Five Best Practices for Math Instruction

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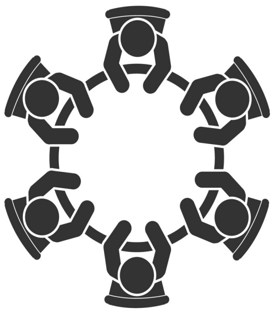
MATH



Introduce yourself.

Describe your strengths in supporting mathematics.

Describe an opportunity for growth.

This Year

September 20	Five Best Practices for Math Instruction • Systematic and explicit instruction • Math vocabulary • Multiple representations • Fluency • Word-Problem Solving
January 30	VIRTUAL Word Problems? No Problem!
April 11	VIRTUAL Mathematics Manipulatives Mania

Today

8:30-11:30	Five Best Practices for Math Instruction
11:30-12:30	LUNCH
12:30-3:00	Five Best Practices for Math Instruction

Mathematical Pathways

Five Best Practices for Math Instruction

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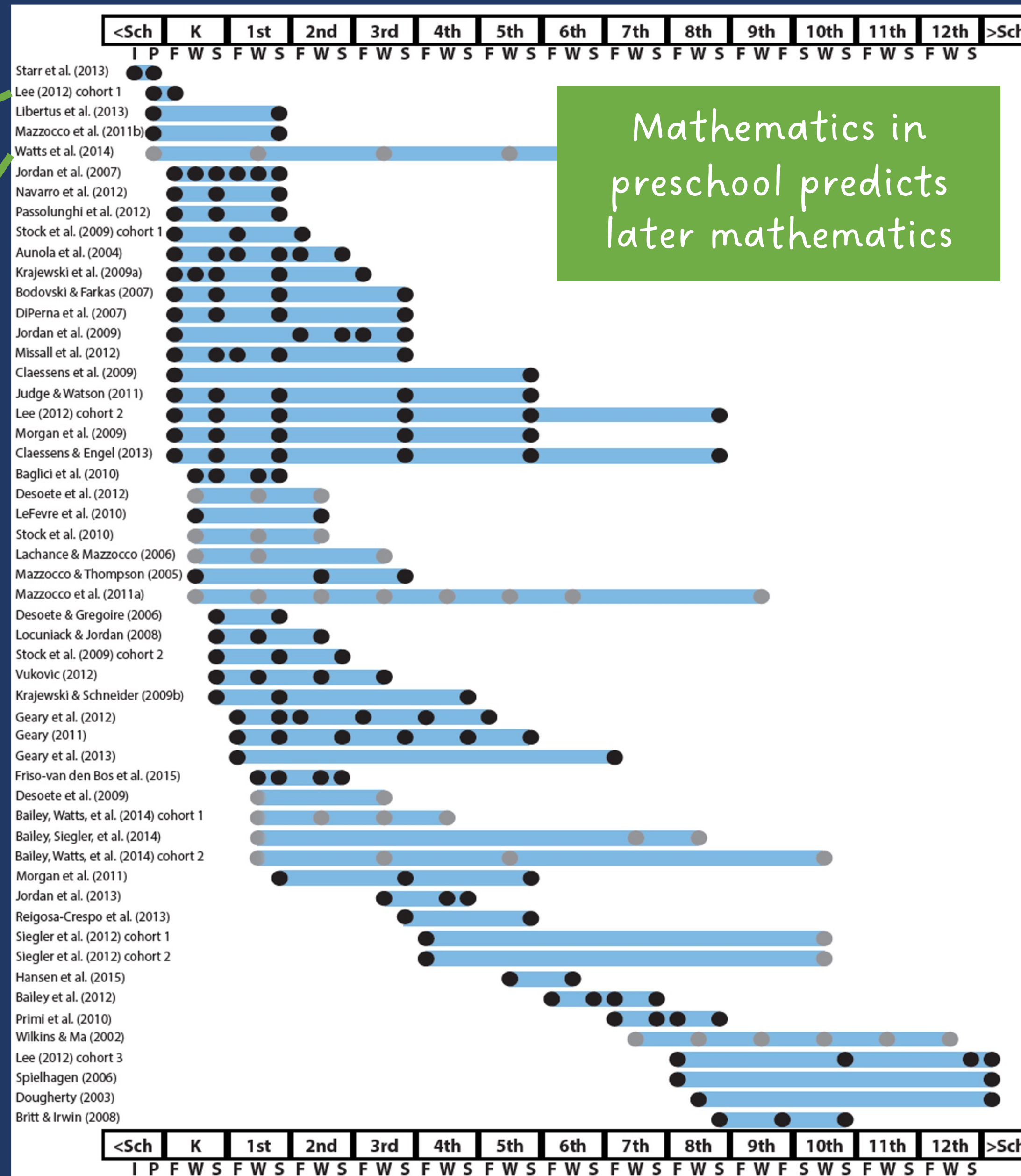
Mathematical Trajectories

Critical Content

Broad math in prek predicted K broad math

Broad math in prek predicted grade 10 broad math



Mathematics in preschool predicts later mathematics

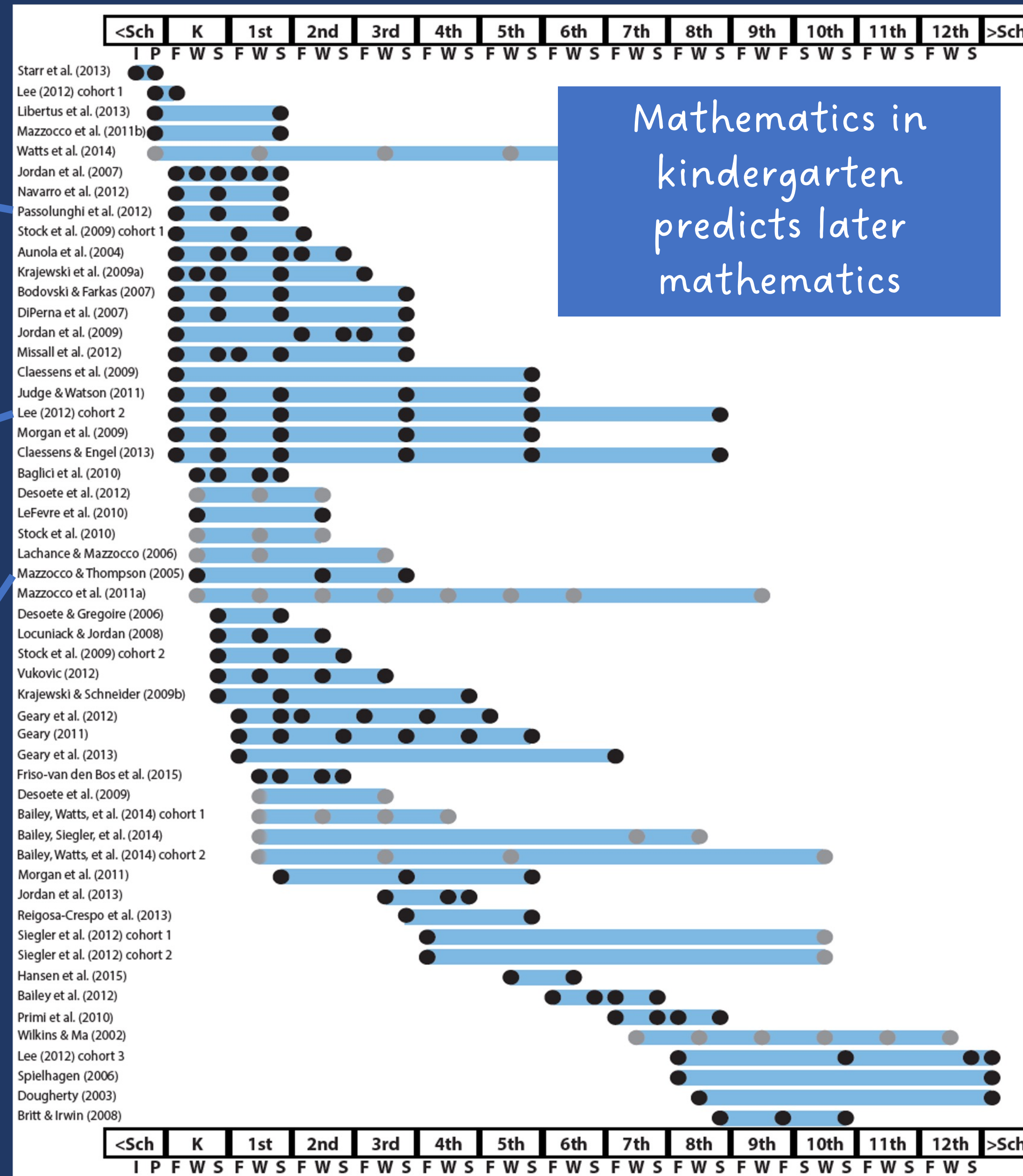
http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Mathematics in kindergarten predicts later mathematics

Counting in K predicted grade 1 broad math

Broad math in K predicted grade 8 broad math

K math accurately predicted math performance below 10th percentile in grades 2 and 3 with 84% correct classification



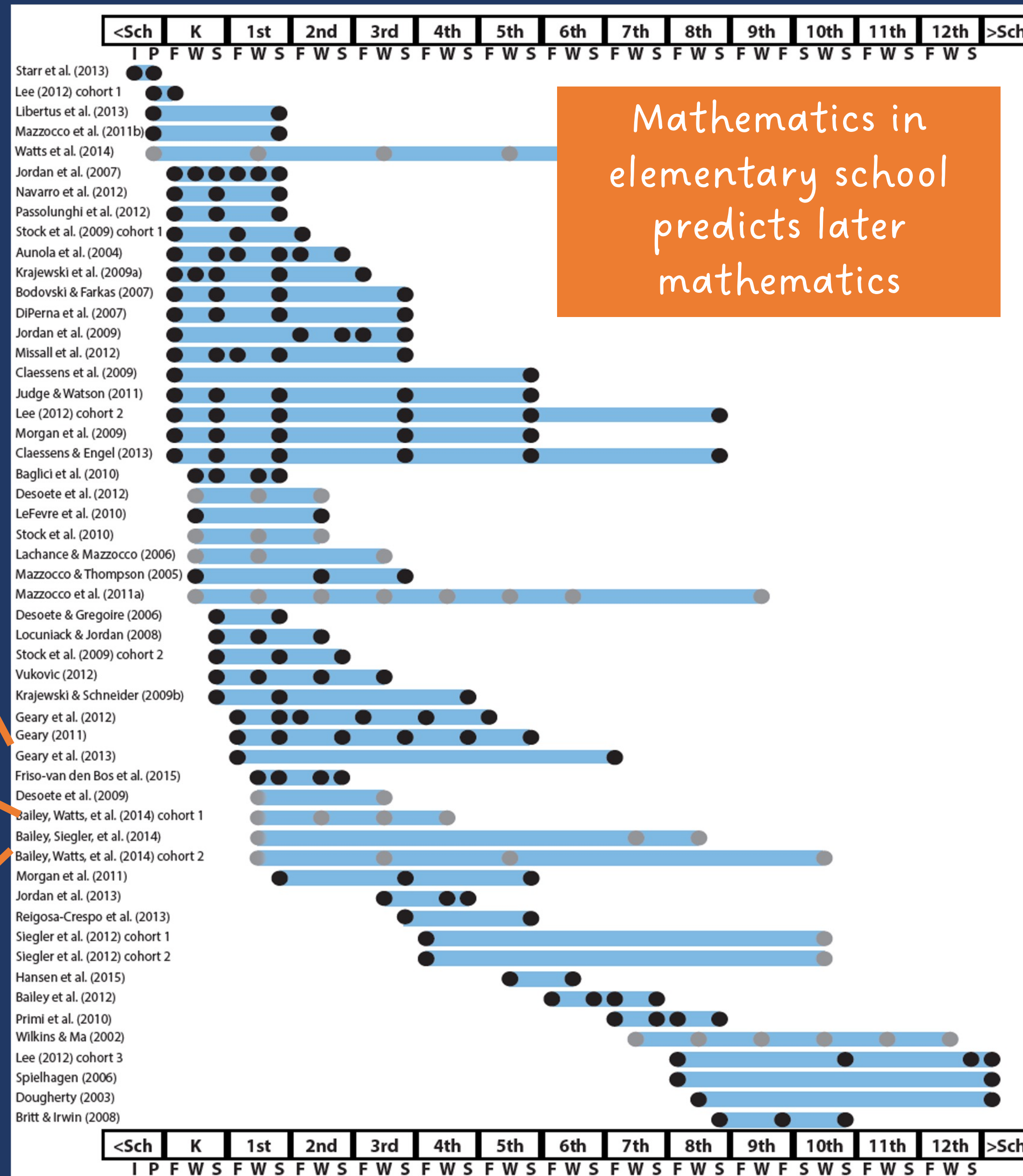
http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Addition influenced arithmetic with increasing importance from grades 1 to 5

Grade 1 arithmetic predicted arithmetic at grades 2, 3, and 4

Grade 1 broad math predicted broad math at grades 3, 5, and 10

Mathematics in elementary school predicts later mathematics

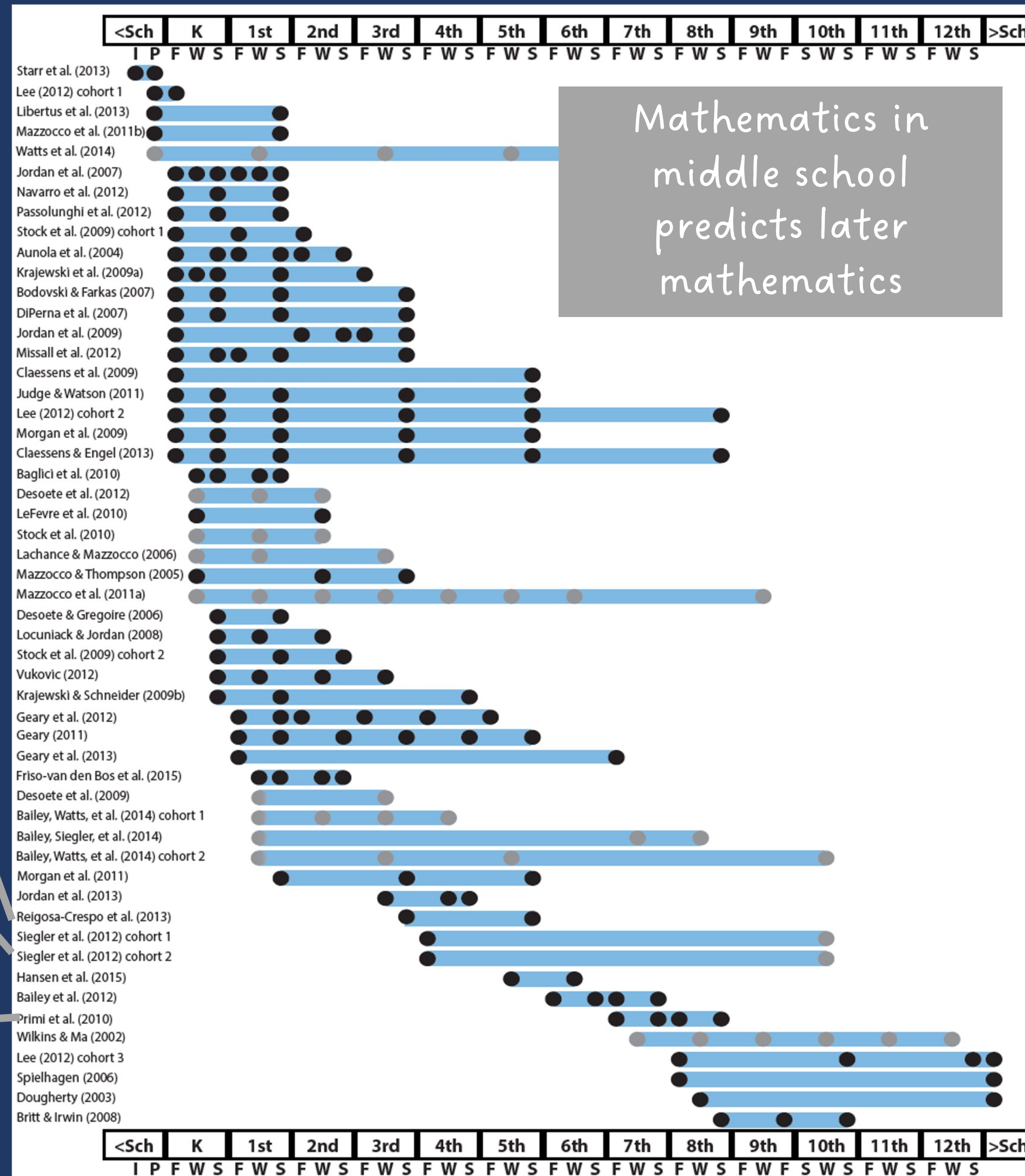


http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Counting and comparison in grades 2 or 4 predicted broad math 1 year later

