)	n	(%)	n	(%)	n	(%)	n	(%)
Gender										
Female	35	(27.3)	6	(4.7)	8	(6.3)	7	(5.5)	5	(3.9)
Male	35	(27.3)	10	(7.8)	10	(7.8)	3	(2.3)	9	(7.0)
Race										
African American	6	(4.7)	4	(3.1)	1	(0.8)	3	(2.3)	1	(0.8)
Caucasian	30	(23.4)	4	(3.1)	5	(3.9)	2	(1.6)	6	(4.7)
Hispanic	28	(21.9)	5	(3.9)	11	(8.6)	5	(3.9)	6	(4.7)
Other	6	(4.7)	3	(2.3)	1	(0.8)	0	(0.0)	1	(0.8)
English learner	12	(9.4)	3	(2.3)	6	(4.7)	1	(0.8)	2	(1.6)
Retained	0	(0.0)	1	(0.8)	1	(0.8)	0	(0.0)	3	(2.3)
Special education	0	(0.0)	1	(0.8)	2	(1.6)	3	(2.3)	2	(1.6)
	М	(SD)	М	(SD)	М	(SD)	М	(SD)	М	(SD)
WRAT Math Computation	40.14	(3.22)	32.50	(1.79)	38.61	(2.06)	31.60	(2.46)	36.93	(2.27)
GMRT Vocabulary	29.46	(5.33)	23.19	(4.17)	13.50	(3.05)	15.10	(2.13)	21.64	(3.10)
Mathematics Vocabulary	68.13	(15.12)	46.00	(19.37)	46.56	(16.10)	21.90	(10.37)	57.07	(13.27)

Typical $>$ MD $=$ RD $>$ MDRD						
MD > MDRD	ES = 1.41					
RD > MDRD	ES = 1.67					
Typical > MDRD	ES = 3.12					
MD < Typical	ES = 1.37					
RD < Typical	ES = 1.40					

AN INVESTIGATION OF THE MATHEMATICS-VOCABULARY **KNOWLEDGE OF FIRST-GRADE** STUDENTS

measures exist in which the primary function is to gauge

We also administered standardized measures of general

word knowledge and mathematics fluency to investigate

the validity of the mathematics-vocabulary measure. Re-

sults indicated a wide variability in how first-grade stu-

dents interpret mathematics-vocabulary terms but strong

reliability for the mathematics-vocabulary measure.

Contents lists available at ScienceDirect Learning and Individual Differences journal homepage: www.elsevier.com/locate/lindif

Learning and Individual Differences 57 (2017) 22-32

An analysis of the mathematics vocabulary knowledge of third- and fifthgrade students: Connections to general vocabulary and mathematics computation*

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ARTICLE INFO ABSTRACT

Keywords: Mathematics Vocabulary Computation Language

To read mathematics textbooks, answer questions on mathematics assessments, and understand educator student communication, students must develop an understanding of the academic language of mathematics. primary aspect of academic language is vocabulary. In this study, we focused on the mathematics-vocabul performance of students in 3rd and 5th grade. We designed and implemented a measure of key mathemat vocabulary in the late elementary grades, and we compared performance on this measure to scores from gener vocabulary and mathematics computation measures. Student performance at both grades was variable, with 62-point range at 3rd grade and a 95-point range at 5th grade. General vocabulary and mathematics com tation were significant predictors of mathematics vocabulary, but the influence of these predictors differed

1. Introduction

On mathematics assessments, students are regularly prompted to read vocabulary, mathematics computation, and mathematics vocabulary.

1.1. Language and reading demands on mathematics assessments

purpose and research questions of this study.

vocabulary is a component of academic language. Finally, we describe the

* This research was supported by a National Academy of Education/Spencer Postdoctoral Fellowship. Statements do not reflect the position or policy of the funding agencies, university, schools, or persons, and no official endorsement should be inferred. * Corresponding author at: 1 University Station D5300, Austin, TX 78712, United States.

E-mail address: strowell/Raustin stesss edu (S.R. Powell).

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Performance on mathematics assessments has also been related to t

data from a longitudinal survey showed that scores from a set of math

edge) given in high school were stronger predictors of adulthood or

The difficulty with using a mathematics assessment as a determ

number of college acceptances for a student (Lee, 2012) and whether

student would graduate from college (You & Nguyen, 2012). Important

matics assessments (i.e., arithmetic reasoning and mathematics know

comes than scores from a set of reading assessments (i.e., word knowled

and paragraph comprehension; Dougherty, 2003). In a comprehensi

study, Ritchie and Bates (2013) learned that both mathematics and

reading assessments at age 7 predicted higher economic outcomes at a 42, with mathematics scores having a slightly stronger influence. Th

collection of research demonstrates how performance on mathema

nant for mathematics competence is that mathematics assessme

rarely assess the single construct of mathematics. For example, t

assessments is important for success during school and beyond.

