Edexcel GCE

Core Mathematics C4

Silver Level S2

(Mark Scheme)

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Question Number	Scheme	Marks
1.	$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$	B1
	$x \to 1 \qquad 9 = 3B \Rightarrow B = 3$	M1
	$x \to -\frac{1}{2} \qquad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$ Any two of *A*, *B*, *C*	A1
	x^2 terms $\qquad 9 = 2A + C \Rightarrow A = 4$ All three correct	A1 **(4)** **[4]**

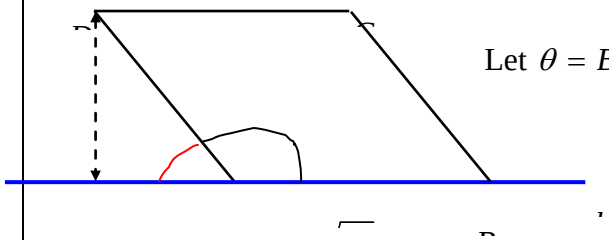
2. (a)	$\int x\sin 3x \, dx = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \, \{dx\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \, \{+c\}$	A1 **[3]**
(b)	$\int x^2\cos 3x \, dx = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \, \{dx\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right) \ \{+c\}$	A1 **isw**
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x \ \{+c\}\right\}$ *Ignore subsequent working*	**[3]** **(6 marks)**

3.	$\frac{d}{dx}\left(2^x\right) = \ln 2.2^x$	B1
	$\ln 2.2^x + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$	M1 A1= A1
	Substituting $(3, 2)$	
	$8\ln 2 + 4\frac{dy}{dx} = 4 + 6\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = 4\ln 2 - 2$ Accept exact equivalents	M1 A1 **(7)** **[7]**

Question Number	Scheme	Marks
4.	(a) $0.0333,\ 1.3596$ awrt $0.0333,\ 1.3596$	B1 B1 **(2)**
	(b) $\text{Area}(R) \approx \frac{1}{2}\times\frac{\sqrt{2}}{4}[\ ...]$	B1
	$\approx\ ...\ \left[0+2\left(0.0333+0.3240+1.3596\right)+3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 **(3)**
	(c) $u = x^2+2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$	B1
	$\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln\left(x^2+2\right)\mathrm{d}x$	B1
	$\int x^3 \ln\left(x^2+2\right)\mathrm{d}x = \int x^2 \ln\left(x^2+2\right)x\,\mathrm{d}x = \int (u-2)(\ln u)\tfrac{1}{2}\mathrm{d}u$	M1
	Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,\mathrm{d}u$ ✱ cso	A1 **(4)**
	(d) $\int (u-2)\ln u\,\mathrm{d}u = \left(\frac{u^2}{2}-2u\right)\ln u - \int\left(\frac{u^2}{2}-2u\right)\frac{1}{u}\mathrm{d}u$	M1 A1
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \int\left(\frac{u}{2}-2\right)\mathrm{d}u$	
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\ \ (+C)$	M1 A1
	$\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\right]_2^4$	
	$= \tfrac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - \left((2-4)\ln 2 - 1 + 4\right)\right]$	M1
	$= \tfrac{1}{2}\left(2\ln 2+1\right)$ $\ln 2+\tfrac{1}{2}$	A1 **(6)**
		[15]

5.	(a) $A = 2$	B1
	$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$	
	$x \to 1 \qquad -3 = 3B \Rightarrow B = -1$	M1 A1
	$x \to -2 \qquad -12 = -3C \Rightarrow C = 4$	A1 **(4)**
	(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\dfrac{x}{2}\right)^{-1}$	M1
	$(1-x)^{-1} = 1 + x + x^2 + \ ...$	B1
	$\left(1+\dfrac{x}{2}\right)^{-1} = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} + \ ...$	B1
	$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\dfrac{1}{2}\right)x^2 + \ ...$	M1
	$= 5 + \ ...$ ft their $A - B + \frac{1}{2}C$	A1 ft
	$= \ ... + \dfrac{3}{2}x^2 + \ ...$ $0x$ stated or implied	A1 A1 **(7)**
		[11]

Question Number	Scheme	Marks
Q6	$\dfrac{\mathrm{d}A}{\mathrm{d}t} = 1.5$	B1
	$A = \pi r^2 \Rightarrow \dfrac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$	B1
	When $A = 2$	
	$2 = \pi r^2 \Rightarrow r = \sqrt{\dfrac{2}{\pi}} \ (= 0.797\,884\ ...\ )$	M1
	$\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{\mathrm{d}A}{\mathrm{d}r} \times \dfrac{\mathrm{d}r}{\mathrm{d}t}$	
	$1.5 = 2\pi r \dfrac{\mathrm{d}r}{\mathrm{d}t}$	M1
	$\dfrac{\mathrm{d}r}{\mathrm{d}t} = \dfrac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$ awrt 0.299	A1
		[5]