Fractions at 10-12 years old predicted broad math 5 years later

Broad math in grade 7 predicted broad math in grade 8

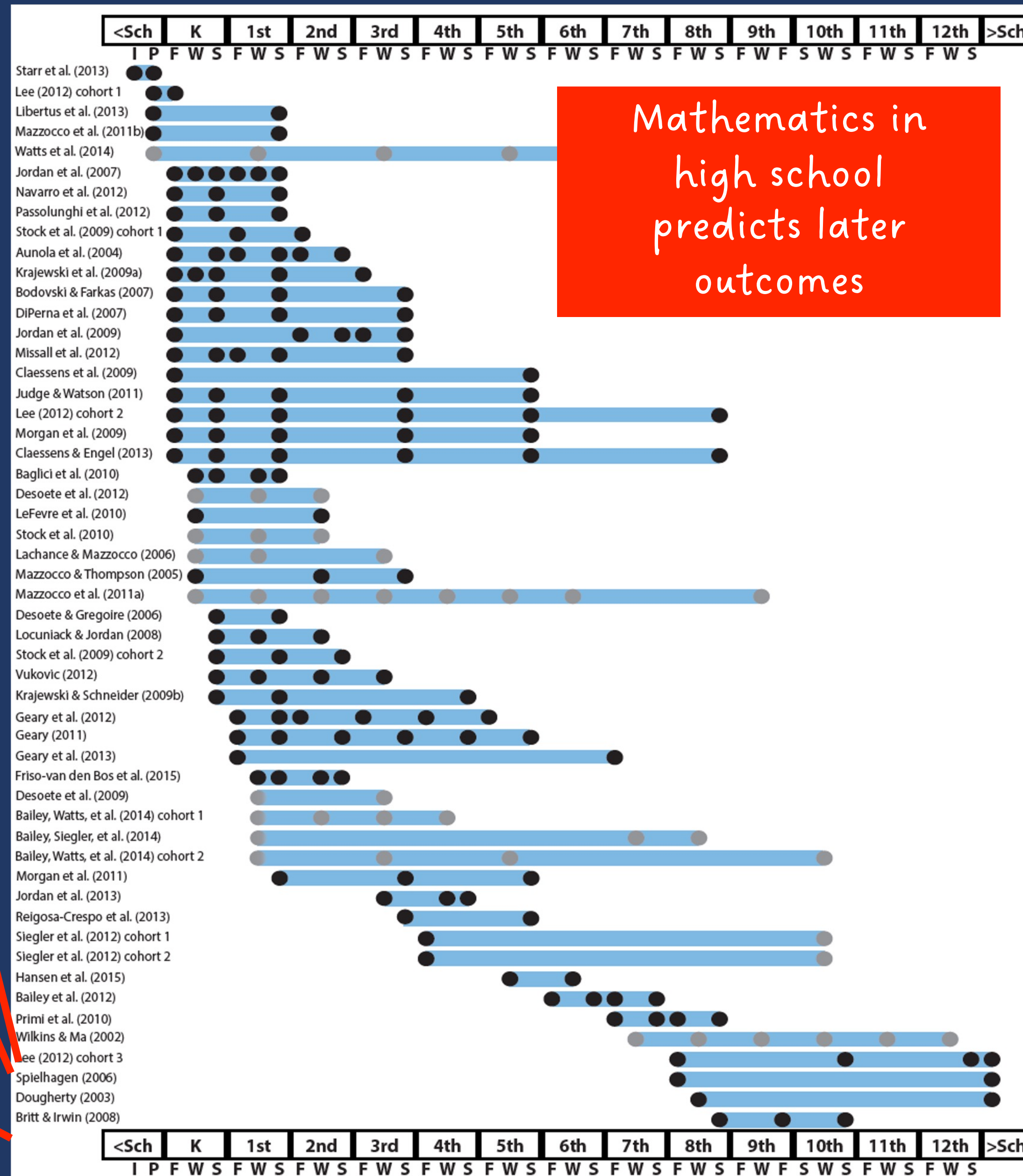


http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

Broad math in grade 8 predicted completion of 4-year college degree

Students who took algebra in grades 8 took more advanced math courses and enrolled in 4-year colleges more often than students who took algebra in grade 9

Numeracy measured in adolescence impacted hourly earnings 7 to 15 years later



Mathematics in high school predicts later outcomes

http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/

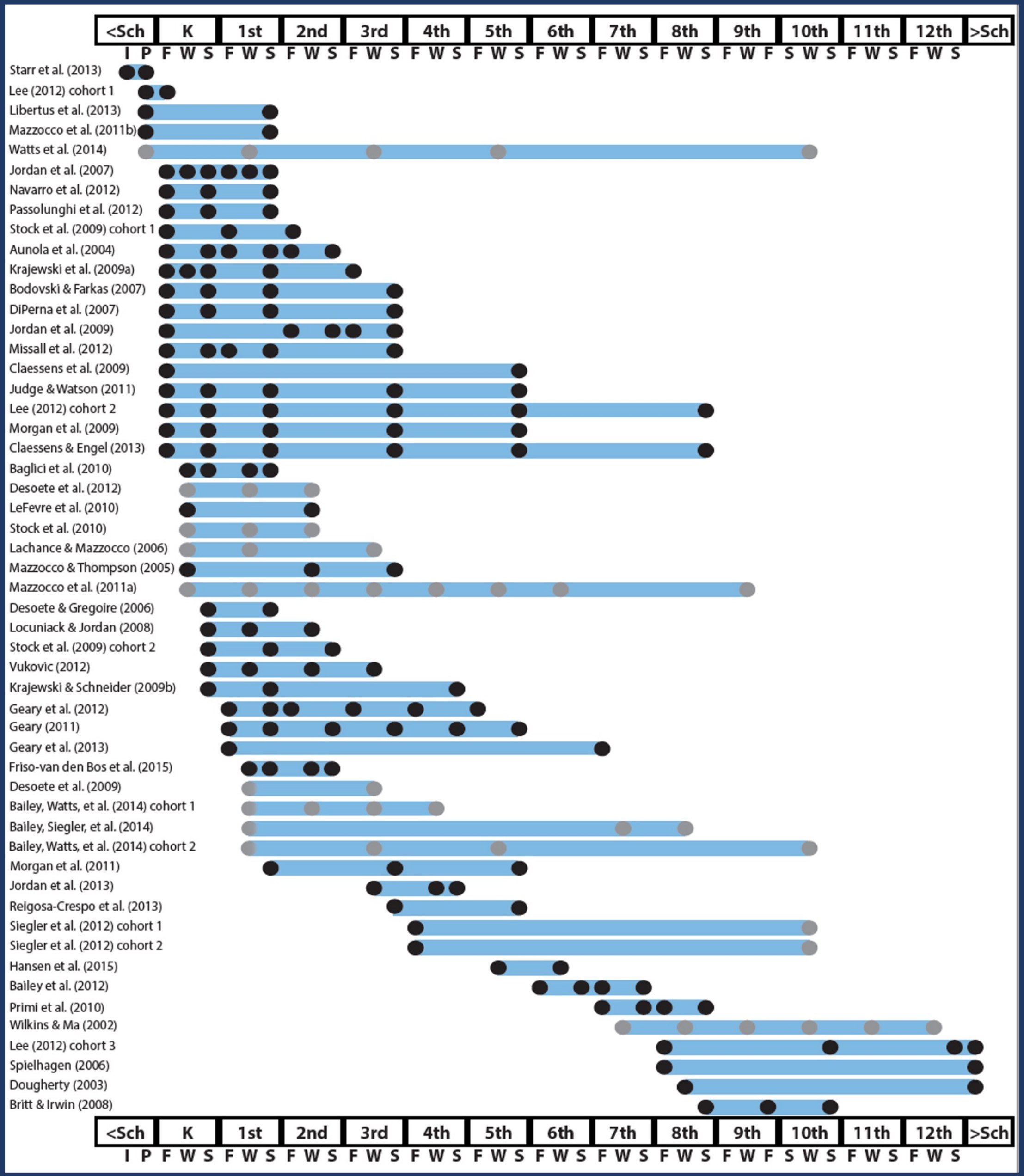
Mathematics in preschool predicts later mathematics

Mathematics in kindergarten predicts later mathematics

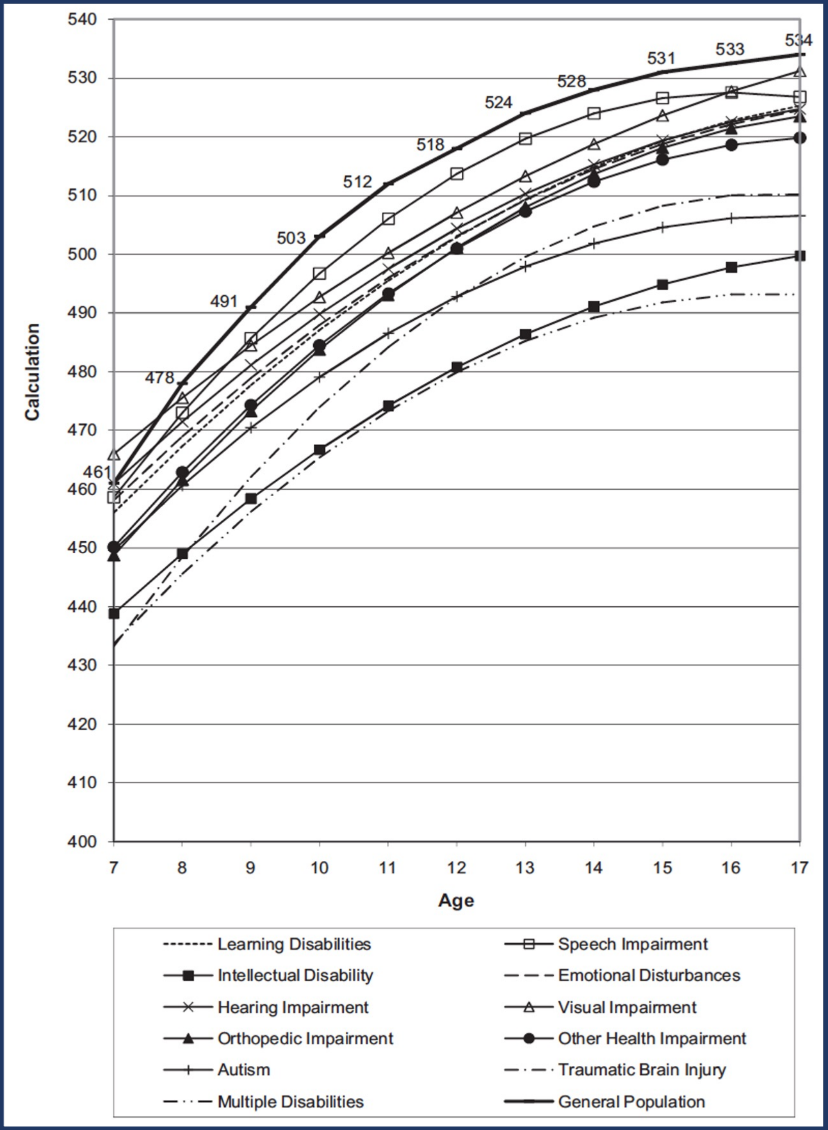
Mathematics in elementary school predicts later mathematics

Mathematics in middle school predicts later mathematics

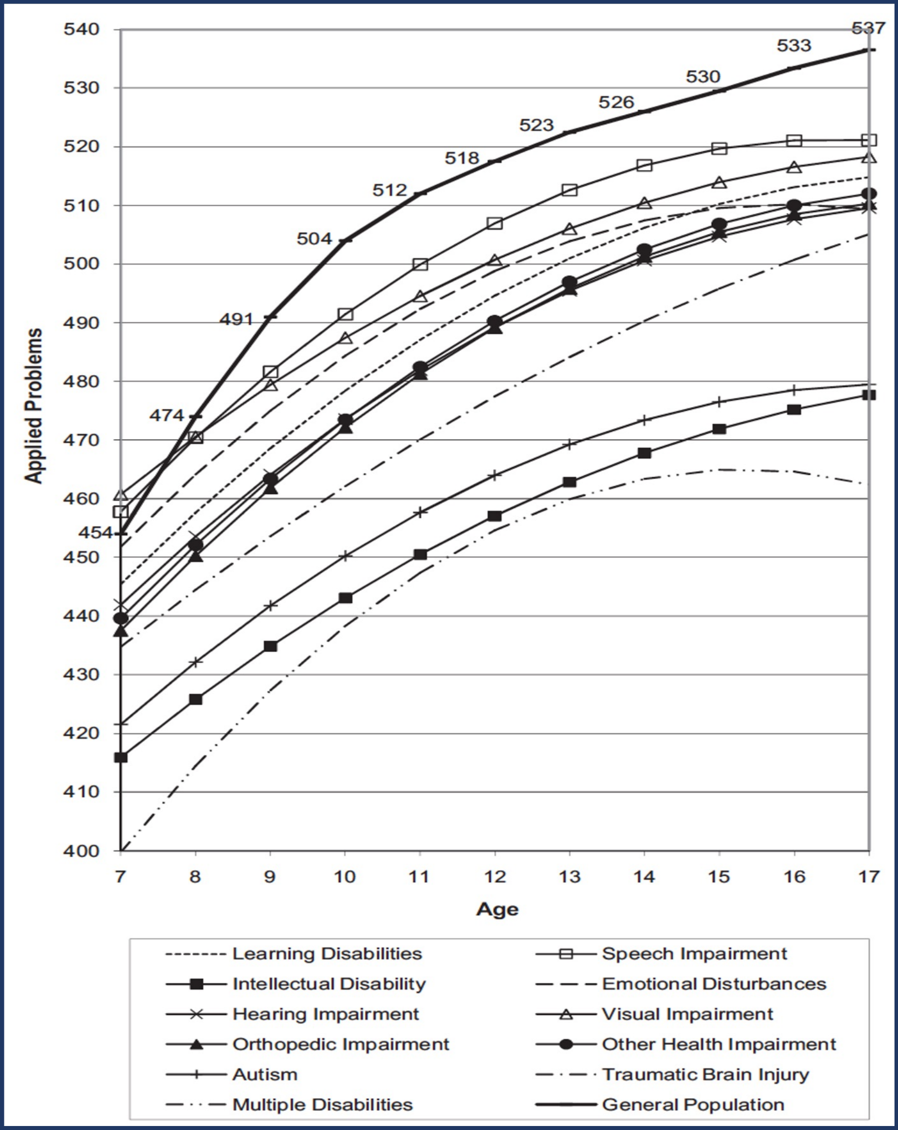
Mathematics in high school predicts later outcomes



http://www.greatertexasfoundation.org/trajectories-of-mathematics-performance/



Computation



Problem Solving

Wei, Lenz, & Blackorby (2013)

Provide examples of how you see earlier math skills relating to later math skills.

Five Best Practices for Math Instruction

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Mathematical Trajectories

Critical Content

continuum of mathematics learning

Fluently add and subtract multi-digit whole numbers

Fluently multiply and divide within 100

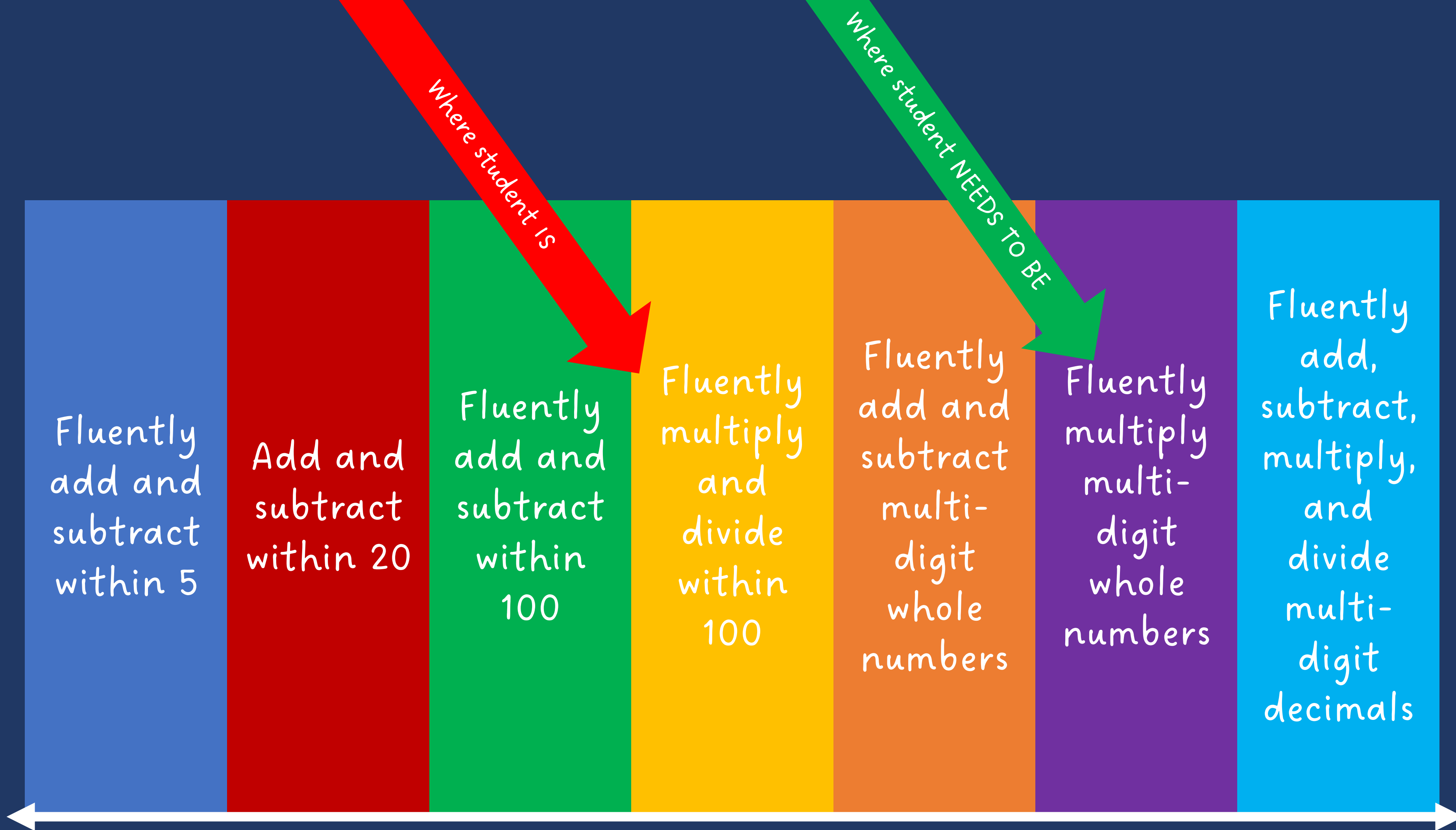
Fluently multiply multi-digit whole numbers

