

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise A, Question 1

#### Question:

Write down whether or not each of the following is a discrete random variable.

Give a reason for your answer.

- a** The average lifetime of a battery.
- b** The number of days in a week.
- c** The number of moves it takes to complete a game of chess.

#### Solution:

- i) This **is not** a discrete random variable. Time is a continuous variable.
- ii) This **is not** a discrete random variable. It is always 7, so does not vary
- iii) This **is** a discrete random variable. It is always a whole number but it does vary.

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### Discrete random variables

#### Exercise A, Question 2

#### Question:

A fair die is thrown four times and the number of times it falls with a 6 on the top,  $Y$ , is noted. Write down all the possible values of  $y$ .

#### Solution:

$y: 0 \ 1 \ 2 \ 3 \ 4$

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### Discrete random variables

#### Exercise A, Question 3

#### Question:

A bag contains two discs with the number 2 on them and two discs with the number 3 on them. A disc is drawn at random from the bag and the number noted. The disc is returned to the bag. A second disc is then drawn from the bag and the number noted.

a Write down the sample space.

The discrete random variable  $X$  is the sum of the two numbers.

b Write down the probability distribution for  $X$ .

c Write down the probability function for  $X$ .

#### Solution:

a  $S = (2,2), (2,3), (3,2), (3,3)$

b

$x$	4	5	6
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$c \ P(X = x) = \begin{cases} \frac{1}{4} & x = 4,6, \\ \frac{1}{2} & x = 5. \end{cases}$$

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#### Exercise A, Question 4

#### Question:

A discrete random variable  $X$  has the following probability distribution:

$x$	1	2	3	4
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{3}$	$k$	$\frac{1}{4}$

Find the value of  $k$ .

#### Solution:

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{4} + k = 1$$

$$k = 1 - \left( \frac{1}{3} + \frac{1}{3} + \frac{1}{4} \right)$$

$$= 1 - \frac{11}{12} = \frac{1}{12}$$

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#### Exercise A, Question 5

#### Question:

The random variable  $X$  has a probability function

$$P(X = x) = kx \quad x = 1, 2, 3, 4.$$

Show that  $k = \frac{1}{10}$ .

#### Solution:

$x$	1	2	3	4
$P(X = x)$	$k$	$2k$	$3k$	$4k$

$$k + 2k + 3k + 4k = 1$$

$$10k = 1$$

$$k = \frac{1}{10}$$

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## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise A, Question 6

#### Question:

The random variable  $X$  has the probability function

$$P(X = x) = \frac{x-1}{10} \quad x = 1, 2, 3, 4, 5.$$

Construct a table giving the probability distribution of  $X$ .

#### Solution:

$x$	1	2	3	4	5
$P(X = x)$	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

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### Discrete random variables

#### Exercise A, Question 7

#### Question:

The random variable  $X$  has a probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3 \\ k(x-1) & x = 2, 4 \end{cases}$$

where  $k$  is a constant.

**a** Find the value of  $k$ .

**b** Construct a table giving the probability distribution of  $X$ .

#### Solution:

**a**

$x$	1	2	3	4
$P(X = x)$	$k$	$k$	$3k$	$3k$

Using the fact that the probabilities add up to 1:

$$k + k + 3k + 3k = 1$$

$$8k = 1$$

$$k = \frac{1}{8}$$

**b**

$x$	1	2	3	4
$P(X = x)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$

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#### Exercise A, Question 8

#### Question:

The discrete random variable  $X$  has a probability function

$$P(X = x) = \begin{cases} 0.1 & x = -2, -1 \\ \beta & x = 0, 1 \\ 0.2 & x = 2 \end{cases}$$

- a** Find the value of  $\beta$ .
- b** Construct a table giving the probability distribution of  $X$ .

#### Solution:

<b><math>x</math></b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>
<b><math>P(X = x)</math></b>	<b>0.1</b>	<b>0.1</b>	<b><math>\beta</math></b>	<b><math>\beta</math></b>	<b>0.2</b>

The probabilities add up to 1.

$$0.1 + 0.1 + \beta + \beta + 0.2 = 1$$

$$2\beta = 1 - 0.4$$

$$\beta = 0.3$$

**b**

<b><math>x</math></b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>
<b><math>P(X = x)</math></b>	<b>0.1</b>	<b>0.1</b>	<b>0.3</b>	<b>0.3</b>	<b>0.2</b>



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#### Exercise A, Question 9

#### Question:

A discrete random variable has the probability distribution shown in the table below.

$x$	0	1	2
$P(X = x)$	$\frac{1}{4} - a$	$a$	$\frac{1}{2} + a$

Find the value of  $a$ .

#### Solution:

$$\frac{1}{4} - a + a + \frac{1}{2} + a = 1$$

$$a = 1 - \frac{3}{4} = \frac{1}{4}$$

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### Discrete random variables

#### Exercise B, Question 1

#### Question:

A discrete random variable  $X$  has probability distribution

$x$	0	1	2	3	4	5
$P(X = x)$	0.1	0.1	0.3	0.3	0.1	0.1

**a** Find the probability that  $X < 3$ .

**b** Find the probability that  $X > 3$ .

**c** Find the probability that  $1 < X < 4$ .

#### Solution:

**a**  $P(X < 3) = P(0) + P(1) + P(2) = 0.1 + 0.1 + 0.3 = \mathbf{0.5}$

**b**  $P(X > 3) = P(4) + P(5) = 0.1 + 0.1 = \mathbf{0.2}$

**c**  $P(1 < X < 4) = P(2) + P(3) = 0.3 + 0.3 = \mathbf{0.6}$

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#### Exercise B, Question 2

#### Question:

A discrete random variable  $X$  has probability distribution

$x$	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$

Find

**a**  $P(1 < X \leq 3)$ ,                      **b**  $P(X < 2)$ .

#### Solution:

**a**  $P(1 < X \leq 3) = P(2) + P(3) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

**b**  $P(X < 2) = P(0) + P(1) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

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#### Exercise B, Question 3

#### Question:

A discrete random variable  $X$  has a probability distribution

$x$	1	2	3	4	5	6
$P(X = x)$	0.1	0.1	0.15	0.25	0.3	0.1

**a** Draw up a table to show the cumulative distribution function  $F(x)$ .

**b** Write down  $F(5)$ .      **c** Write down  $F(2.2)$ .

#### Solution:

**a**

$x$	1	2	3	4	5	6
$F(x_0)$	0.1	0.2	0.35	0.60	0.9	1

**b**  $F(5) = 0.9$

**c**  $F(2.2) = 0.2$

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#### Exercise B, Question 4

#### Question:

A discrete random variable has a cumulative distribution function  $F(x)$  given in the table.

