Operations

November 11, 2022

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Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin

www.sarahpowellphd.com

srpowell@utexas.edu

@sarahpowellphd

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Say hello.

Describe the mathematics you support.

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November 2022

Operations

- Addition and subtraction concepts
- Multiplication and division concepts
- Computation with addition, subtraction, multiplication, and division

January 2023

Fractions

- Length, area, and set models
- Comparison of fractions
- Ordering of fractions
- Computation of fractions

March 2023

Word-Problem Solving

- Attack strategies
- Schemas

April 2023

Geometry

- Understanding two-dimensional shapes
- Lines and angles
- Understanding three-dimensional shapes

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Instructional Platform

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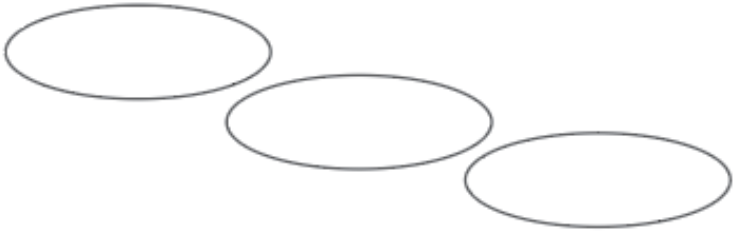
Operations

Sarah R. Powell, Ph.D.
srpowell@utexas.edu
@sarahpowellhd
www.sarahpowellphd.com

Instructional Platform

Instructional Delivery



Instructional Strategies

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Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

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MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

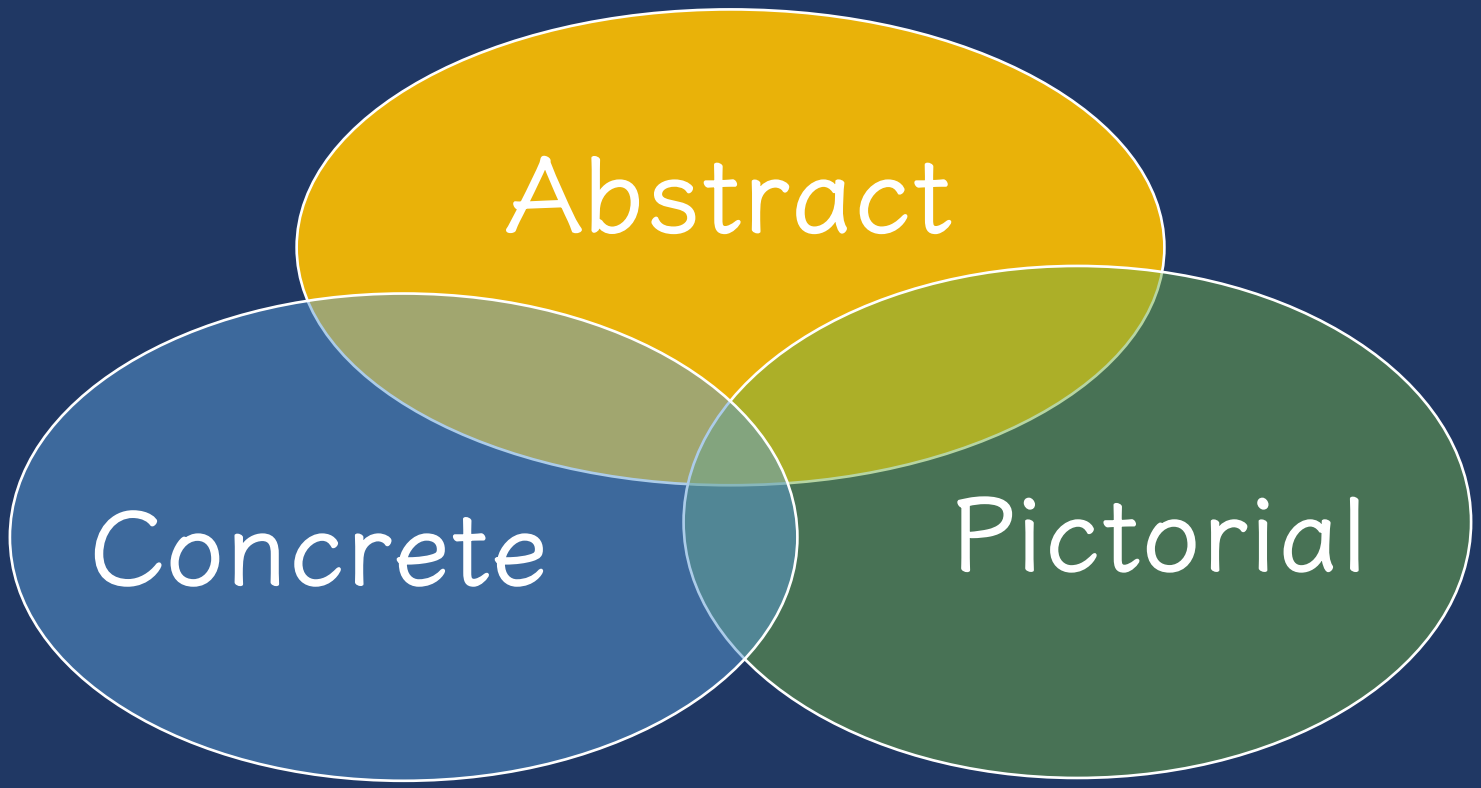
What math content do you model?

How do you engage students in guided practice?

Use formal math language

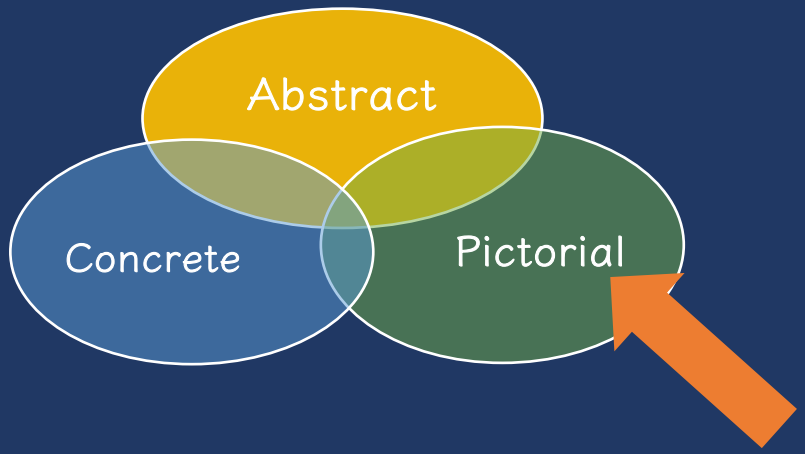
Use terms precisely

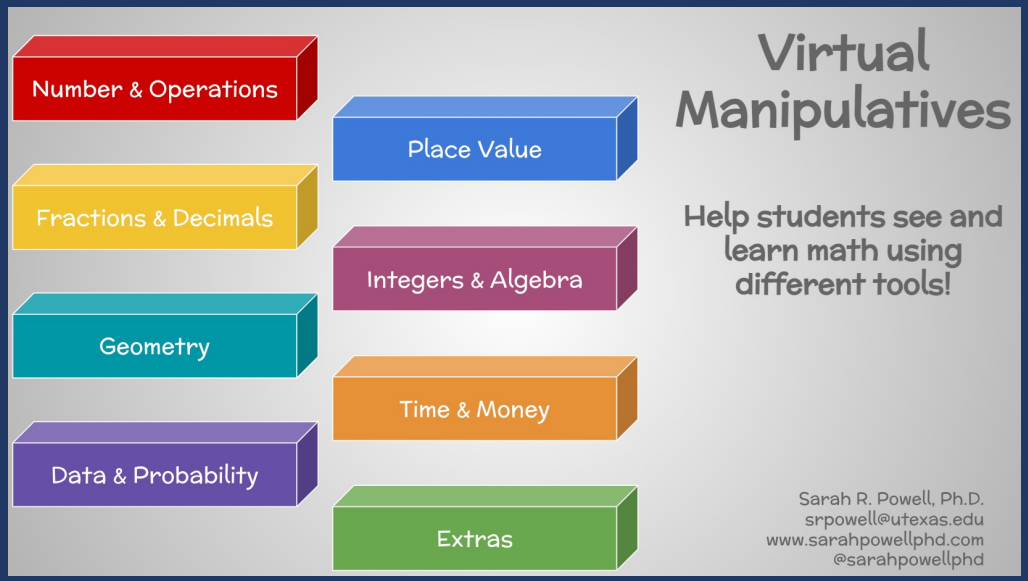
What's one way you support the math vocabulary of students?



What's a hands-on tool you use in your teaching?

What's a virtual manipulative you use?