Fluently add and subtract within 100 using strategies

Fluently add and subtract within 5

Add and subtract within 20

Fluently add, subtract, multiply, and divide multi-digit decimals


Where student IS
Where student NEEDS TO BE
Fluently add and subtract within 5
Add and subtract within 20
Fluently add and subtract within 100
Fluently multiply and divide within 100
Fluently add and subtract multi-digit whole numbers
Fluently multiply multi-digit whole numbers
Fluently add, subtract, multiply, and divide multi-digit decimals

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

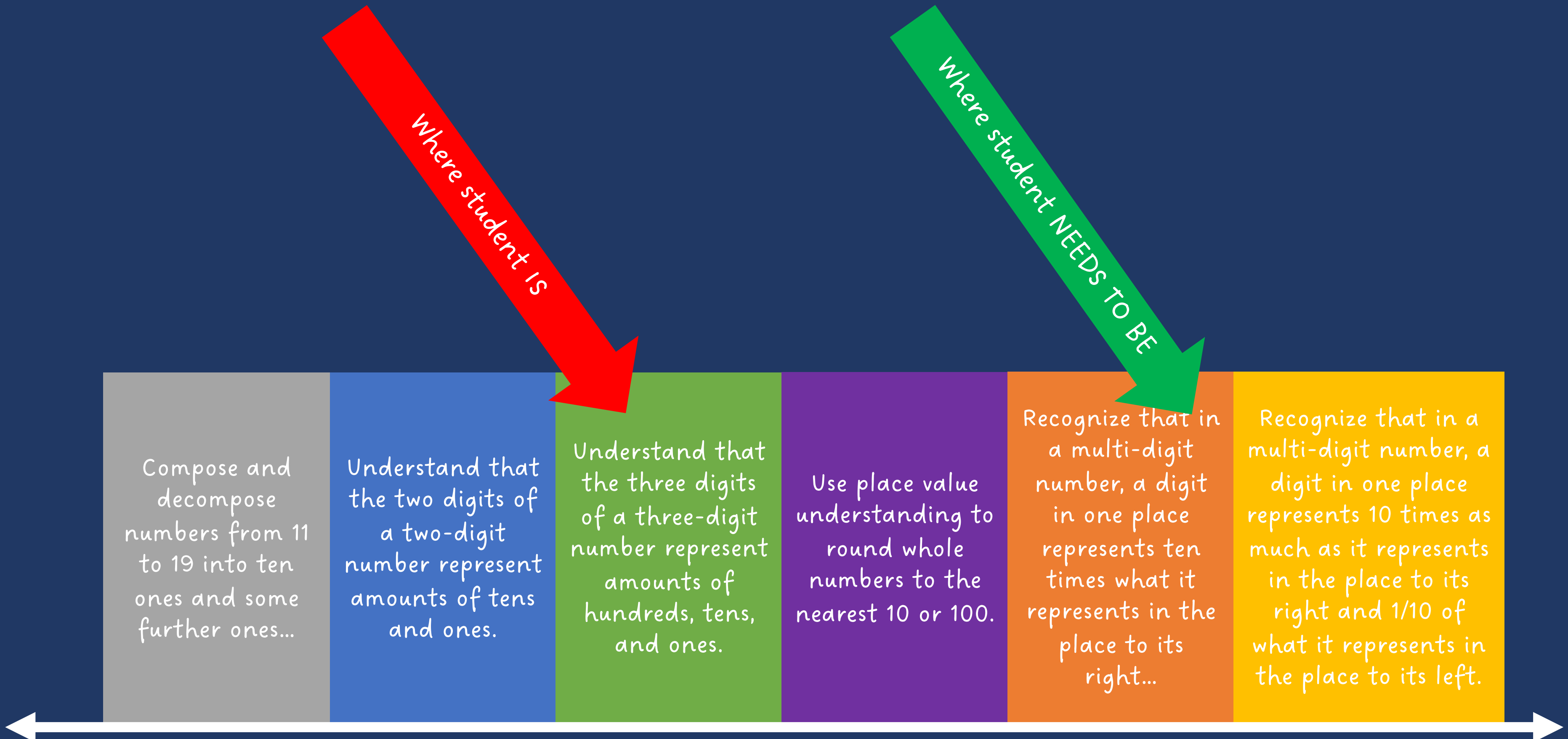
Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones

Compose and decompose numbers from 11 to 19 into ten ones and some further ones

Understand that the two digits of a two-digit number represent amounts of tens and ones

Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right

Use place value understanding to round whole numbers to the nearest 10 or 100



Where student IS
Where student NEEDS TO BE
Compose and decompose numbers from 11 to 19 into ten ones and some further ones…
Understand that the two digits of a two-digit number represent amounts of tens and ones.
Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.
Use place value understanding to round whole numbers to the nearest 10 or 100.
Recognize that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right…
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions...

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20

Solve real-world and math problems involving the four operations with rational numbers.

Use addition and subtraction within 100 to solve one- and two-step word problems

Solve real-world and math problems leading to two linear equations in two variables

Use multiplication and division within 100 to solve word problems

Solve addition and subtraction word problems, and add and subtract within 10

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators

Solve addition and subtraction word problems, and add and subtract within 10

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20

Use addition and subtraction within 100 to solve one- and two-step word problems

Use multiplication and division within 100 to solve word problems

Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators

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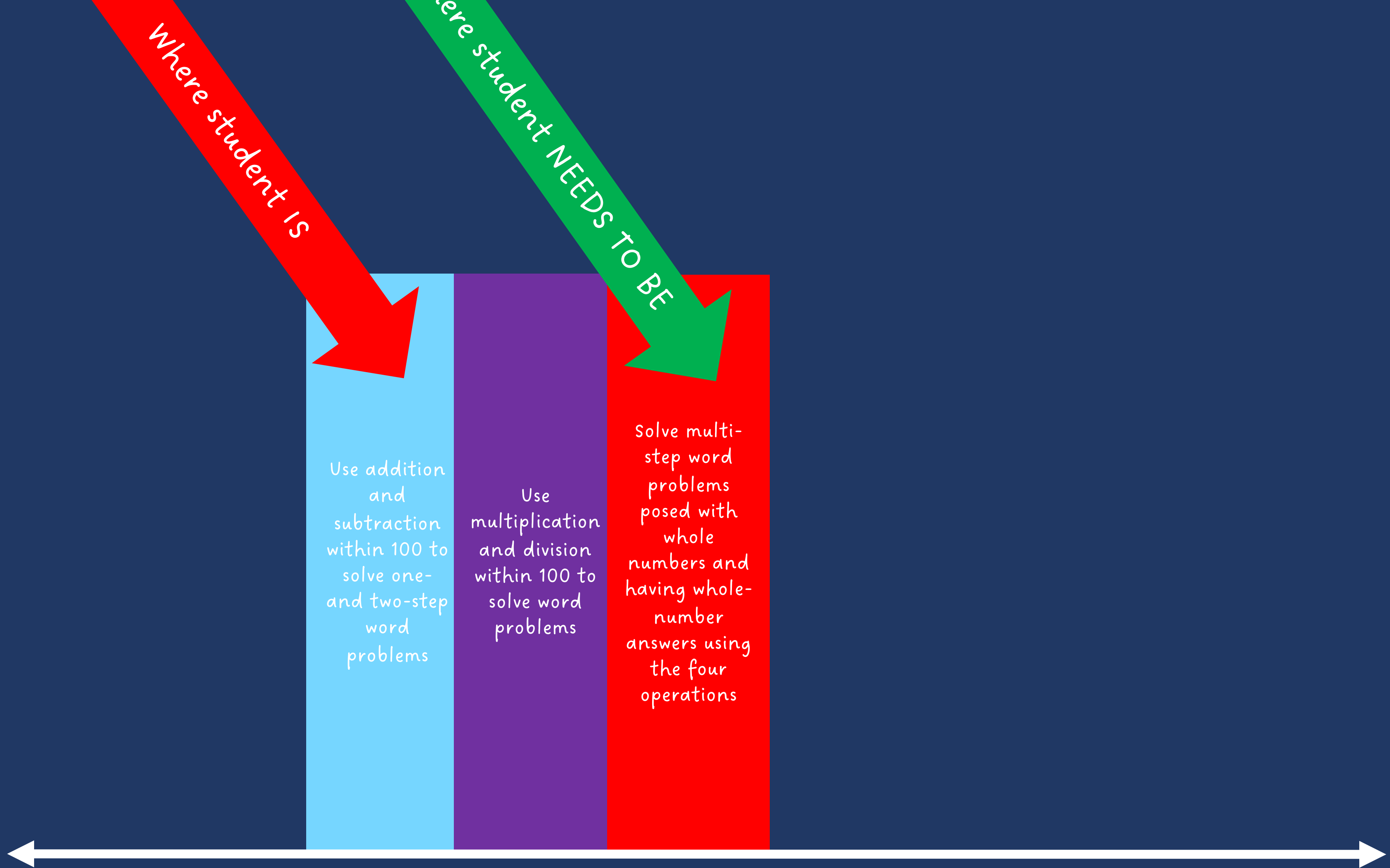
Solve real-world and math problems involving the four operations with rational numbers

Solve real-world and math problems leading to two linear equations in two variables


Where student IS
ere student NEEDS TO BE
Use addition and subtraction within 100 to solve one- and two-step word problems
Use multiplication and division within 100 to solve word problems
Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations


Where student IS
Where student NEEDS TO BE
Understand that the two digits of a two-digit number represent amounts of tens and ones.
Apply properties of operations as strategies to multiply and divide....
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division...
Fluently add and subtract multi-digit whole numbers using the standard algorithm.
Fluently multiply multi-digit whole numbers using the standard algorithm.

Kindergarten	Grade 1	Grade 2	Grade 3
Comparing and Ordering Numbers			
(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:	(2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:
(G) compare sets of objects up to at least 20 in each set using comparative language.	(E) use place value to compare whole numbers up to 120 using comparative language.	(D) use place value to compare and order whole numbers up to 1,200 using comparative language, numbers, and symbols (>, <, or =).	(D) compare and order whole numbers up to 100,000 and represent comparisons using the symbols >, <, or =.
(H) use comparative language to describe two numbers up to 20 presented as written numerals.			
	(F) order whole numbers up to 120 using place value and open number lines.		
	(G) represent the comparison of two numbers to 100 using the symbols >, <, or =.		

https://www.texasgateway.org/resource/vertical-alignment-charts-revised-mathematics-teks

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.
** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

https://achievethecore.org/category/774/mathematics-focus-by-grade-level

Table A.3. Grades 6–8 Curriculum Focal Points and Connections Compared with the Expectations of the Content Standards in *Principles and Standards for School Mathematics*

🟠 Grade 6 🟢 Grade 7 🔴 Grade 8 🟣 Outside grades 6–8 ⚪ Not identified at any level, pre-K–8

Curriculum Focal Points and Connections

Grade 6 Curriculum Focal Points

***Number and Operations:* Developing an understanding of and fluency with multiplication and division of fractions and decimals**

Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.

***Number and Operations:* Connecting ratio and rate to multiplication and division**

Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.

***Algebra:* Writing, interpreting, and using mathematical expressions and equations**

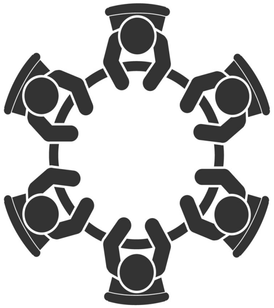
Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that

Expectations of the Content Standards

Number and Operations, Grades 6–8

🟠🟢🔴	Work flexibly with fractions, decimals, and percents to solve problems
🟠🟢	Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line
🟢	Develop meaning for percents greater than 100 and less than 1
🟠🟢	Understand and use ratios and proportions to represent quantitative relationships
🔴🟣	Develop an understanding of large numbers [identified in Grades 4 and 5 Curriculum Focal Points] and recognize and appropriately use exponential, scientific, and calculator notation
🟢	Use factors, multiples, prime factorization, and relatively prime numbers to solve problems
🟢	Develop meaning for integers and represent and compare quantities with them
🟠🟢🔴	Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
🟠🟢🔴	Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals
🟠🟢🔴	Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems
🟠🟢🔴	Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods

https://www.nctm.org/curriculumfocalpoints/



What is the critical math content for your students?

Instructional Platform

What's the continuum of mathematics support?

novice learners or struggling learners

explicit instruction

learners with acquired knowledge and skills

discovery or inquiry

Anita Archer (2019) facebook/com/watch/?v=320845308601739

Instructional Platform

evidence-based practice

A practice that has shown consistent and positive results

An intervention (i.e., packaged program) that has shown consistent and positive results

A method or strategy that has shown **consistent and positive results**

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

udents should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Instructional Platform

Instructional Delivery

Instructional Strategies

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit

Vocabulary

Representations

INSTRUCTIONAL STRATEGIES

Fluency

Word Problems

Be Explicit

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit

INSTRUCTIONAL STRATEGIES

Explicit Instruction

Research and Information

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Over a half century of research supports explicit (i.e., direct, systematic) instruction.
(Stockard et al., 2018)

When compared to discovery approaches, explicit instruction demonstrates higher outcomes.
(Alfieri et al., 2011; Kroesbergen et al., 2004; Poncy et al., 2010)

Numerous meta-analyses and large-scale studies have identified explicit instruction as essential for the teaching and learning of mathematics.
(Chodura et al., 2015; Ennis & Losinski, 2019; Jitendra et al., 2018; Kong et al., 2021; Morgan et al., 2015; Nelson & McMaster, 2019; Powell et al., 2021).

Explicit Instruction

MODELING

PRACTICE

SUPPORTS

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Modeling is a dialogue between the teacher and students.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Modeling includes a step-by-step explanation of how to do a math problem.

A teacher may do 1 modeled problem or several.

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Today, we are learning about addition. This is important because sometimes you have different amounts – like money – and you want to know how much money you have altogether."

"Let's solve this problem. What's the problem?"

"26 plus 79."

"To solve 26 plus 79, first decide about the operation. Should we add, subtract, multiply, or divide?"

"Add."

"How did you know we want to add?"

"There's a plus sign."

$$\begin{array}{r} 26 \\ +\ 79 \\ \hline \end{array}$$

"The plus sign tells us we want to add. To add, let's use the partial sums strategy. What strategy?"

"Partial sums."

"What might partial mean?"

"Part of."

"We'll find parts – or partial sums - then add them together. With the partial sums strategy, we start adding in the greatest place value. What's the greatest place value in this problem?"

"The tens."

$$\begin{array}{r} 26 \\ +\ 79 \\ \hline \end{array}$$

"So, let's add the tens. What's 20 plus 70? Use your base-10 blocks or other tools."

"90."

"20 plus 70 equals 90. Let's write 90 right here below the equal line. What will we write?"

"90."

"90 is the partial sum when you add the tens. What does 90 represent?"

"It's the partial sum of adding 20 plus 70."

"Now, let's add the ones. What should we add?"

"6 plus 9."

"6 plus 9 equals what? Use your base-10 blocks or other tools."

"15."

"How did you get 15?"

"We knew we had 9, then we added on 6."

"Let's write 15 below the 90. Where do we write the 15?"

"Below the 90."

"15 is the partial sum when you add the ones. Now, let's add the partial sums together. What will we add?"

"90 plus 15."

"What's 90 plus 15? Use your go-to strategy."

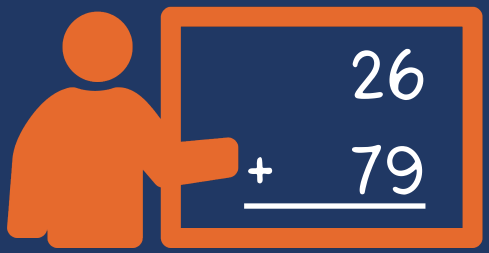
"105."

"How did you add those addends?"

"I added 90 plus 10 then added 5 more."

"So, when you add 26 plus 79, the sum is 105. Who can share how we solved this problem?"

"We used the partial sums strategy. We added the tens then added the ones. Then we added the partial sums."



What did you observe?

How would you improve this example?

Modeling needs to include planned examples.

These examples should be sequenced so easier skills lead to more difficult skills.