$x$	0	1	2	3	4	5	6
$F(x)$	0	0.1	0.2	0.45	0.5	0.9	1

**a** Draw up a table to show the probability distribution  $X$ .

**b** Write down  $P(X < 5)$ .    **c** Find  $P(2 \leq X < 5)$ .

#### Solution:

**a**

$x$	1	2	3	4	5	6
$P(X = x)$	0.1	0.1	0.25	0.05	0.4	0.1

**b**  $P(X < 5) = 0.5$

**c**  $P(2 \leq X < 5) = 0.1 + 0.25 + 0.05 = 0.4$

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#### Exercise B, Question 5

#### Question:

5 The random variable  $X$  has a probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5 \\ k(x-1) & x = 2, 4, 6 \end{cases}$$

where  $k$  is a constant.

**a** Find the value of  $k$ .

**b** Draw a table giving the probability distribution of  $X$ .

**c** Find  $P(2 \leq X < 5)$ .

**d** Find  $F(4)$ .

**e** Find  $F(1.6)$ .

#### Solution:

**a**

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>P(X = x)</b>	$k$	$k$	$3k$	$3k$	$5k$	$5k$

$$k + k + 3k + 3k + 5k + 5k = 1$$

$$k = \frac{1}{18}$$

**b**

<b>x</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>P(X = x)</b>	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{5}{18}$

$$\mathbf{c} \quad P(2 \leq X < 5) = P(2) + P(3) + P(4) = \frac{1}{18} + \frac{1}{6} + \frac{1}{6} = \frac{7}{18}$$

**d** Remember  $F$  means the cumulative function

$$F(4) = 1 - (P(6) + P(5)) = 1 - \left(\frac{5}{18} + \frac{5}{18}\right) = \frac{8}{18} \text{ or } \frac{4}{9}$$

(This could also be done by adding  $P(1)$   $P(2)$   $P(3)$  and  $P(4)$ .)

**e** 1.6 lies below 2 but above 1. Because this is a **discrete** random variable  $F(1.6)$  is the same as  $F(1)$  which is  $\frac{1}{18}$



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#### Exercise B, Question 6

#### Question:

The discrete random variable  $X$  has the probability function

$$P(X = x) = \begin{cases} 0.1 & x = -2, -1 \\ \alpha & x = 0, 1 \\ 0.3 & x = 2 \end{cases}$$

- a** Find the value of  $\alpha$
- b** Draw a table giving the probability distribution of  $X$ .
- c** Write down the value of  $F(0.3)$ .

#### Solution:

**a**

$x$	-2	-1	0	1	2
$P(X = x)$	0.1	0.1	$\alpha$	$\alpha$	0.3

$$\begin{aligned} 0.1 + 0.1 + \alpha + \alpha + 0.3 &= 1 \\ 2\alpha &= 0.6 \\ \alpha &= 0.3 \end{aligned}$$

**b**

$x$	-2	-1	0	1	2
$P(X = x)$	0.1	0.1	0.25	0.25	0.3

**c**  $F(0.3) = F(0) = 0.1 + 0.1 + 0.25 = 0.45$



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#### Exercise B, Question 7

#### Question:

The discrete random variable  $X$  has a cumulative distribution function  $F(x)$  defined by

$$F(x) = \begin{cases} 0 & x = 0 \\ \frac{1+x}{6} & x = 1, 2, 3, 4, 5 \\ 1 & x > 5 \end{cases}$$

**a** Find  $P(X \leq 4)$ .

**b** Show that  $P(X = 4)$  is  $\frac{1}{6}$ .

**c** Find the probability distribution for  $X$ .

#### Solution:

**a**

$x$	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
$F(x)$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	<b>1</b>

$$P(X \leq 4) = \frac{5}{6}$$

$$\mathbf{b} \quad P(X = 4) = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$$

**c**

$x$	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
$P(X = x)$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

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#### Exercise B, Question 8

#### Question:

The discrete random variable  $X$  has a cumulative distribution function  $F(x)$  defined by  $F(x) = \begin{cases} 0 & x = 0 \\ \frac{(x+k)^2}{16} & x = 1, 2 \text{ and } 3 \\ 1 & x > 3 \end{cases}$

**a** Find the value of  $k$ .

**b** Find the probability distribution for  $X$ .

#### Solution:

$$\mathbf{a} \quad \frac{(x+k)^2}{16} = 1 \text{ when } x = 3$$

$$\frac{(3+k)^2}{16} = 1$$

$$(3+k)^2 = 16$$

$$3+k = \pm 4$$

$k = 1$  (negative probabilities do not exist)

**b**

$x$	1	2	3
$F(x)$	$\frac{4}{16}$	$\frac{9}{16}$	1

So Probability distribution is

$x$	1	2	3
$P(X = x)$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{7}{16}$

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### Discrete random variables

#### Exercise C, Question 1

#### Question:

Find  $E(X)$  and  $E(X^2)$  for the following distributions of  $x$ .

**a**

$x$	2	4	6	8
$P(X = x)$	0.3	0.3	0.2	0.2

**b**

$x$	1	2	3	4
$P(X = x)$	0.1	0.4	0.1	0.4

#### Solution:

$$\begin{aligned} \mathbf{a} \quad E(X) &= (2 \times 0.3) + (4 \times 0.3) + (6 \times 0.2) + (8 \times 0.2) \\ &= 0.6 + 1.2 + 1.2 + 1.6 = \mathbf{4.6} \end{aligned}$$

$$\begin{aligned} E(X^2) &= (4 \times 0.3) + (16 \times 0.3) + (36 \times 0.2) + (64 \times 0.2) \\ &= 1.2 + 4.8 + 7.2 + 12.8 = \mathbf{26} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad E(X) &= (1 \times 0.1) + (2 \times 0.4) + (3 \times 0.1) + (4 \times 0.4) \\ &= 0.1 + 0.8 + 0.3 + 1.6 = \mathbf{2.8} \end{aligned}$$

$$\begin{aligned} E(X^2) &= (1 \times 0.1) + (4 \times 0.4) + (9 \times 0.1) + (16 \times 0.4) \\ &= 0.1 + 1.6 + 0.9 + 6.4 = \mathbf{9} \end{aligned}$$

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#### Exercise C, Question 2

#### Question:

A biased die has the probability distribution

$x$	1	2	3	4	5	6
$P(X = x)$	0.1	0.1	0.1	0.2	0.4	0.1

Find  $E(X)$  and  $E(X^2)$ .