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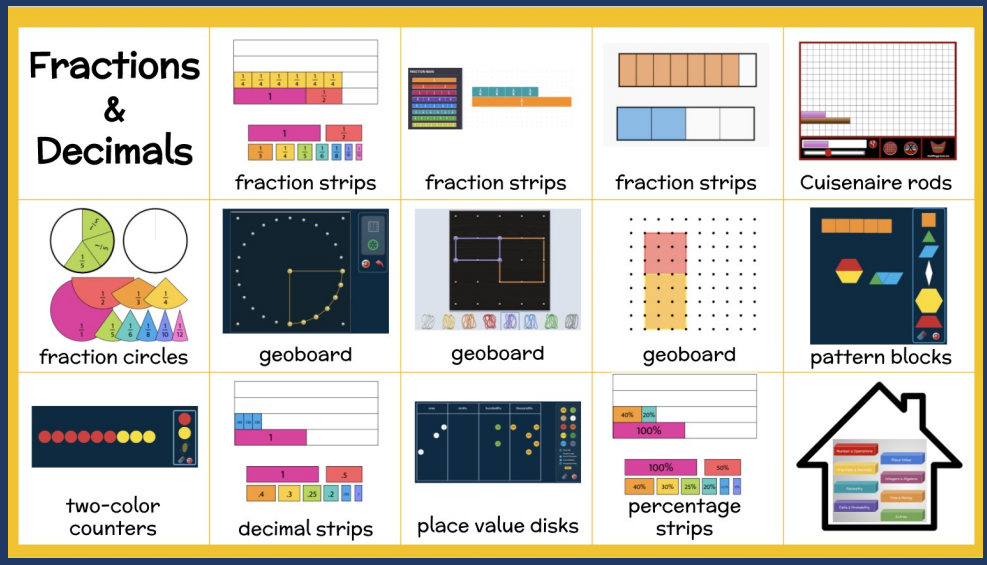

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Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Knowing multiples

Identifying shapes

Knowing formulas

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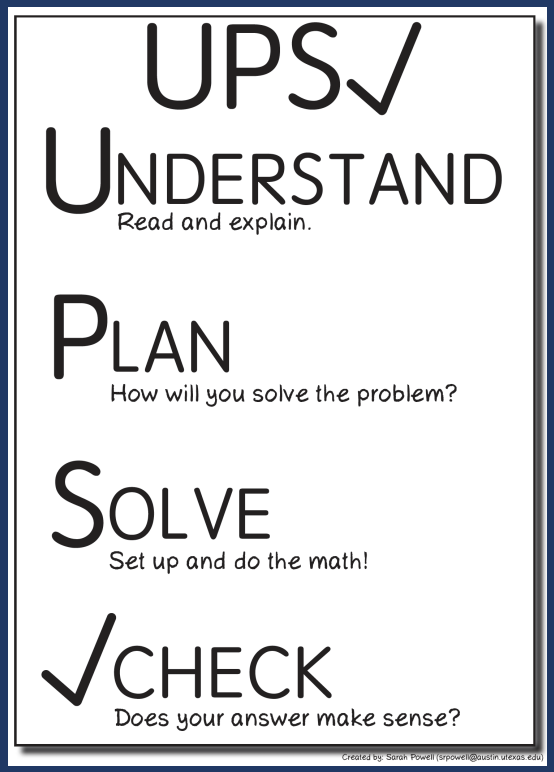
Addition	Subtraction
Multiplication	Division

How do you support students with fact fluency?



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

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Addition and Subtraction Concepts

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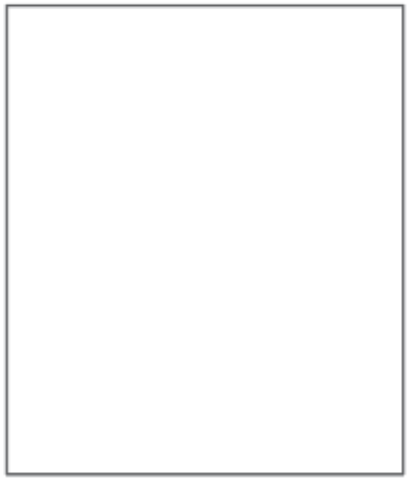
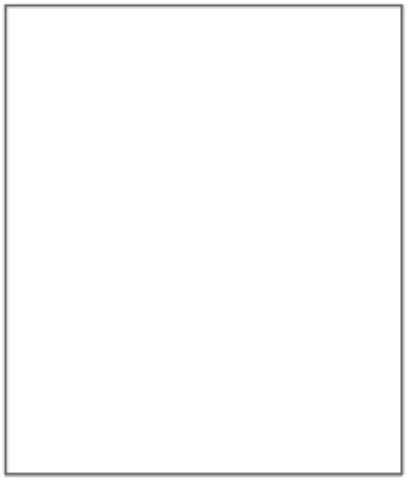
Addition





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What are the difficulties your students have with addition?

Addition

100 addition facts

Single-digit addends sum to a single- or double-digit number

$$\begin{array}{rl} 5 & \text{(addend)} \\ +\ 4 & \text{(addend)} \\ \hline 9 & \text{(sum)} \end{array}$$

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Total

Addition

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

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Total

Addition

Count one set, count another set, put sets together, count sum

Model:

4 + 5

9 + 3

Change

Addition

Start with a set, add the other set, count sum

$$2 + 3 = 5$$

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Change

Addition

Start with a set, add the other set, count sum

Model:

4 + 5

9 + 3

Total

Addition

Parts put together into a total

Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?

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Total

Addition

Parts put together into a total

Write a total story.

Change

Addition

An amount that **increases** or decreases

Premila had $4. Then they earned $5 for cleaning their room. How much money does Premila have now?

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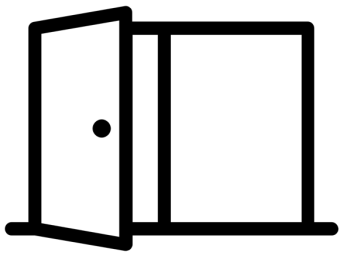
Change

Addition

An amount that **increases** or decreases

Write a change (increase) story.

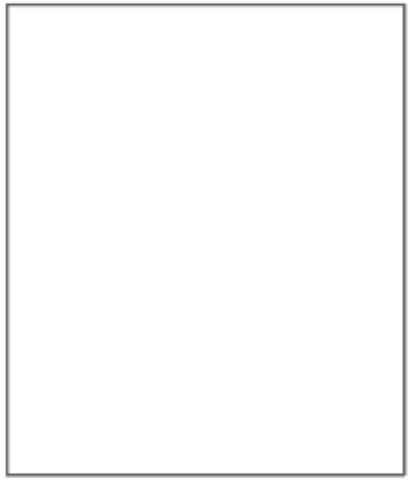


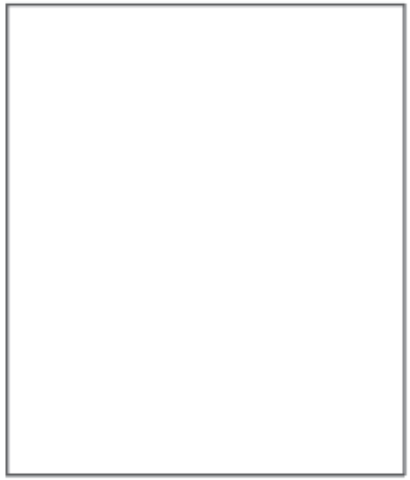
(1) Model 3 + 9 as a total problem.

(2) Model 3 + 9 as a change problem.

(3) Discuss how to distinguish between total and change.

Subtraction





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What are the difficulties your students have with subtraction?

Subtraction

100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{rl} 16 & \text{(minuend)} \\ -\ \ 8 & \text{(subtrahend)} \\ \hline 8 & \text{(difference)} \end{array}$$

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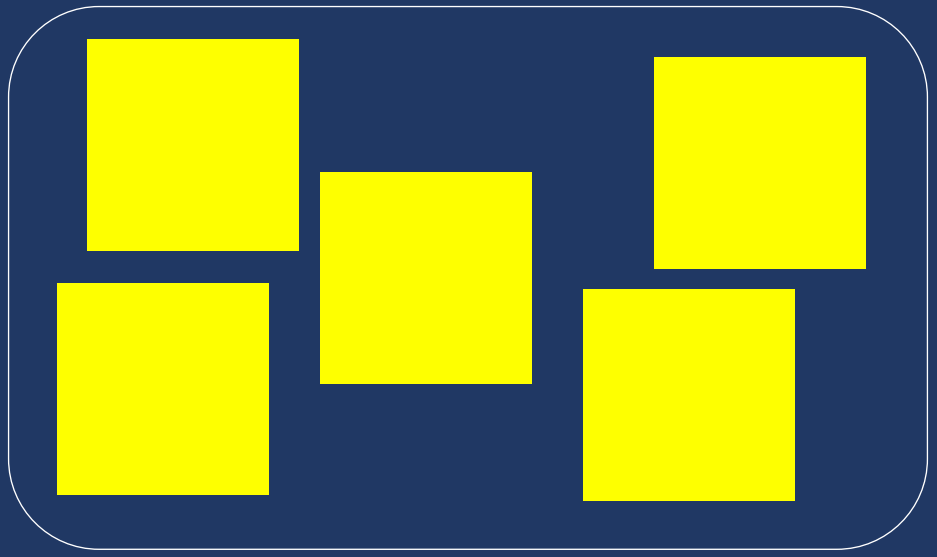
Change