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

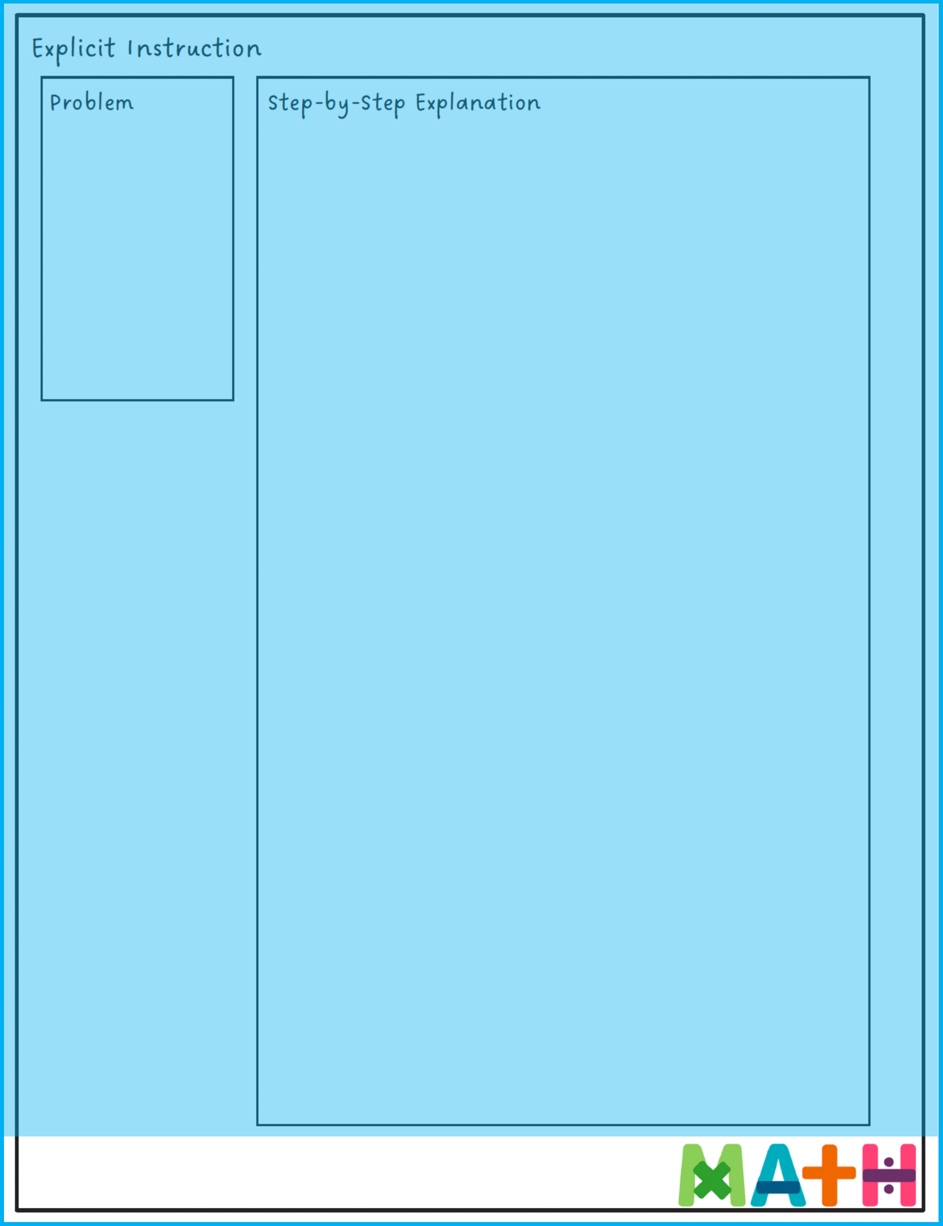
Independent practice

SUPPORTS

Ask high-level and low-level questions

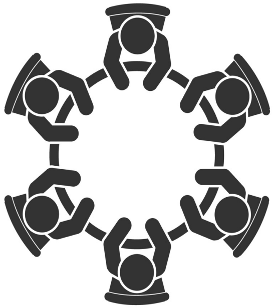
Eliciting frequent responses

Providing affirmative and corrective feedback


Explicit Instruction

Problem

Step-by-Step Explanation



Select a math problem.

Work with a partner to outline a step-by-step explanation.

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Practice continues as a dialogue between the teacher and students.

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Guided practice is practice in which the teacher and students practice problems together.

"Let's work on a problem together."

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Independent practice is practice in which the students practice independently with teacher support.

"Now, you'll practice a problem on your own. Use your attack strategy!"

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Explicit Instruction

Problem

Practice Opportunities

High-Level Questions

Low-Level Questions

Affirmative Feedback

Corrective Feedback

Describe how you would engage students in practice.

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

These **Supports** should be used in both **Modeling** and **Practice**.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, it is essential to engage students and check for understanding.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"What is 7 times 9?"

"63."

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Why do you use zero pairs?"

"Because a positive 1 and a negative 1 equal 0. I use the zero pair to help me subtract."

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, students should frequently respond. The frequent responses keeps student attention and keeps student learning active.

MODELING

Step-by-step explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

During **Modeling** and **Practice**, students should receive immediate feedback on their responses.

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Nice work using your word problem attack strategy."

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

"Let's look at that again. Tell me how you added in the hundreds column."

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

Explicit Instruction

Problem

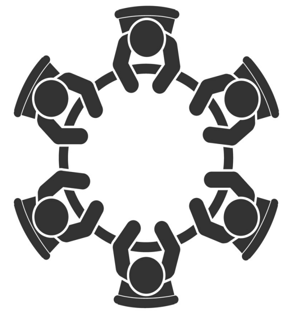
Practice Opportunities

High-Level Questions

Low-Level Questions

Affirmative Feedback

Corrective Feedback



Provide several of your questions.

Provide examples of your feedback.

Explicit Instruction

Research and Information

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

What are your strengths with explicit instruction?

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Focus on Vocabulary

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit

Vocabulary

INSTRUCTIONAL STRATEGIES

Math Vocabulary

Research and Information

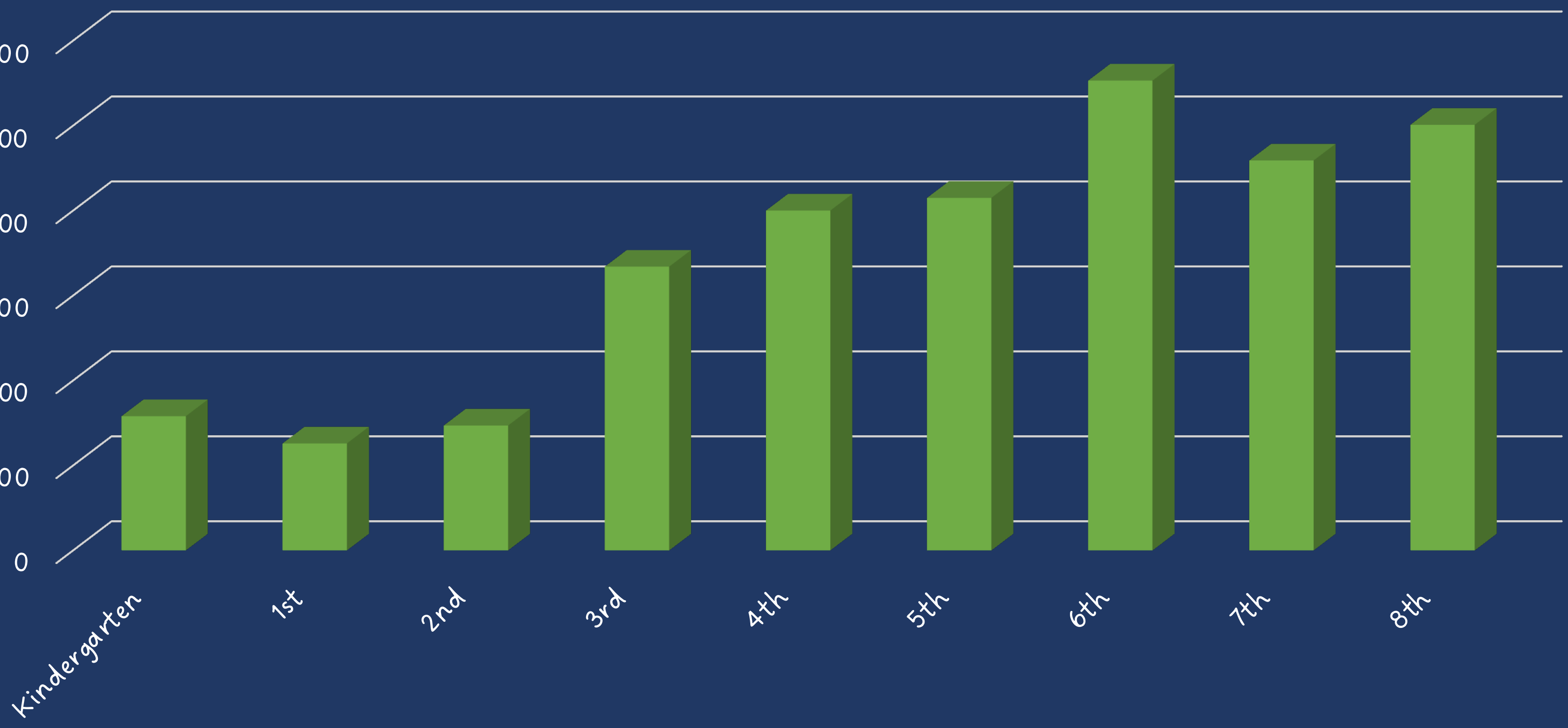
Use Formal Mathematics Language

Instead of that...	Say this...

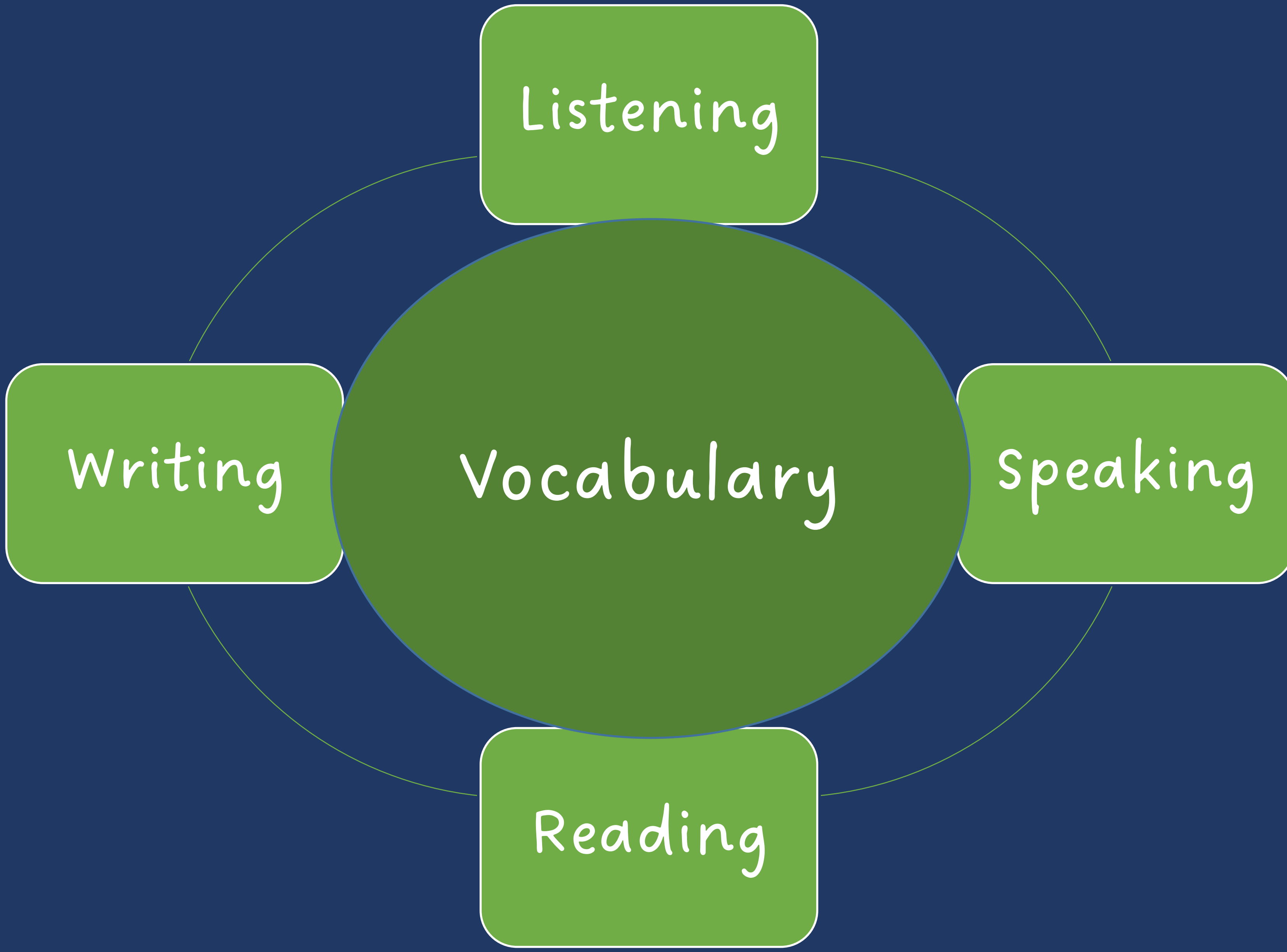
Significant correlation (r = .49) between mathematics vocabulary and mathematics performance. Mathematics vocabulary appears most important for word-problem performance (r = .58).
(Lin et al., 2021)

Early mathematics vocabulary related to mathematics and literacy.
(Hornburg et al., 2018; Purpura et al., 2017)

Students who experience difficulty with mathematics demonstrate lower mathematics vocabulary performance.
(Hughes et al., 2020; Powell & Nelson, 2017; Powell et al., 2017; Unal et al., 2021)



Powell, Bos, & Lin (2019)


Listening
Writing
Vocabulary
Speaking
Reading

1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)

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2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

divide vs. Continental Divide

variable vs. variably cloudy

Rubenstein & Thompson (2002)

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   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

sum vs. some

base vs. bass

eight vs. ate

rows vs. rose

Rubenstein & Thompson (2002)

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7. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)

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6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)

MATH

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2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

skip count vs. multiples

one-fourth vs. one quarter

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

11. Informal terms may be used for formal math terms

rhombus vs. diamond

vertex vs. corner

Use formal math language

Use terms precisely

What number is in the tens place?

What digit is in the tens place?
What is the value of the digit in the tens place?

135

Why this is important…

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.

The alligator eats the bigger number

is less than
OR
is greater than

Why this is important…

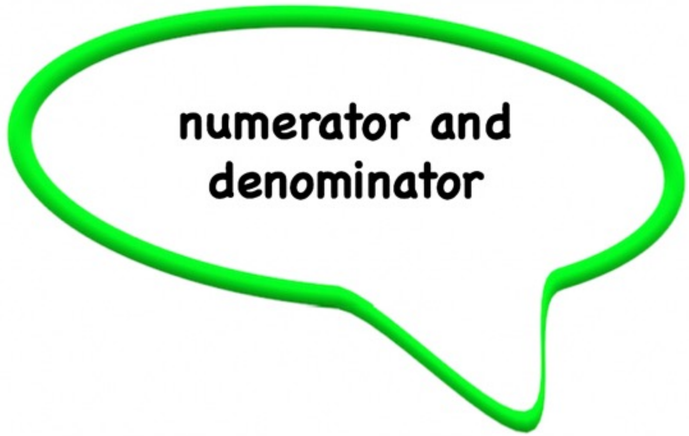
- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.

$$\begin{array}{r} 167 \\ +\ \ 294 \\ \hline \end{array}$$

Why this is important...

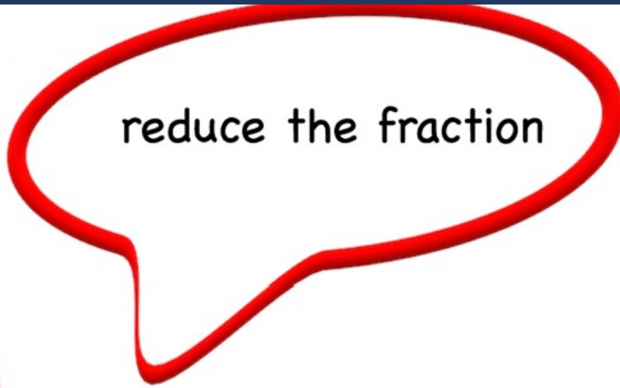
- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.

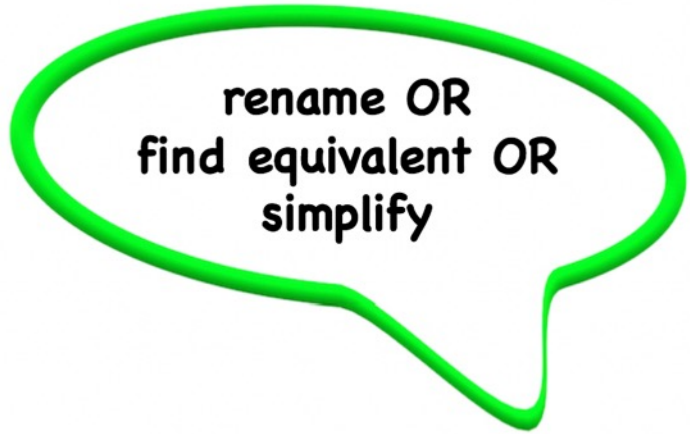
MATH



Why this is important…

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.





