Six Effective Components of Mathematics Instruction for Students with Learning Difficulties

HELP Group Summit

October 19, 2018

sarahpowellphd.com

Evidence-based mathematics resources for educators

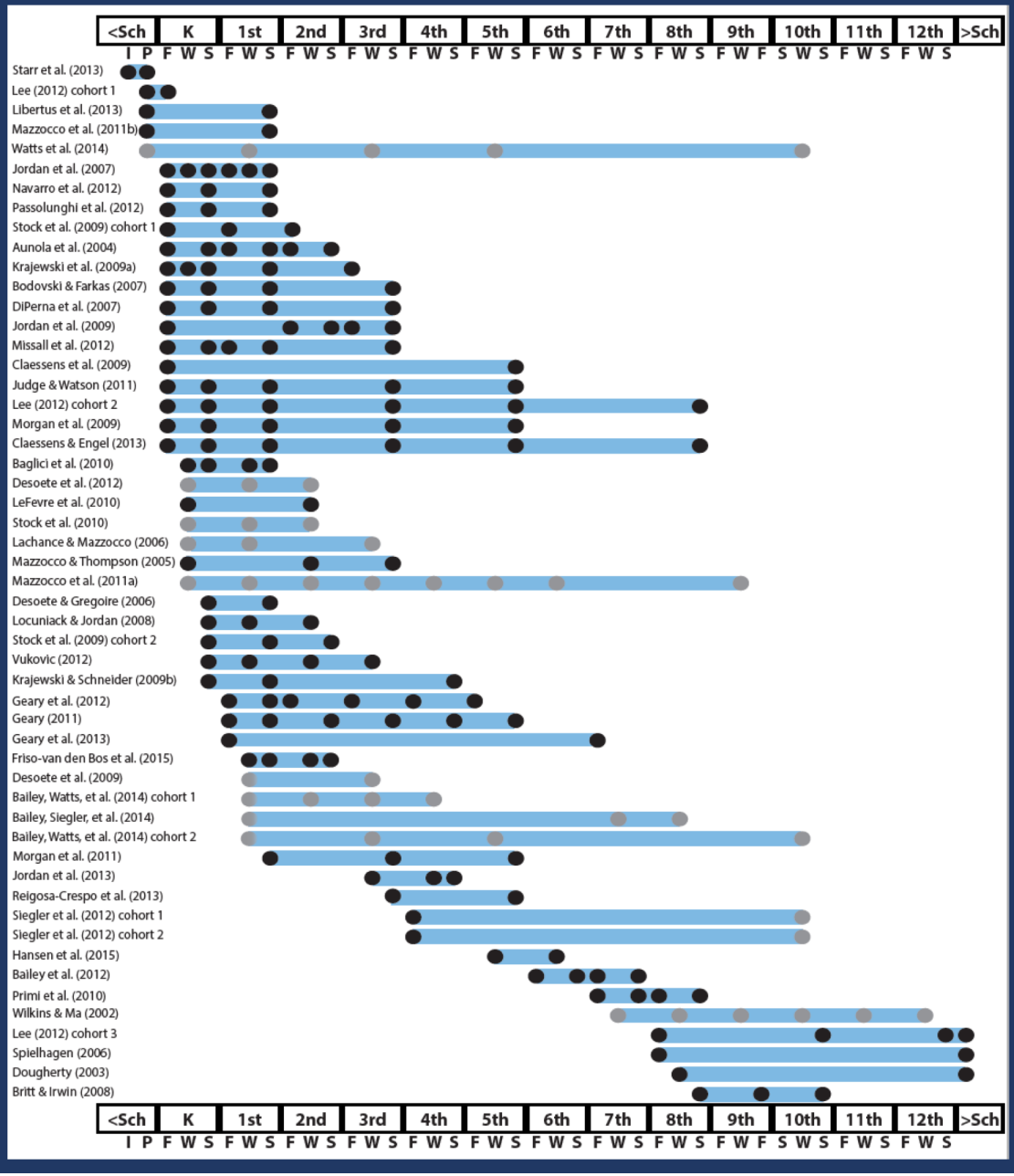
Contact Information

Sarah Powell, Ph.D

The University of Texas at Austin

srpowell@austin.utexas.edu

@sarahpowellphd

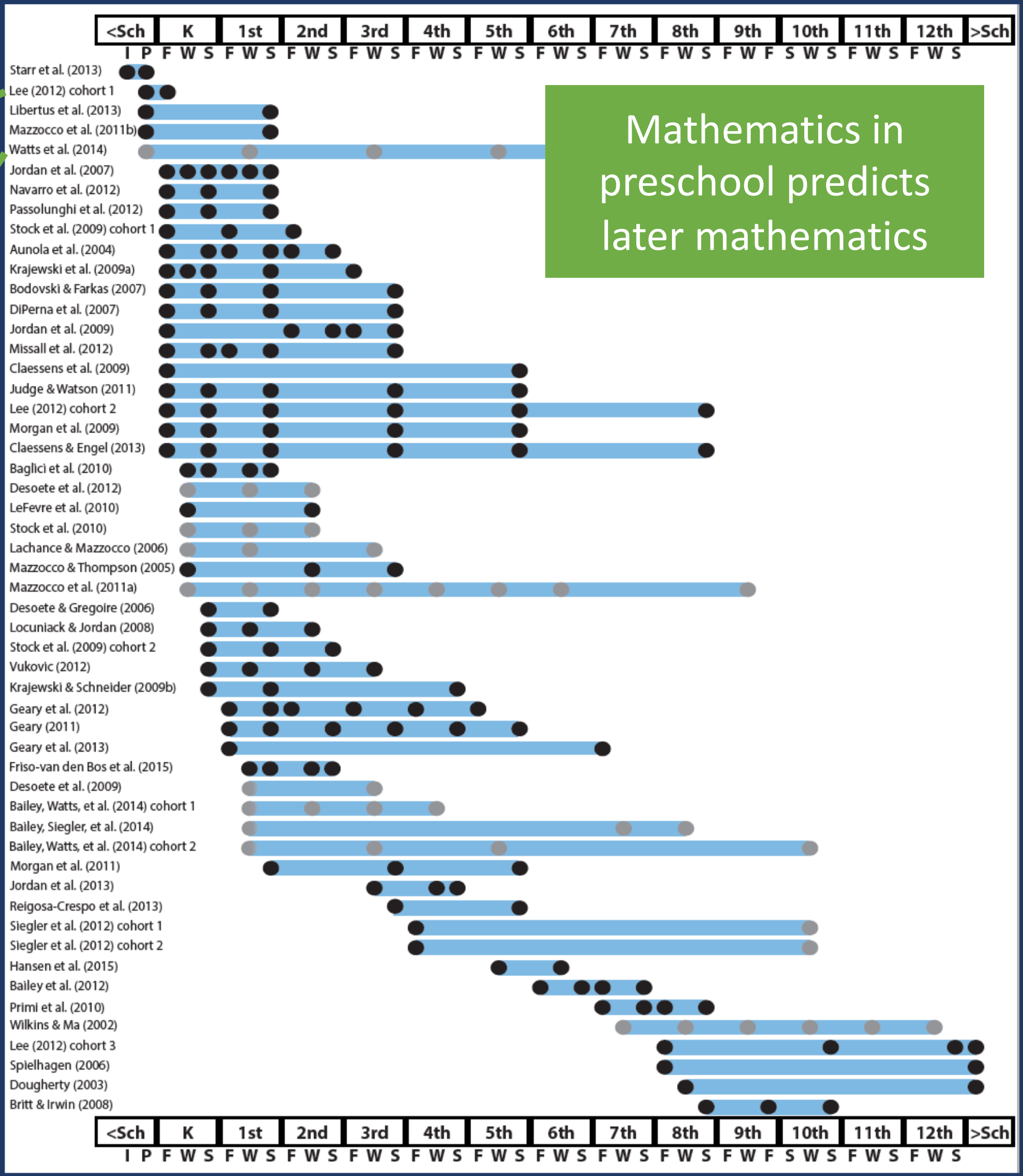


http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Broad math in preK predicted K broad math

Broad math in preK predicted grade 10 broad math

Mathematics in preschool predicts later mathematics



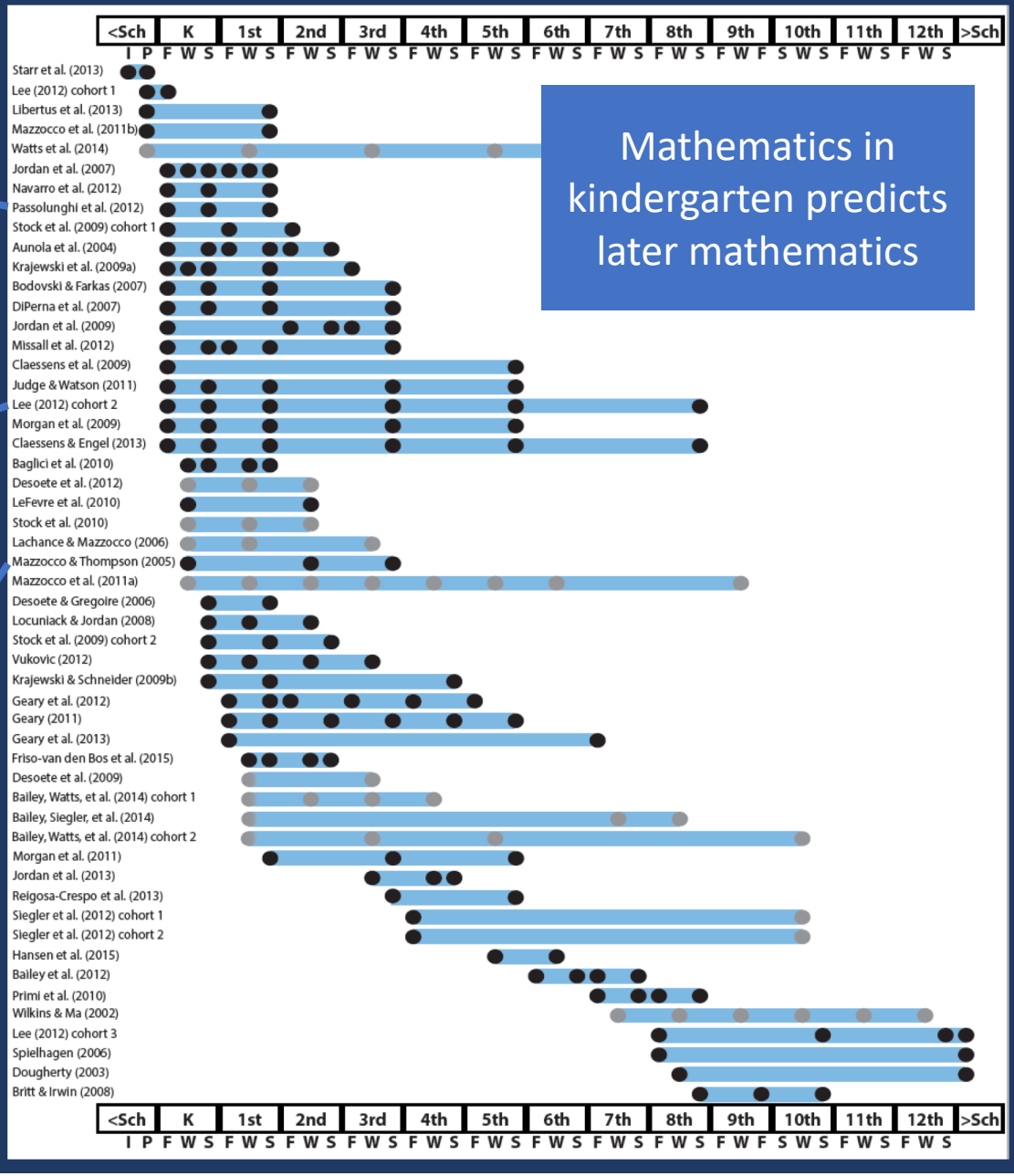
http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