Subtraction

Start with a set, take away from that set, count difference



$$5 - 3 = 2$$

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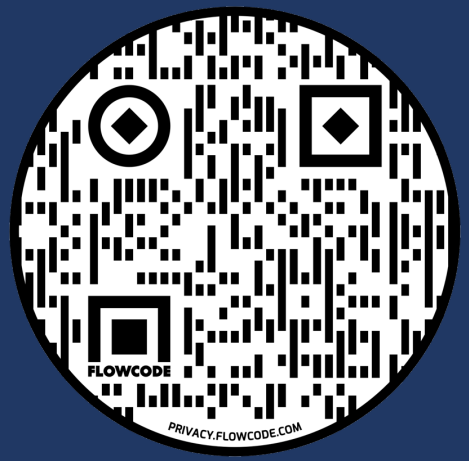
Change

Subtraction

Start with a set, take away from that set, count difference



Model:

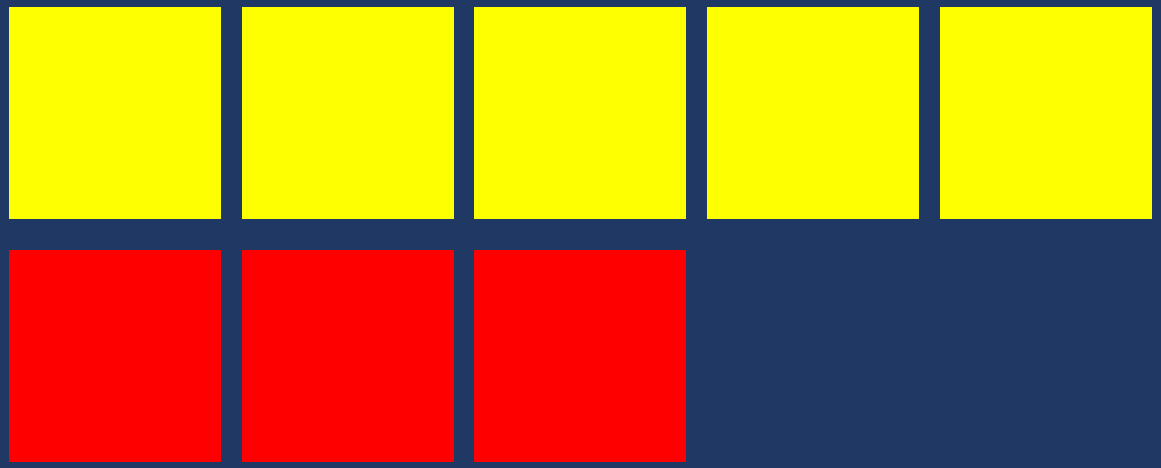
$9 - 3$

$11 - 7$

Difference

Subtraction

Compare two sets, count difference



$$5 - 3 = 2$$

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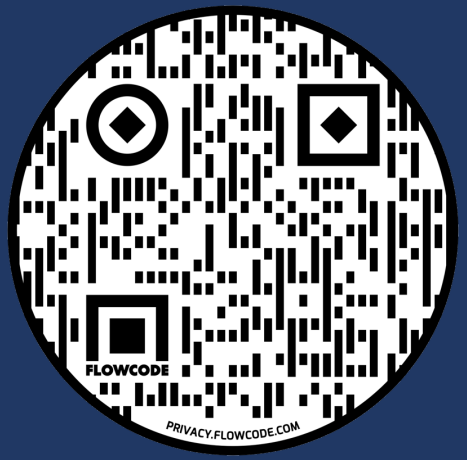
Difference

Subtraction

Compare two sets, count difference



Model:

$9 - 3$

$11 - 7$

Change

Subtraction

An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

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Change

Subtraction

An amount that increases or **decreases**

Write a change (decrease) story.

Difference

Subtraction

Greater and lesser amounts compared for a difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

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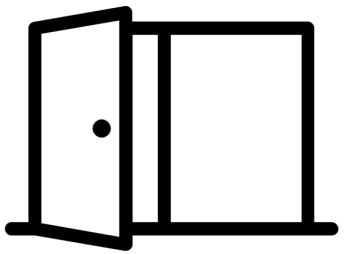
Difference

Subtraction

Greater and lesser amounts compared for a difference

Write a difference story.



(1) Model 12 – 5 as a change problem.

(2) Model 12 – 5 as a difference problem.

(3) Discuss how to distinguish between change and difference.

Multiplication

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What are the difficulties your students have with multiplication?

Multiplication

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{rl} 2 & \text{(factor)} \\ \times\ 3 & \text{(factor)} \\ \hline 6 & \text{(product)} \end{array}$$

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Equal Groups

Multiplication

Show the groups, show the amount for each group, count product

$$3 \times 2 = 6$$

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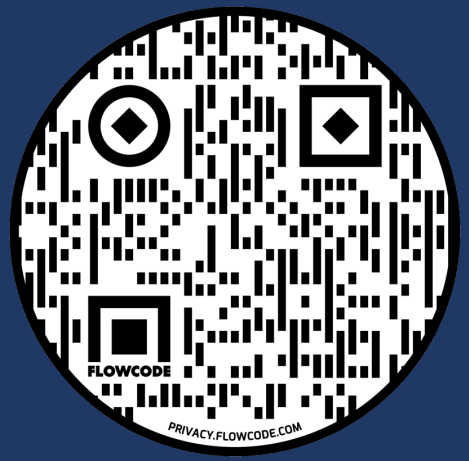
Equal Groups

Multiplication

Show the groups, show the amount for each group, count product



Model:

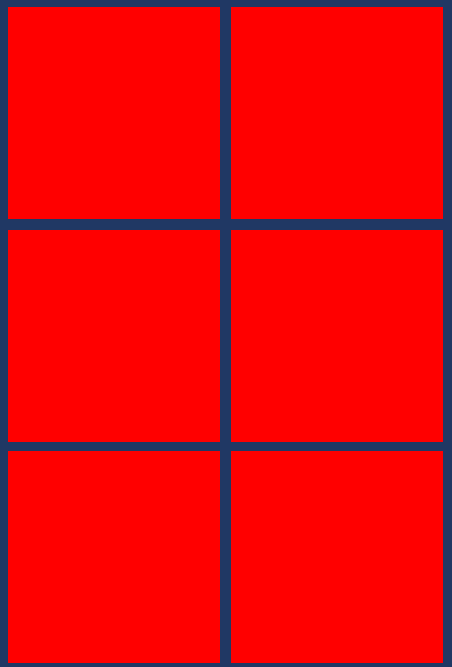
5×3

4×2

Equal Groups

Multiplication

Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

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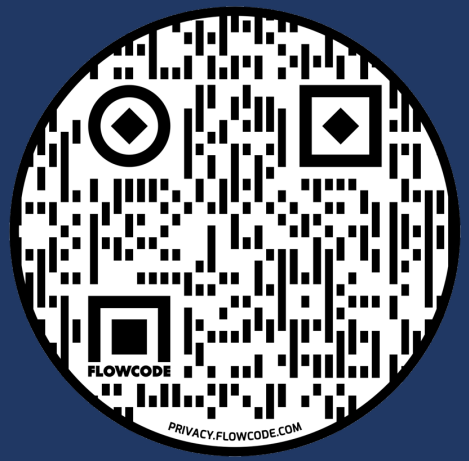
Equal Groups

Multiplication

Show the groups, show the amount for each group, count product



Model:

5×3

4×2

Comparison

Multiplication

Show a set, then multiply the set

$$3 \times 2 = 6$$

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Comparison

Multiplication

Show a set, then multiply the set

Model:

5×3

4×2

Equal Groups

Multiplication

Groups multiplied by number in each group for a product

Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?

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Equal Groups

Multiplication

Groups multiplied by number in each group for a product

Write an equal groups story.

Comparison

Multiplication

Set multiplied by a number of times for a product

Vivienne picked 12 apples. Jessica picked 2 times as many apples as Vivienne. How many apples did Jessica pick?

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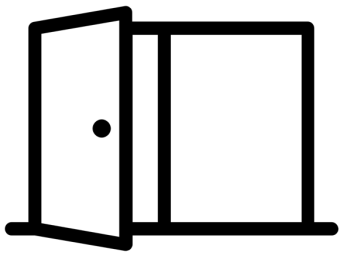
Comparison

Multiplication

Set multiplied by a number of times for a product

Write a comparison story.



(1) Model 2 × 5 as an equal groups problem.

(2) Model 2 × 5 as a comparison problem.

(3) Discuss how to distinguish between equal groups and comparison.