#### Solution:

$$E(X) = (1 \times 0.1) + (2 \times 0.1) + (3 \times 0.1) + (4 \times 0.2) + (5 \times 0.4) + (6 \times 0.1)$$

$$= 0.1 + 0.2 + 0.3 + 0.8 + 2.0 + 0.6 = \mathbf{4}$$

$$E(X^2) = (1 \times 0.1) + (4 \times 0.1) + (9 \times 0.1) + (16 \times 0.2) + (25 \times 0.4) + (36 \times 0.1)$$

$$= 0.1 + 0.4 + 0.9 + 3.2 + 10 + 3.6 = \mathbf{18.2}$$

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#### Exercise C, Question 3

#### Question:

The random variable  $X$  has a probability function

$$P(X = x) = \begin{cases} \frac{1}{x} & x = 2, 3, 6 \\ 0 & \text{all other values} \end{cases}$$

**a** Construct a table giving the probability distribution of  $X$ .

**b** Work out  $E(X)$  and  $E(X^2)$ .

**c** State with a reason whether or not  $(E(X))^2 = E(X^2)$ .

#### Solution:

**a**

$x$	2	3	6
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

**b**

$$\begin{aligned} E(X) &= \left(2 \times \frac{1}{2}\right) + \left(3 \times \frac{1}{3}\right) + \left(6 \times \frac{1}{6}\right) \\ &= 1 + 1 + 1 = \mathbf{3} \end{aligned}$$

$$\begin{aligned} E(X^2) &= \left(4 \times \frac{1}{2}\right) + \left(9 \times \frac{1}{3}\right) + \left(36 \times \frac{1}{6}\right) \\ &= 2 + 3 + 6 = \mathbf{11} \end{aligned}$$

**c**

$$(E(X))^2 = 3 \times 3 = 9$$

$$E(X^2) = 11$$

Therefore  $(E(X))^2$  does not equal  $E(X^2)$

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#### Exercise C, Question 4

#### Question:

Two coins are tossed 50 times. The number of heads is noted each time.

**a** Construct a probability distribution table for the number of heads when the two coins are tossed once, assuming that the two coins are unbiased.

**b** Work out how many times you would expect to get 0, 1 or 2 heads.

The following table shows the actual results.

<b>Number of heads (<math>h</math>)</b>	0	1	2
<b>Frequency (<math>f</math>)</b>	7	22	21

**c** State whether or not the actual frequencies obtained support the statement that the coins are unbiased. Give a reason for your answer.

#### Solution:

**a**

Number of heads ( $h$ )	0	1	2
$P(H = h)$	0.25	0.5	0.25

**b**

$$0.25 \times 50 = 12.5$$

$$0.5 \times 50 = 25$$

We would expect to get **1 head 25 times** and **0 or 2 heads 12.5 times each**.

**c** The coins would appear to be biased. There were far too many times when 2 heads appeared and not enough when 0 heads appeared.

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#### Exercise C, Question 5

#### Question:

The random variable  $X$  has the following probability distribution.

$x$	1	2	3	4	5
$P(X = x)$	0.1	$a$	$b$	0.2	0.1

Given  $E(X) = 2.9$  find the value of  $a$  and the value of  $b$ .

#### Solution:

$x$	1	2	3	4	5
$P(X = x)$	0.1	$a$	$b$	0.2	0.1

The probabilities add up to 1 so

$$0.1 + a + b + 0.2 + 0.1 = 1$$

$$a + b = 0.6 \quad (1)$$

and

$$2.9 = (1 \times 0.1) + (2 \times a) + (3 \times b) + (4 \times 0.2) + (5 \times 0.1)$$

$$2.9 = 0.1 + 2a + 3b + 0.8 + 0.5$$

$$2a + 3b = 1.5 \quad (2)$$

multiply (1) by (2)

$$2a + 2b = 1.2 \quad (3)$$

(2) minus (3)

$$b = 0.3$$

so from 3  $2a + 0.6 = 1.2$

$$a = 0.3$$

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#### Exercise C, Question 6

#### Question:

A fair spinner with equal sections numbered 1 to 5 is thrown 500 times. Work out how many times it can be expected to land on 3.

#### Solution:

$x$	1	2	3	4	5
$P(X = x)$	0.2	0.2	0.2	0.2	0.2

$$0.2 \times 500 = 100$$

We can expect it to land on 3 100 times.

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#### Exercise D, Question 1

#### Question:

For the following probability distribution

$x$	-1	0	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

**a** write down  $E(X)$ .

**b** find  $\text{Var}(X)$ .

#### Solution:

**a** By symmetry  $E(X) = 1$

**b**  $\text{Var } X = E(X^2) - (E(X))^2$

$$E(X^2) = \frac{1}{5} + 0 + \frac{1}{5} + \frac{4}{5} + \frac{9}{5} = 3$$

$$(E(X))^2 = 1^2 = 1$$

$$\text{Var } X = 3 - 1 = \mathbf{2}$$

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#### Exercise D, Question 2

#### Question:

Find the expectation and variance of each of the following distributions of  $X$ .

**a**

$x$	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

**b**

$x$	-1	0	1
$P(x = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

**c**

$x$	-2	-1	1	2
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

#### Solution:

$$\mathbf{a} \text{ Mean} = E(X) = \left(1 \times \frac{1}{3}\right) + \left(2 \times \frac{1}{2}\right) + \left(3 \times \frac{1}{6}\right) = \frac{1}{3} + 1 + \frac{1}{2} = \mathbf{1\frac{5}{6}}$$

$$E(X^2) = \frac{1}{3} + 2 + \frac{9}{6} = 3\frac{5}{6}$$

$$\text{Var } X = 3\frac{5}{6} - \left(1\frac{5}{6}\right)^2 = \frac{138}{36} - \frac{121}{36} = \frac{17}{36}$$

$$\mathbf{b} \text{ Mean} = E(X) = \left(-1 \times \frac{1}{4}\right) + \left(0 \times \frac{1}{2}\right) + \left(1 \times \frac{1}{4}\right) = 0 \text{ (or by symmetry = 0)}$$

$$E(X^2) = \left(1 \times \frac{1}{4}\right) + \left(0 \times \frac{1}{2}\right) + \left(1 \times \frac{1}{4}\right) = \frac{1}{2}$$

$$\text{Var } X = \frac{1}{2} - 0 = \frac{1}{2}$$

$$\mathbf{c} \text{ Mean} = E(X) = \left(-2 \times \frac{1}{3}\right) + \left(-1 \times \frac{1}{3}\right) + \left(1 \times \frac{1}{6}\right) + \left(2 \times \frac{1}{6}\right) = -\frac{1}{2}$$

$$E(X^2) = \left(4 \times \frac{1}{3}\right) + \left(1 \times \frac{1}{3}\right) + \left(1 \times \frac{1}{6}\right) + \left(4 \times \frac{1}{6}\right) = 2\frac{1}{2}$$

$$\text{Var } X = 2\frac{1}{2} - \left(-\frac{1}{2}\right)^2 = 2\frac{1}{4}$$

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#### Exercise D, Question 3

#### Question:

Given that  $Y$  is the score when a single unbiased die is rolled, find  $E(Y)$  and  $\text{Var}(Y)$ .