sarahpowellphd.com
Evidence-based mathematics resources for educators

Counting in K predicted grade 1 broad math

Broad math in K predicted grade 8 broad math

K math accurately predicted math performance below 10th percentile in grades 2 and 3 with 84% correct classification



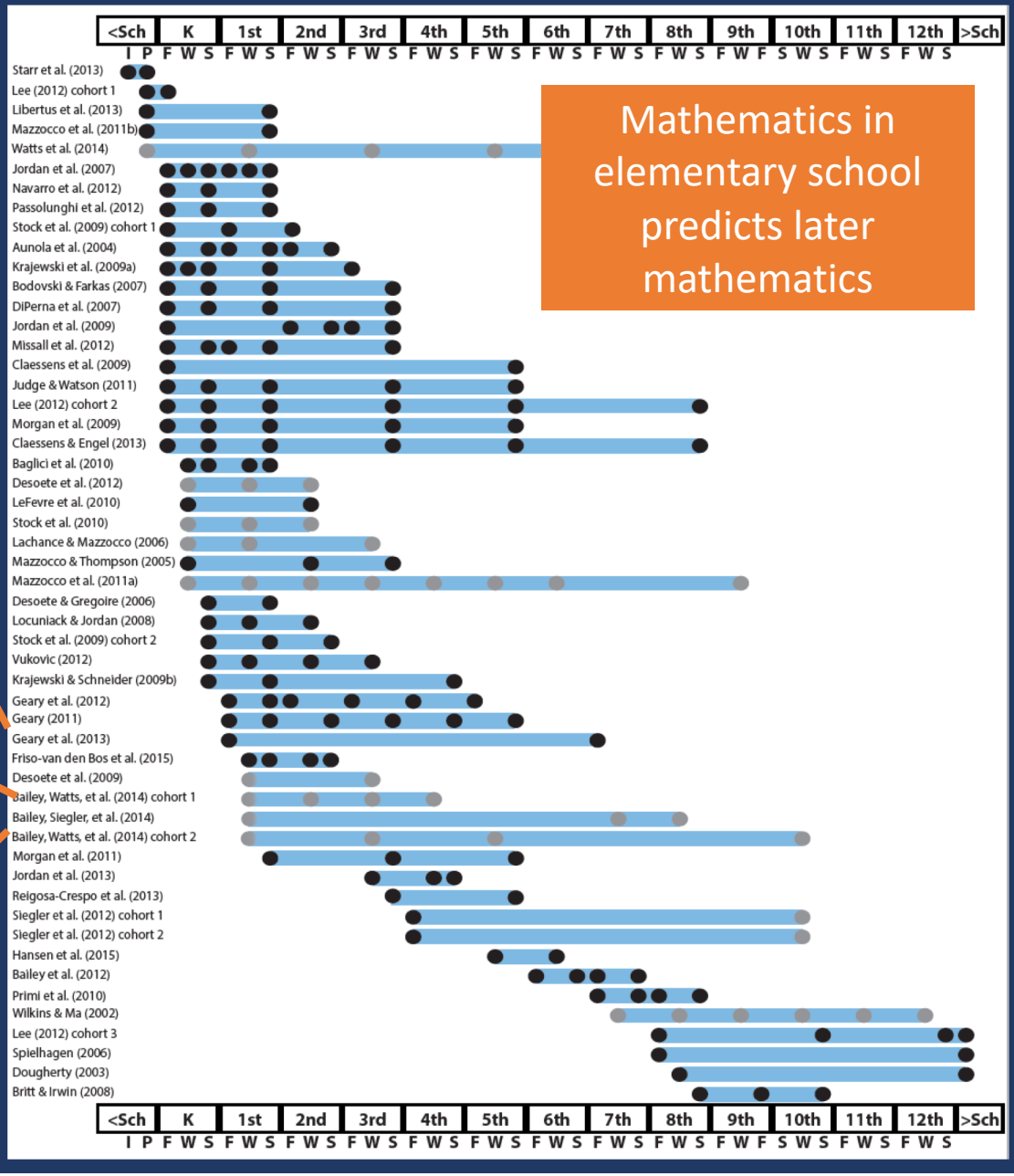
Mathematics in kindergarten predicts later mathematics

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Addition influenced arithmetic with increasing importance from grades 1 to 5