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

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MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

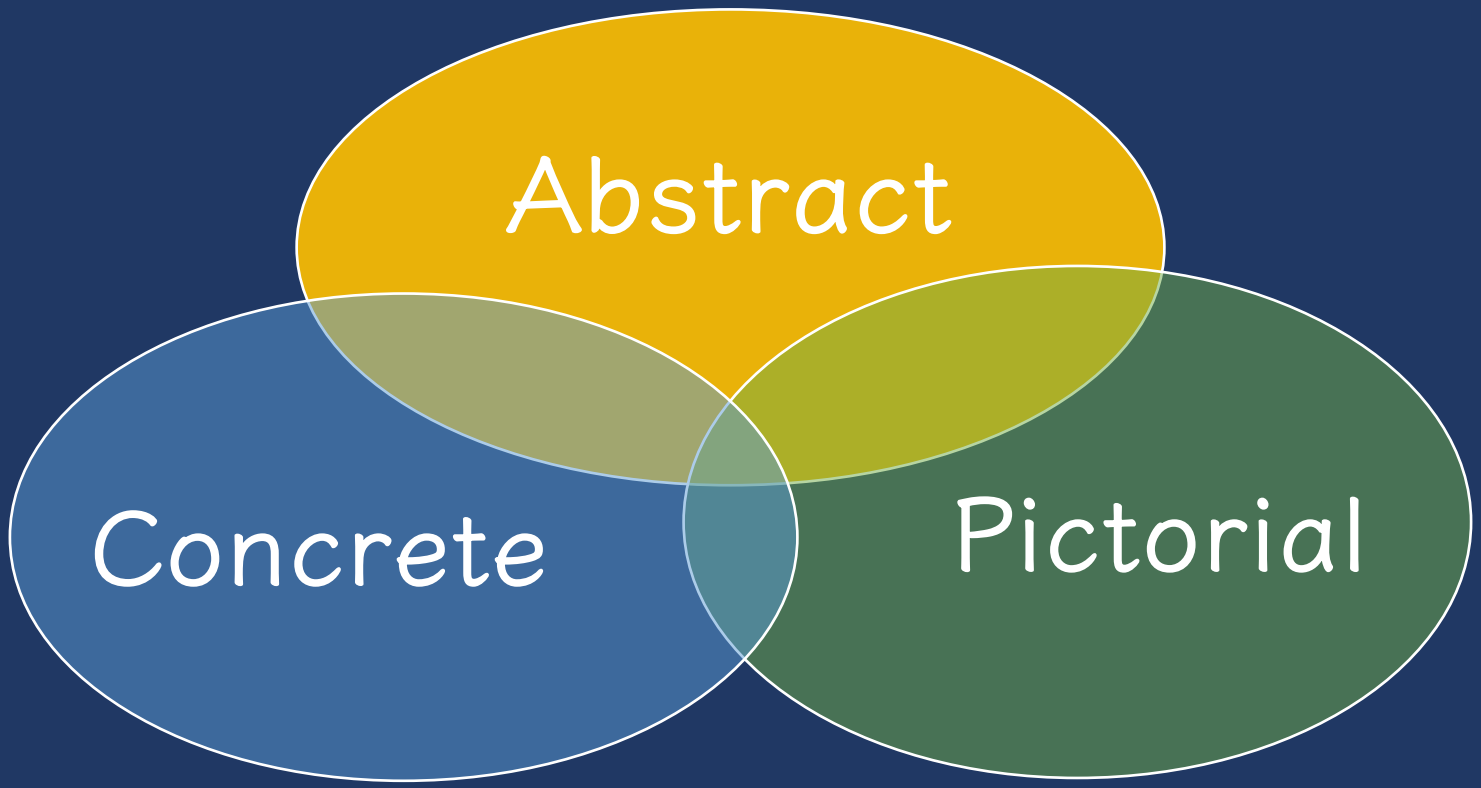
What are your strengths with modeling addition and subtraction?

What are your opportunities for growth?

Use formal math language

Use terms precisely

What are five essential math vocabulary for addition and subtraction?



What are the representations you'll use to teach addition and subtraction?

Division

90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

8	÷	4	=	2
(dividend)		(divisor)		(quotient)

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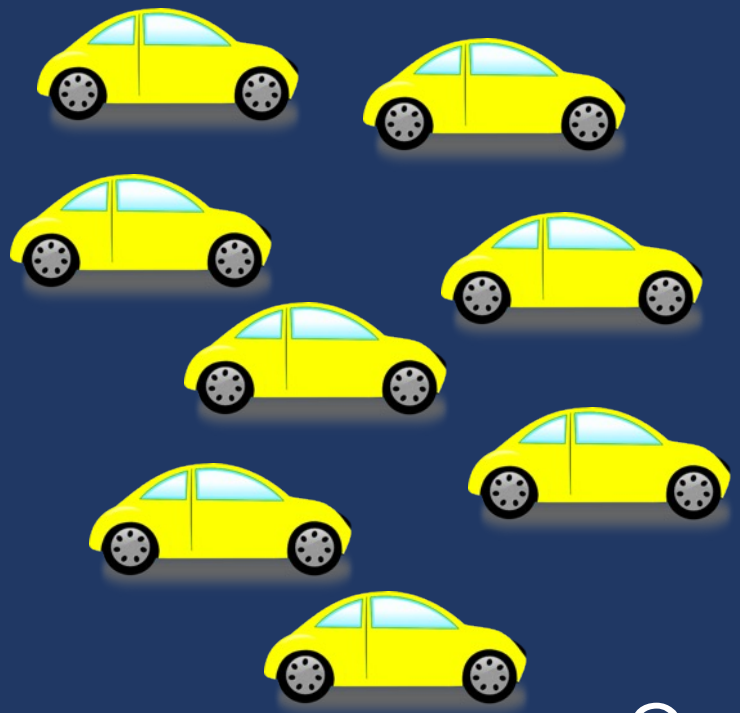
Equal Groups (Partitive Division)

Division

Show the dividend, divide equally among divisor, count quotient



$$8 \div 2 = 4$$

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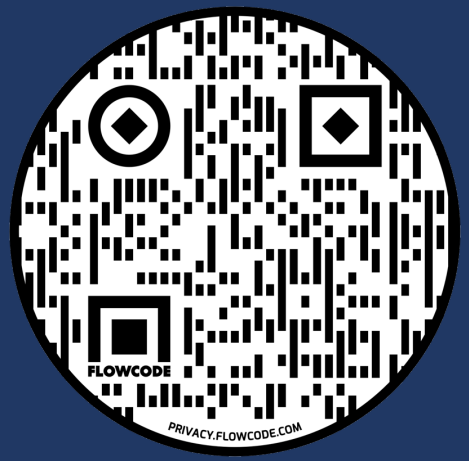
Equal Groups

(Partitive Division)

Division

Show the dividend, divide equally among divisor, count quotient



Model:

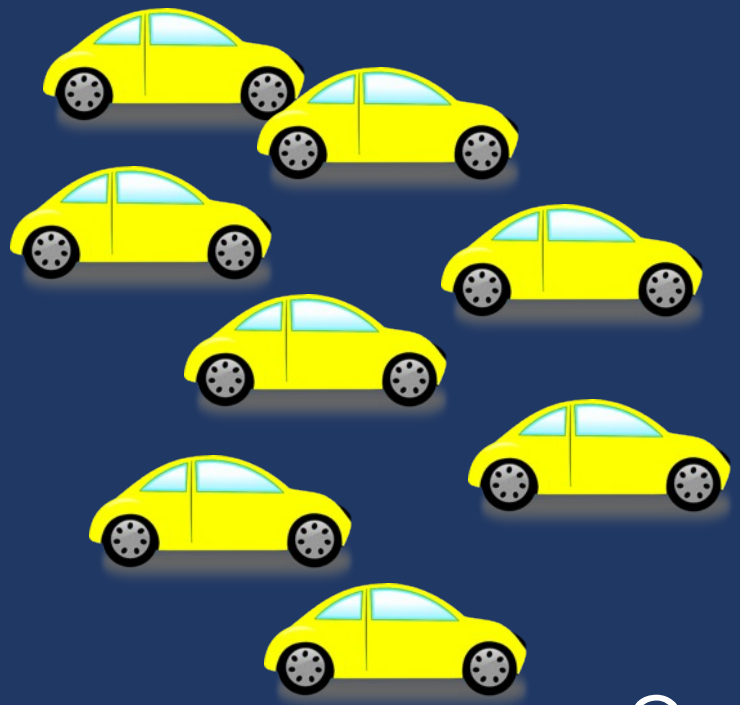
$10 \div 5$

$12 \div 3$

Equal Groups (Quotative Division)

Division

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

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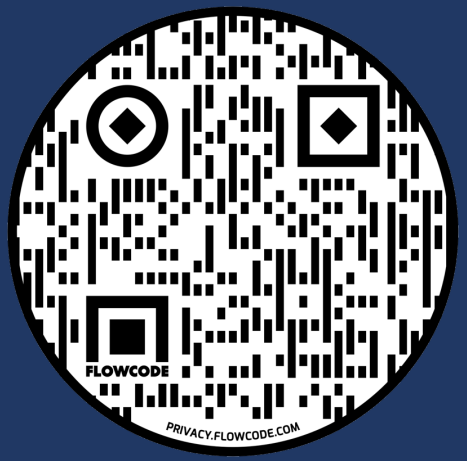
Equal Groups

(Quotative Division)

Division

Show the dividend, make groups of the divisor, count groups



Model:

$10 \div 5$

$12 \div 3$

Equal Groups (Partitive Division)

Groups multiplied by number in each group for a product

Stefanie has 12 apples. She wants to share them equally among her 2 friends. How many apples will each friend receive?

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Equal Groups (Partitive Division)

Groups multiplied by number in each group for a product

Write a partitive story.

Equal Groups (Quotative Division)

Groups multiplied by number in each group for a product

Nicole has 12 apples. She put them into bags with 6 apples each. How many bags did Nicole use?

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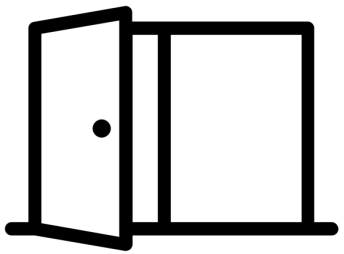
Equal Groups (Quotative Division)

Groups multiplied by number in each group for a product

Write a quotative story.