#### Solution:

$y$	1	2	3	4	5	6
$P(Y = y)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(Y) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = 3\frac{1}{2}$$

$$E(X^2) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} = 15\frac{1}{6}$$

$$\text{Var } X = 15\frac{1}{6} - \left(3\frac{1}{2}\right)^2 = 2\frac{11}{12}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise D, Question 4

#### Question:

Two fair cubical dice are rolled and  $S$  is the sum of their scores.

**a** Find the distribution of  $S$ .

**b** Find  $E(S)$ .

**c** Find  $\text{Var}(S)$ .

#### Solution:

**a**

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$s$	2	3	4	5	6	7	8	9	10	11	12
$P(S = s)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\mathbf{b} \quad E(S) = \frac{2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12}{36} = 7 \text{ (or by symmetry = 7)}$$

$$\mathbf{c} \quad E(S^2) = \frac{4 + 18 + 48 + 100 + 180 + 294 + 320 + 324 + 300 + 242 + 144}{36}$$

$$= 54\frac{5}{6}$$

$$\text{Var } S = 54\frac{5}{6} - 7^2 = 5\frac{5}{6}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise D, Question 5

#### Question:

Two fair tetrahedral (four-sided) dice are rolled and  $D$  is the difference between their scores.

**a** Find the distribution of  $D$  and show that  $P(D = 3) = \frac{1}{8}$ .

**b** Find  $E(D)$ .

**c** Find  $\text{Var}(D)$ .

#### Solution:

**a**

Diff.	1	2	3	4
1	0	1	2	3
2	1	0	1	2
3	2	1	0	1
4	3	2	1	0

$d$	0	1	2	3
$P(D = d)$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

From Distribution Table it can be seen that  $P(D = 3) = \frac{1}{8}$

$$\mathbf{b} \quad E(D) = 0 + \frac{3}{8} + \frac{1}{2} + \frac{3}{8} = 1\frac{1}{4}$$

$$\mathbf{c} \quad E(D^2) = 0 + \frac{3}{8} + 1 + 1\frac{1}{8} = 2\frac{1}{2}$$

$$\text{Var } D = 2\frac{1}{2} - \left(1\frac{1}{4}\right)^2 = \frac{15}{16}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise D, Question 6

#### Question:

A fair coin is tossed repeatedly until a head appears or three tosses have been made. The random variable  $T$  represents the number of tosses of the coin.

**a** Show that the distribution of  $T$  is

$t$	1	2	3
$P(T = t)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

**b** Find the expectation and variance of  $T$ .

#### Solution:

$$\mathbf{a} \ P(H) = \frac{1}{2}$$

$$P(TH) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(TTH) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(TTT) = \frac{1}{8}$$

$$P(T = 1) = \frac{1}{2}$$

$$P(T = 2) = \frac{1}{4}$$

$$P(T = 3) = \frac{1}{4}$$

$$\mathbf{b} \ E(T) = 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{4} = \mathbf{1\frac{3}{4}}$$

$$\text{Var}(T) = 1 \times \frac{1}{2} + 4 \times \frac{1}{4} + 9 \times \frac{1}{4} - \left(1\frac{3}{4}\right)^2 = \frac{\mathbf{11}}{\mathbf{16}}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise D, Question 7

#### Question:

The random variable  $X$  has the following distribution:

$x$	1	2	3
$P(X = x)$	$a$	$b$	$a$

where  $a$  and  $b$  are constants.

**a** Write down  $E(X)$ .

**b** Given that  $\text{Var}(X) = 0.75$ , find the values of  $a$  and  $b$ .

#### Solution:

**a**  $E(X) = 2$  by symmetry

$$\mathbf{b} \sum p(x) = 2a + b = 1 \quad (1)$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= 10a + 4b - 2^2 \\ &= 10a + 4b - 4 = \frac{3}{4} \quad (2) \end{aligned}$$

$$10a + 4b = 4\frac{3}{4} \quad \text{from (2)}$$

$$\underline{8a + 4b = 4} \quad \text{from (1)} \times 4$$

$$2a = \frac{3}{4}$$

$$a = \frac{3}{8}$$

$$b = 1 - 2a$$

$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise E, Question 1

#### Question:

$$E(X) = 4, \text{Var}(X) = 10$$

Find

**a**  $E(2X)$ ,

**b**  $\text{Var}(2X)$ .

#### Solution:

Remember mean is  $E(X)$  and variance is  $\text{Var } X$ .

**a**  $E(2X) = 2 E(X) = 2 \times 4 = \mathbf{8}$

**b**  $\text{Var}(2X) = 2^2 \text{Var } X = 4 \times 10 = \mathbf{40}$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise E, Question 2

#### Question:

$$E(X) = 2, \text{Var}(X) = 6$$

Find

**a**  $E(3X)$ ,

**b**  $E(3X + 1)$ ,

**c**  $E(X - 1)$ ,

**d**  $E(4 - 2X)$ ,

**e**  $\text{Var}(3X)$ ,

**f**  $\text{Var}(3X + 1)$ ,

**g**  $\text{Var}(X - 1)$ .

#### Solution:

**a**  $E(3X) = 3 E(X) = 3 \times 2 = \mathbf{6}$

**b**  $E(3X + 1) = 3 E(X) + 1 = (3 \times 2) + 1 = \mathbf{7}$

**c**  $E(X - 1) = E(X) - 1 = 2 - 1 = \mathbf{1}$

**d**  $E(4 - 2X) = 4 - 2E(X) = 4 - 2 \times 2 = \mathbf{0}$

**e**  $\text{Var}(3X) = 3^2 \text{Var} X = 9 \times 6 = \mathbf{54}$

**f**  $\text{Var}(3X + 1) = 3^2 \text{Var} X = \mathbf{54}$

**g**  $\text{Var}(X - 1) = \text{Var} X = \mathbf{6}$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise E, Question 3

#### Question:

The random variable  $X$  has an expectation of 3 and a variance of 9.