Grade 1 arithmetic predicted arithmetic at grades 2, 3, and 4

Grade 1 broad math predicted broad math at grades 3, 5, and 10

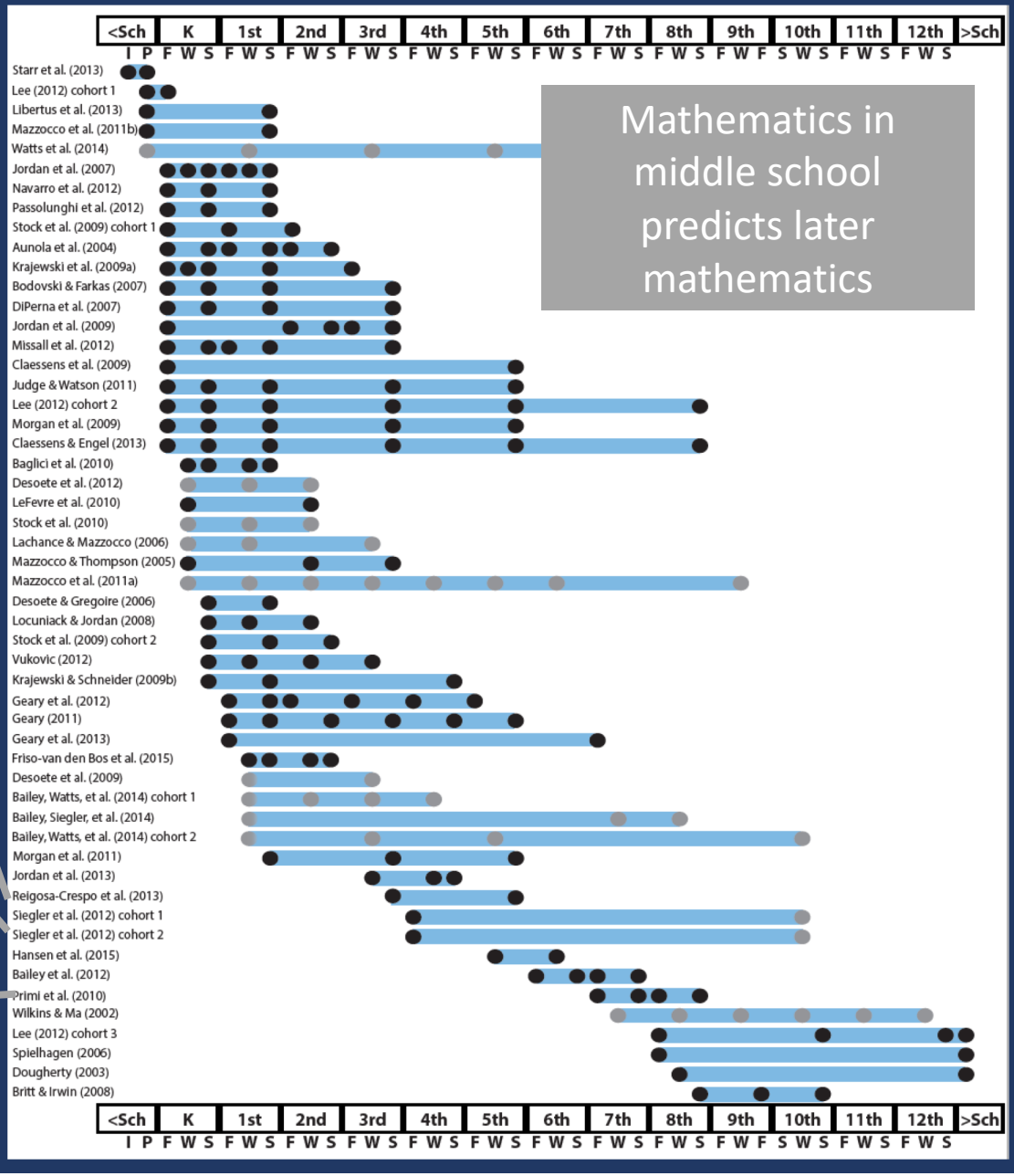


sarahpowellphd.com
Evidence-based mathematics resources for educators

Counting and comparison in grades 2 or 4 predicted broad math 1 year later

Fractions at 10-12 years old predicted broad math 5 years later

Broad math in grade 7 predicted broad math in grade 8



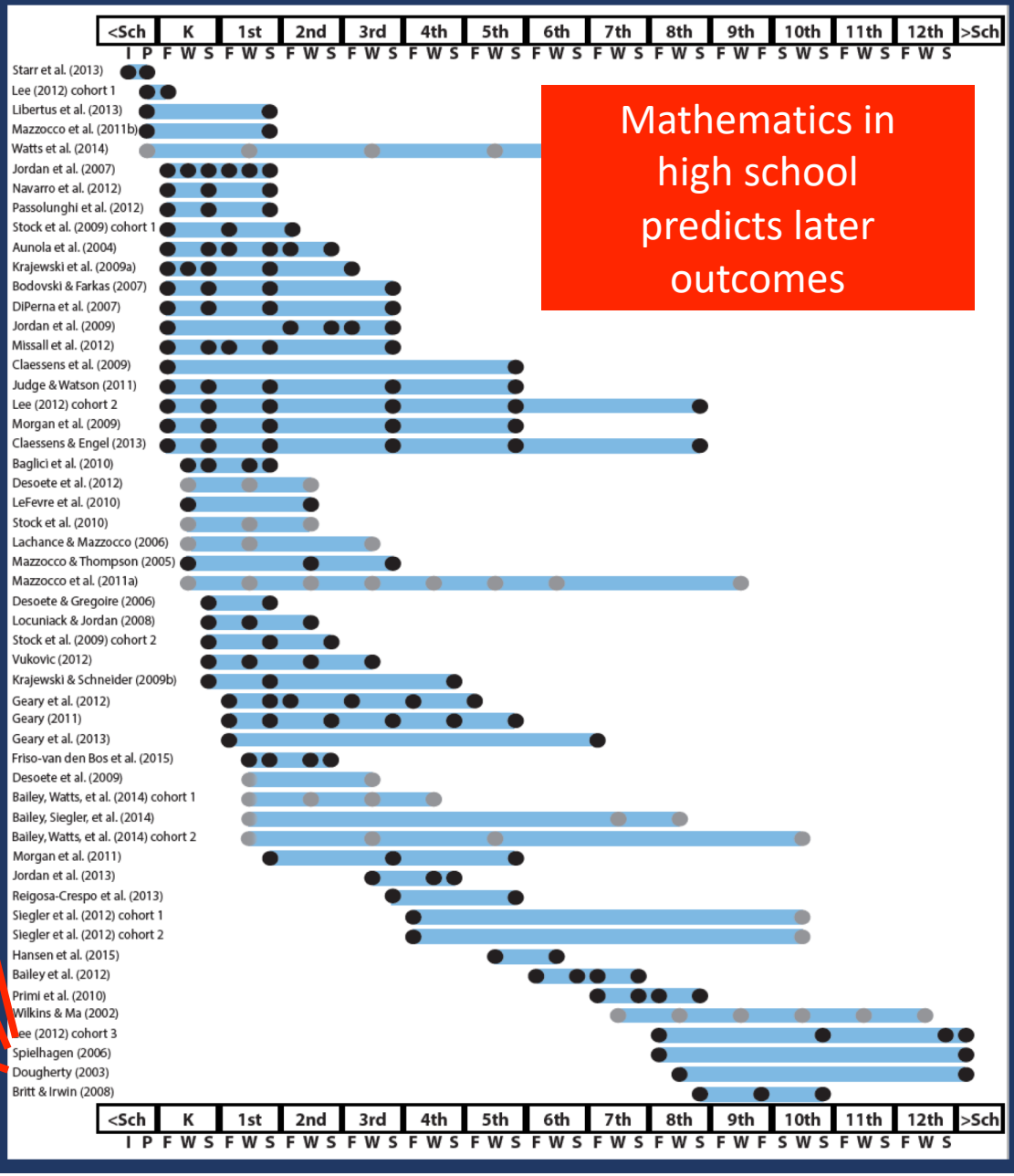
http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

sarahpowellphd.com
Evidence-based mathematics resources for educators

Broad math in grade 8 predicted completion of 4-year college degree

Students who took algebra in grades 8 took more advanced math courses and enrolled in 4-year colleges more often than students who took algebra in grade 9

Numeracy measured in adolescence impacted hourly earnings 7 to 15 years later



http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

sarahpowellphd.com
Evidence-based mathematics resources for educators

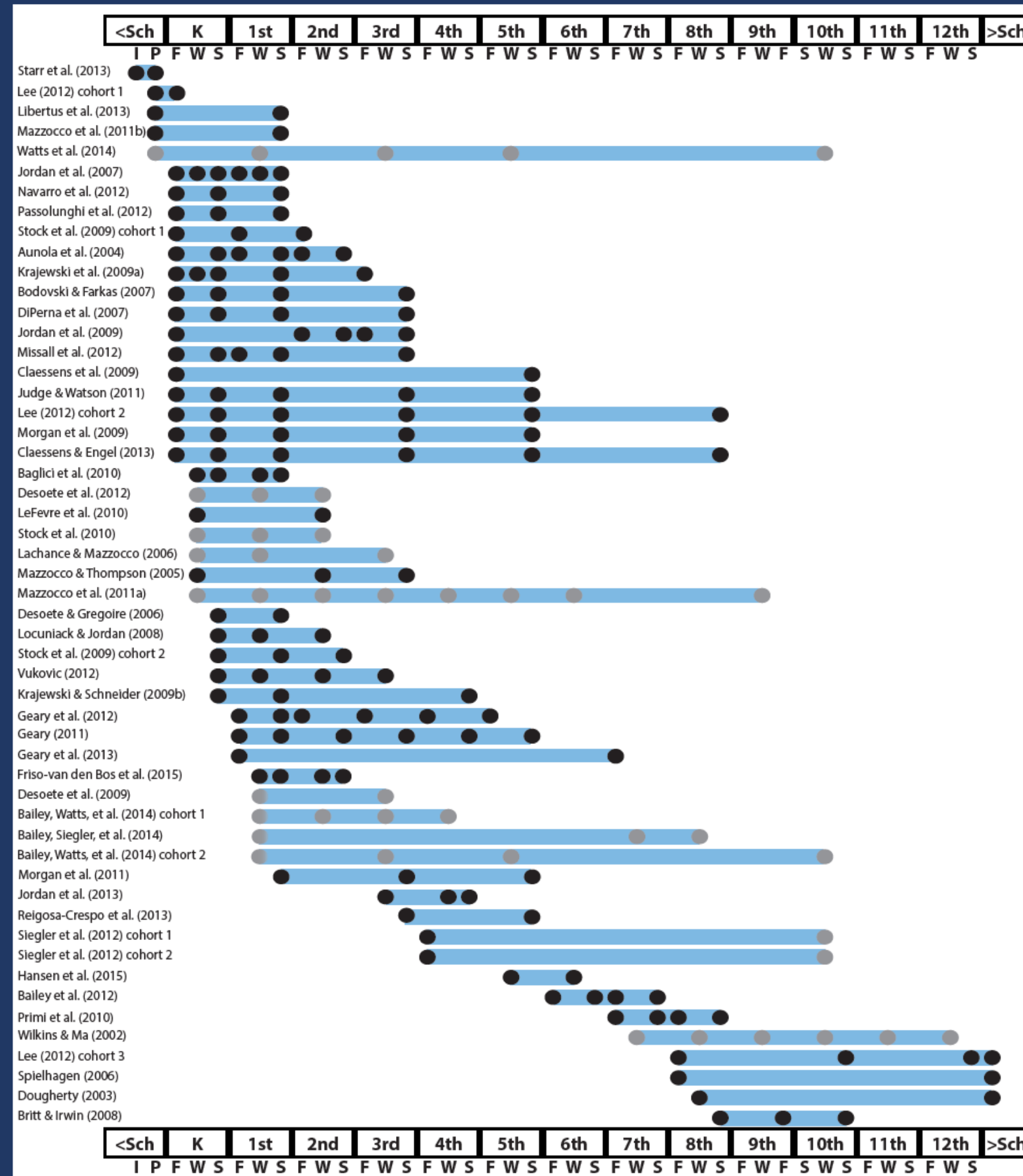
Mathematics in preschool predicts later mathematics

Mathematics in kindergarten predicts later mathematics

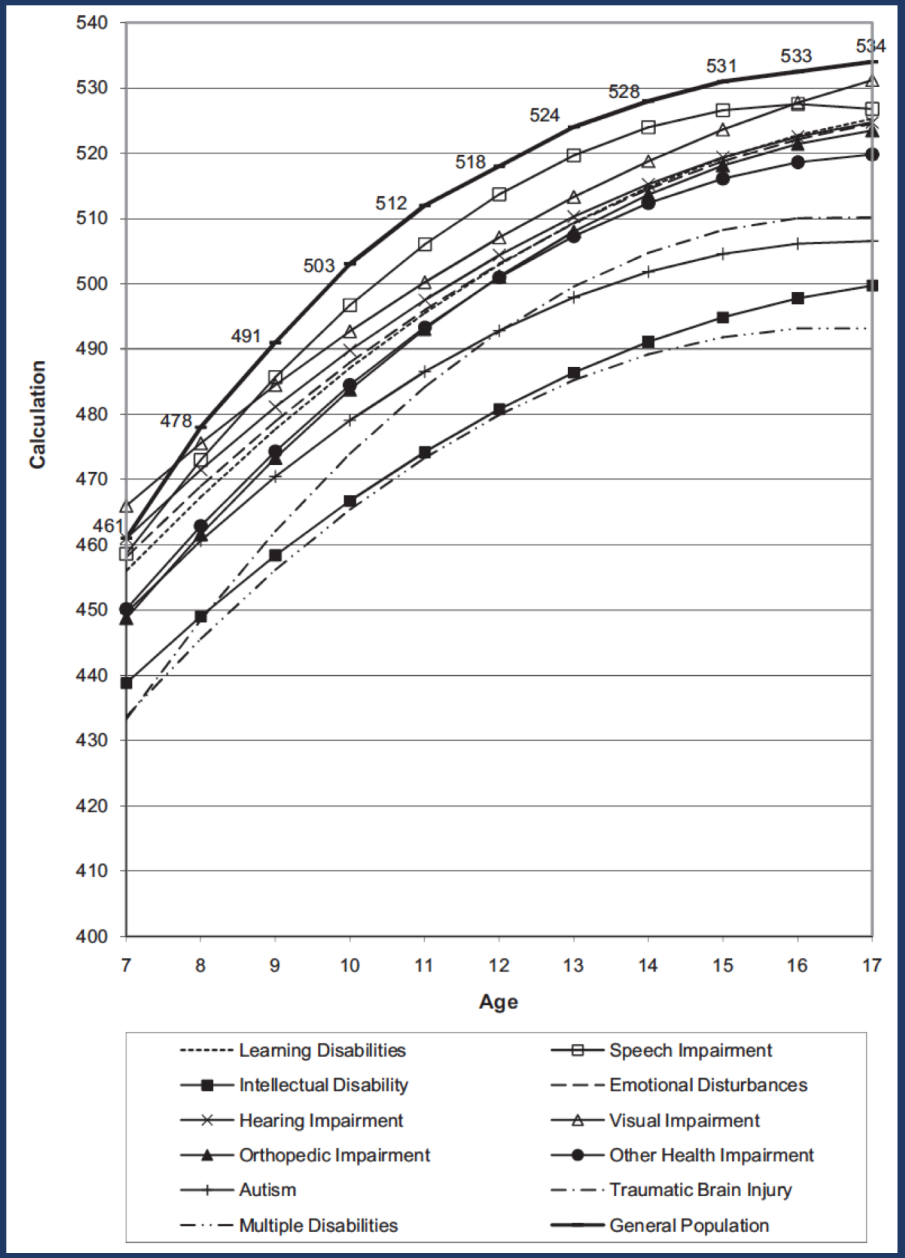
Mathematics in elementary school predicts later mathematics

