Simple Harmonic Motion

1) Horizontal spring-mass system

   i) single spring

   ii) springs in parallel

   iii) springs in series

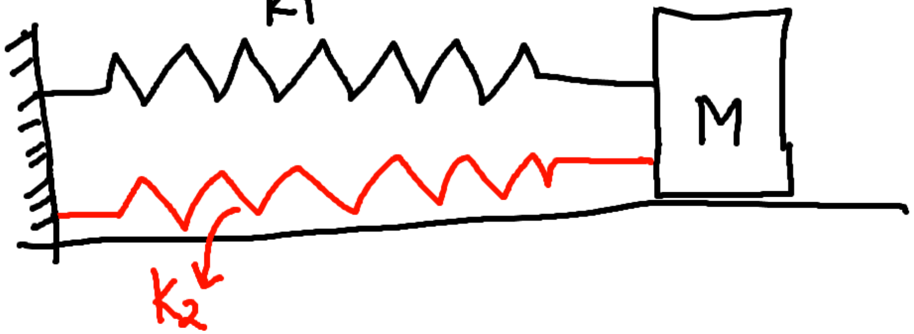
$\omega ; T$

2) Vertical spring-mass system

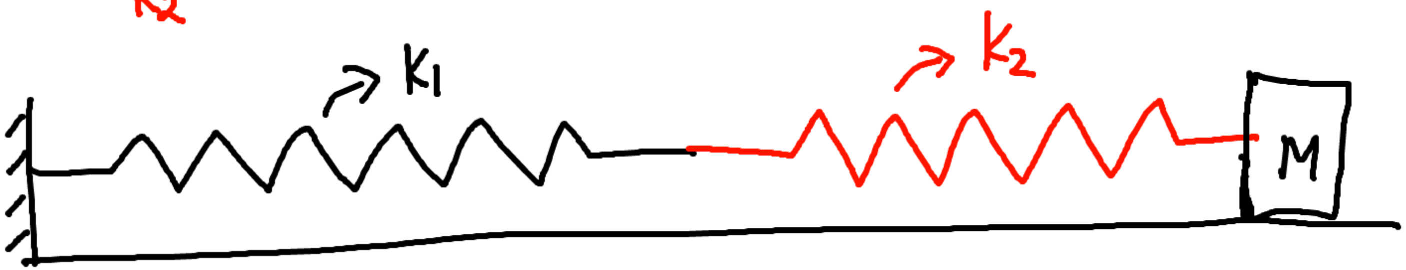
i)

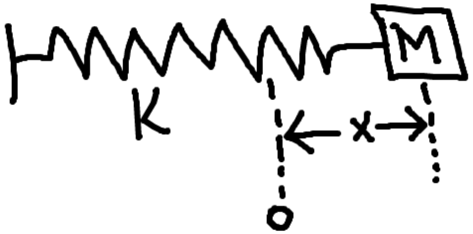
ω ; T

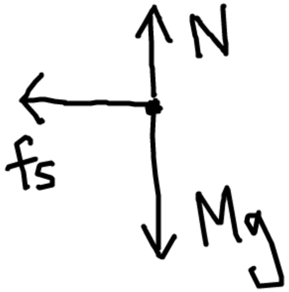
ii)



iii)







$$x = A\cos(\omega t + \phi)$$

$$ma = f_s = -kx$$

$$a = \frac{-kx}{M}$$

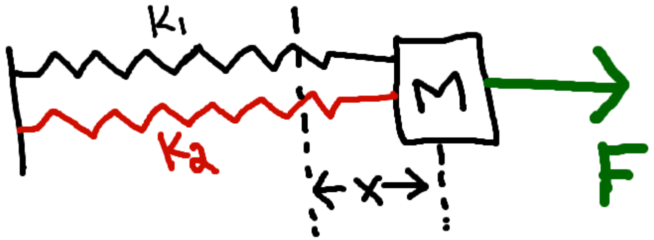
$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi)$$

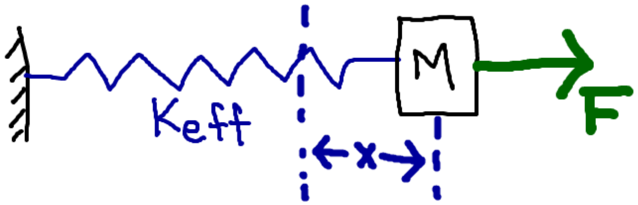
$$\boxed{\frac{d^2x}{dt^2} = \frac{-k}{M}x}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

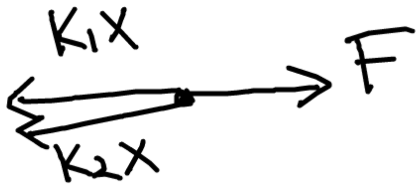
$$+\omega^2 x = +\frac{k}{M}x$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$