Why this is important…

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

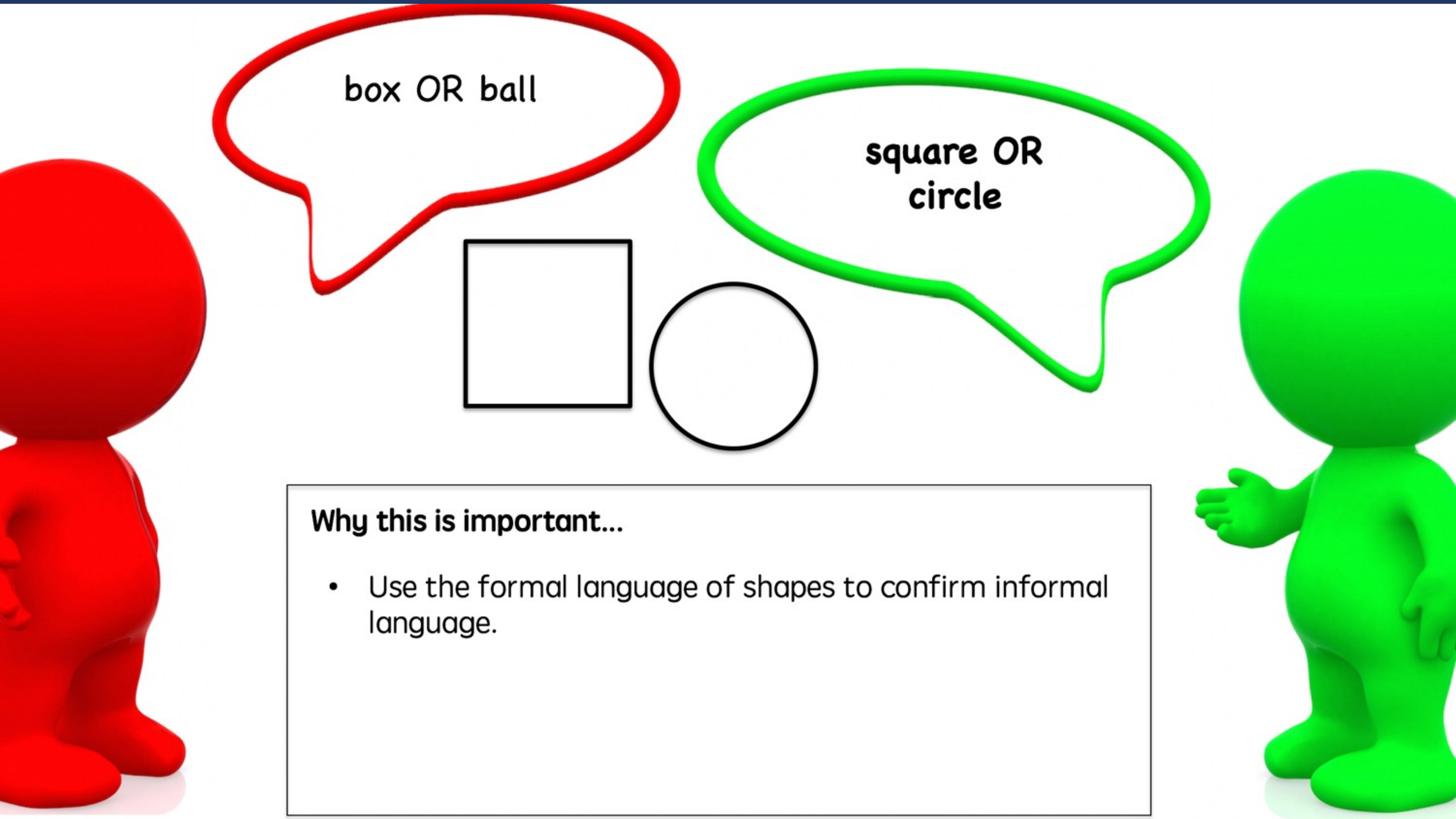
Four point seven
Four point oh seven

Four and seven tenths
Four and seven hundredths

4.7
4.07

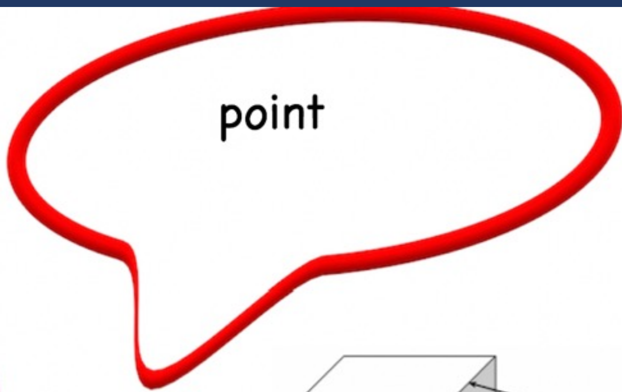
Why this is important...

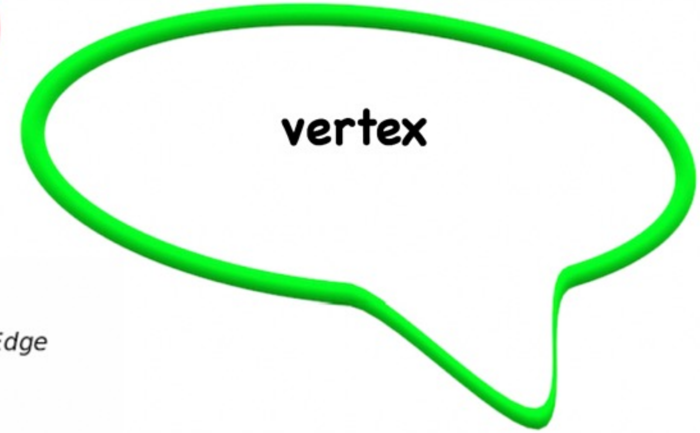
- Accurately shares the magnitude of the decimal.
- Emphasizes place value.

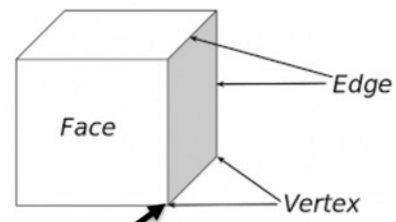


Why this is important…

- Use the formal language of shapes to confirm informal language.

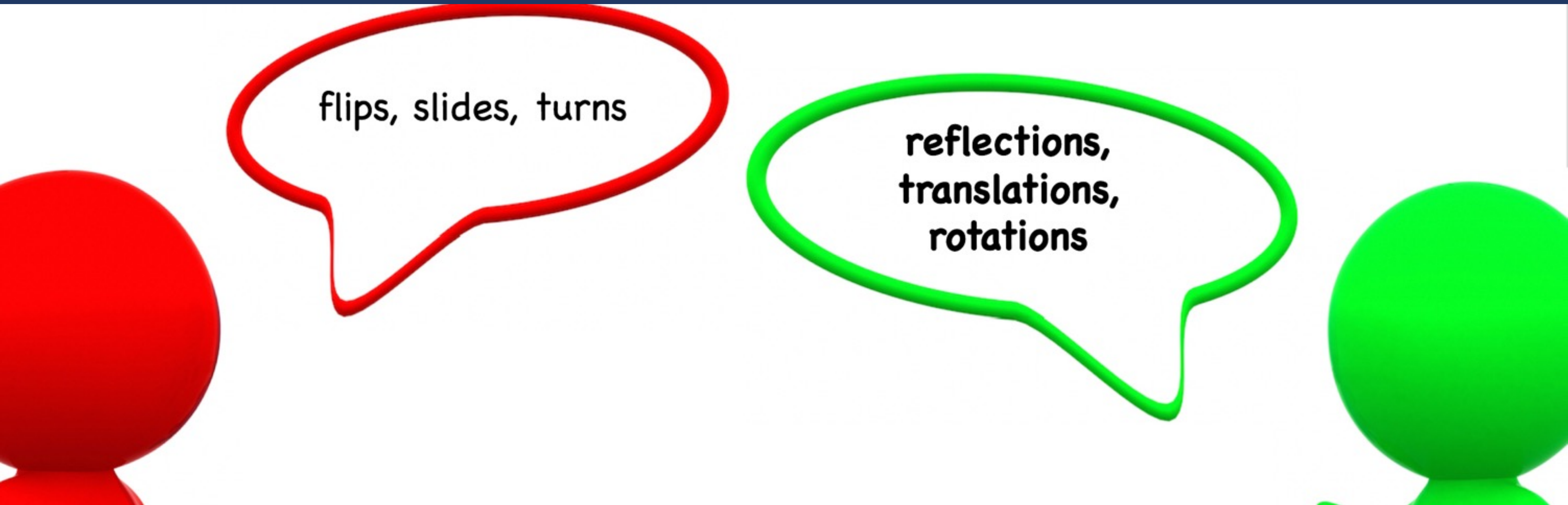






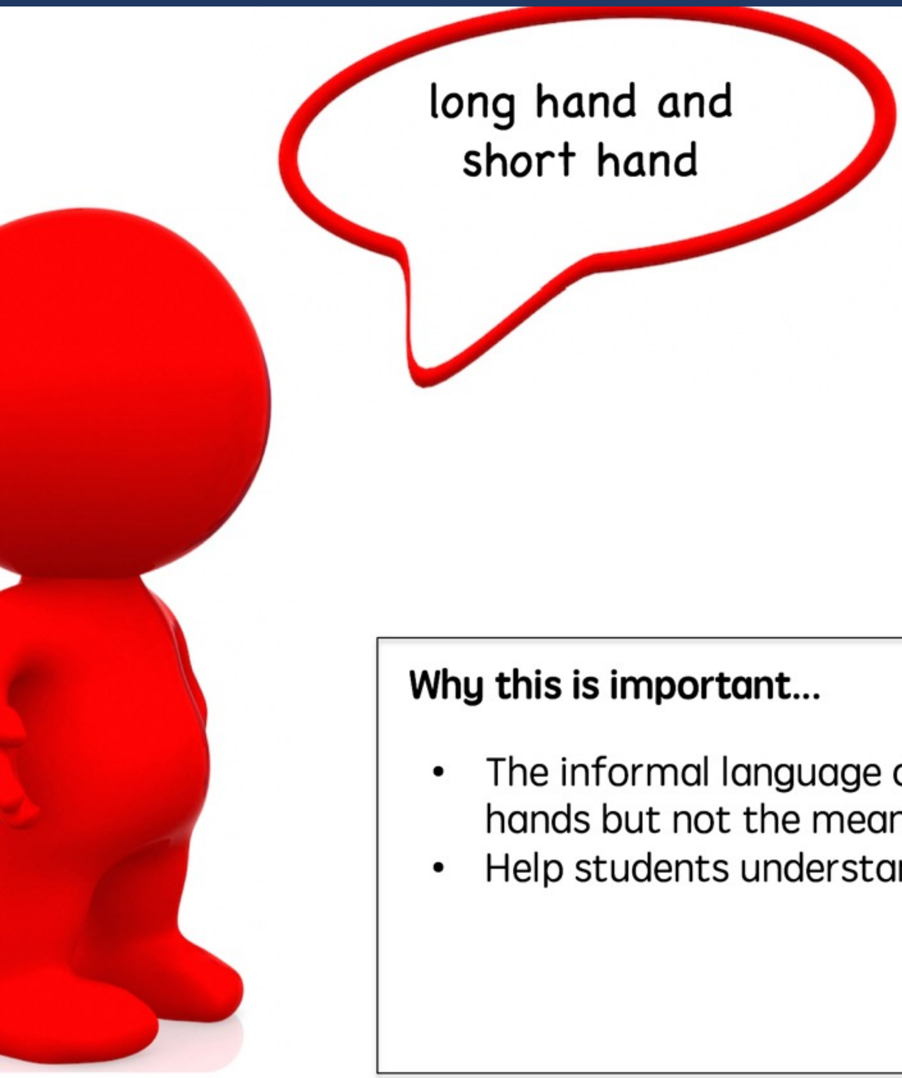
Why this is important...

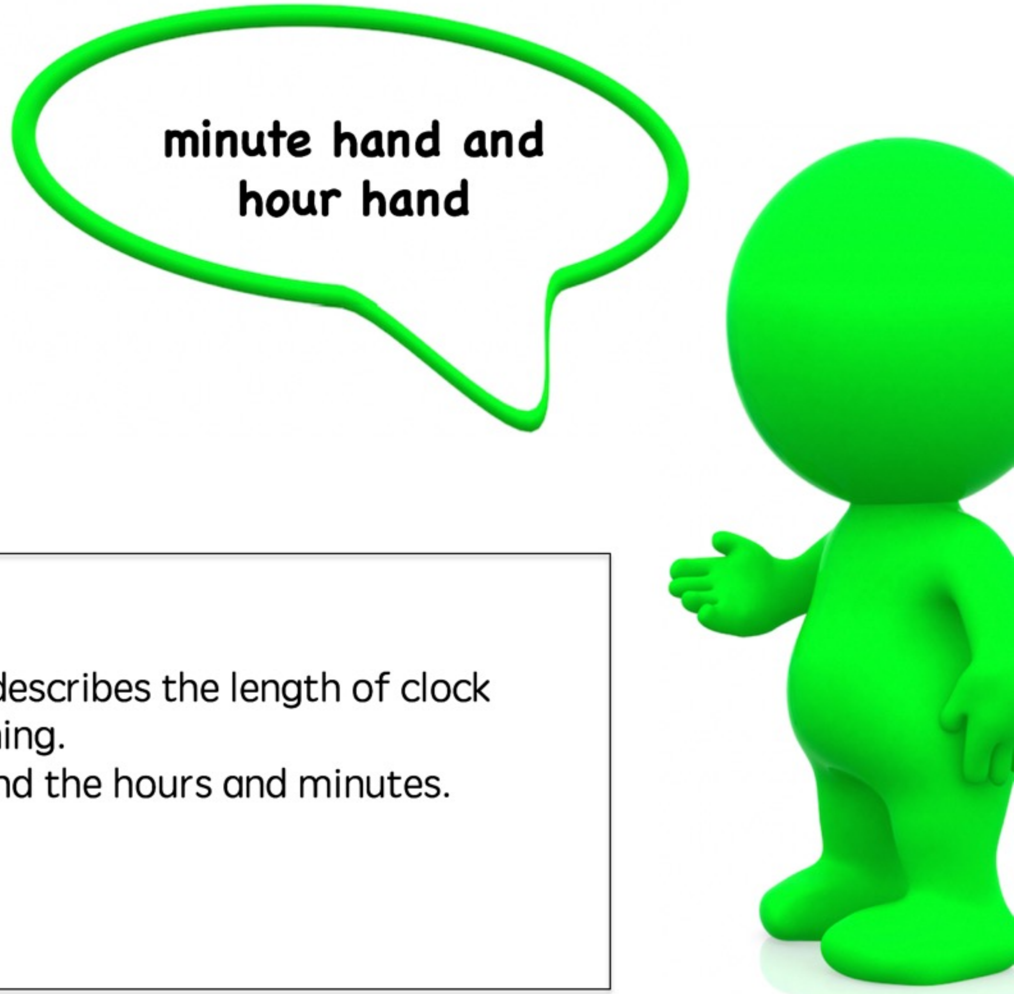
- This is the endpoint where two or more line segments or rays meet.



Why this is important…

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.





Why this is important…

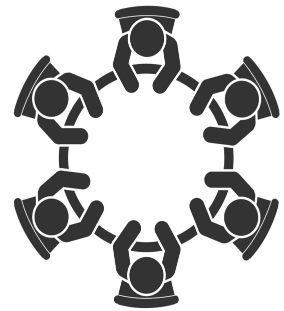
- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.

Math Vocabulary

Research and Information

Use Formal Mathematics Language

Instead of that...	Say this...



Identify examples of "Instead of ___, say ___."

Use formal math language

Use terms precisely

Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Factor

1 x 8 = 8

2 x 4 = 8

factor factor

Multiple

8 x 1 = 8

8 x 2 = 16

multiples of 8

E

Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

4:3

Proper fraction

$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

D

Coefficient

Constant

Term

Variable

$$\overbrace{2x^2}^{\text{term}} + \overbrace{x}^{\text{term}} - \overbrace{3}^{\text{term}}$$