Mathematics in middle school predicts later mathematics

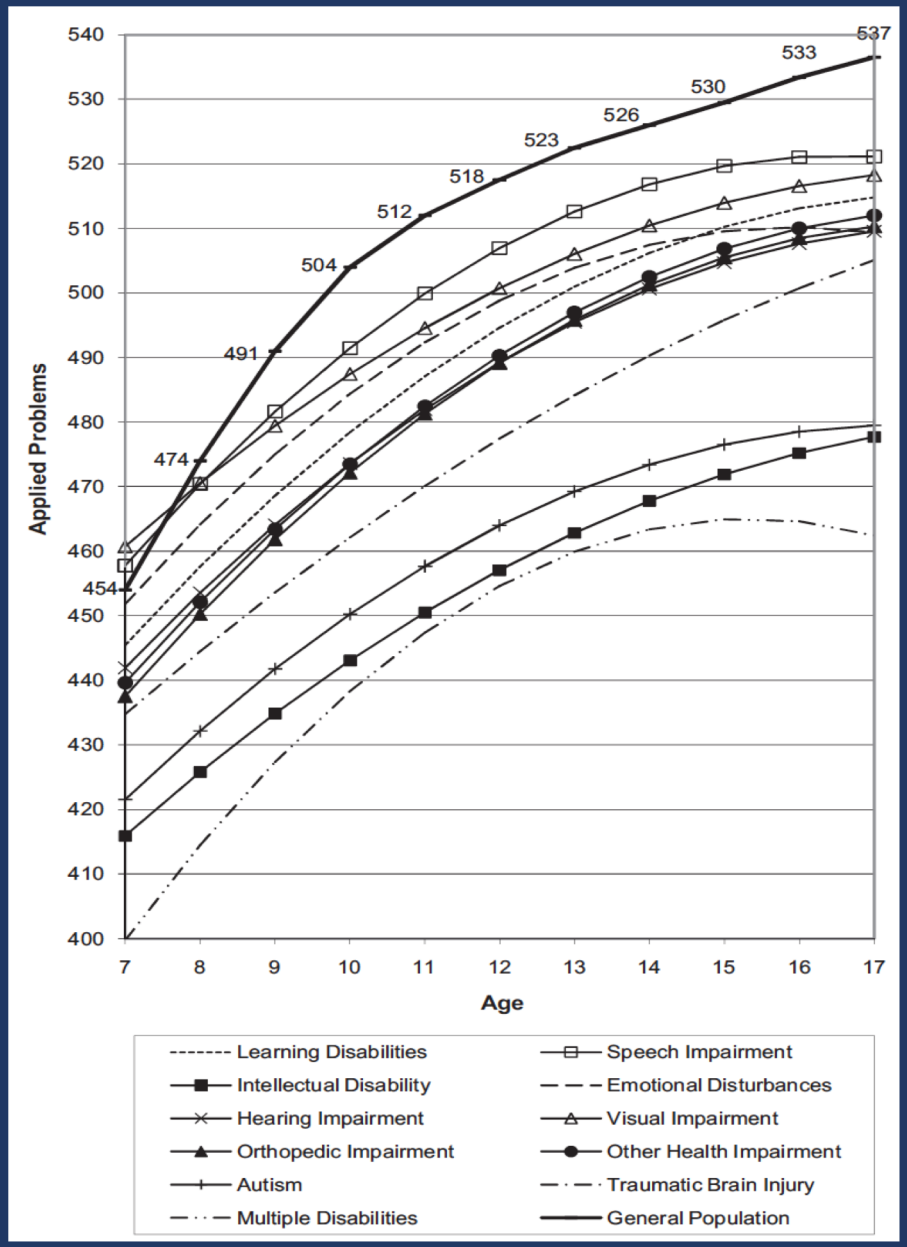
Mathematics in high school predicts later outcomes



http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

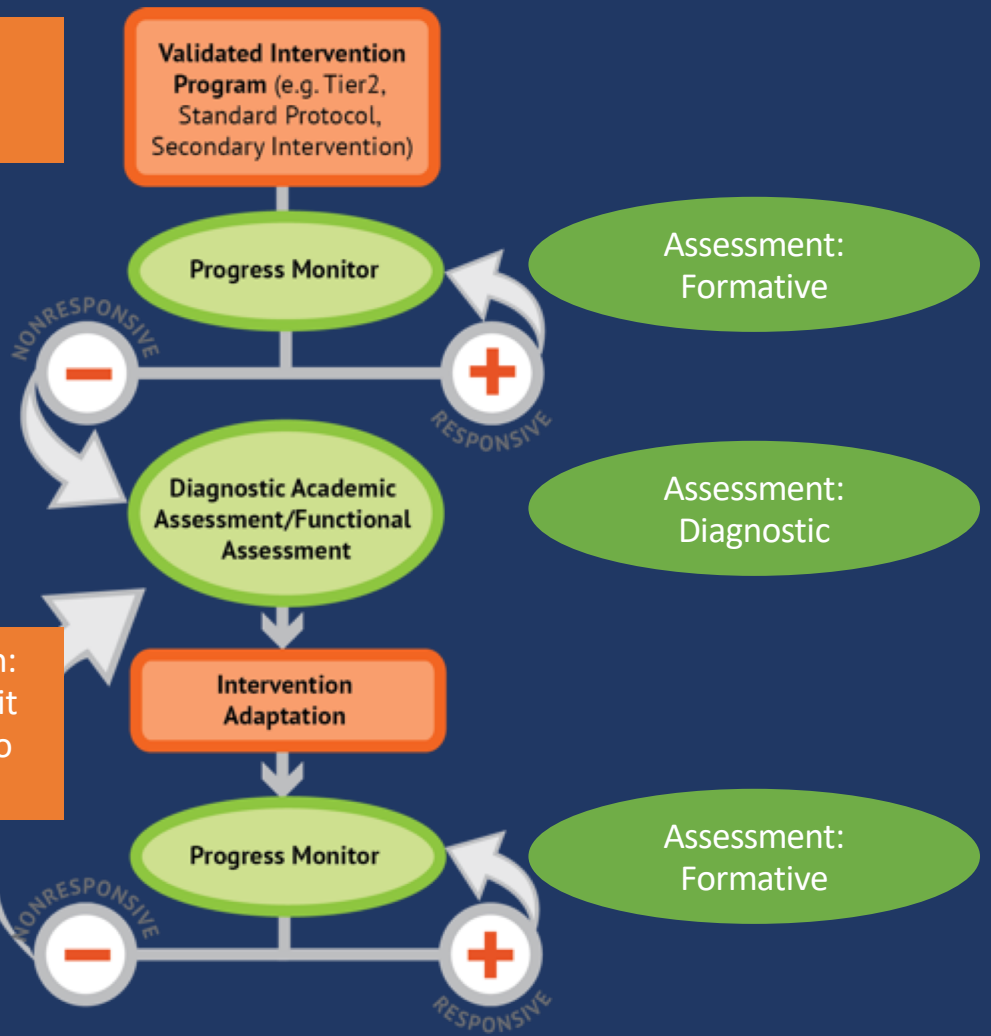


Wei, Lenz, & Blackorby (2013)



Wei, Lenz, & Blackorby (2013)

Intervention: Occurs frequently and with intensity

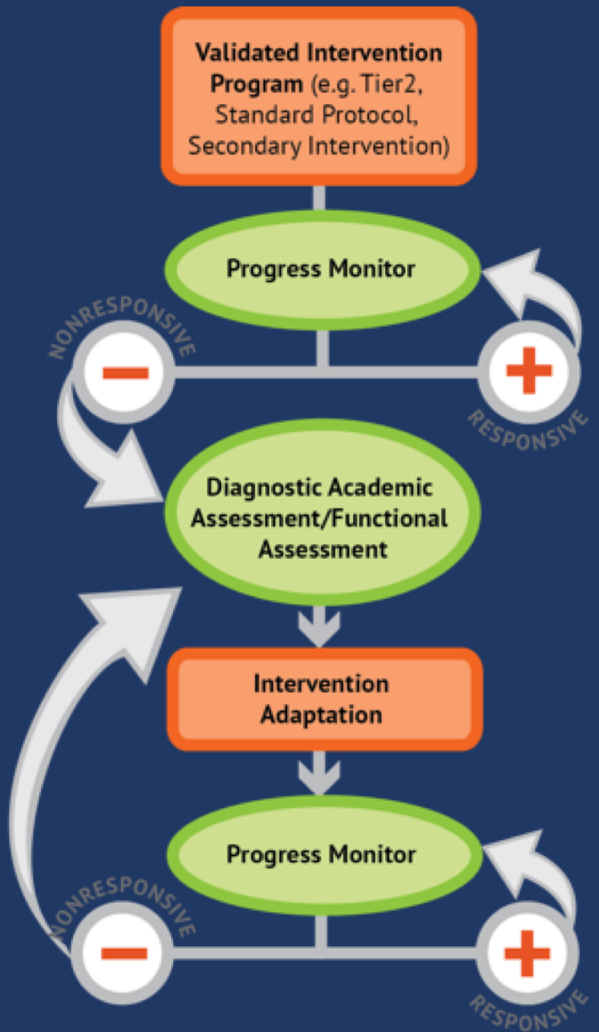


Assessment: Formative

Assessment: Diagnostic

Intervention adaptation: Content, dosage, explicit instruction, attention to transfer

Assessment: Formative



Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Concise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction
- Motivation component

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Modeling

Clear Explanation

Planned Examples

Goal and importance

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

"Let's continue working with our three-dimensional shapes and volume. Understanding volume and calculating volume helps with measuring capacity."

