Math 1496 Calc I

Derivatives

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Standard Derv

$\frac{d(1)}{dx} = 0$ $\frac{d \sin x}{dx} = \cos x$ $\frac{d \cot x}{dx} = -\csc^2 x$

$\frac{d x^n}{dx} = n x^{n-1}$ $\frac{d \cos x}{dx} = -\sin x$ $\frac{d \sec x}{dx} = \sec x \tan x$

$\frac{d e^x}{dx} = e^x$ $\frac{d \tan x}{dx} = \sec^2 x$ $\frac{d \csc x}{dx} = -\csc x \cot x$

Rules

$\frac{d(f \pm g)}{dx} = \frac{df}{dx} \pm \frac{dg}{dx}$

$\frac{d(cf)}{dx} = c\frac{df}{dx}$

$(fg)' = f'g + fg'$ product

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ quotient

ex

$f(x) = e^x \sin x$

$f' = e^x \sin x + e^x \cos x$

$y = \frac{x+1}{2x-1}$

$y' = \frac{1(2x-1) - 2(x+1)}{(2x-1)^2}$

Note: $y = f(x)$ f', y' $\frac{df}{dx}$, $\frac{dy}{dx}$ all mean the same!

Consider

$y = (x^2+1)^2$ composition

thinking $y' = 2(x^2+1)$

$y = (x^2+1)(x^2+1) = x^4+2x^2+1$

$y' = 4x^3+4x^2$

$= 4x(x^2+1)$

missing $2x$

$y' = 2(x^2+1)(2x)$

$y = \sin(2x)$

$y' = \cos(2x)$?

$y = 2\sin x\cos x$

$y' = 2\cos x\cos x + 2\sin x(-\sin x)$

$= 2(\cos^2 x - \sin^2 x)$

no 2

$= \cos 2x \cdot 2$

$y = \sin 3x$ Dont really want to expand this

Going to make this into problems we can do

$$y = (x^2+1)^2$$
$\underbrace{\hphantom{x^2+1}}$ hard part

let $u = x^2+1$ so $y = u^2$

want $\frac{dy}{dx}$

But $\frac{dy}{du} = 2u$ $\frac{du}{dx} = 2x$

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$ ← chain Rule

so $\frac{dy}{dx} = 2u \cdot 2x$ ← what is u: $u = x^2+1$

$= 2(x^2+1) \cdot 2x$ like earlier

Ex

$y = \sin 2x$ let $u = 2x$ so $y = \sin u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos u \cdot 2$

$= \cos(2x) \cdot 2$

ex if $y = \sqrt{x^2+1}$ find y'

so let $u = x^2+1$ so $y = \sqrt{u} = u^{1/2}$

$$u' = 2x, \quad y' = \frac{1}{2}(u)^{-1/2}$$

$$y' = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{\cancel{2}} u^{-1/2} \cdot \cancel{2}x$$

$$= \frac{x}{\sqrt{u}} = \frac{x}{\sqrt{x^2+1}}$$

so $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{x^2+1}}$

ex $y = \left(\dfrac{x+1}{x-1}\right)^5$ ← could expand.

$$y' = 5\left(\frac{x+1}{x-1}\right)^4 \left[\frac{1(x-1) - 1(x+1)}{(x-1)^2}\right]$$

Ex $y = e^{\sin(3x)}$ 3 Level of compositi

(1) let $u = 3x$

so $y = e^{\sin u}$

(2) let $v = \sin u$ so $y = e^{v}$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dv} = e^{v}$$

$$\frac{dv}{dx} = \cos u$$

$$\frac{du}{dx} = 3$$

$$= e^{v} \cdot \cos u \cdot 3$$

$$= e^{\sin u} \cdot \cos u \cdot 3$$

$$= e^{\sin 3x} \cdot \cos 3x \cdot 3 \leftarrow \text{answer.}$$