coefficient

variable

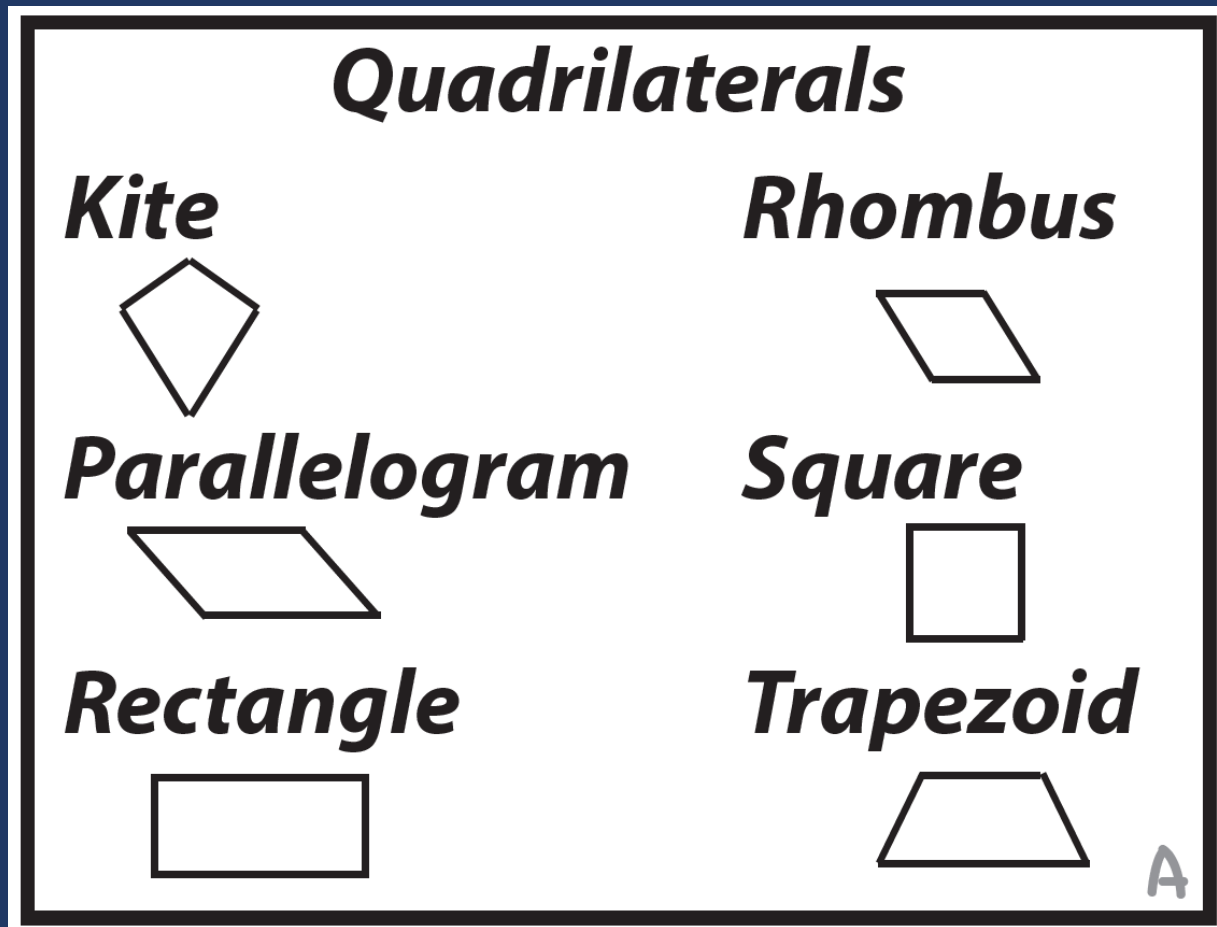
variable

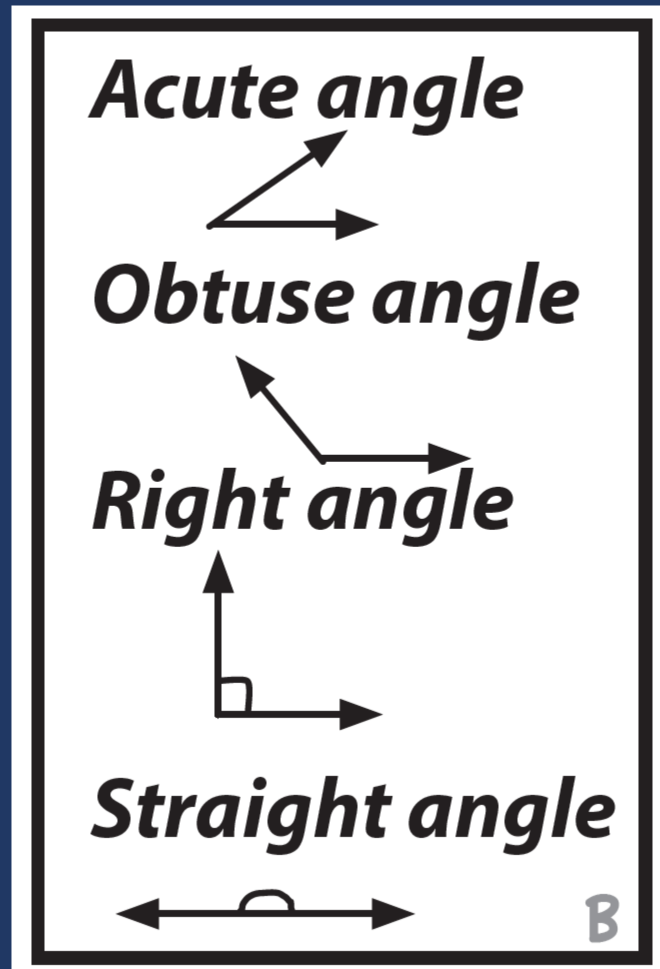
constant

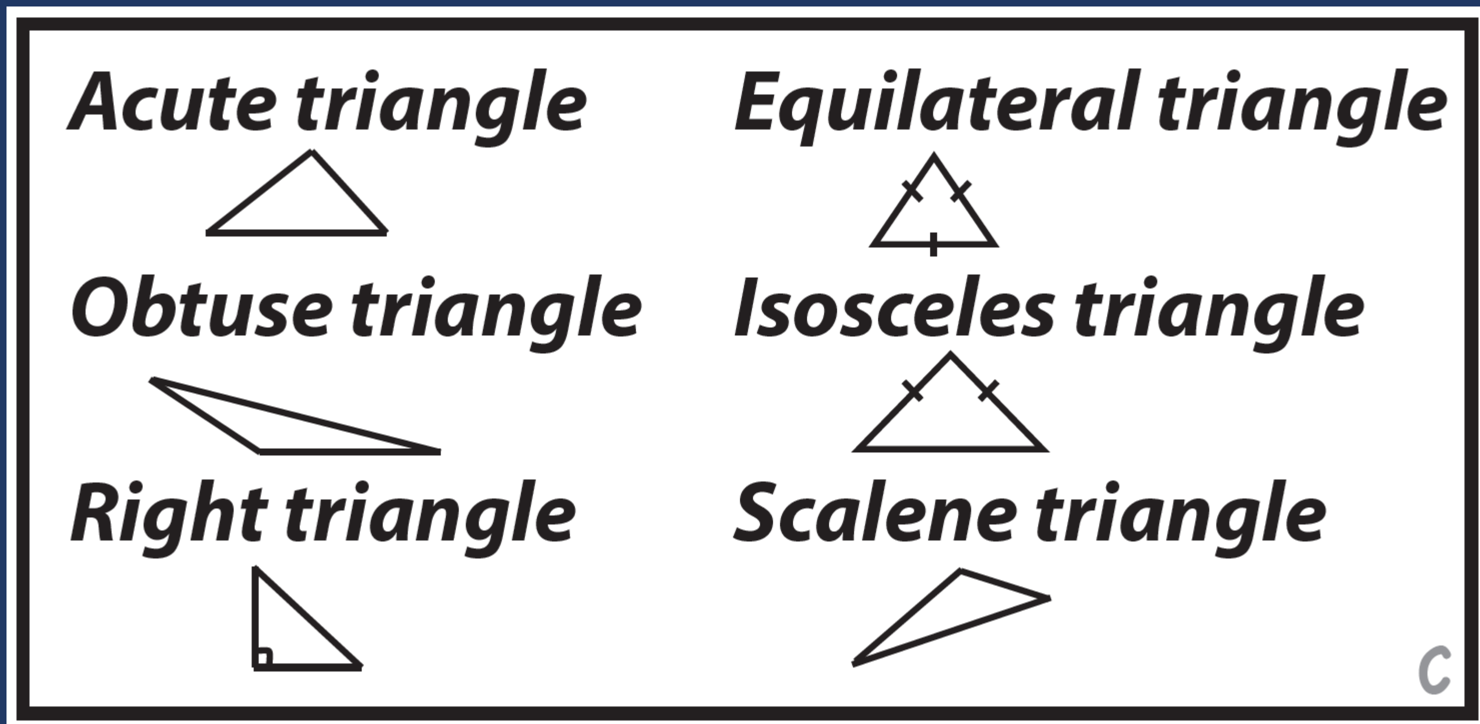
A

Equation	$9x - 4 = 7x$
Expression	$9x - 4$
Formula	$a^2 + b^2 = c^2$
Function	$f(x)$
Inequality	$9x - 4 > 6x$

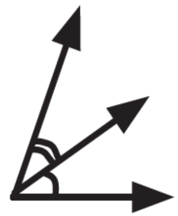
C


Quadrilaterals
Kite
Rhombus
Parallelogram
Square
Rectangle
Trapezoid
A

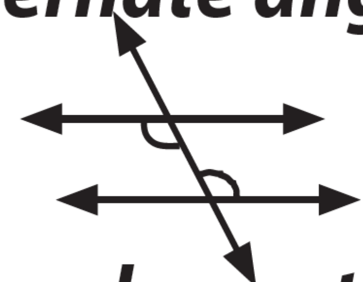

Acute angle
Obtuse angle
Right angle
Straight angle
B


Acute triangle
Equilateral triangle
Obtuse triangle
Isosceles triangle
Right triangle
Scalene triangle
C

Adjacent angles

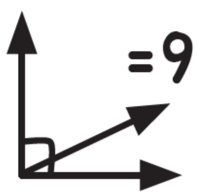


Alternate angles

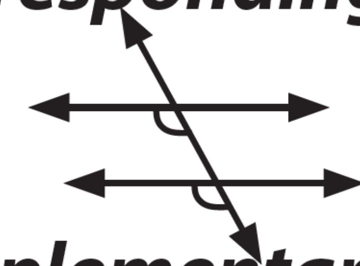


Complementary angles

=90°



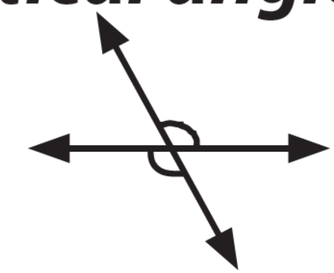
Corresponding angles



Supplementary angles

=180°

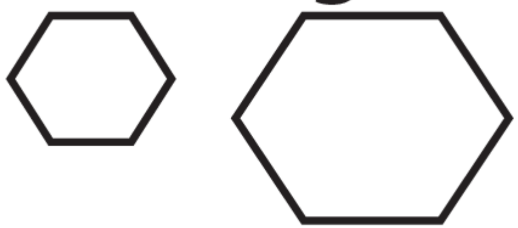
Vertical angles

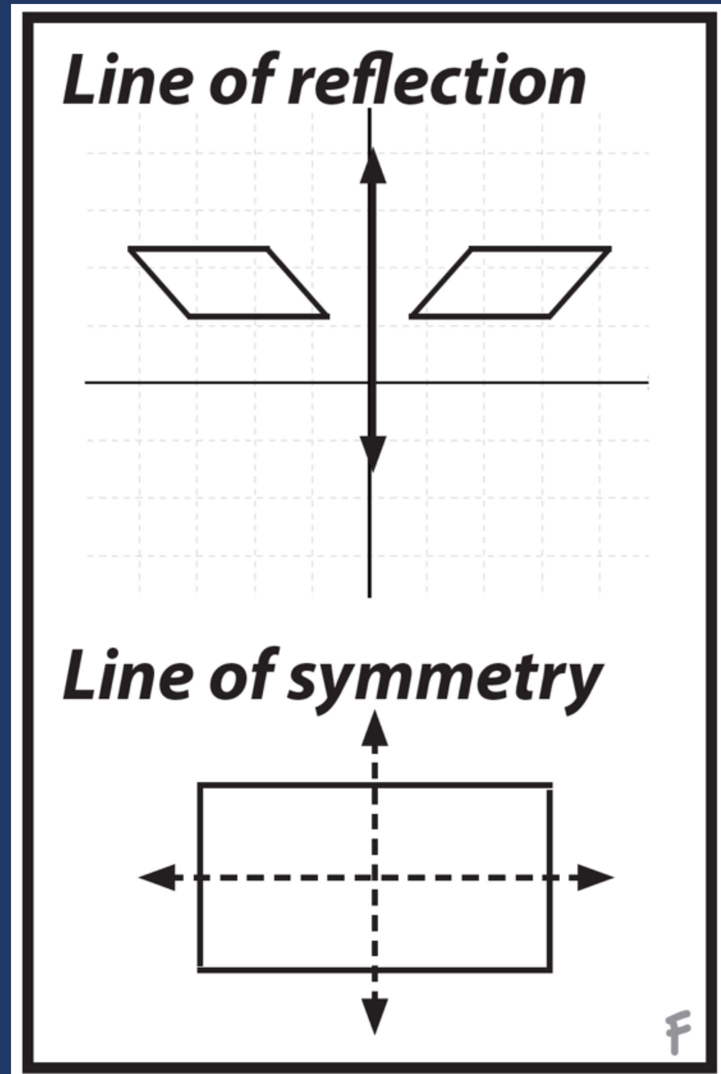


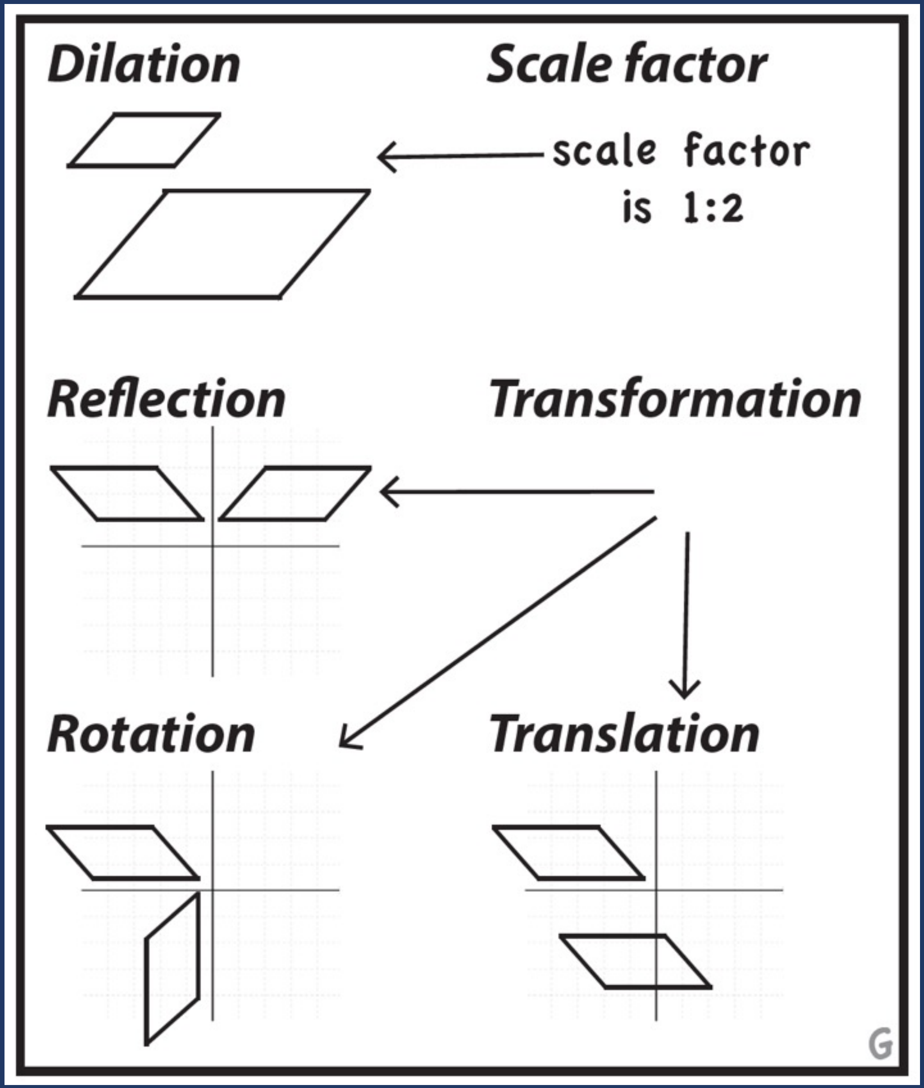
D

Congruent figures

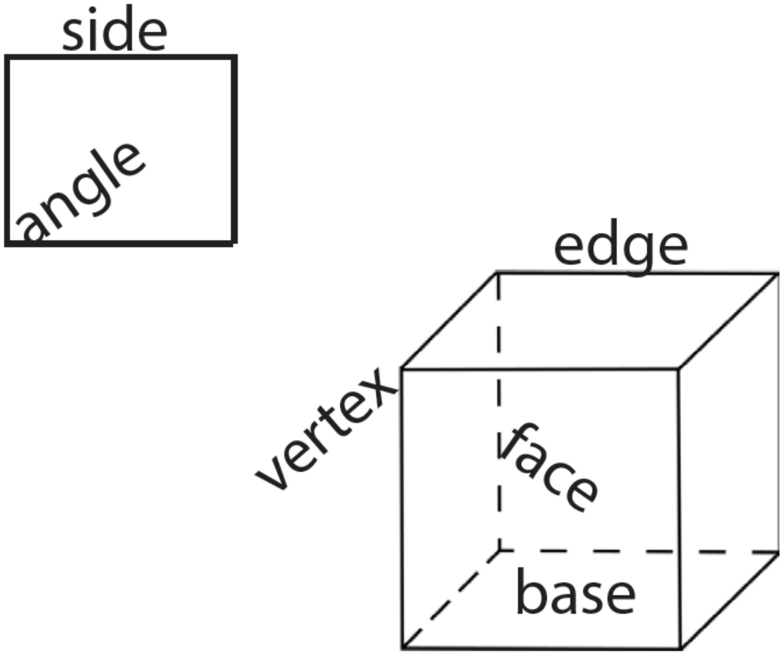
Similar figures

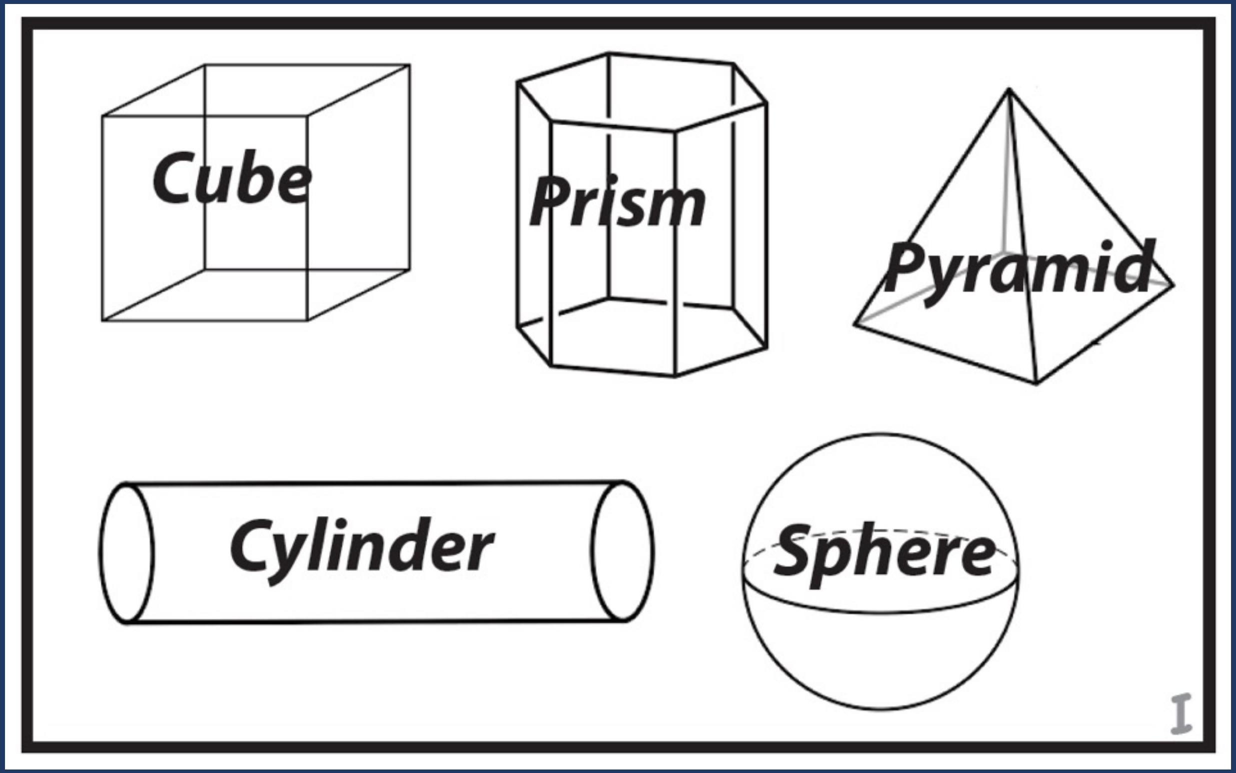



Line of reflection
Line of symmetry
F


Dilation
Scale factor
scale factor
is 1:2
Reflection
Transformation
Rotation
Translation
G