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

"To solve 26 plus 79, I first decide about the operation. Do I add, subtract, multiply or divide?"

"The plus sign tells me to add. So, I'll add 26 plus 79. I'll use the partial sums strategy. First, I add 20 plus 70. What's 20 plus 70?"

"20 plus 70 is 90. I write 90 right here."

"Then I add 6 plus 9. What's 6 plus 9?"

"6 plus 9 is 15. So, I write 15 here."

"Finally, we add the partial sums. What do we add?"

"So, we add the partial sums of 90 and 15. 90 plus 15 is 105. So, 26 plus 79 equals 105."

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

Concise language

"To solve 26 plus 79, I first decide about the **operation**. Do I **add**, **subtract**, **multiply**, or **divide**?"

"The **plus sign** tells me to **add**. So, I'll **add** 26 **plus** 79. I'll use the **partial sums** strategy. First, I **add** 20 **plus** 70. What's 20 **plus** 70?"

"20 **plus** 70 is 90. I write 90 right here."

"Then I **add** 6 **plus** 9. What's 6 **plus** 9?"

"6 **plus** 9 is 15. So, I write 15 here."

"Finally, we **add** the **partial sums**. Why do we **add** the **partial sums**?"

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

Concise language

Examples

"Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends."

$24 / 6$ $28 \div 7$ $35 \overline{)\,5}$

Modeling

Clear Explanation

Planned Examples

Goal and importance

Model steps

Concise language

Examples

With non-examples

$32 \div 8$ $42 \div 7$ $25 - 5$

Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supporting Practices

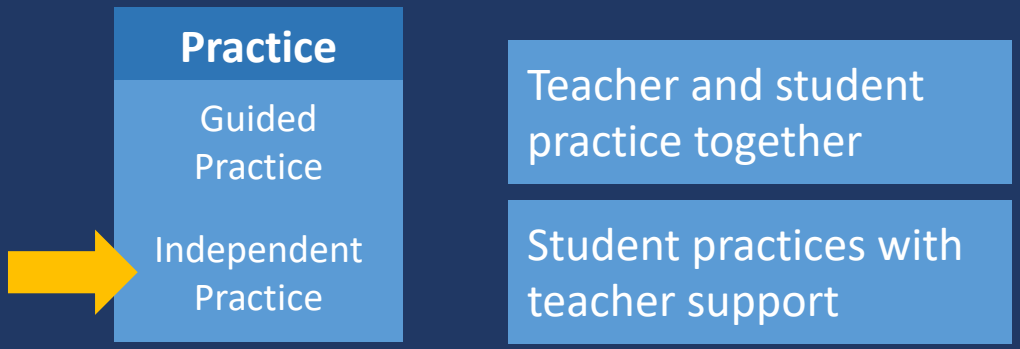
- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Practice

Guided Practice

Independent Practice

Teacher and student practice together



Modeling	Practice
Clear Explanation	Guided Practice
Planned Examples	Independent Practice

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

"What is 7 times 9?"

"Which shape has 6 sides?"

"What do you do when you see a word problem?"

"Why do you have to regroup?"

"How would you solve this problem?"

"Why do you have to use zero pairs?"

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

"Turn and discuss the formula for perimeter with your partner."

"Write the multiplication problem on your whiteboard."

"In your math journal, draw a picture to help you remember to term *parallelogram*."

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

"Good work using your word-problem attack strategy."

"Let's look at that again. Tell me how you added in the hundreds column."

Supporting Practices

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.

Affirmative and corrective

Planned and organized

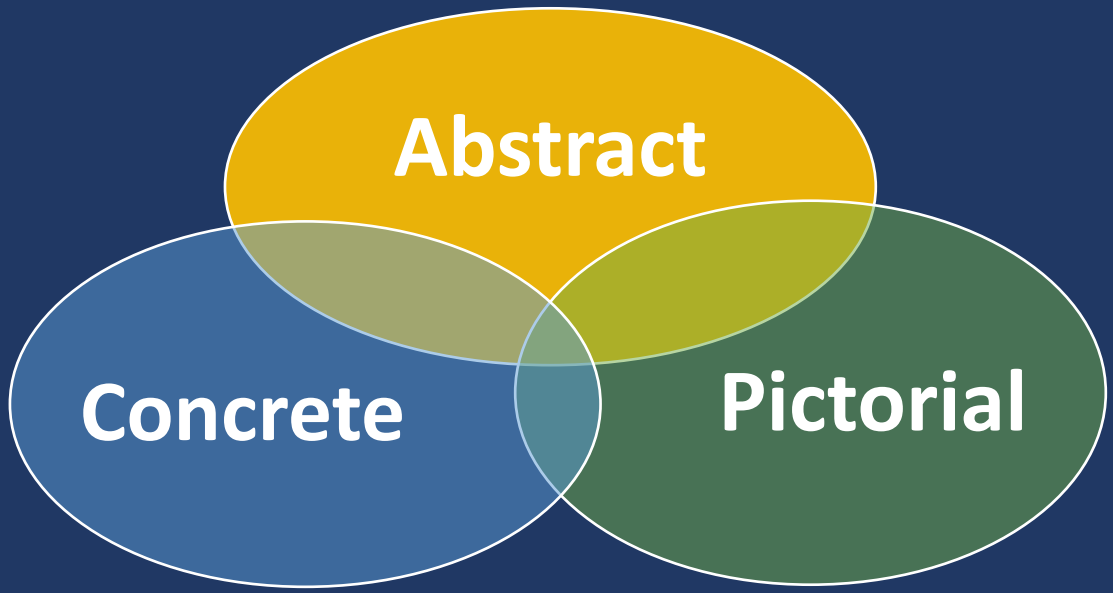
Introduction of material

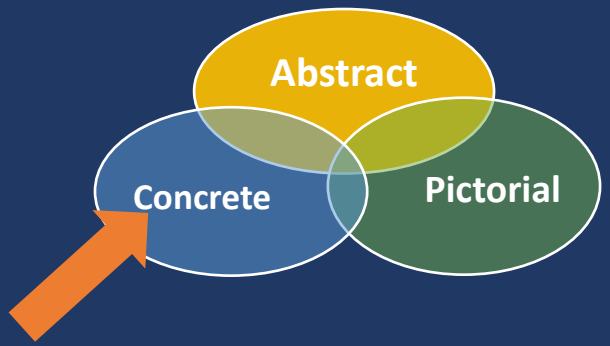
Review of material

How do you use explicit instruction within intensive intervention?

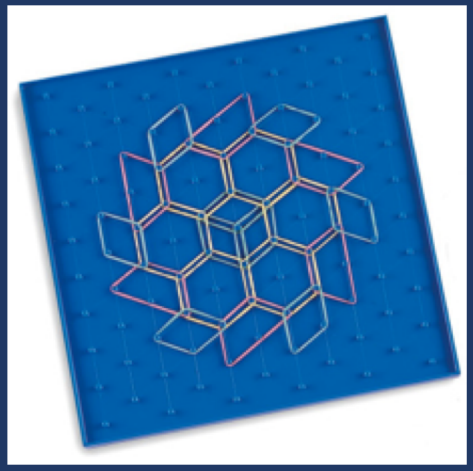
- ❑ Model steps using concise language
- ❑ Provide guided practice opportunities
- ❑ Provide independent practice opportunities
- ❑ Use supporting practices during modeling and practice
  - ❑ Ask the right questions
  - ❑ Elicit frequent responses
  - ❑ Provide feedback
  - ❑ Be planned and organized

