Allocating Indivisible Resources under Price Rigidities in Polynomial Time

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M-PREF'13

Outline

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Dynamic Mechanisms

- Dynamic mechanisms for resource allocation:
  - price adjustment processes;
  - market-clearing prices;
  - Walrasian equilibrium.
- Main issue of a dynamic mechanism:
  - Can a procedure lead to an equilibrium state?

Price Rigidities

- “Good” allocation= Economy efficiency + Social equality + computation tractable.
- Price rigidities may play a key role, for example:
  - Capped-price housing;
  - Limit price of stock;
  - Minimum wage.

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Main Contributions

- Computational problems of dynamic auction proposed by [Talman and Yang, 2008]
- Proof about the existence of constrained Walrasian equilibrium
- Strategical issues based on "expected profits" and "expected prices"

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Assumptions

- Valuation functions and prices are all integers.
- Buyers can afford the maximum price of any item.
- Every item has a lower bound price $\underline{\mathbf{p}}_a$ and a upper bound price $\overline{\mathbf{p}}_a$.
- All buyers report their favorite items(making them get most profits) at every stage.
- Every buyer could only get one item.

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Constrained Walrasian Equilibrium

Criteria of *constrained Walrasian equilibrium* $\langle \mathbf{p}, R, \pi \rangle$:

- $\mathbf{p} \in P$, R is a rationing system
- π is an equilibrium allocation
- If an item is not allocated to any buyer, price of the item should be lower bounder price.
- If $R(i, a) = 0$, then price of item a should be the upper bound price, and there is some buyer has it.
- If $R(i, a) = 0$, we change value of $R(i, a)$ to 1, then buyer i will demand item a.

Example

- $R(1, c) = R(3, c) = 0$
- Lower bound price $\underline{\mathbf{p}} = (0, 5, 4, 1, 5)$, Upper bound price $\overline{\mathbf{p}} = (0, 6, 6, 4, 7)$,
- Price vector $\mathbf{p} = (0, 5, 4, 4, 7)$,
- Allocation π: $\pi(1) = o$, $\pi(2) = c$, $\pi(3) = b$, $\pi(4) = a$, and $\pi(5) = d$.

$\langle \mathbf{p}, R, \pi \rangle$ is a constrained Walrasian equilibrium.

Table: Values, Profits, and Constrained Demand

buyer i	$u_i(o)$	$u_i(a)$	$u_i(b)$	$u_i(c)$	$u_i(d)$	$V_i(\mathbf{p}, R)$	$D_i(\mathbf{p}, R)$
1	0	4	3	5	7	0	{o,d}
2	0	7	6	8	3	4	{c}
3	0	5	5	8	7	1	{b}
4	0	9	4	3	2	4	{a}
5	0	6	2	4	10	3	{d}

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Over-demanded Set

- A set X' is *over-demanded*, if $|\{i \in N | D_i \subseteq X'\}| > |X'|$.
- X' is *minimal* if no strict subset of X' is over-demanded.

Theorem 1

There exists an over-demanded set of items in $\mathcal{D} = (D_i)_{i \in N}$ if and only if there does not exist an equilibrium allocation.

- We proposed an algorithm called MODS to find a minimal over-demanded set.

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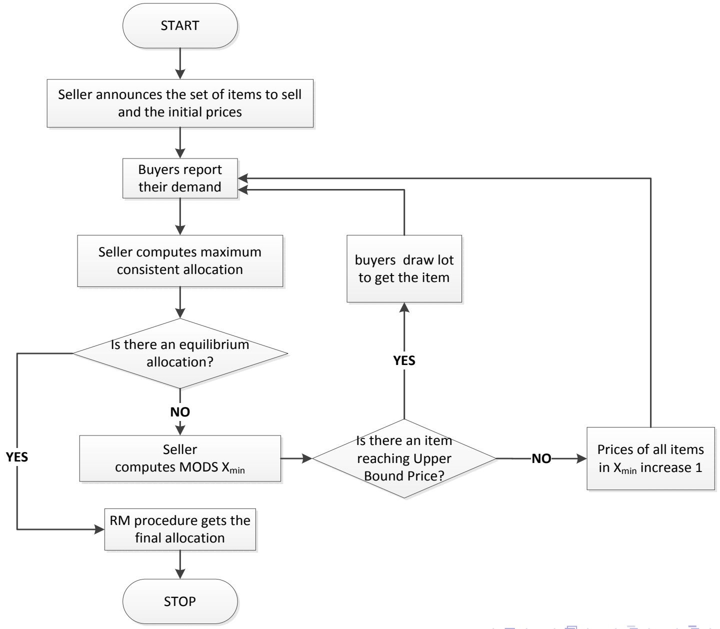
MAPR


START
Seller announces the set of items to sell and the initial prices
Buyers report their demand
Seller computes maximum consistent allocation
Is there an equilibrium allocation?
NO
YES
Seller computes MODS X_{min}
Is there an item reaching Upper Bound Price?
YES
NO
buyers draw lot to get the item
Prices of all items in X_{min} increase 1
RM procedure gets the final allocation
STOP

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Properties of MAPR

- MAPR always terminates and is polynomial in $|N|$, $|X|$, and $\sum_{a \in X} (\overline{\mathbf{p}}_a - \underline{\mathbf{p}}_a)$.
- MAPR leads to a constrained Walrasian equilibrium.

Strategy Issues

- Since the history of MAPR is nondeterministic, we defined buyers' *expected profits* and items' *expected prices*.
- MAPR is generally not *strategyproof* (in the sense of expected profits).
- Two interesting questions:
  1. Is MAPR strategyproof for some restricted domains?
  2. When it is not, how hard is it for an buyer who knows the valuations of the others to compute an optimal strategy?

Strategy Issues(continued)

- When there are two buyers, MAPR is strategyproof.
- We conjecture that the manipulation problem of MAPR is NP-hard.

Conclusion

- There exists an over-demanded set if and only if there is no equilibrium allocation.
- We proposed an algorithm to compute the Over-demanded set.
- We proposed an algorithm MAPR which can find a constrained Walrasian Equilibrium in polynomial time.
- We proved that when there are two agents, MAPR is strategyproof.

Future Work

- Complexity of manipulation by one buyer.
- Problem of manipulation by two or more buyers.
- Collusion problem: manipulating prices between some buyers and the seller.
- Problem of allocating divisible goods and sharable goods under price rigidities.