Find

**a**  $E(2X + 1)$ ,

**b**  $E(2 + X)$ ,

**c**  $\text{Var}(2X + 1)$ ,

**d**  $\text{Var}(2 + X)$ .

#### Solution:

**a**  $E(2X + 1) = 2 E(X) + 1 = (2 \times 3) + 1 = 7$

**b**  $E(2 + X) = E(X + 2) = E(X) + 2 = 3 + 2 = 5$

**c**  $\text{Var}(2X + 1) = 2^2 \text{Var}(X) = 4 \times 9 = 36$

**d**  $\text{Var}(2 + X) = \text{Var}(X + 2) = \text{Var}(X) = 9$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise E, Question 4

#### Question:

The random variable  $X$  has a mean  $\mu$  and standard deviation  $\sigma$ .

Find, in terms of  $\mu$  and  $\sigma$

**a**  $E(4X)$ ,

**b**  $E(2X + 2)$ ,

**c**  $E(2X - 2)$ ,

**d**  $\text{Var}(2X + 2)$ ,

**e**  $\text{Var}(2X - 2)$ .

#### Solution:

**a**  $E(4X) = 4 E(X) = 4\mu$

**b**  $E(2X + 2) = 2 E(X) + 2 = 2\mu + 2$

**c**  $E(2X - 2) = 2 E(X) - 2 = 2\mu - 2$

**d**  $\text{Var}(2X + 2) = 2^2 \text{Var}(X) = 4\sigma^2$  (Remember Standard deviation is  $\sigma$  so variance is  $\sigma^2$ )

**e**  $\text{Var}(2X - 2) = 2^2 \text{Var}(X) = 4\sigma^2$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise E, Question 5

#### Question:

The random variable  $Y$  has mean 2 and variance 9.

Find:

**a**  $E(3Y + 1)$ ,

**b**  $E(2 - 3Y)$ ,

**c**  $\text{Var}(3Y + 1)$ ,

**d**  $\text{Var}(2 - 3Y)$ ,

**e**  $E(Y^2)$ ,

**f**  $E[(Y - 1)(Y + 1)]$ .

#### Solution:

**a**  $E(3Y + 1) = 3E(Y) + 1 = 3 \times 2 + 1 = 7$

**b**  $E(2 - 3Y) = 2 - 3E(Y) = 2 - 3 \times 2 = -4$

**c**  $\text{Var}(3Y + 1) = 3^2 \text{Var}(Y) = 9 \times 9 = 81$

**d**  $\text{Var}(2 - 3Y) = (-3)^2 \text{Var}(Y) = 9 \times 9 = 81$

**e**  $E(Y^2) = \text{Var}(Y) + [E(Y)]^2 = 9 + 2^2 = 13$

**f**  $E[(Y - 1)(Y + 1)] = E(Y^2 - 1) = E(Y^2) - 1 = 13 - 1 = 12$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise E, Question 6

#### Question:

The random variable  $T$  has a mean of 20 and a standard deviation of 5.

It is required to scale  $T$  by using the transformation  $S = 3T + 4$ .

Find  $E(S)$  and  $\text{Var}(S)$ .

#### Solution:

$$E(S) = E(3T + 4) = 3 E(T) + 4 = (3 \times 20) + 4 = \mathbf{64}$$

$$\text{Var}(T) = 5 \times 5 = 25$$

$$\text{Var}(S) = \text{Var}(3T + 4) = 3^2 \text{Var} T = 9 \times 25 = \mathbf{225}$$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise E, Question 7

#### Question:

A fair spinner is made from the disc in the diagram and the random variable  $X$  represents the number it lands on after being spun.

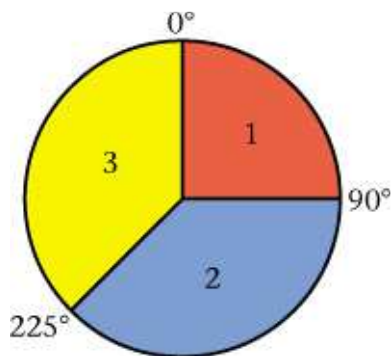
**a** Write down the distribution of  $X$ .

**b** Work out  $E(X)$ .

**c** Find  $\text{Var}(X)$ .

**d** Find  $E(2X + 1)$ .

**e** Find  $\text{Var}(3X - 1)$ .



#### Solution:

**a**

$x$	1	2	3
$P(X = x)$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$

$$\mathbf{b} \quad E(X) = \left(1 \times \frac{1}{4}\right) + \left(2 \times \frac{3}{8}\right) + \left(3 \times \frac{3}{8}\right) = 2\frac{1}{8}$$

$$\mathbf{c} \quad E(X^2) = \left(1 \times \frac{1}{4}\right) + \left(4 \times \frac{3}{8}\right) + \left(9 \times \frac{3}{8}\right) = 5\frac{1}{8}$$

$$\text{Var}(X) = 5\frac{1}{8} - \left(2\frac{1}{8}\right)^2 = \frac{39}{64}$$

$$\mathbf{d} \quad E(2X + 1) = 2E(X) + 1 = \left(2 \times 2\frac{1}{8}\right) + 1 = 5\frac{1}{4}$$

$$\mathbf{e} \quad \text{Var}(3X - 1) = 3^2 \text{Var} X = 9 \times \frac{39}{64} = 5\frac{31}{64}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise E, Question 8

#### Question:

The discrete variable  $X$  has the probability distribution

$x$	-1	0	1	2
$P(X = x)$	0.2	0.5	0.2	0.1

**a** Find  $E(X)$ ,

**b** Find  $\text{Var}(X)$ ,

**c** Find  $E(\frac{1}{3}X + 1)$ ,

**d** Find  $\text{Var}(\frac{1}{3}X + 1)$ .

#### Solution:

$$\mathbf{a} \ E(X) = -0.2 + 0 + 0.2 + 0.2 = \mathbf{0.2}$$

$$\mathbf{b} \ E(X^2) = 0.2 + 0 + 0.2 + 0.4 = \mathbf{0.8}$$

$$\text{Var}(X) = 0.8 - 0.2^2 = 0.8 - 0.04 = \mathbf{0.76}$$

$$\mathbf{c} \ E\left(\frac{1}{3}X + 1\right) = \frac{1}{3}E(X) + 1 = \left(\frac{1}{3} \times 0.2\right) + 1 = \left(\frac{1}{3} \times \frac{1}{5}\right) + 1 = \mathbf{1\frac{1}{15}} (1.0\bar{6})$$

$$\mathbf{d} \ \text{Var}\left(\frac{1}{3}X + 1\right) = \left(\frac{1}{3}\right)^2 \text{Var}X = \frac{1}{9} \times 0.76 = \frac{1}{9} \times \frac{19}{25} = \frac{\mathbf{19}}{\mathbf{225}} (0.08\bar{4})$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise F, Question 1

#### Question:

$X$  is a discrete uniform distribution over the numbers 1, 2, 3, 4 and 5. Work out the expectation and variance of  $X$ .