(1) Model 15 ÷ 3 as a partitive problem.

(2) Model 15 ÷ 3 as a quotative problem.

(3) Discuss how to distinguish between partitive and quotative.

Building Fluency

Fluency is doing mathematics *easily and accurately.*

Fluency in mathematics *makes mathematics easier.*

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both *conceptual learning and procedural learning.*

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Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Knowing multiples

Identifying shapes

Knowing formulas

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Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ 8 \\ \hline \end{array}$$

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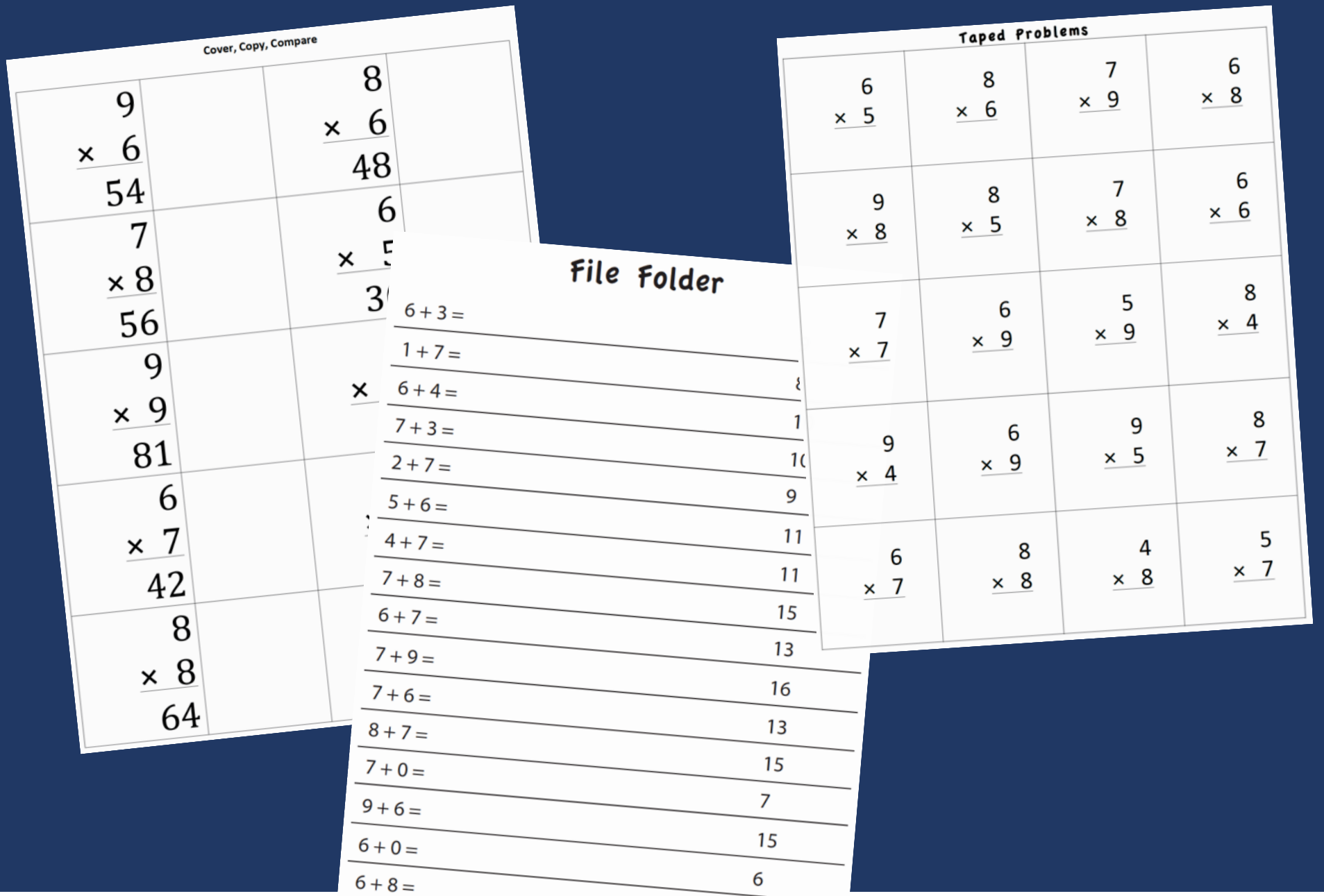

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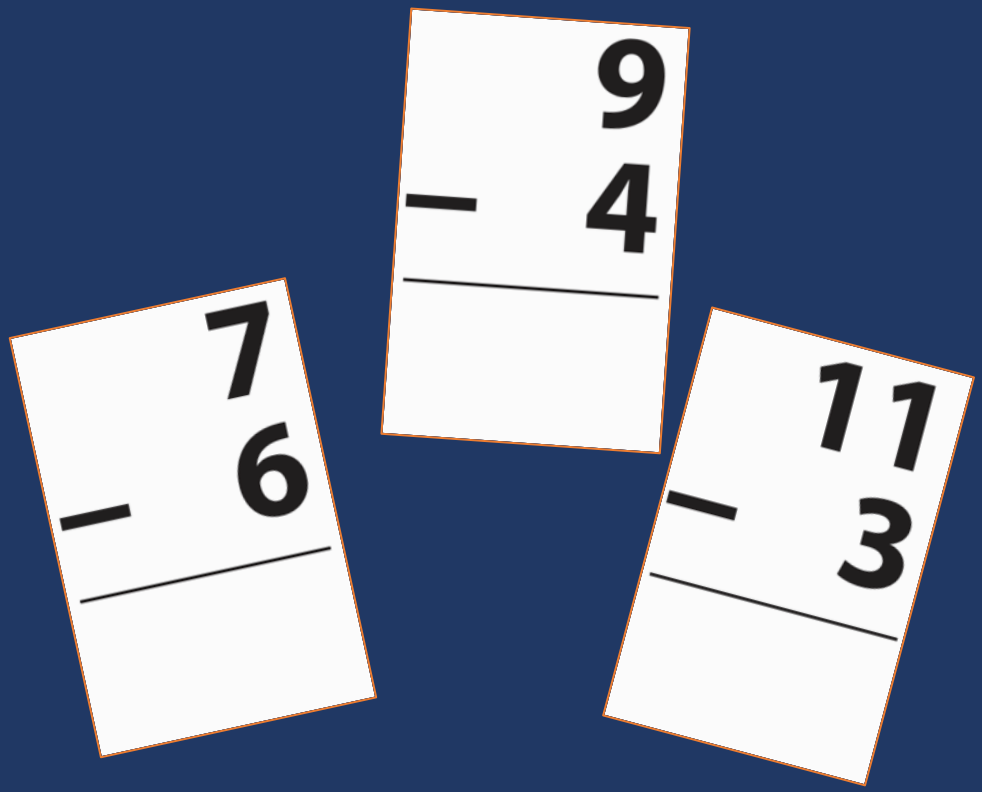

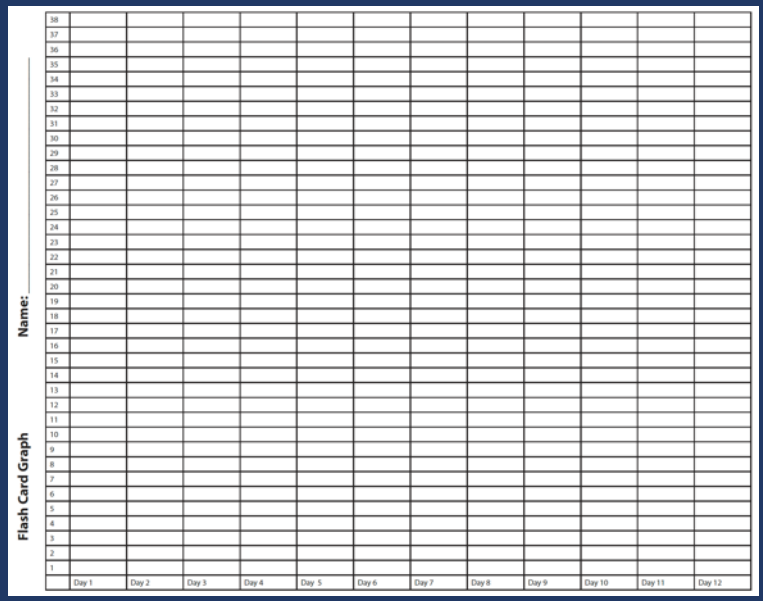

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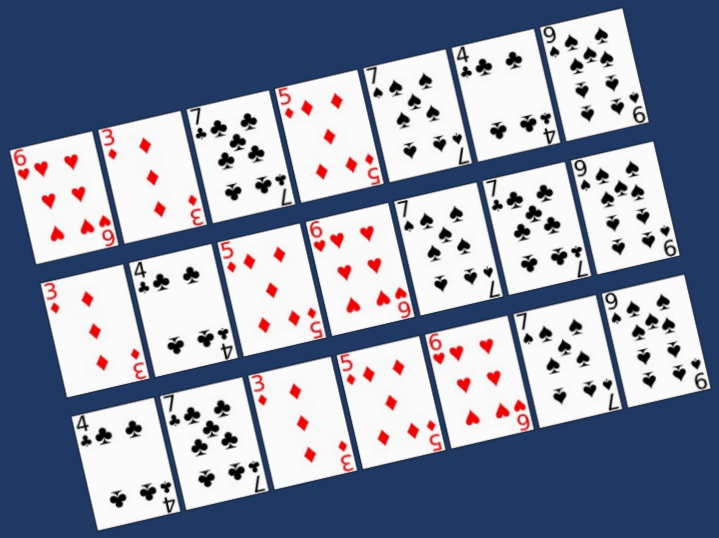