F → Applied Force



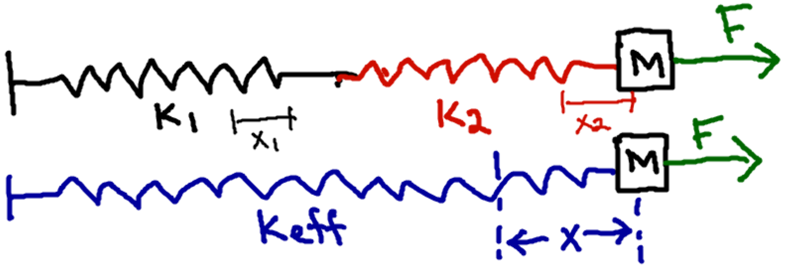
$$F = K_1 x + k_2 x$$

$$K_{eff}\, x = (K_1 + K_2)\, x$$

$$\boxed{K_{eff} = K_1 + K_2}$$

$$F = K_{eff}\, x$$

$$\omega = \sqrt{\frac{K_{eff}}{M}} = \sqrt{\frac{K_1 + k_2}{M}}$$



$$F = K_2 X_2 = K_1 X_1$$

$$F = K_{eff} X$$

$$X = X_1 + X_2$$

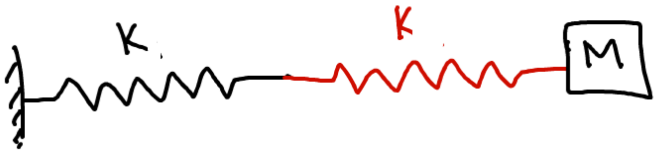
$$\frac{F}{K_{eff}} = \frac{F}{K_1} + \frac{F}{K_2}$$

$$\frac{1}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K_{eff} = \frac{K_1 K_2}{K_1 + K_2}$$

$$W = \sqrt{\frac{K_{eff}}{M}}$$

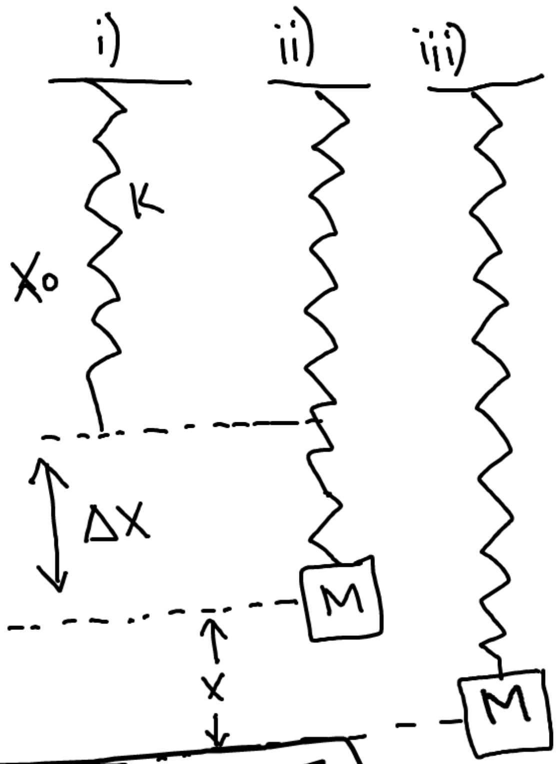
$$\boxed{W = \sqrt{\frac{K_1 K_2}{M(K_1 + K_2)}}}$$

	K_{eff}	ω (rad/s)	T (sec)
	K	$\sqrt{\frac{K}{M}}$	$2\pi\sqrt{\frac{M}{K}}$
	$2K$	$\sqrt{\frac{2K}{M}}$	$2\pi\sqrt{\frac{M}{2K}}$
	$\frac{K^2}{2K} = \frac{K}{2}$	$\sqrt{\frac{K}{2M}}$	$2\pi\sqrt{\frac{2M}{K}}$

All spring constants are K

$$\omega = \frac{2\pi}{T}$$

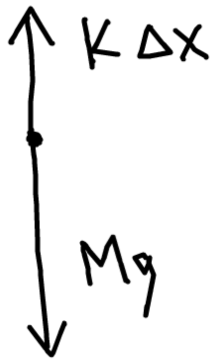
Vertical Spring



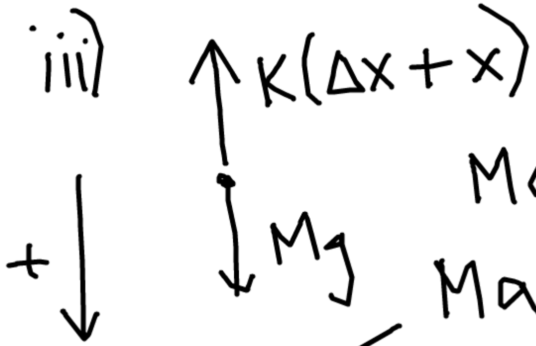
$$\boxed{K\Delta x = Mg}$$

$$\boxed{\Delta x = \frac{Mg}{K}}$$

ii)



iii)



$$Ma = Mg - K(\Delta x + x)$$

$$Ma = \underbrace{\cancel{Mg} - \cancel{K\Delta x}} - Kx$$

$$\boxed{Ma = -Kx}$$ → same as horizontal

$$\boxed{\omega = \sqrt{\frac{K}{M}}}$$

What happens when $M \to 0$?

$$\omega = \sqrt{\frac{K}{M}}$$

$M = 0$ $\quad$ $\omega = \infty$