#### Solution:

$$\text{Expectation} = \frac{n+1}{2} = \frac{5+1}{2} = 3$$

$$\text{Variance} = \frac{(n+1)(n-1)}{12} = \frac{(5+1)(5-1)}{12} = 2$$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise F, Question 2

#### Question:

Seven similar balls are placed in a bag. The balls have the numbers 1 to 7 on them. A ball is drawn out of the bag. The variable  $X$  represents the number on the ball.

**a** Find  $E(X)$ .

**b** Work out  $\text{Var}(X)$ .

#### Solution:

**a**  $n = 7$

$$E(X) = \frac{n+1}{2} = 4$$

**b**  $\text{Var}(X) = \frac{(n+1)(n-1)}{12} = 4$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise F, Question 3

#### Question:

A fair die is thrown once and the random variable  $X$  represents the value on the upper face.

**a** Find the expectation and variance of  $X$ .

**b** Calculate the probability that  $X$  is within one standard deviation of the expectation.

#### Solution:

$$\mathbf{a} \text{ Expectation} = \frac{n+1}{2} = \frac{6+1}{2} = 3\frac{1}{2}$$

$$\text{Var}(X) = \frac{(n+1)(n-1)}{12} = \frac{7 \times 5}{12} = 2\frac{11}{12} = 2.91\dot{6}$$

#### **b**

$x$	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\sigma = \sqrt{2.91\dot{6}} = 1.7078$$

Therefore we want between  $3.5 - 1.7078 = 1.7922$  and  $3.5 + 1.7078 = 5.2078$

$$P(1.7922 \leq X \leq 5.2078) = p(2) + p(3) + p(4) + p(5) = \frac{2}{3}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise F, Question 4

#### Question:

A card is selected at random from a pack of cards containing the even numbers 2, 4, 6, ..., 20. The variable  $X$  represents the number on the card.

**a** Find  $P(X > 15)$ .

**b** Find the expectation and variance of  $X$ .

#### Solution:

**a** This is a uniform distribution.

$x$	2	4	6	8	10	12	14	16	18	20
$P(X = x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$$P(X > 15) = P(16) + P(18) + P(20) = \frac{3}{10}$$

**b** Let  $R$  be a uniform distribution over the numbers 1, 2, ..., 10.

Then  $X = 2R$

$$E(R) = \frac{n+1}{2} = 5.5$$

$$\text{Var}(R) = \frac{(n+1)(n-1)}{12} = \frac{99}{12} = 8\frac{1}{4} = 8.25$$

$$E(X) = 2E(R) = 2 \times 5.5 = \mathbf{11}$$

$$\text{Var}(X) = \text{Var}(2R) = 2^2 \text{Var}(R) = 4 \times 8.25 = \mathbf{33}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise F, Question 5

#### Question:

A card is selected at random from a pack of cards containing the odd numbers 1, 3, 5, ..., 19. The variable  $X$  represents the number on the card.

**a** Find  $P(X > 15)$ .

**b** Find the expectation and variance of  $X$ .

#### Solution:

$y$	1	3	5	7	9	11	13	15	17	19
$P(Y = y)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$$\mathbf{a} \quad P(X > 15) = P(17) + P(19) = \frac{1}{5}$$

$$\mathbf{b} \quad Y = X - 1 \quad (X \text{ as in previous question})$$

$$E(Y) = E(X - 1) = E(X) - 1 = 11 - 1 = \mathbf{10}$$

$$\text{Var}(Y) = \text{Var}(X - 1) = \text{Var}(X) = \mathbf{33}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise F, Question 6

#### Question:

A straight line is drawn on a piece of paper. The line is divided into four equal lengths and the segments are marked 1, 2, 3 and 4. In a party game a person is blindfolded and asked to mark a point on the line and the number of the segment is recorded. A discrete uniform distribution over the set  $(1, 2, 3, 4)$  is suggested as model for this distribution. Comment on this suggestion.

#### Solution:

A discrete uniform distribution is not likely to be a good model for this distribution. The game depends on the skill of the player. The points are quite likely to cluster around the middle.

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

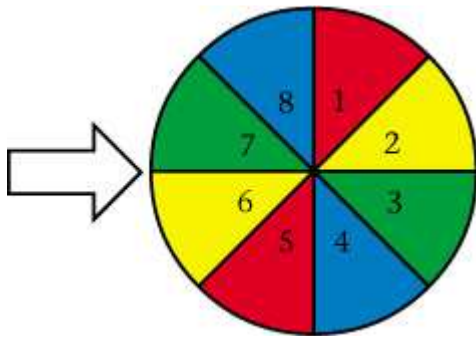
#### Exercise F, Question 7

#### Question:

In a fairground game the spinner shown is used.

It cost 5p to have a go on the spinner.

The spinner is spun and the number of pence shown is returned to the contestant.



If  $X$  is the number which comes up on the next spin,

**a** name a suitable model for  $X$ ,

**b** find  $E(X)$ ,

**c** find  $\text{Var}(X)$ ,

**d** explain why you should not expect to make money at this game if you have a large number of goes.

#### Solution:

**a** A discrete uniform distribution

**b**  $n = 8$

$$E(X) = \frac{n+1}{2} = 4.5$$

$$\text{cVar}(X) = \frac{(n+1)(n-1)}{12} = \frac{63}{12} = 5\frac{1}{4} \text{ or } 5.25$$

**d** The expected winnings are less than the 5p stake.

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 1

#### Question:

The random variable  $X$  has probability function

$$P(X = x) = \frac{x}{21}, x = 1, 2, 3, 4, 5, 6.$$

**a** Construct a table giving the probability distribution of  $X$ .

Find

**b**  $P(2 < X \leq 5)$ ,

**c**  $E(X)$ ,

**d**  $\text{Var}(X)$ ,

**e**  $\text{Var}(3 - 2X)$ .

#### Solution:

**a**

<b>x</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>P(X = x)</b>	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$

**b**  $P(3) + P(4) + P(5) = \frac{3}{21} + \frac{4}{21} + \frac{5}{21} = \frac{12}{21}$

**c**  $E(X) = \left(1 \times \frac{1}{21}\right) + \left(2 \times \frac{2}{21}\right) + \left(3 \times \frac{3}{21}\right) + \left(4 \times \frac{4}{21}\right) + \left(5 \times \frac{5}{21}\right) + \left(6 \times \frac{6}{21}\right)$   
 $= \frac{91}{21} = 4\frac{1}{3}$

**d**  $E(X^2) = \left(1 \times \frac{1}{21}\right) + \left(4 \times \frac{2}{21}\right) + \left(9 \times \frac{3}{21}\right) + \left(16 \times \frac{4}{21}\right) + \left(25 \times \frac{5}{21}\right) + \left(36 \times \frac{6}{21}\right)$   
 $= \frac{441}{21} = 21$

$$\text{Var}(X) = 21 - \left(4\frac{1}{3}\right)^2 = 21 - 18\frac{7}{9} = 2\frac{2}{9}$$

**e**  $\text{Var}(3 - 2X) = \text{Var}(-2X + 3) = (-2)^2 \text{Var}(X) = 4 \times 2\frac{2}{9} = 8\frac{8}{9}$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 2

#### Question:

The discrete random variable  $X$  has the probability distribution shown.