Angle
Base
Edge
Face
Side
Vertex




Cube
Prism
Pyramid
Cylinder
Sphere

Coordinate plane

Ordered pair

Quadrants

x

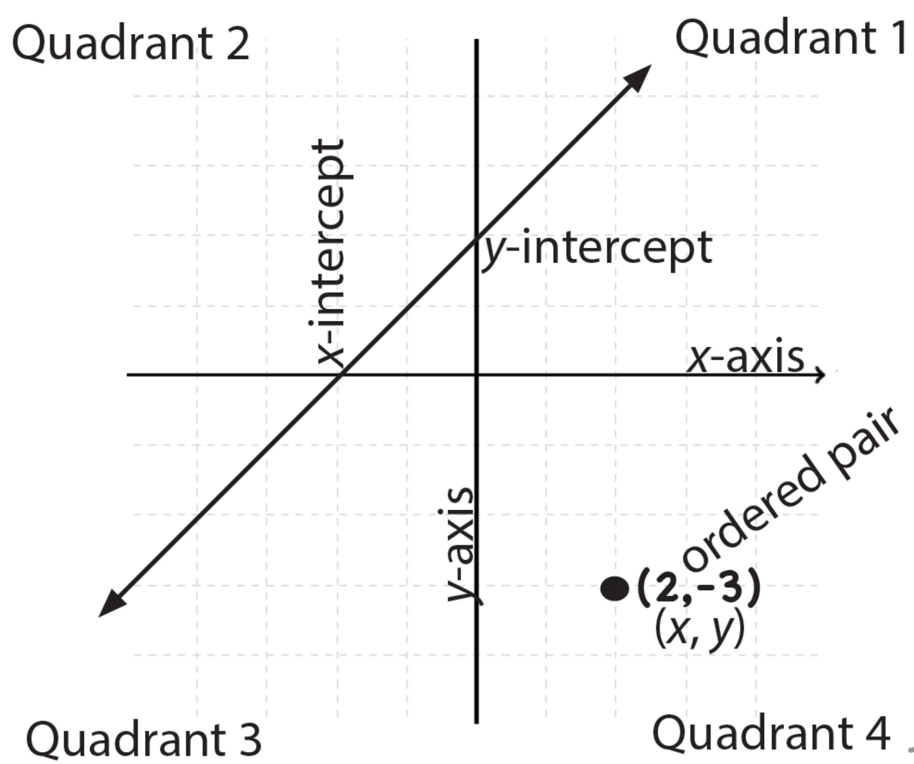
x-axis

x-intercept

y

y-axis

y-intercept



Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Discuss terms you want your students to use with precision.

Use formal math language

Use terms precisely

Math Vocabulary

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Strategies for Teaching Mathematics Language

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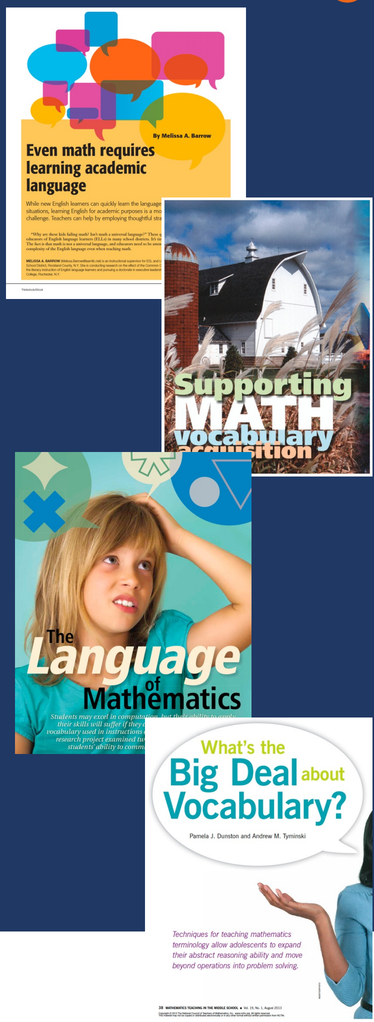

Use explicit instruction.
(Powell & Driver, 2015; Stevens et al., 2022)

Use semantic maps.
(Stevens et al., 2022)

Use visuals.
(Powell & Driver, 2015)

Use flashcards with spaced practice.
(Petersen-Brown et al., 2019)

Use read-alouds.
(Purpura et al., 2017)

Use explicit instruction.
Use multiple representations.
Create opportunities for discussion and feedback.
Monitor student progress.
Coordinate vocabulary instruction across settings.
Create additional practice opportunities.
(Nelson et al., 2020)

Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language

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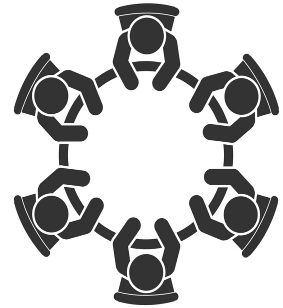

Discuss your strategy for focusing on mathematical vocabulary in your teaching.

Math Vocabulary

Use Terms With Precision

Strategies for Teaching Mathematics Language

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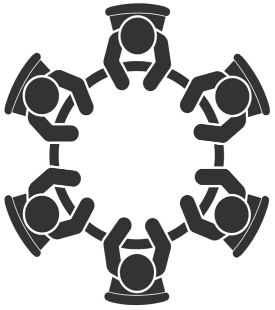

What are your strengths with mathematical language?

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Use Representations

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit

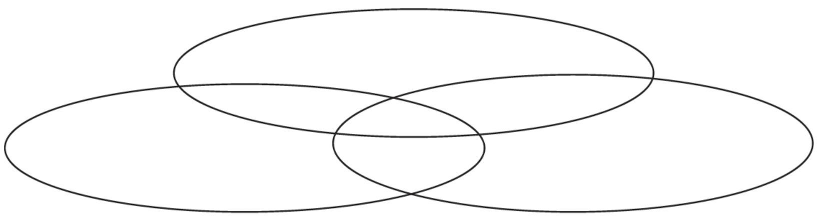
Vocabulary

Representations

INSTRUCTIONAL STRATEGIES

Representations

Research and Information



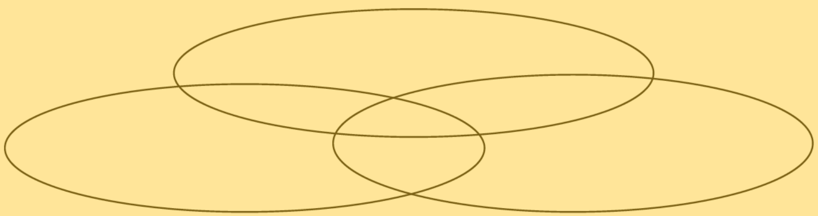
Hands-on manipulatives contribute to increases in mathematics performance.
(Bouck & Park, 2018; Carbonneau et al., 2013; Namkung & Bricko, 2021; Sherman & Bisanz, 2009; Strickland & Maccini, 2012)

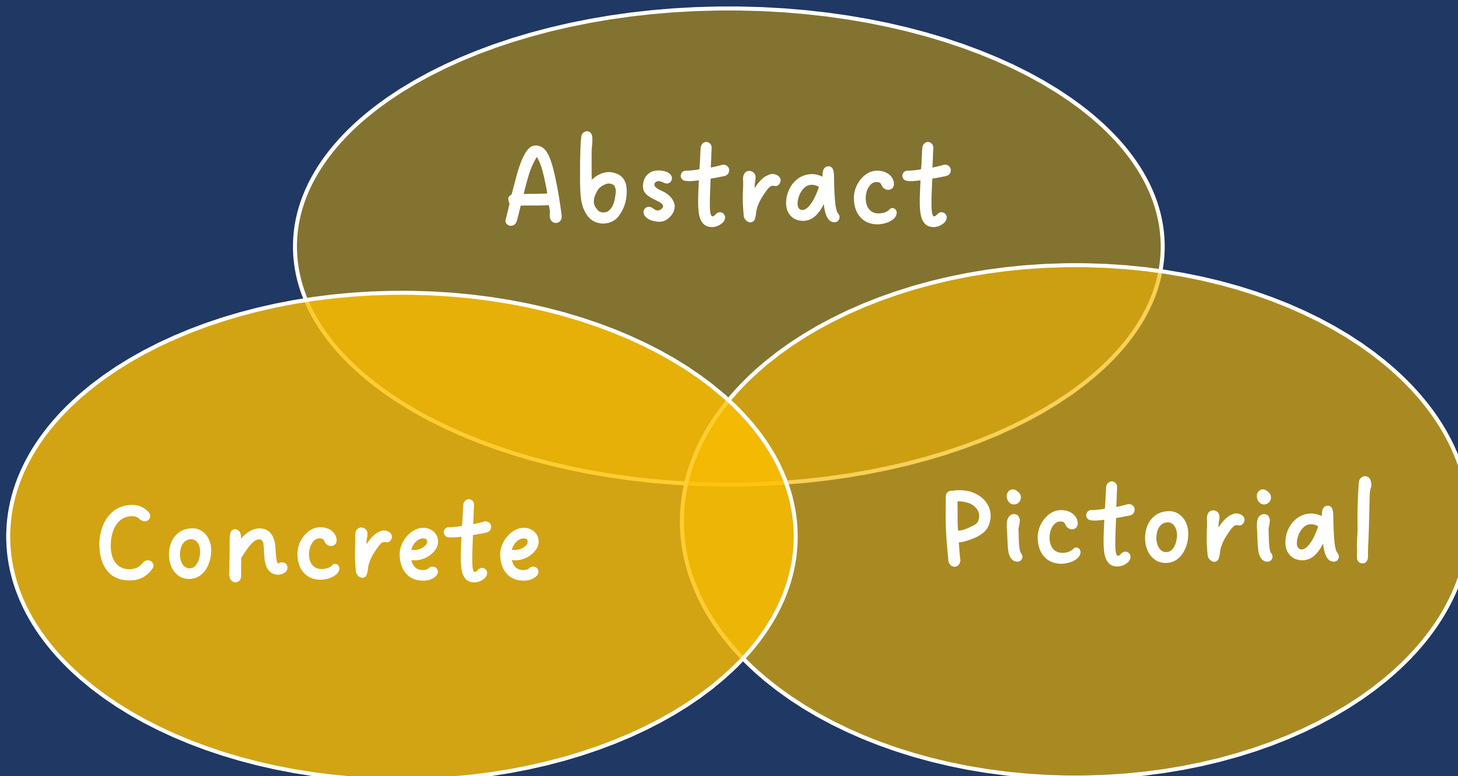
Virtual manipulatives contribute to increases in mathematics performance.
(Bouck et al., 2020; Satsangi et al., 2016)

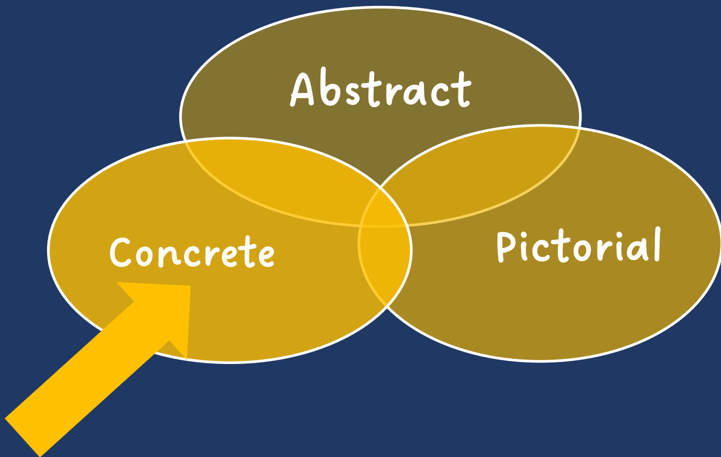
Other visuals (e.g., graphic organizers) contribute to increases in mathematics performance.
(Jitendra et al., 2009; Sharp & Dennis, 2017; van Garderen, 2007; Xin, 2008)