Question Number	Scheme		Marks
7.	$\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$, $\{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\}$ & $\overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$		
(a)	$\overrightarrow{AB} = = \pm((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})); = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$		M1; A1 **[2]**
(b)	$l: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$		M1 A1ft **[2]**
	 Let $\theta = B\hat{A}D$	Let d be the shortest distance from C to l.	
(c)	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$		M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{AD}\rvert} = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}$	Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{AD}$ or $\overrightarrow{DA})$.	M1
	$\cos\theta = \pm\left(\dfrac{-9 + 6 - 5}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$	Correct followed through expression or **equation**.	A1$\sqrt{}$
	$\cos\theta = \dfrac{-8}{\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544... = 109$ (nearest °)	awrt 109	A1 **cso AG** **[4]**
(d)	$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$		M1
	So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$		A1 **[2]**
(e)	Area $ABCD = \left(\frac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$	awrt 23.2	M1; dM1 A1 **[3]**
(f)	$\dfrac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.19894905...$		M1
	$\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$	awrt 3.54	A1 **[2]** **(15 marks)**

Question Number	Scheme	Marks
8.	$\frac{\mathrm{d}x}{\mathrm{d}t} = k(M - x)$, where M is a constant	
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}$ is the rate of increase of the mass of waste products. Any one correct explanation.	B1
	M is the total mass of unburned fuel and waste fuel (or the initial mass of unburned fuel) Both explanations are correct.	B1
		(2)
(b)	$\int \frac{1}{M-x}\,\mathrm{d}x = \int k\,\mathrm{d}t$ or $\int \frac{1}{k(M-x)}\,\mathrm{d}x = \int \mathrm{d}t$	B1
	$-\ln(M - x) = kt \ \{+ c\}$ or $-\frac{1}{k}\ln(M - x) = t \ \{+ c\}$	M1 A1
	$\{t = 0,\ x = 0 \Rightarrow\}\ -\ln(M - 0) = k(0) + c$	M1
	$c = -\ln M \Rightarrow -\ln(M - x) = kt - \ln M$	
	then either... $-kt = \ln(M - x) - \ln M$; $-kt = \ln\left(\frac{M-x}{M}\right)$; $\mathrm{e}^{-kt} = \frac{M-x}{M}$; $M\mathrm{e}^{-kt} = M - x$ ***or...*** $kt = \ln M - \ln(M - x)$; $kt = \ln\left(\frac{M}{M-x}\right)$; $\mathrm{e}^{kt} = \frac{M}{M-x}$; $(M - x)\mathrm{e}^{kt} = M$; $M - x = M\mathrm{e}^{-kt}$	ddM1
	leading to $x = M - M\mathrm{e}^{-kt}$ or $x = M(1 - \mathrm{e}^{-kt})$ oe	A1 * **cso**
		(6)
(c)	$\left\{x = \frac{1}{2}M,\ t = \ln 4 \Rightarrow\right\}\ \frac{1}{2}M = M(1 - \mathrm{e}^{-k\ln 4})$	M1
	$\Rightarrow \frac{1}{2} = 1 - \mathrm{e}^{-k\ln 4} \Rightarrow e^{-k\ln 4} = \frac{1}{2} \Rightarrow -k\ln 4 = -\ln 2$	
	So $k = \frac{1}{2}$	A1
	$x = M\left(1 - \mathrm{e}^{-\frac{1}{2}\ln 9}\right)$	dM1
	$x = \frac{2}{3}M$ $x = \frac{2}{3}M$	A1 **cso**
		(4) **[12]**

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	4		84	3.34	3.91	3.70	3.45	3.17	2.79	2.36	1.66
2	6		71	4.28	5.83	5.13	4.10	3.12	2.24	1.46	0.51
3	7		74	5.20	6.72	6.02	5.43	4.70	3.95	2.91	1.45
4	15		67	9.99	14.23	12.43	10.19	7.93	5.88	4.41	2.94
5	11		68	7.49	10.31	8.79	7.52	6.39	5.34	4.19	2.57
6	5		65	3.23		4.34	3.12	2.26	1.59	0.86	0.56
7	15		60	9.04	13.94	10.95	7.98	6.17	4.54	3.55	1.61
8	12		41	4.93	9.74	5.48	3.21	1.63	1.20	0.95	0.33
	75		**63**	**47.50**		**56.84**	**45.00**	**35.37**	**27.53**	**20.69**	**11.63**