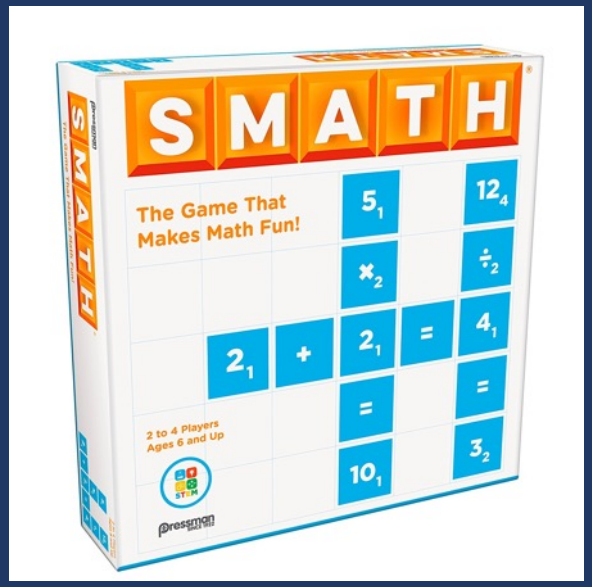

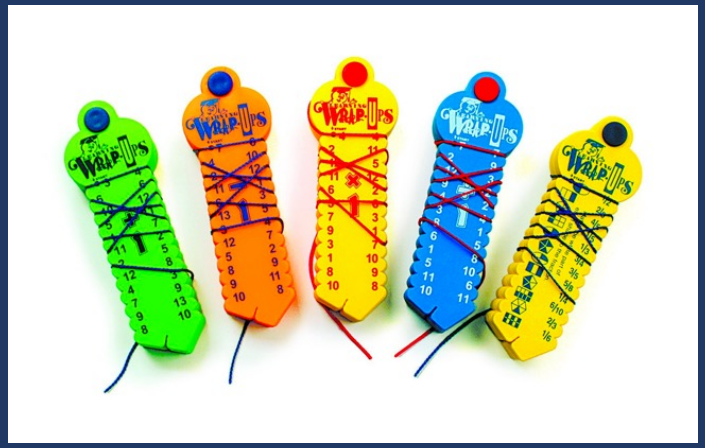

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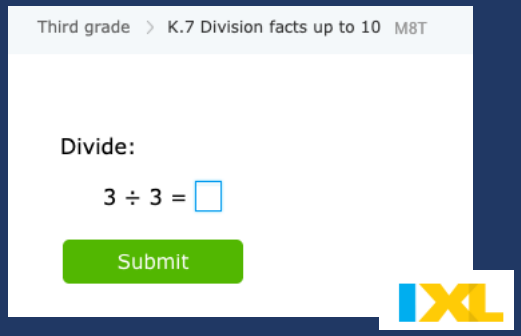

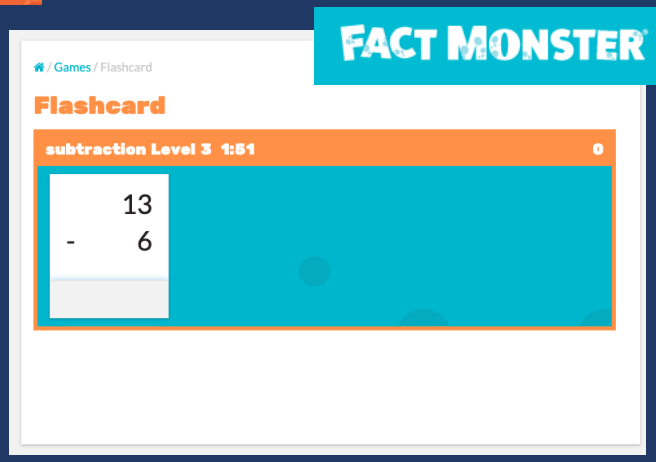

DAILY and BRIEF

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Addition	Subtraction
Multiplication	Division

Describe three activities to help students with fact fluency.

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

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MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

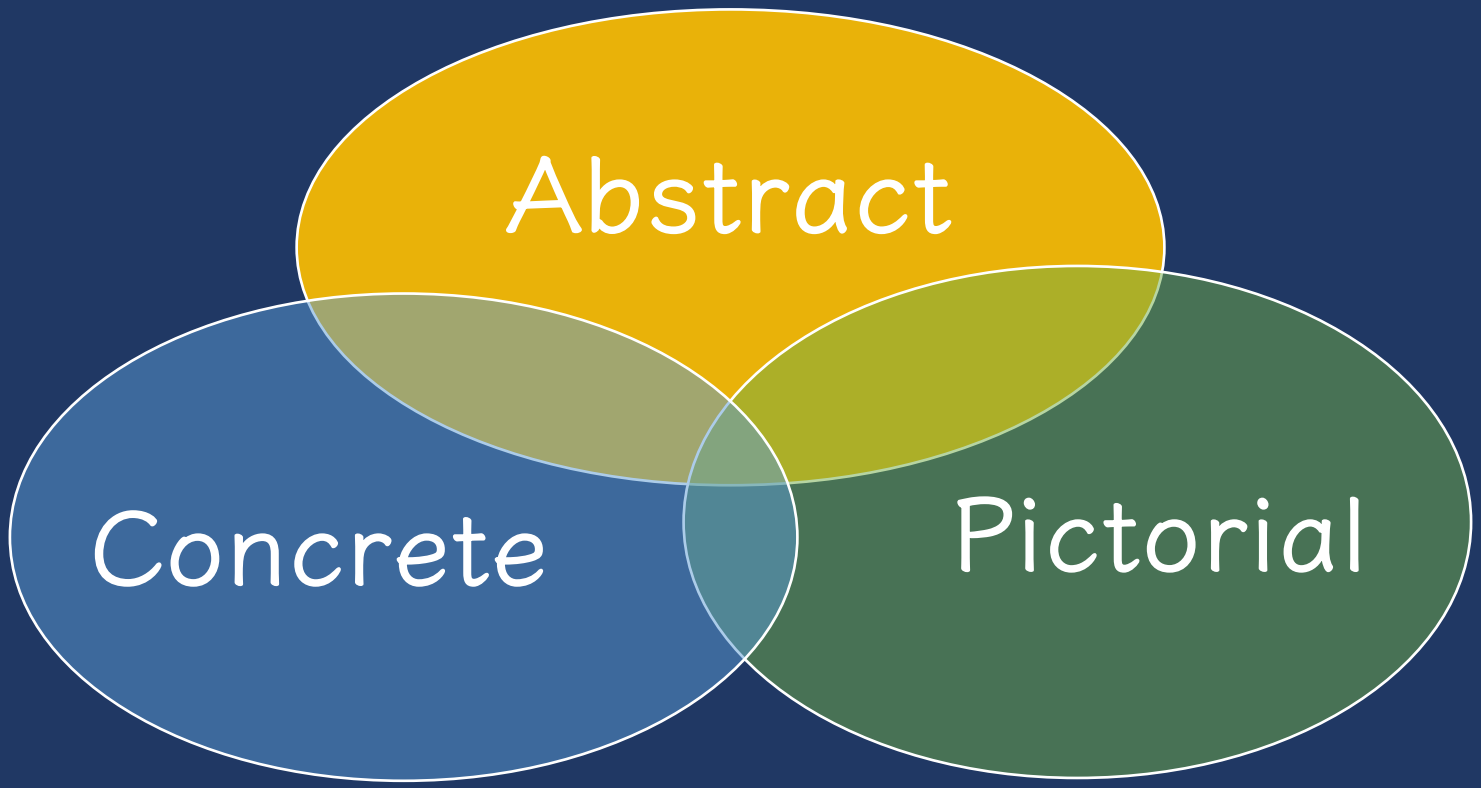
What are your strengths with modeling multiplication and division?

What are your opportunities for growth?

Use formal math language

Use terms precisely

What are five essential math vocabulary for multiplication and division?



What are the representations you'll use to teach multiplication and division?

