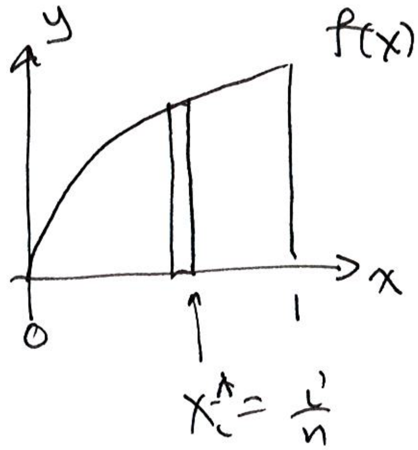
Math 1496 Calc 1

11/8/22

1–5

Last Class



$f(x) = \sqrt{x}$

(1) $\Delta x = \frac{1}{n}$

(2) $x_i^* = \frac{i}{n}$

(3) $h_i = \sqrt{\frac{i}{n}}$

(4) $A_i = \sqrt{\frac{i}{n}} \cdot \frac{1}{n}$

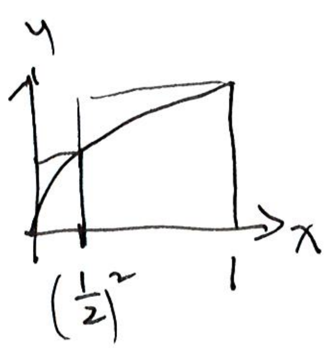
$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{i}{n}} \cdot \frac{1}{n} = \lim_{n\to\infty} \frac{1}{n\sqrt{n}} \sum_{i=1}^{n} \sqrt{i}$$

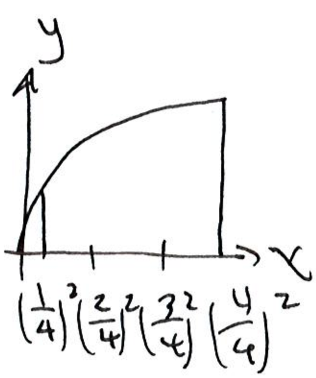
we have no formula for $\sum_{i=1}^{n} \sqrt{i}$

New Idea

change the location of x_i^*

ex 2 rectangle





4 rectangles

why? $h_i = \sqrt{x_i^*} = \sqrt{\left(\frac{3}{4}\right)^2} = \frac{3}{4}$ for 3rd rectangle in doing so, each rectangle has a different thickness

$$\Delta x_1 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

$$\Delta x_2 = \left(\frac{2}{4}\right)^2 - \left(\frac{1}{4}\right)^2 = \frac{4}{16} - \frac{1}{16} = \frac{3}{16}$$

$$\Delta x_3 = \left(\frac{3}{4}\right)^2 - \left(\frac{2}{4}\right)^2 = \frac{9-4}{16} = \frac{5}{16}$$

$$\Delta x_4 = \left(\frac{4}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = \frac{16-9}{16} = \frac{7}{16}$$

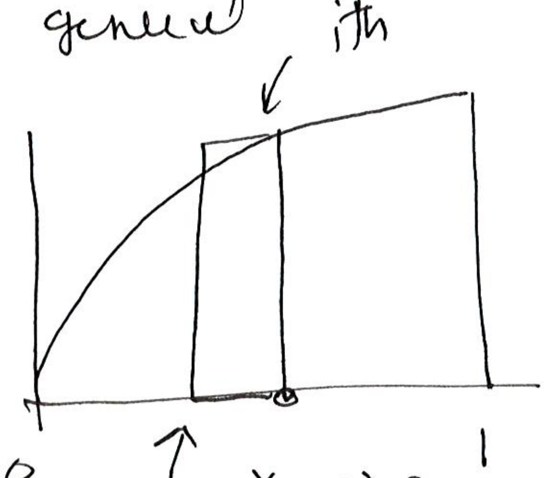
So for example

$A = h_i \Delta x_i$

$$A_1 + A_2 + A_3 + A_4$$

$$\left(\frac{1}{4}\right)\left(\frac{1}{16}\right) + \frac{2}{4}\left(\frac{3}{16}\right) + \frac{3}{4}\left(\frac{5}{16}\right) + \frac{4}{4}\left(\frac{7}{16}\right)$$