$x$	-2	-1	0	1	2	3
$P(X = x)$	0.1	0.2	0.3	$r$	0.1	0.1

Find

**a**  $r$ ,

**b**  $P(-1 \leq X < 2)$ ,

**c**  $F(0.6)$ ,

**d**  $E(2X + 3)$ ,

**e**  $\text{Var}(2X + 3)$ .

#### Solution:

$$\mathbf{a} \quad 0.1 + 0.2 + 0.3 + r + 0.1 + 0.1 = 1$$

$$r = 1 - 0.8 = \mathbf{0.2}$$

$$\mathbf{b} \quad P(-1) + P(0) + P(1) = 0.2 + 0.3 + 0.2 = \mathbf{0.7}$$

**c**

$x$	-2	-1	0	1	2	3
$F(X)$	0.1	0.3	0.6	0.8	0.9	1

$$F(0.6) = F(0) = \mathbf{0.6}$$

$$\mathbf{d} \quad E(X) = (-0.2) + (-0.2) + 0 + 0.2 + 0.2 + 0.3 = 0.3$$

$$E(2X + 3) = 2E(X) + 3 = (2 \times 0.3) + 3 = \mathbf{3.6}$$

$$\mathbf{e} \quad E(X^2) = 0.4 + 0.2 + 0 + 0.2 + 0.4 + 0.9 = 2.1$$

$$\text{Var}(X) = 2.1 - 0.3^2 = 2.01$$

$$\text{Var}(2X + 3) = 2^2 \text{Var} X = 4 \times 2.01 = \mathbf{8.04}$$



# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 3

#### Question:

A discrete random variable  $X$  has the probability distribution shown in the table below.

$x$	0	1	2
$P(X = x)$	$\frac{1}{5}$	$b$	$\frac{1}{5} + b$

**a** Find the value of  $b$ .

**b** Show that  $E(X) = 1.3$ .

**c** Find the exact value of  $\text{Var}(X)$ .

**d** Find the exact value of  $P(X \leq 1.5)$ .

#### Solution:

**a**

$x$	0	1	2
$P(X = x)$	$\frac{1}{5}$	$b$	$b + \frac{1}{5}$

$$\frac{1}{5} + b + b + \frac{1}{5} = 1$$

$$2b = 1 - \frac{2}{5} = \frac{3}{5}$$

$$b = \frac{3}{10}$$

$$\mathbf{b} \quad E(X) = \left(0 \times \frac{1}{5}\right) + \left(1 \times \frac{3}{10}\right) + \left(2 \times \frac{5}{10}\right) = 0 + \frac{3}{10} + 1 = 1.3$$

$$\mathbf{c} \quad E(X^2) = \left(0 \times \frac{1}{5}\right) + \left(1 \times \frac{3}{10}\right) + \left(4 \times \frac{5}{10}\right) = 0 + \frac{3}{10} + 2 = 2.3$$

$$\text{Var}(X) = 2.3 - 1.3^2 = 0.61$$

$$\mathbf{d} \quad P(0) + P(1) = \frac{1}{5} + \frac{3}{10} = 0.5$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 4

#### Question:

The discrete random variable  $X$  has a probability function

$$P(X = x) = \begin{cases} k(1-x) & x = 0,1 \\ k(x-1) & x = 2,3 \\ 0 & \text{otherwise} \end{cases}$$

where  $k$  is a constant.

**a** Show that  $k = \frac{1}{4}$ .

**b** Find  $E(X)$  and show that  $E(X^2) = 5.5$ .

**c** Find  $\text{Var}(2X - 2)$ .

#### Solution:

**a**  $k(1 - 0) + k(1 - 1) + k(2 - 1) + k(3 - 1) = 1$

$$k + k + 2k = 1$$

$$4k = 1$$

$$k = \frac{1}{4}$$

**b**

$x$	0	1	2	3
$P(X = x)$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$

$$E(X) = 0 + 0 + \frac{1}{2} + \frac{3}{2} = 2$$

$$E(X^2) = \left(0 \times \frac{1}{4}\right) + (1 \times 0) + \left(4 \times \frac{1}{4}\right) + \left(9 \times \frac{1}{2}\right) = 1 + 4.5 = 5.5$$

**c**  $\text{Var}(X) = 5.5 - 4 = 1.5$

$$\text{Var}(2X - 2) = 4 \times 1.5 = 6$$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 5

#### Question:

A discrete random variable  $X$  has the probability distribution,

$x$	0	1	2	3
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$

Find

**a**  $P(1 < X \leq 2)$ ,

**b**  $F(1.5)$ ,

**c**  $E(X)$ ,

**d**  $E(3X - 1)$ ,

**e**  $\text{Var}(X)$ .

#### Solution:

**a**  $P(1 < X \leq 2) = P(2) = \frac{1}{8}$

**b**  $F(1.5) = F(1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

**c**  $E(X) = 0 + \frac{1}{2} + \frac{1}{4} + \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$

**d**  $E(3X - 1) = 3E(X) - 1 = 3\frac{9}{8} - 1 = 2\frac{3}{8}$

**e**  $E(X^2) = 0 + \frac{1}{2} + \frac{1}{2} + \frac{9}{8} = 2\frac{1}{8}$

$$\text{Var}(X) = 2\frac{1}{8} - \left(1\frac{1}{8}\right)^2 = \frac{55}{64}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 6

#### Question:

A discrete random variable is such that each of its values is assumed to be equally likely.

**a** Write the name of the distribution.

**b** Give an example of such a distribution.

A discrete random variable  $X$  as defined above can take values 0, 1, 2, 3, and 4.