Building Fluency with Computation

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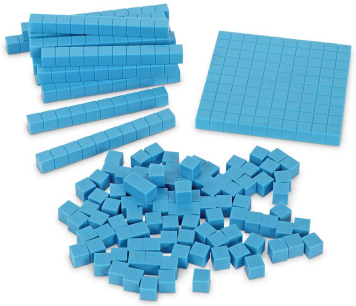
Addition and Subtraction Computation

227 + 185 =	232 – 164 =
	

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Partial Sums

A.

$$\begin{array}{r} 74 \\ +\ 18 \\ \hline 80 \\ +12 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ +\ 365 \\ \hline 1{,}000 \\ 80 \\ +\ 10 \\ \hline 1{,}090 \end{array}$$

$$\begin{array}{r} 227 \\ +\ 185 \\ \hline \end{array}$$

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Opposite Change

A.

$$\begin{array}{r} 74 \xrightarrow{-4} 70 \\ +\ 18 \xrightarrow{+4} +22 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \xrightarrow{+5} 730 \\ +\ 365 \xrightarrow{-5} +360 \\ \hline 1{,}090 \end{array}$$

$$\begin{array}{r} 227 \\ +\ 185 \\ \hline \end{array}$$

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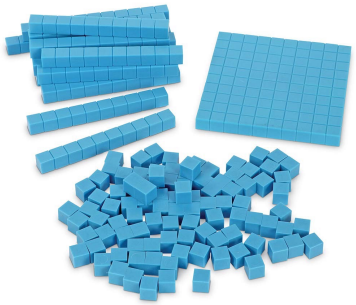
Addition and Subtraction Computation

227 + 185 =	232 – 164 =
	

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Partial Differences

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline +50 \\ -\ 5 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline +300 \\ -90 \\ -1 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

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Same Change

A.

$$\begin{array}{r} 62 \xrightarrow{+3} 65 \\ -\ 17 \xrightarrow{+3} -20 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \xrightarrow{+4} 309 \\ -\ 96 \xrightarrow{+4} -100 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

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Add Up

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 20 \\ 60 \\ 62 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ 40 \\ +\ 2 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ 100 \\ 300 \\ 305 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ 200 \\ +\ 5 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

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(1) Model an addition problem.

(2) Model a subtraction problem.

Multiplication and Division Computation

13 x 7 =	135 ÷ 5 =
13 x 47 =	804 ÷ 12 =

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Partial Products

A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$

$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

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Area (Array)

A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$

20	4	
800	160	40
60	12	3

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$

100	30	2	
5000	1500	100	50
300	90	6	3

$$\begin{array}{r} 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$

$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

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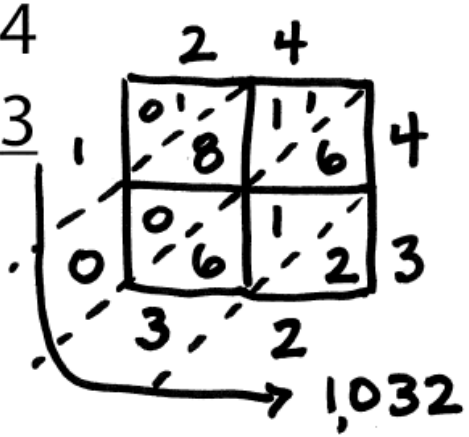
Lattice

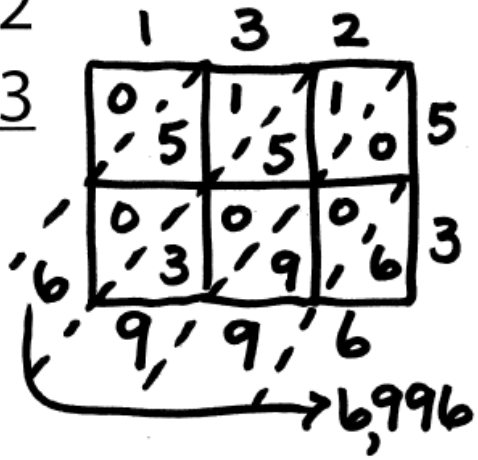
A.

$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline \end{array}$$



B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline \end{array}$$



$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

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Multiplication and Division Computation

13 x 7 =	135 ÷ 5 =
13 x 47 =	804 ÷ 12 =

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Partial Quotients

A.

$$
\begin{array}{r|l}
12\overline{)158} & \\
-120 & 10 \\
\hline
38 & \\
-36 & +3 \\
\hline
2 & 13\ \text{R}2
\end{array}
$$

B.

$$
\begin{array}{r|l}
34\overline{)970} & \\
-680 & 20 \\
\hline
290 & \\
-170 & 5 \\
\hline
120 & \\
102 & +3 \\
\hline
18 & 28\ \text{R}18
\end{array}
$$

$$
\begin{array}{r}
804 \\
\div\ \ 12 \\
\hline
\end{array}
$$

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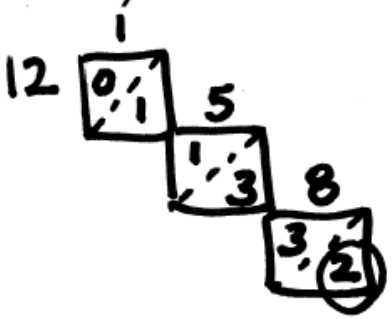
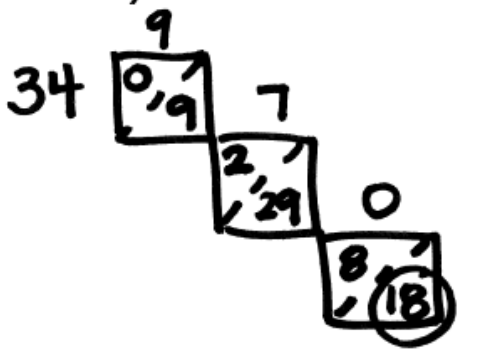
Lattice

A. $12\overline{)158}$ = 13 R2



B. $34\overline{)970}$ = 28 R 18



$$804 \div 12$$

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Division as Fractions

A. $12\overline{)158}$

$\frac{100}{12}$ $8\frac{4}{12}$

$\frac{50}{12}$ $4\frac{2}{12}$

$\frac{8}{12} + \frac{8}{12} = 12\frac{14}{12} = 13\frac{2}{12}$

B. $34\overline{)970}$

$\frac{900}{34}$ $26\frac{16}{34}$

$\frac{70}{34}$ $+2\ \frac{2}{34}$

$\frac{0}{34}$ $= 28\frac{18}{34}$

$804 \div 12$

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(1) Model a multiplication problem.

(2) Model a division problem.