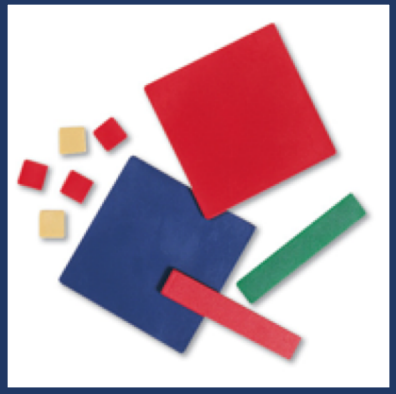
Multiple Representations

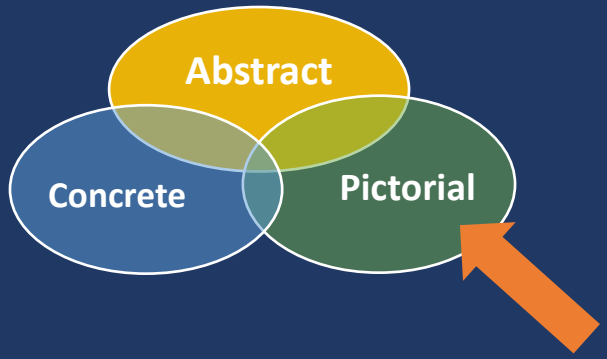




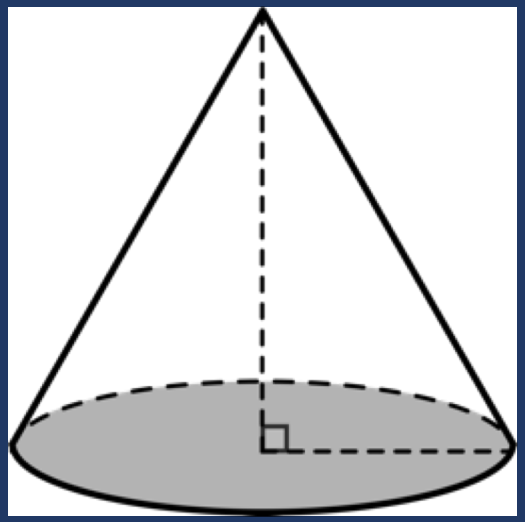
Three-dimensional objects

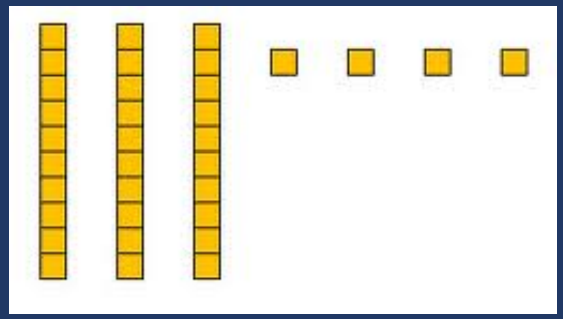


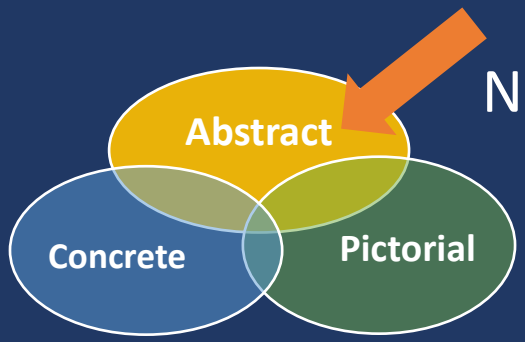




Two-dimensional images







Numerals, symbols, and words

$2 + 8 = 10$

34 = 3 tens and 4 ones

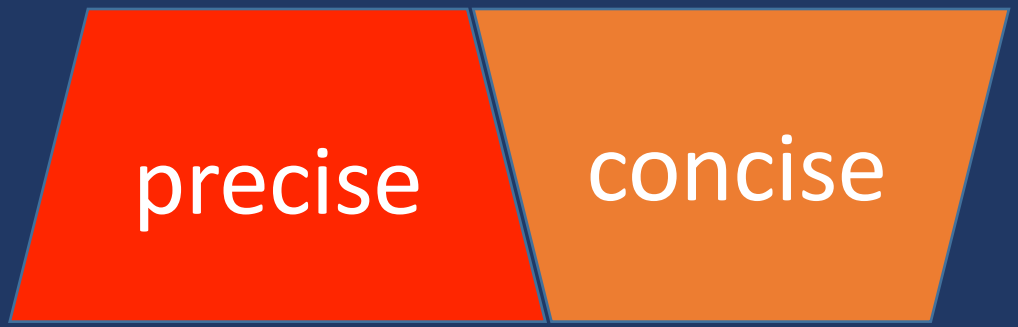
$x - 6 = 8$

$$\begin{array}{r} 4{,}179 \\ +\ \ 569 \\ \hline \end{array}$$

How should multiple representations be used within intensive intervention?

- Use three-dimensional concrete materials to teach concepts and procedures
- Use two-dimensional representations to teach concepts and procedures
- Ensure students understand mathematics with numbers and symbols (i.e., the abstract)

Focus on Language



Language of Mathematics

- Technical terms

trapezoid	rhombus	numerator	addend	subtract

- Subtechnical terms

base	degrees	cube	plane	arc

- Symbolic terms

plus	zero	twelve	dollars	and

- General terms

above	measure	answer	longest	outside

Instead of…	Say…
"And the last one is 10."	"8, 9, 10. We'll stop counting there but we could count more."
"What number is in the tens place?"	"What digit is in the tens place?"
"Six hundred and forty-eight"	"Six hundred forty-eight"
"Bigger number and smaller number"	"Number that is greater and the number that is less"

Hughes et al. (2016)

Instead of…	Say…
"Numbers in the fraction"	"This fraction is one number."
"Top number and bottom number"	"Numerator and denominator"
"Reduce"	"Find an equivalent fraction"
"One point two nine"	"One and twenty-nine hundredths"

Hughes et al. (2016)

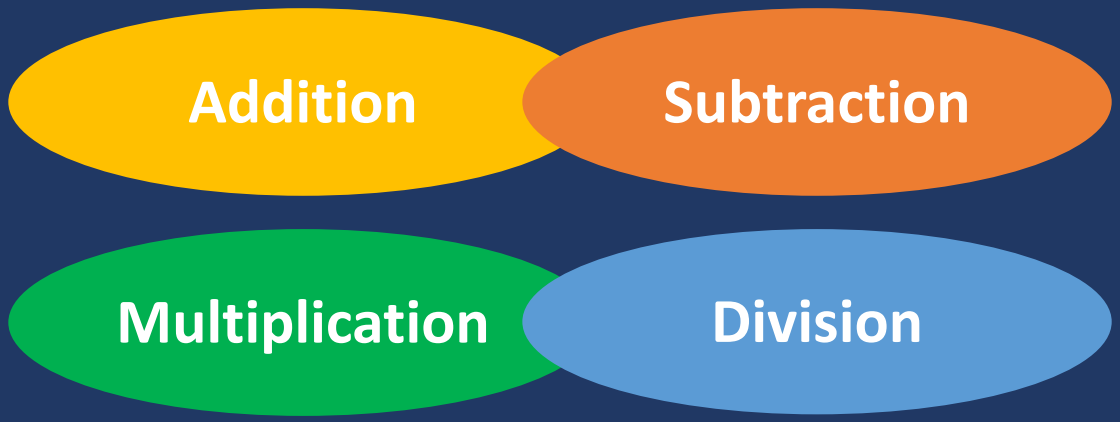
Instead of…	Say…
“Corner”	“Angle”
“Flips, slides, and turns”	“Reflections, translations, and rotations”
“Box or ball”	“Cube or sphere”
“Long hand and short hand”	“Minute hand and hour hand”

Hughes et al. (2016)

How do you attend to language within intensive intervention?

- ❑ Understand why formal mathematical language is important
- ❑ Plan for mathematical language to be precise
- ❑ Plan for mathematical language to be concise

Fluency



Addition

100 addition basic facts

- Single-digit addends sum to a single- or double-digit number

	5	**(addend)**
+	4	**(addend)**
	9	**(sum)**

Addition: Part-Part-Whole (Total)

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

Addition: Join (Change Increase)

Start with a set, add the other set, count sum

2 + 3 = 5

Subtraction

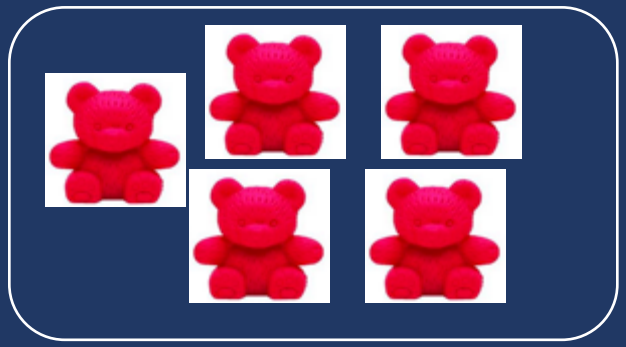
100 subtraction basic facts

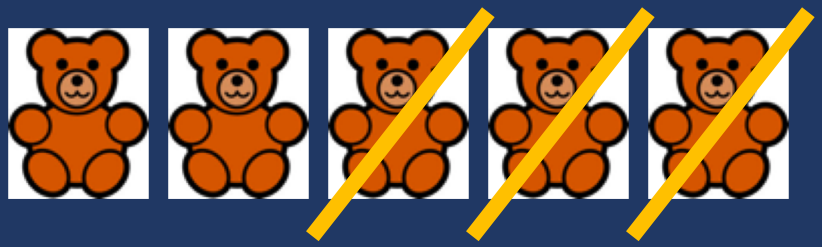
- Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

	16	**(minuend)**
–	8	**(subtrahend)**
	8	**(difference)**

Subtraction: Separate (Change Decrease)

Start with a set, take away from that set, count difference

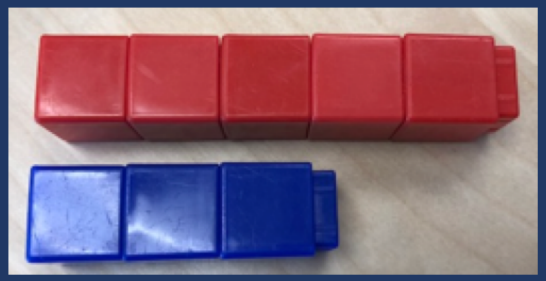


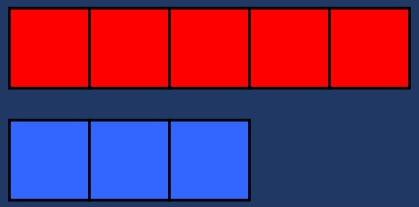


$$5 - 3 = 2$$

Subtraction: Compare (Difference)

Compare two sets, count difference





$$5 - 3 = 2$$

Multiplication

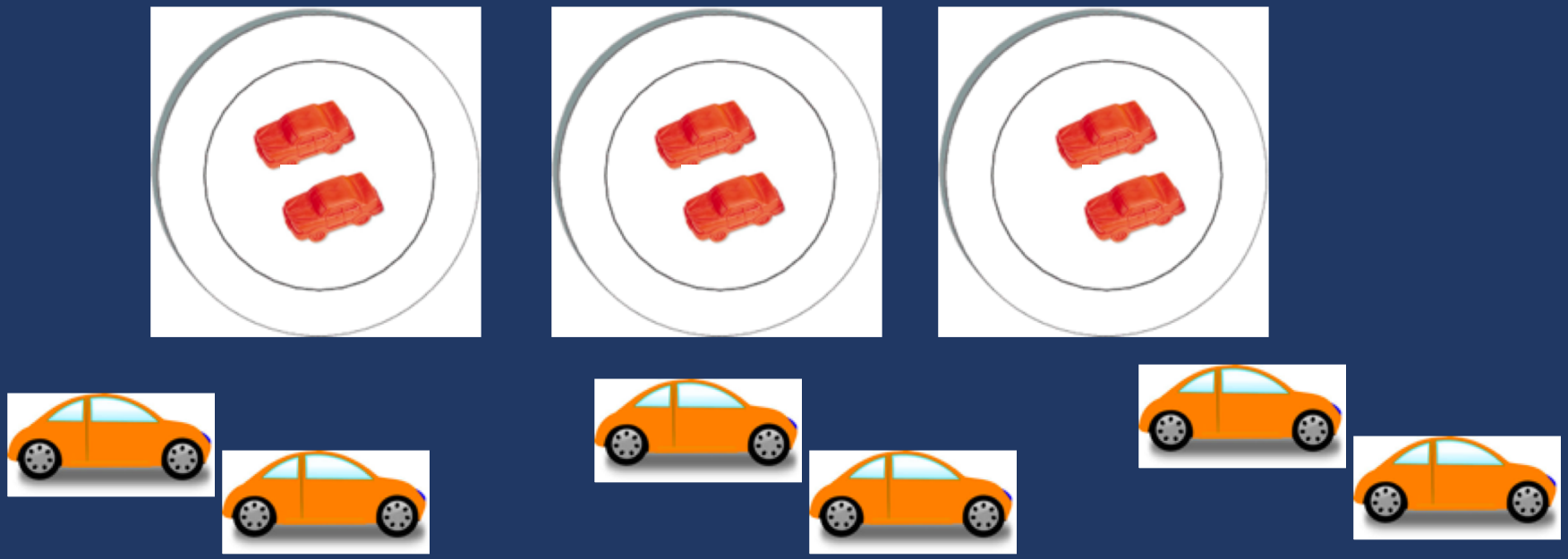
100 multiplication basic facts

- Multiplication of single-digit factors results in a single- or double-digit product