In general



$x_{i-1} = \left(\frac{i-1}{n}\right)^2$ $\quad x_i = \left(\frac{i}{n}\right)^2$

$$\Delta x_i = \left(\frac{i}{n}\right)^2 - \left(\frac{i-1}{n}\right)^2$$

$$= \frac{i^2 - (i^2 - 2i + 1)}{n^2}$$

$$= \frac{2i - 1}{n^2}$$

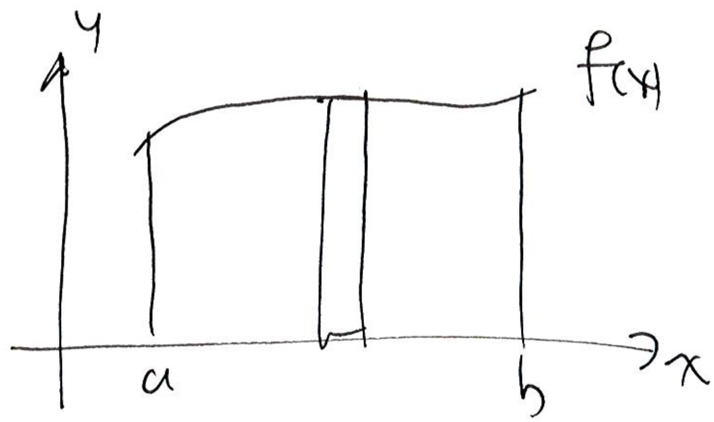
$$h_i = \sqrt{\left(\frac{i}{n}\right)^2} = \frac{i}{n}$$

so $A_i = h_i \Delta x_i$

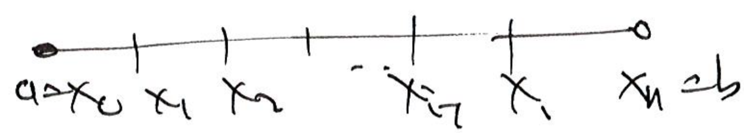
$$= \frac{i}{n}\left(\frac{2i-1}{n^2}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i(2i-1)}{n^3}$$ ← use formula now!

In general



(1) subdivide interval



$a = x_0 < x_1 < x_2 \cdots x_i < \cdots x_n = b$

(2) thickness i^{th} rect.

$$\Delta x_i = x_i - x_{i-1}$$

(3) pick x_i^* in $[x_{i-1}, x_i]$

$$h_i = f(x_i^*)$$

(4) Area of i^{th} rectangle

$$A_i = f(x_i^*)\Delta x_i$$

(5) $$\lim_{\substack{n\to\infty \\ \Delta x_i \to 0}} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$ ← Called a Riemann Sum.

Definite Integral

$$\int_a^b f(x)\,dx = \lim_{\substack{n\to\infty \\ \Delta x_i \to 0}} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

Fundamental Thm Calc

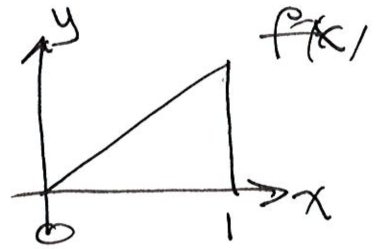
If $f(x)$ is continuous on $[a,b]$

then $$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is the anti derivative of $f(x)$

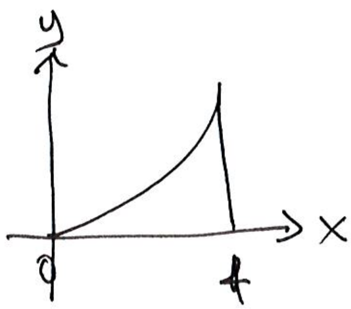
ie. $F'(x) = f(x)$

Ex 1



$$A = \int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$$

Ex 2



$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

Ex 3

$$\int_0^1 x^{1/2}\,dx = \frac{x^{3/2}}{3/2}\Big|_0^1 = \frac{1}{3/2} - 0 = 2/3$$