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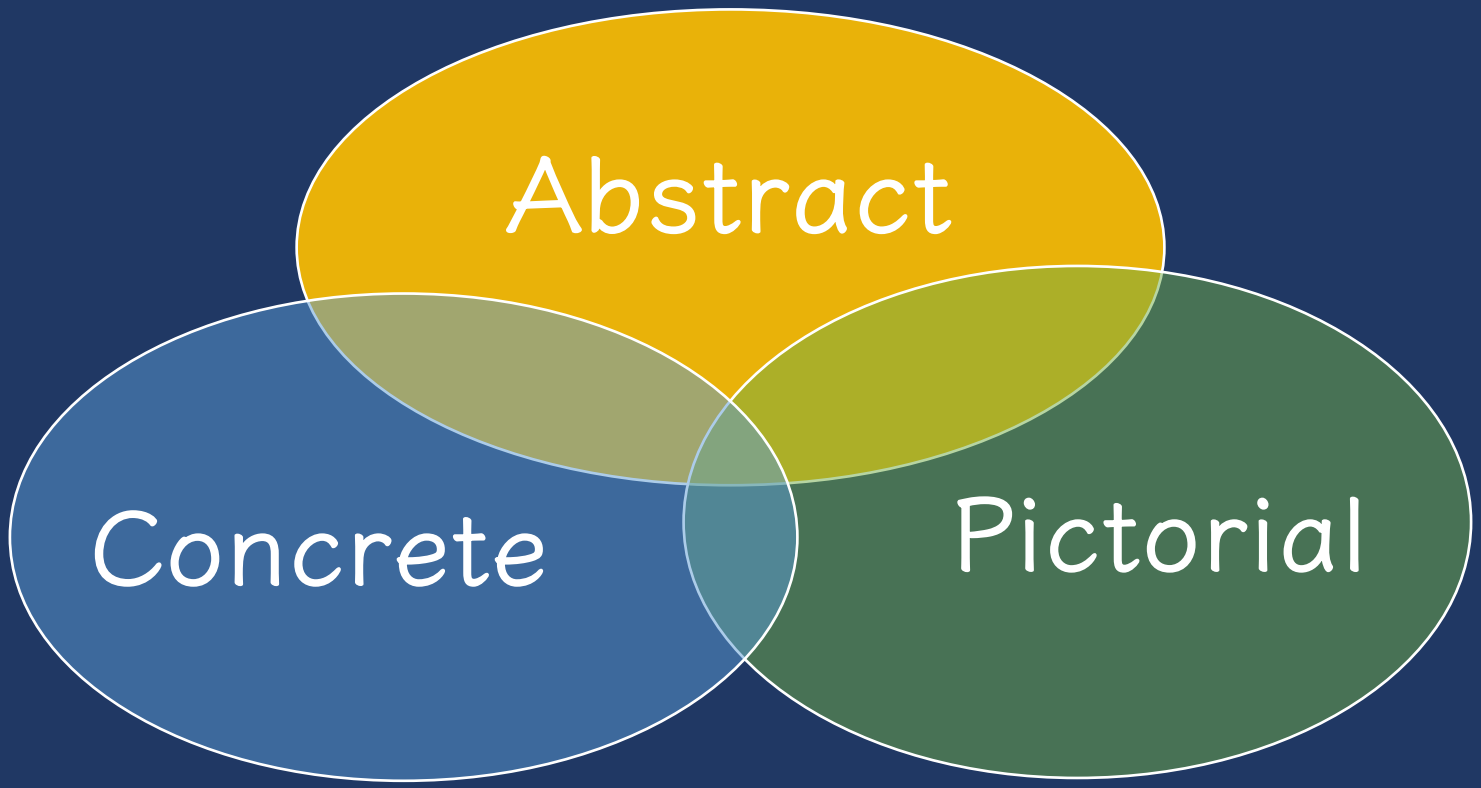
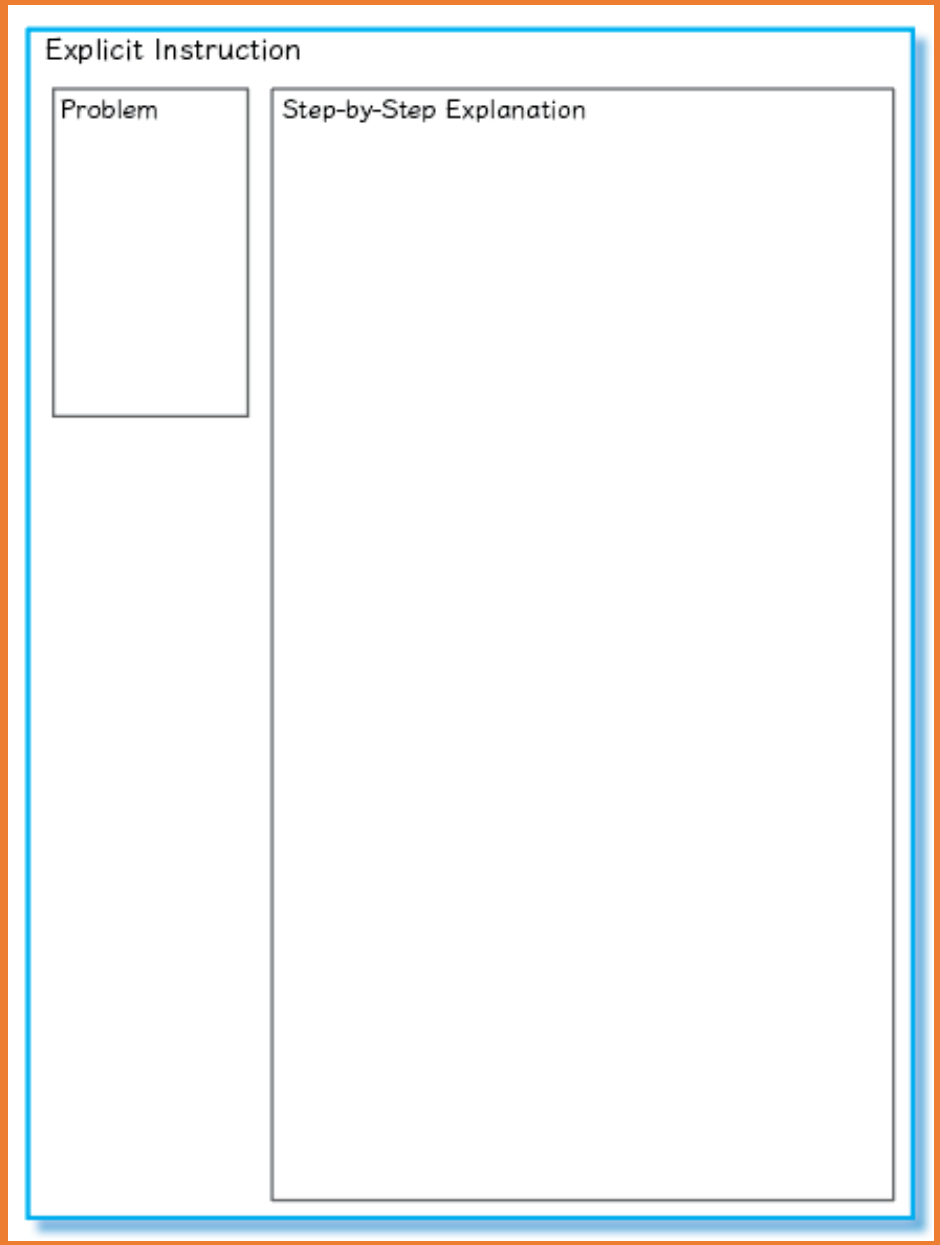


What are the representations you'll use to teach computation?



1. Choose a math problem.
2. Write a step-by-step explanation. Focus on the language of math in your explanation. Consider the representations you will use.

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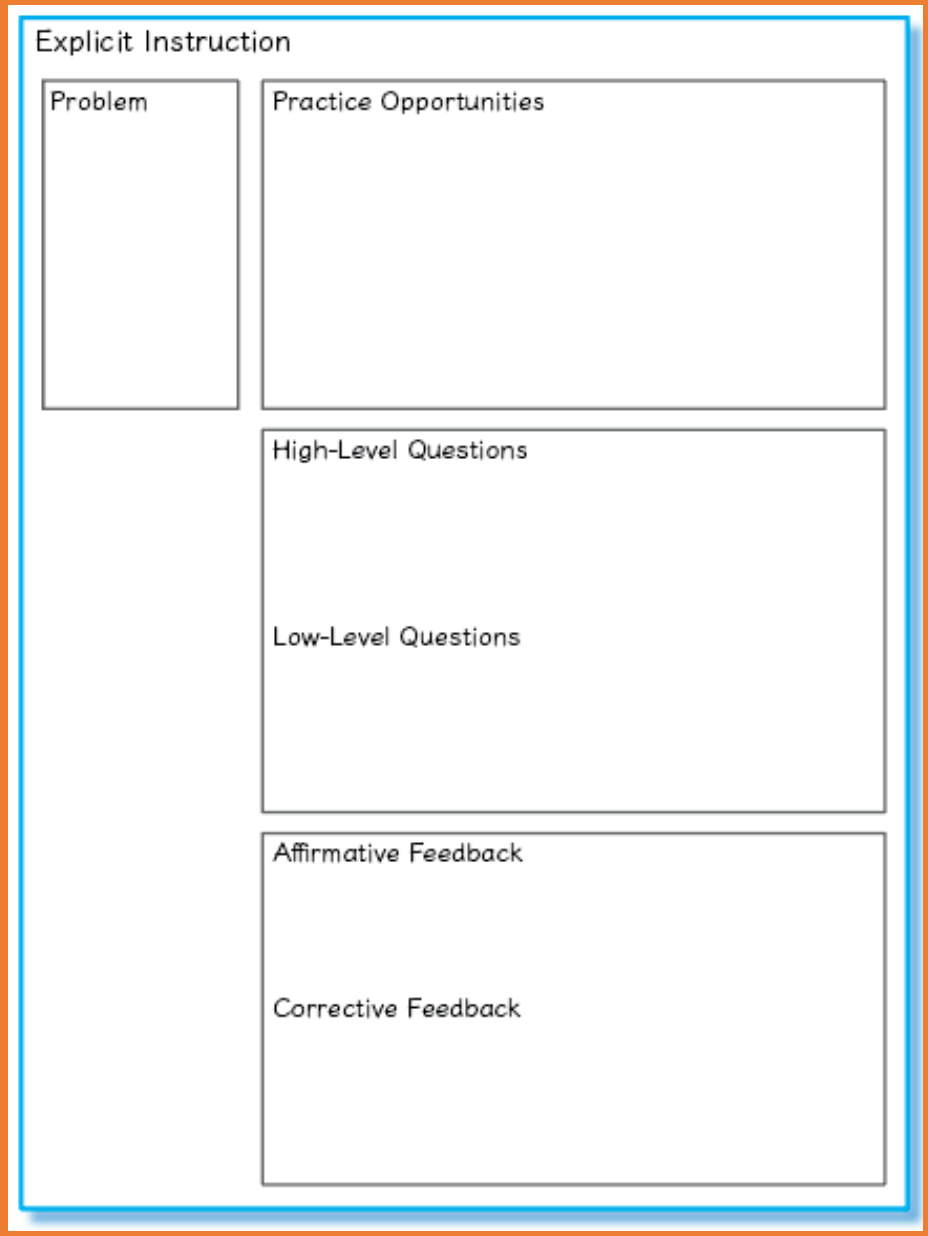

1. Describe the practice opportunities you will use.
2. Write 3 high-level questions.
3. Write 3 low-level questions.
4. Write 2 ways to provide affirmative feedback.
5. Write 2 ways to provide corrective feedback.

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Explicit Instruction

Problem	Step-by-Step Explanation

1. Teach your problem.

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What were your strengths with your teaching?

What are your opportunities for growth?

November 2022

Operations

- Addition and subtraction concepts
- Multiplication and division concepts
- Computation with addition, subtraction, multiplication, and division

January 2023

Fractions

- Length, area, and set models
- Comparison of fractions
- Ordering of fractions
- Computation of fractions

March 2023

Word-Problem Solving

- Attack strategies
- Schemas

April 2023

Geometry

- Understanding two-dimensional shapes
- Lines and angles
- Understanding three-dimensional shapes

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Sarah R. Powell, Ph.D.

Associate Professor
The University of Texas at Austin

www.sarahpowellphd.com

srpowell@utexas.edu

@sarahpowellphd

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