	2	(factor)
×	3	(factor)
	6	(product)

Multiplication: Equal Groups

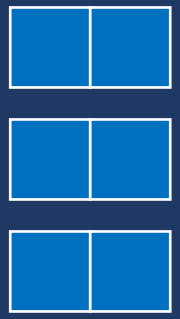
Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

Multiplication: Array/Area

Make the array, count product



$$3 \times 2 = 6$$

Multiplication: Comparison

Show a set, then multiply the set

$$3 \times 2 = 6$$

Division

90 division basic facts

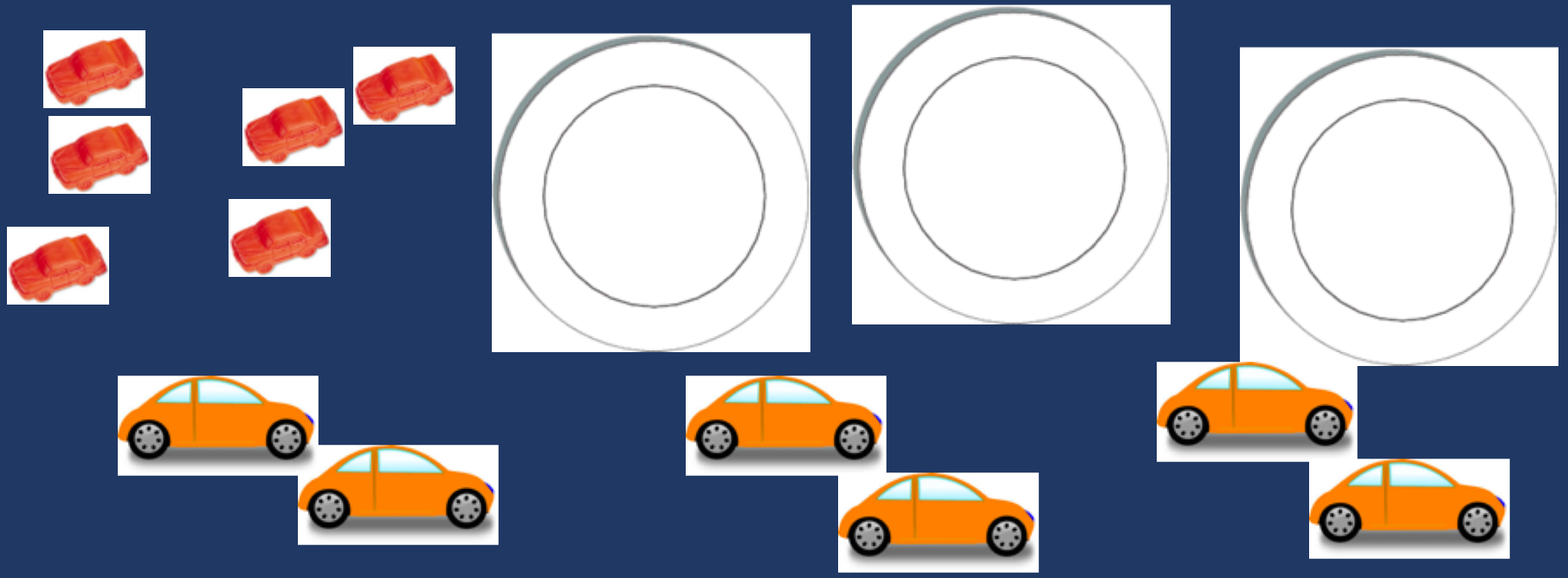
- Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

8 ÷ 4 = 2

(dividend) (divisor) (quotient)

Division: Equal Groups (Partitive Division)

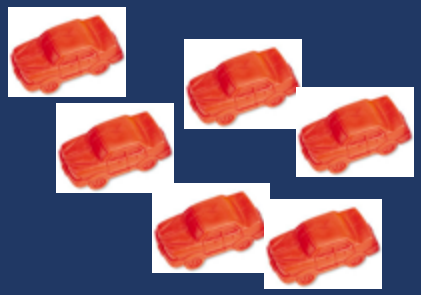
Show the dividend, divide equally among divisor, count quotient

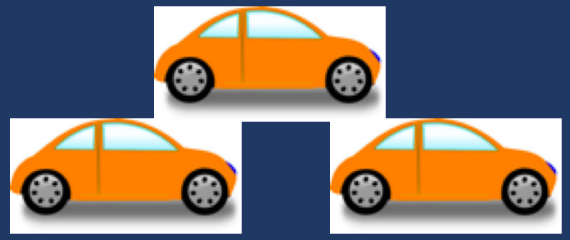


$$6 \div 3 = 2$$

Division: Equal Groups (Measurement Division)

Show the dividend, make groups of the divisor, count groups





$$6 \div 3 = 2$$

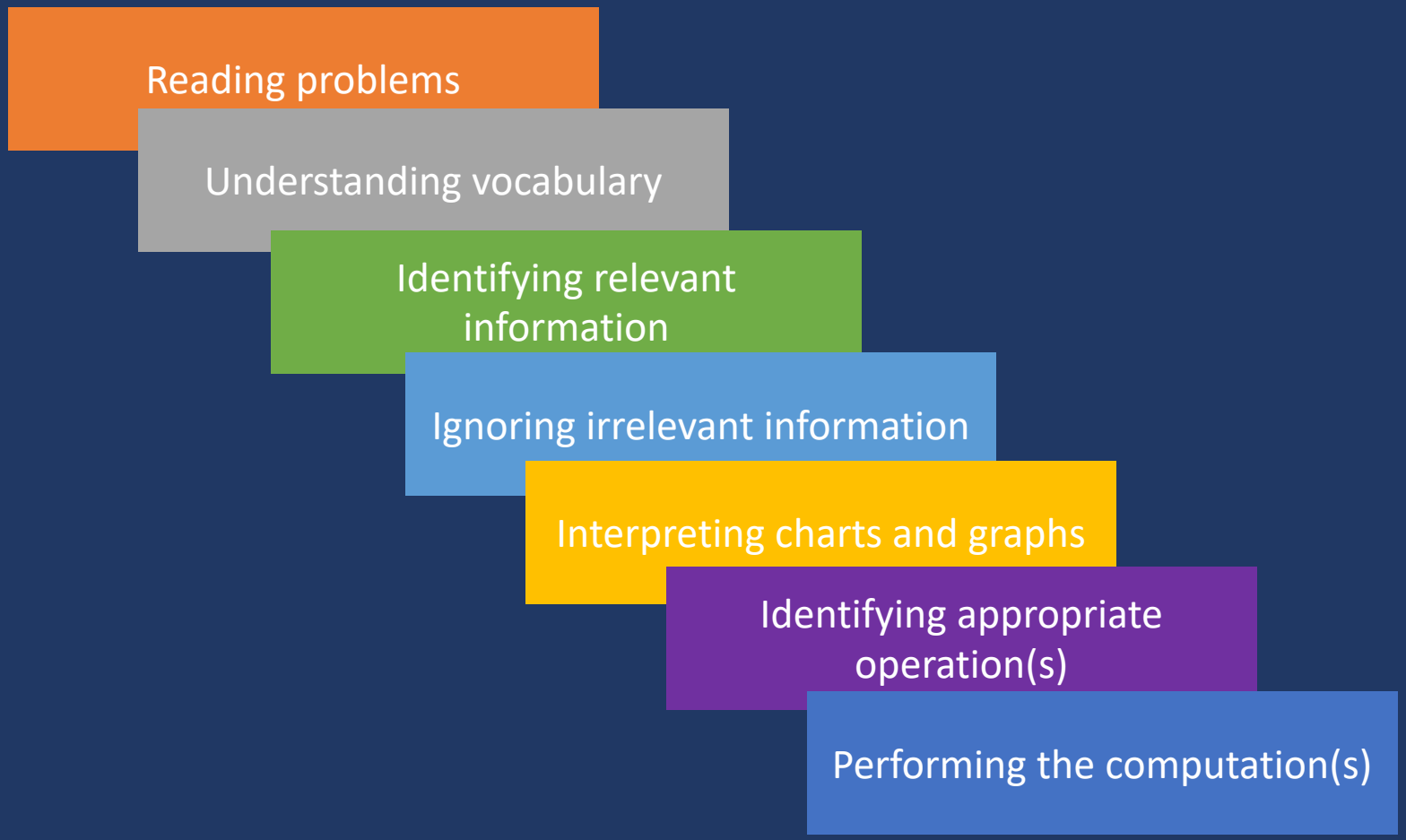
X	1	2	3	4	5

-3 + (-4) = ___ 5 – (-6) = ___

How to build fact fluency within intensive intervention?