Find

**c**  $E(X)$ ,

**d**  $\text{Var}(X)$ .

#### Solution:

**a** A discrete uniform distribution

**b** Any distribution where all the probabilities are the same. An example is throwing a fair die.

**c**

$x$	0	1	2	3	4
$P(X = x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$$E(X) = 0 + \frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} = 2 \text{ (or } \frac{5+1}{2} - 1 = 2) \text{ (OR use symmetry)}$$

$$\mathbf{d} \ E(X^2) = 0 + \frac{1}{5} + \frac{4}{5} + \frac{9}{5} + \frac{16}{5} = \mathbf{6} \text{ (or } \text{Var}(X) = \frac{(5+1)(5-1)}{12} = 2)$$

$$\text{Var}(X) = 6 - 4 = \mathbf{2}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 7

#### Question:

The random variable  $X$  has a probability distribution

$x$	1	2	3	4	5
$P(X = x)$	0.1	$p$	$q$	0.3	0.1

**a** Given that  $E(X) = 3.1$ , write down two equations involving  $p$  and  $q$ .

Find

**b** the value of  $p$  and the value of  $q$ ,

**c**  $\text{Var}(X)$ ,

**d**  $\text{Var}(2X - 3)$ .

#### Solution:

$$\mathbf{a} \quad 0.1 + p + q + 0.3 + 0.1 = 1$$

$$p + q = 0.5 \quad \mathbf{1}$$

$$0.1 + 2p + 3q + 1.2 + 0.5 = 3.1$$

$$\mathbf{2p + 3q = 1.3} \quad \mathbf{2}$$

$$\mathbf{b} \quad 2p + 3q = 1.3 \quad (\text{equation 2})$$

$$2p + 2q = 1 \quad (\text{equation 1 times 2}) \quad \mathbf{3}$$

$$\mathbf{q = 0.3} \quad (\text{equation 2-3})$$

$$p + \mathbf{0.3} = 0.5$$

$$p = \mathbf{0.2}$$

**c**

$x$	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.3	0.3	0.1

$$E(X) = 0.1 + 0.4 + 0.9 + 1.2 + 0.5 = 3.1$$

$$E(X^2) = 0.1 + 0.8 + 2.7 + 4.8 + 2.5 = 10.9$$

$$\text{Var}(X) = 10.9 - 3.1^2 = \mathbf{1.29}$$

**d**  $\text{Var}(2X + 3) = 4 \text{Var}(X) = 4 \times 1.29 = \mathbf{5.16}$

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# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 8

#### Question:

The random variable  $X$  has probability function

$$P(X = x) = \begin{cases} kx & x = 1, 2 \\ k(x-2) & x = 3, 4, 5 \end{cases}$$

where  $k$  is a constant.

- Find the value of  $k$ .
- Find the exact value of  $E(X)$ .
- Show that, to three significant figures,  $\text{Var}(X) = 2.02$ .
- Find, to one decimal place,  $\text{Var}(3 - 2X)$ .

#### Solution:

**a**

$x$	1	2	3	4	5
$P(X = x)$	$k$	$2k$	$k$	$2k$	$3k$

$$k + 2k + k + 2k + 3k = 1$$

$$9k = 1$$

$$k = \frac{1}{9}$$

$$\mathbf{b} \quad E(X) = \frac{1}{9} + \frac{4}{9} + \frac{3}{9} + \frac{8}{9} + \frac{15}{9} = \frac{31}{9} = \mathbf{3\frac{4}{9}}$$

$$\mathbf{c} \quad E(X^2) = \frac{1}{9} + \frac{8}{9} + 1 + \frac{32}{9} + \frac{75}{9} = \frac{125}{9} = 13\frac{8}{9}$$

$$\text{Var}(X) = 13\frac{8}{9} - \left(3\frac{4}{9}\right)^2 = 13.888 - 11.864 = \mathbf{2.02} \text{ to 3 sig figs}$$

$$\mathbf{d} \quad \text{Var}(3 - 2X) = (-2)^2 \text{Var}(X) = 4 \times 2.02 = \mathbf{8.1} \text{ to 1 dp}$$

# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 9

#### Question:

The random variable  $X$  has the discrete uniform distribution

$$P(X = x) = \frac{1}{6} \quad x = 1, 2, 3, 4, 5, 6.$$

**a** Write down  $E(X)$  and show that  $\text{Var}(X) = \frac{35}{12}$ .

**b** Find  $E(2X - 1)$ .

**c** Find  $\text{Var}(3 - 2X)$ .

#### Solution:

**a**  $E(X) = 3.5 = 3\frac{1}{2}$

$$E(X^2) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} = \frac{91}{6} = 15\frac{1}{6}$$

$$\text{Var}(X) = 15\frac{1}{6} - \left(3\frac{1}{2}\right)^2 = \frac{91}{6} - \frac{49}{4} = \frac{35}{12}$$

**b**  $E(2X - 1) = 2E(X) - 1 = 7 - 1 = \mathbf{6}$

**c**  $\text{Var}(3 - 2X) = 4 \text{Var}(X) = 4 \times \frac{35}{12} = \frac{35}{3} = 11\frac{2}{3}$  or **11.67** to 2 dp



# Solutionbank S1

## Edexcel AS and A Level Modular Mathematics

### Discrete random variables

#### Exercise G, Question 10

#### Question:

The random variable  $X$  has probability function

$$p(x) = \frac{(3x-1)}{26} \quad x = 1, 2, 3, 4.$$

**a** Construct a table giving the probability distribution of  $X$ .

Find

**b**  $P(2 < X \leq 4)$ ,

**c** the exact value of  $E(X)$ .

**d** Show that  $\text{Var}(X) = 0.92$  to two significant figures.

**e** Find  $\text{Var}(1 - 3X)$ .

#### Solution:

**a**

<b>x</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>P(X = x)</b>	$\frac{2}{26}$	$\frac{5}{26}$	$\frac{8}{26}$	$\frac{11}{26}$

$$\mathbf{b} \quad P(2 < X \leq 4) = P(3) + P(4) = \frac{19}{26}$$

$$\mathbf{c} \quad E(X) = \frac{2}{26} + \frac{10}{26} + \frac{24}{26} + \frac{44}{26} = \frac{80}{26} = \mathbf{3\frac{1}{13}}$$

$$\mathbf{d} \quad E(X^2) = \frac{2}{26} + \frac{20}{26} + \frac{72}{26} + \frac{176}{26} = \frac{270}{26} = 10\frac{10}{26} = \mathbf{10\frac{5}{13}}$$

$$\text{Var}(X) = 10\frac{5}{13} - \left(3\frac{1}{13}\right)^2 = 10.385 \dots - 9.467 \dots = \mathbf{0.92}$$

$$\mathbf{e} \quad \text{Var}(1 - 3X) = (-3)^2 \text{Var}(X) = 9 \times 0.92 = \mathbf{8.28}$$