National Assessment of Educational Progress (NAEP; U. S. Dep

Learning Disabilities Research & Practice, 32(4), 231-245 © 2017 The Division for Learning Disabilities of the Council for Exceptional Children DOI: 10.1111/Jdrp.12144

Differences in the Mathematics-Vocabulary Knowledge of Fifth-Grade Students With and Without Learning Difficulties

Suzanne R. Forsyth (D) and Sarah R. Powell The University of Texas at Austin

The purpose of this pilot study was to explore the impact of mathematics and reading learning difficulties on the mathematics-vocabulary understanding of fifth-grade students. Students (n = 114) completed three measures: mathematics computation, general vocabulary, and mathematics vocabulary. Based on performance on the mathematics computation and general vocabulary measures, students were categorized with no learning difficulty (i.e., typical), mathematics difficulty without reading difficulty (MD-only), reading difficulty without mathematics difficulty (RD-only), and combined mathematics and reading difficulties (MDRD). On the mathematics-vocabulary measure, students with MD-only or RD-only scored significantly lower than typical students, and students with MDRD demonstrated significantly lower performance than students with MD-only or RD-only.

INTRODUCTION

In the United States, federal legislation requires annual accountability testing to measure student progress and mandates that schools use evidence-based practices to mitigate performance gaps among student groups (Every Child Succeeds Act, 2015; Individuals with Disabilities Education Improvement Act, 2004). Many students, however, continue to fall behind their peers, failing to meet minimum standards on high-stakes assessments, even when receiving evidencebased instruction. One reason for persistent failure on mathematics assessments may be that students with learning difficulties, with or without a diagnosed disability, struggle with the mathematics language used for classroom instruction, in textbooks, and on assessments.

In this pilot study, we explored how students with mathematics difficulty without reading difficulty (MD-only), reading difficulty without mathematics difficulty (RD-only), and combined mathematics and reading difficulty (MDRD) performed on a measure of mathematics-vocabulary terms. This measure included mathematics vocabulary terms frequently encountered on high-stakes assessments throughout the elementary grades. We compared the performance of students with MD-only, RD-only, and MDRD to students without difficulty in mathematics or reading. In this introduction, we discuss the language of mathematics encountered in classroom learning and on high-stakes assessments. We then describe the difficulties that students with MD-only, RD-only, and MDRD encounter when answering questions on such assessments. Finally, we state the purpose and

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Most preschool language and vocabulary skills are learned through oral interactions, and by the time children enter elementary school, significant differences in vocabulary acquisition and language comprehension are apparent (Beck, McKeown, & Kucan, 2013; Hart & Risley, 1999). Knowledge of specific mathematics language (i.e., quantity and spatial words) influences early numeracy development (Purpura, Hume, Sims, & Lonigan, 2011; Toll & Van Luit, 2014). These early numeracy skills are more predictive of later academic outcomes in both mathematics and reading than attentionrelated, behavioral, social, and early reading skills (Duncan et al., 2007).

The Language of Mathematics

As students progress through elementary school, linguistic skills relate to the ability to master a variety of mathematics skills, including numeration, calculation, geometry, measurement, number line concepts, and magnitude comparison (Krajewski & Schneider, 2009; LeFevre et al., 2010). At the third-grade level, Vukovic and Lesaux (2013) determined that phonological skills and general language abilities were more strongly correlated to calculation and word-problem skills than working memory and visuospatial processing, respectively. By late elementary school, oral language skills also impact fraction competence (Chow & Jacobs, 2016; Seethaler, Fuchs, Star, & Bryant, 2011).

Mastering mathematical language has been compared in difficulty to learning a second language (Wakefield, 2000). Conceptual meaning in mathematics is constructed using several systems of communication: (a) symbolic notation, (b) visual representations, and (c) oral and written language (Schleppegrell, 2007). The symbolic notation itself requires knowledge about written numbers, symbols, and their placement (Monroe & Panchyshyn, 1995). For

S _ m measure mathematics competency. Results from such assessments ha

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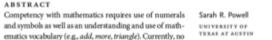
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research questions of this study.

mathematics-vocabulary performance levels. assisted educators in deciding promotion from one grade level to the ne (Maggio & Sayler, 2013) and whether students were prepared to enroll upper-level mathematics coursework in high school (Spielhagen, 200

words and sentences to solve mathematics problems. In order to provide appropriate mathematics instruction to all students, it may be necessary to consider the reading and language demands in mathematics, above and beyond mathematical concepts and procedures. In this study, we administered an assessment of mathematics vocabulary, along with assessments of general vocabulary and mathematics computation. We explored how students in the late elementary grades respond to mathematics-vocabulary items and aimed to understand the connections, if any, among general In this introduction, we describe the language and reading demands on mathematics assessments. Then, we discuss the construct of academic language as it relates to mathematics, and describe why mathematics

All school-age students take mathematics assessments designed to



Gena Nelson mathematics-vocabulary understanding. We created a AMERICAN INSTITUTES 64-item mathematics-vocabulary measure for first grade FOR RESEARCH and piloted the assessment with 104 first-grade students.

N mathematics, students solve problems by manipulating numerals and interpreting symbols, but an understanding of mathematics requires more than knowledge of numerals and symbols. Mathematics also requires a languagebased vocabulary component. Educators and students use language to teach and learn about mathematics, and this oral and written language is filled with vocabulary terms specific to mathematics (e.g., sum, octagon) and vocabulary terms from everyday language that have mathematical meaning (e.g., more, quarters). Furthermore, the majority of standardized mathematics tests require interpretation of written

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ABSTRACT

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