- Teach the *concepts* of the operations
- Teach *strategies* to understand how facts fit together
- Practice building *fluency* with a variety of activities and games

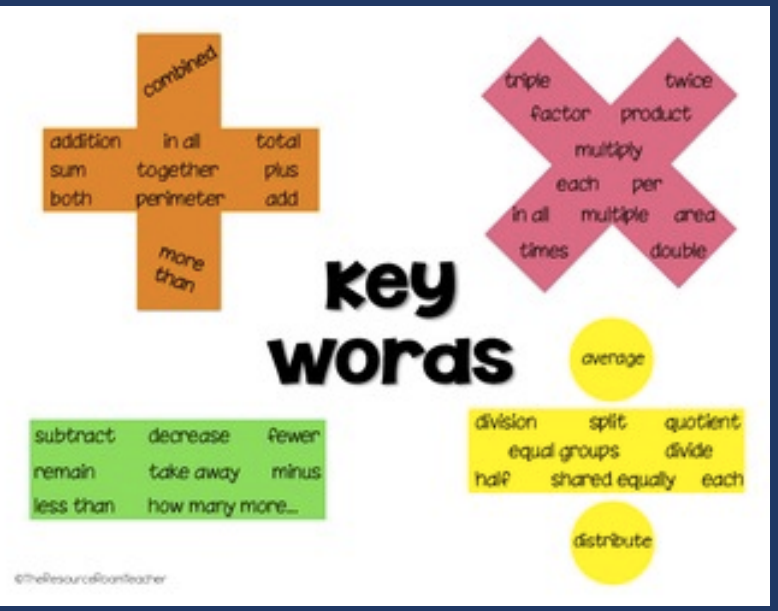
Problem Solving Difficulties



Don't tie **key words** to **operations**

Have an **attack strategy**

Teach word-problem **schemas**


combined
addition
in all
total
sum
together
plus
both
perimeter
add
more than
triple
twice
factor
product
multiply
each
per
in all
multiple
area
times
double
Key words
average
subtract
decrease
fewer
remain
take away
minus
less than
how many more...
division
split
quotient
equal groups
divide
half
shared equally
each
distribute
©TheResourceRoomTeacher


total
plus
in all
Addition
join
sum
together
What it looks like:
1 + 3 = 4
?
minus
Part
left
Subtraction
take away
fewer
What it looks like:
4 - 1 = 3
?


Key Words Used in Math Word Problems
Addition Words
+ add
+ all together or altogether
+ and
+ both
+ combined
+ how many in all
+ how much
+ in all
+ increased by
+ plus
+ sum
+ together
+ total
Subtraction Words
- change
- decreased by
- difference
- fewer or fewer than
- how many are left (or have left)
- how many did not have
- how many (or much) more
- how much longer (shorter, taller, heavier, etc.)
- less or less than
- lost
- minus
- need to
- reduce
- remain
- subtract
- take away
Multiplication Words
× by (dimension)
× double
× each group
× every
× factor of
× increased by
× multiplied by
× of
× product
× times
× triple
Division Words
÷ as much
÷ cut up
÷ each group has
÷ equal sharing
÷ half (or other fractions)
÷ how many in each
÷ parts
÷ per
÷ percent
÷ quotient of
÷ ratio of
÷ separated
÷ share something equally

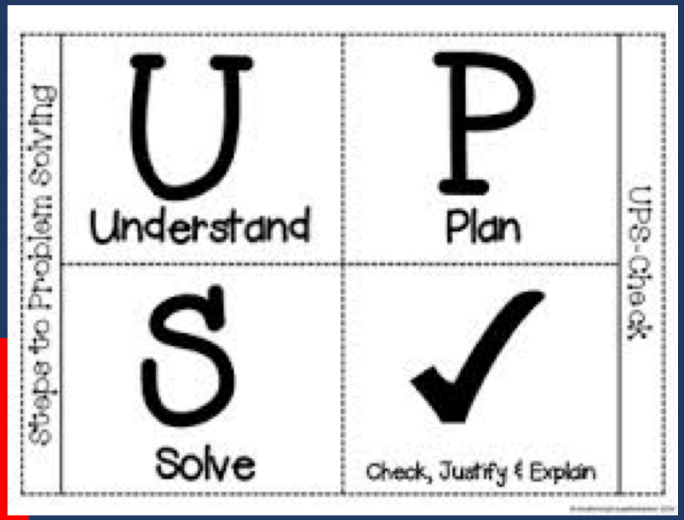
Problem Solving

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

SIGNS

Survey questions
Identify key words
Graphically draw problem
Note operations
Solve and check



Problem Solving

When teaching about word problems, students should learn the *schema* of the word problem.

- Total
- Difference
- Change
- Equal Groups
- Comparison
- Ratios/Proportions

How do you incorporate effective problem-solving strategies within intensive intervention?

- ❑ **Don't** use key words tied to operations
- ❑ **Do** teach students an *attack strategy*
- ❑ **Do** teach students *schemas*

- ❑ **Do** explicitly teach problem solving
- ❑ **Do** provide problem-solving instruction regularly (i.e., several times a week)
- ❑ **Do** practice schemas that students will encounter regularly

Motivation Component

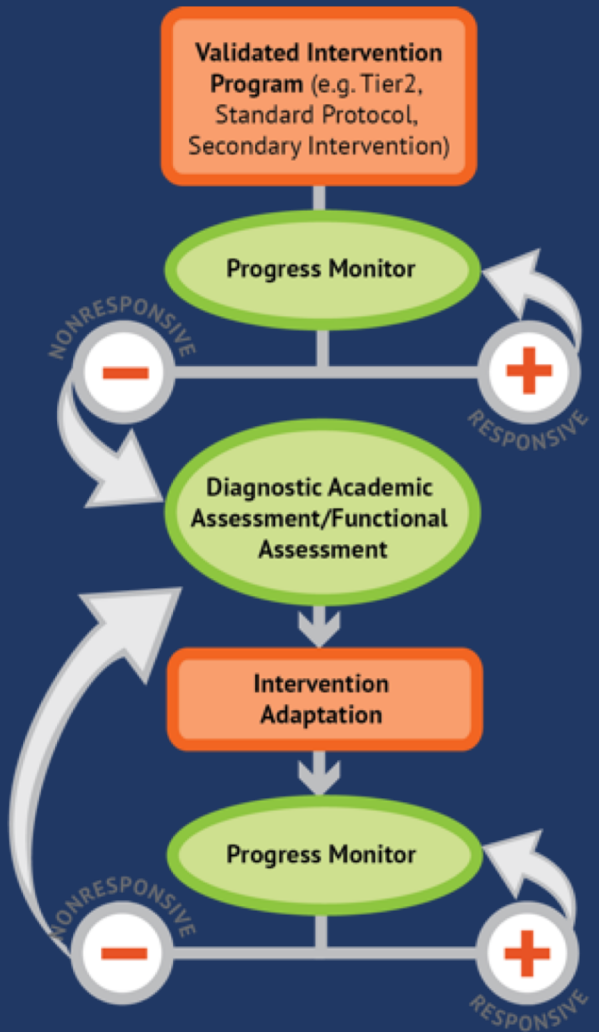
on task

keep attention

regulate behavior

How do you incorporate a motivational component within intensive intervention?

- Utilize a motivational component, when necessary



Instructional Platform

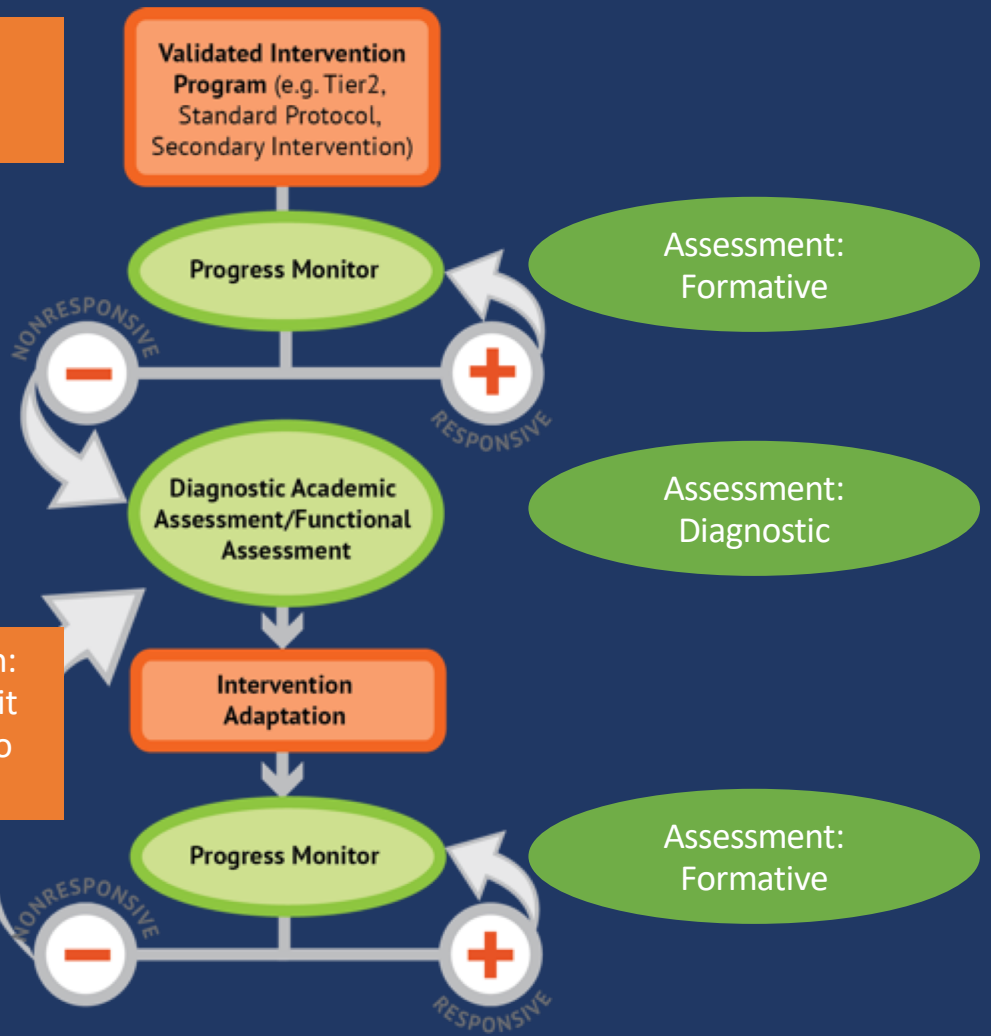
INSTRUCTIONAL DELIVERY

- Explicit instruction
- Concise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction
- Motivation component

Intervention: Occurs frequently and with intensity



Assessment: Formative

Assessment: Diagnostic

Intervention adaptation: Content, dosage, explicit instruction, attention to transfer

Assessment: Formative

Contact Information

Sarah Powell, Ph.D.

The University of Texas at Austin

srpowell@austin.utexas.edu

@sarahpowellphd