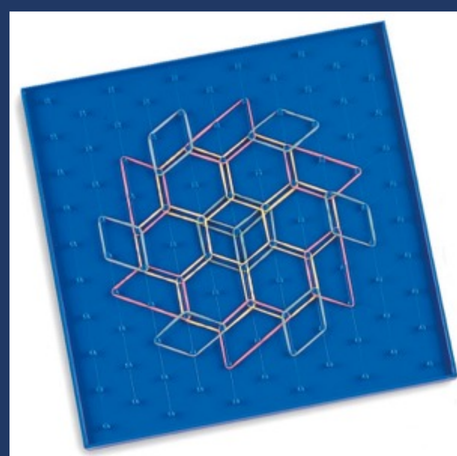
Representations

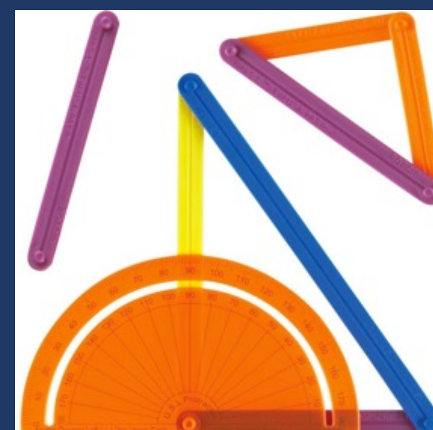
Research and Information

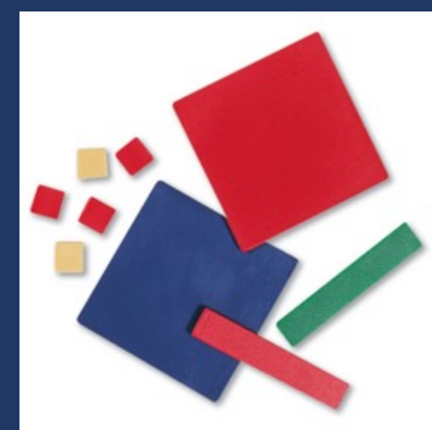


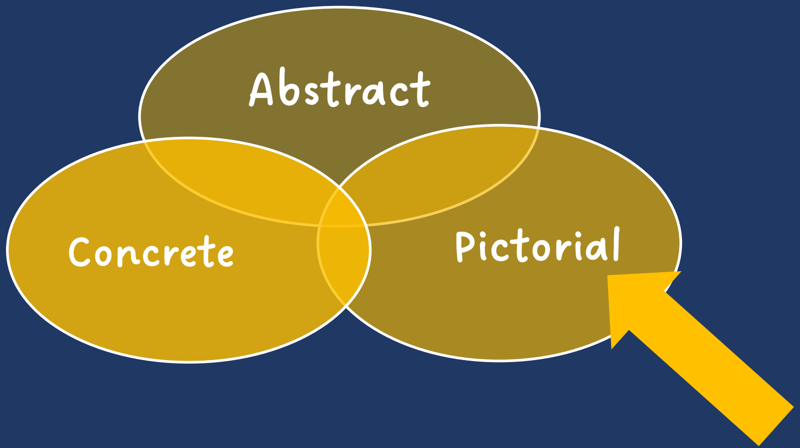

Abstract
Concrete
Pictorial

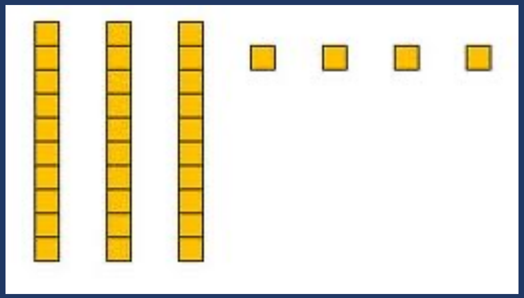

Abstract
Concrete
Pictorial

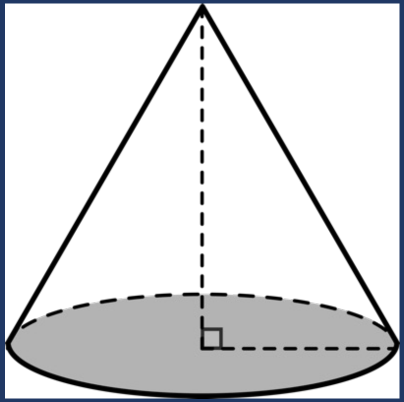


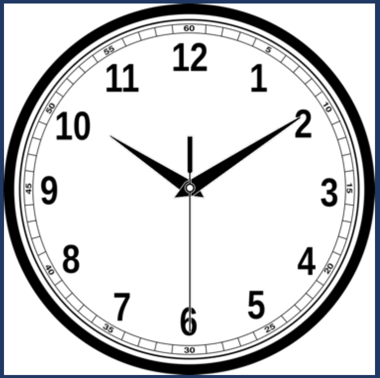





Abstract
Concrete
Pictorial






12
1
2
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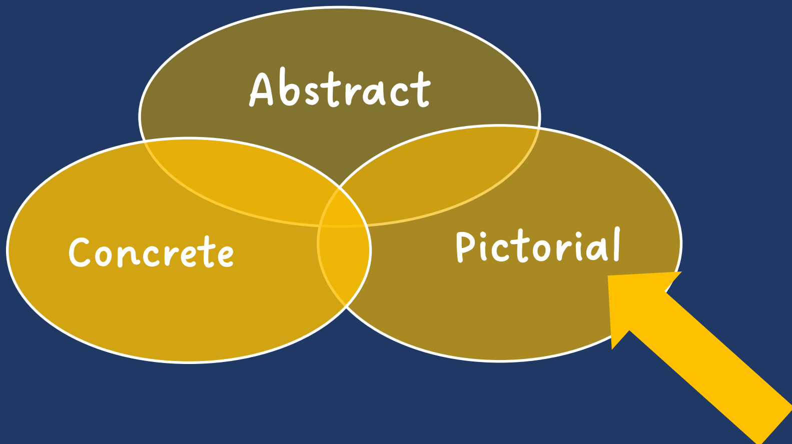
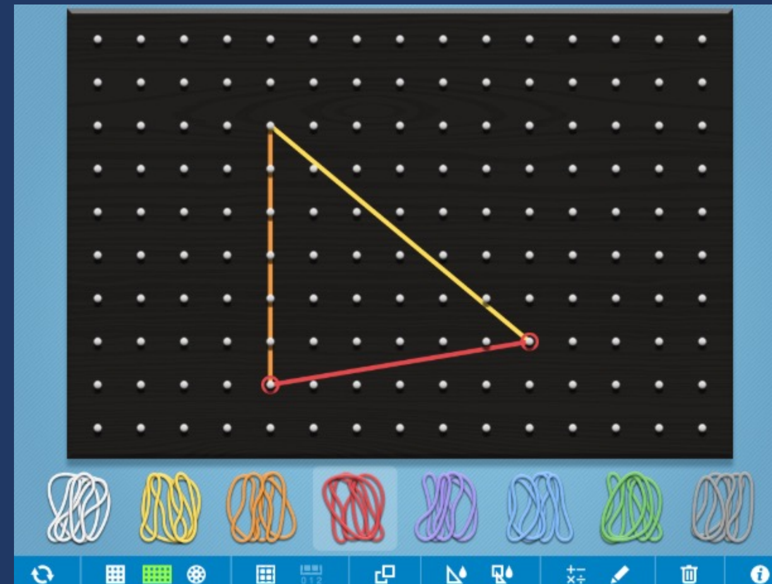
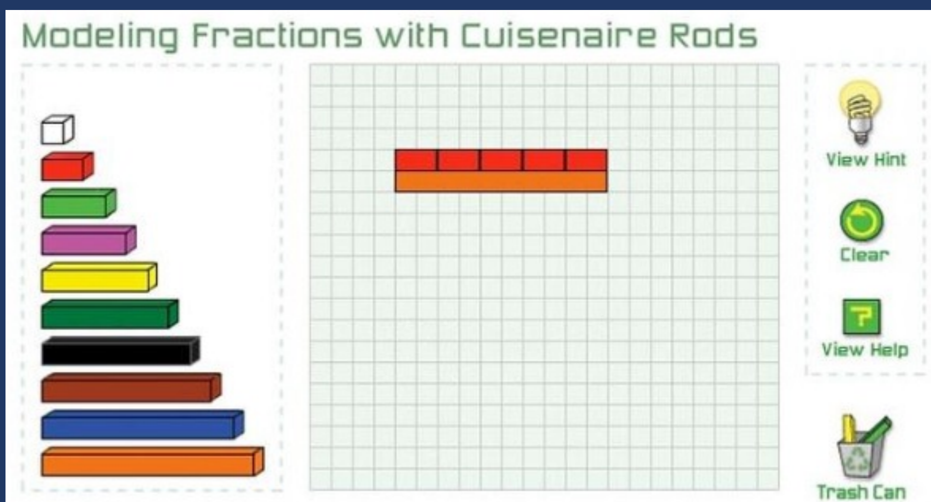
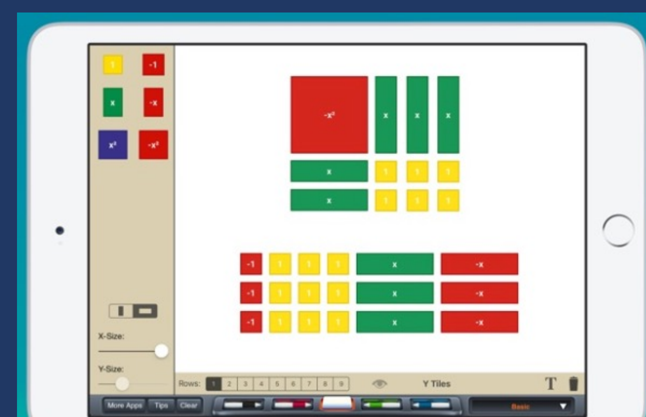
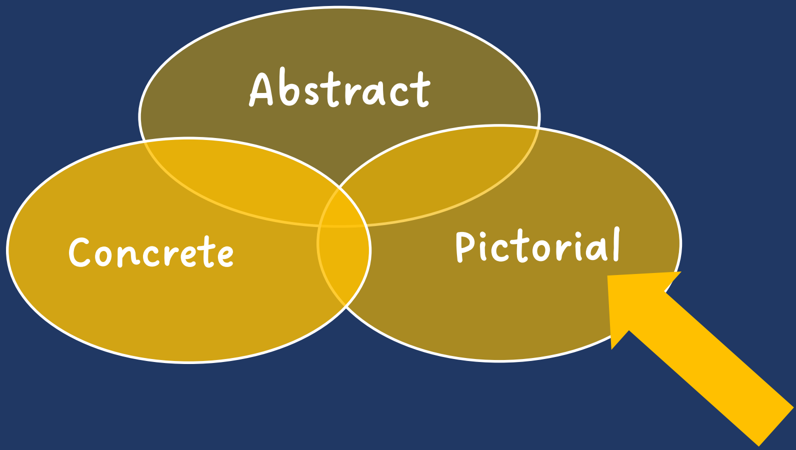
8
9
10
11

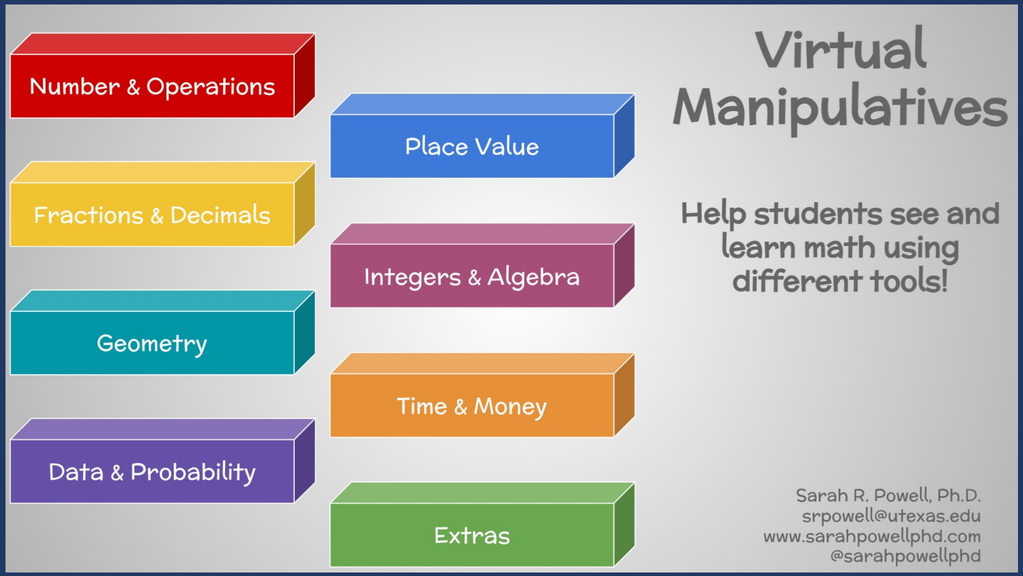

Abstract
Concrete
Pictorial

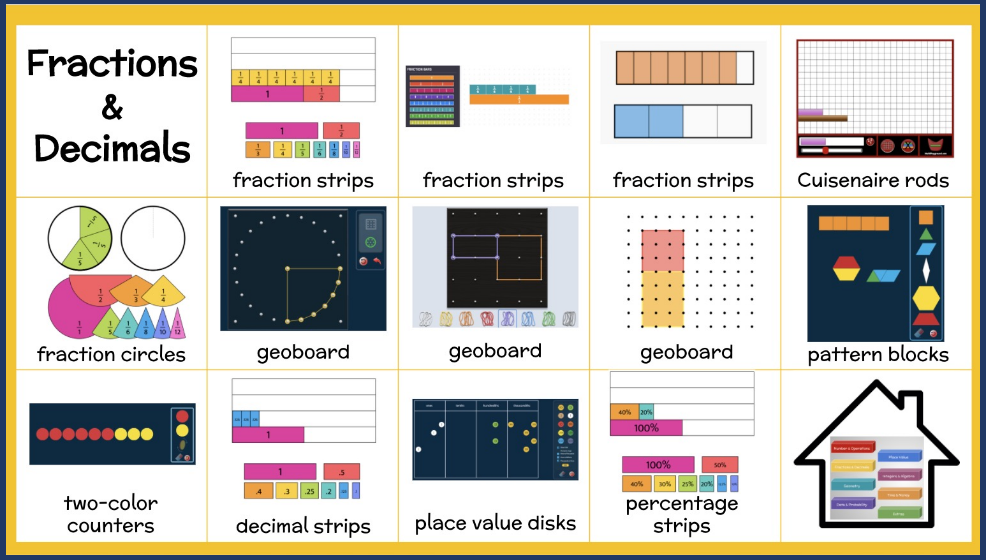


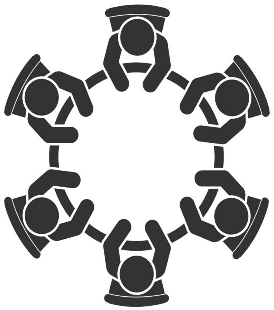

Modeling Fractions with Cuisenaire Rods
View Hint
Clear
View Help
Trash Can




Abstract
Concrete
Pictorial

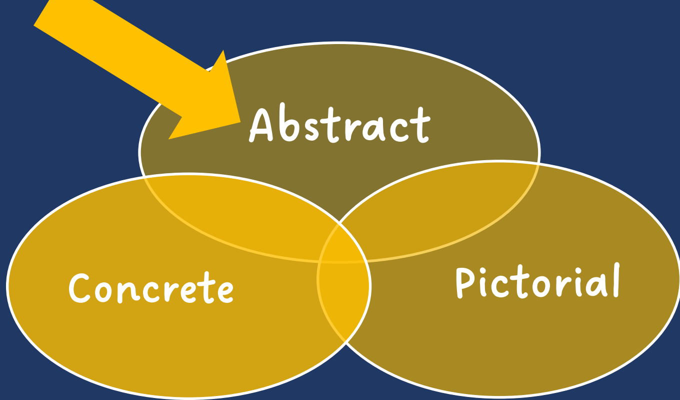

Number & Operations
Place Value
Fractions & Decimals
Integers & Algebra
Geometry
Time & Money
Data & Probability
Extras
Virtual Manipulatives
Help students see and learn math using different tools!
Sarah R. Powell, Ph.D.
srpowell@utexas.edu
www.sarahpowellphd.com
@sarahpowellphd


Fractions & Decimals
fraction strips
fraction strips
fraction strips
Cuisenaire rods
fraction circles
geoboard
geoboard
geoboard
pattern blocks
two-color counters
decimal strips
place value disks
percentage strips



Explore 3 virtual manipulatives.

Share with a partner.



$2 + 8 = 10$

$34 = 3$ tens and 4 ones

$x - 6 = 8$

$$\begin{array}{r} 4{,}179 \\ +\quad 569 \\ \hline \end{array}$$

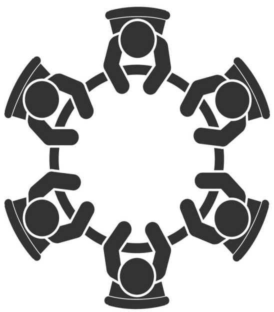
Representations

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Students should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.



What are your strengths with multiple representations?

What are the opportunities for growth?

What materials do you need?

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

udents should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Build Fluency

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit

Vocabulary

Representations

INSTRUCTIONAL STRATEGIES

Fluency

Fluency

Research and Information

Types of Fluency

Type	Memorization?	
	Yes	No

Mathematics fluency, particularly fluency with facts and computation, is related to overall mathematics performance.
(Bailey et al., 2012; Cirino et al., 2019; Koponen et al., 2007, 2017; Vukovic et al., 2014

Fact fluency practice improves mathematics fact performance.
(Burns et al., 2010; Codding et al., 2011; McCallum et al., 2004; Nelson et al., 2013; Poncy et al., 2010; Schutte et al., 2015; Stocker & Kubina, 2017)

Fluency is doing mathematics easily and accurately.

Fluency in mathematics makes mathematics easier.

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to emphasize both conceptual learning and procedural learning.

Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Knowing multiples

Identifying shapes

Knowing formulas

Memorization or automaticity

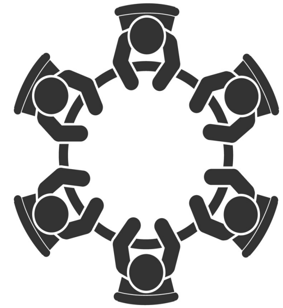
Ease and accuracy

Fluency

Research and Information

Types of Fluency

Type	Memorization?	
	Yes	No



List different types of fluency. Discuss whether they require memorization.

Fluency with Facts

Addition

Subtraction

Multiplication

Division

Addition

100 addition facts

Single-digit addends sum to a single- or double-digit number

$$\begin{array}{r} 5 \\ +\ 4 \\ \hline 9 \end{array} \quad \begin{array}{l} \text{(addend)} \\ \text{(addend)} \\ \text{(sum)} \end{array}$$

Total

Addition

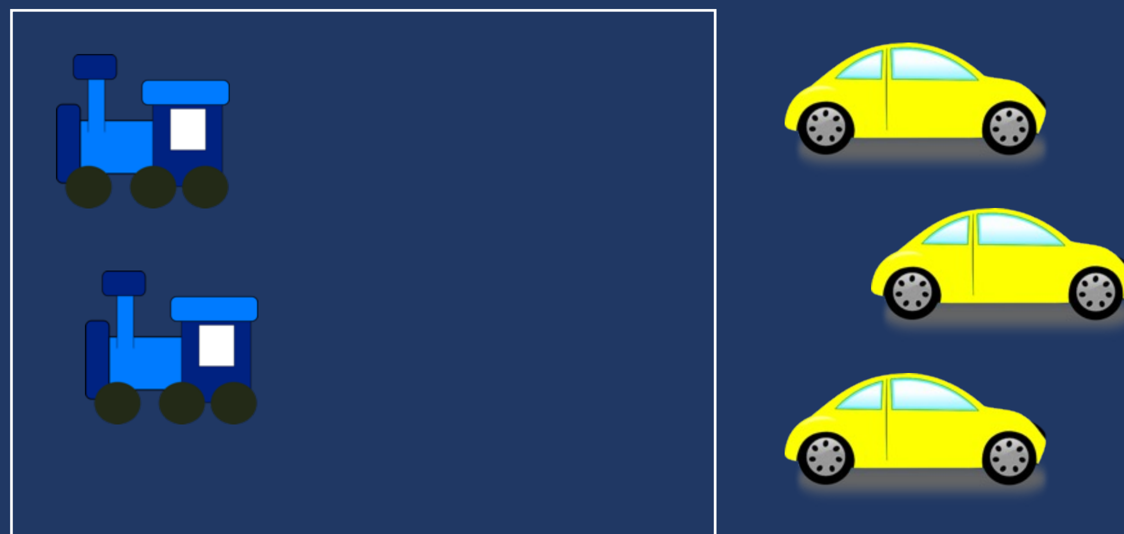
Count one set, count another set, put sets together, count sum

$2 + 3 = 5$

Change

Addition

Start with a set, add the other set, count sum



$$2 + 3 = 5$$

Total

Addition

Parts put together into a total

Karly saw 4 cardinals and 5 blue jays. How many birds did Karly see?

Change

Addition

An amount that **increases** or **decreases**

Premila had $4. Then they earned $5 for cleaning their room. How much money does Premila have now?

$3 + 9 =$ __

Addition

If you have brown eyes:

What's a Total story to show addition?

If you don't have brown eyes:

What's a Change/Join story to show addition?

100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

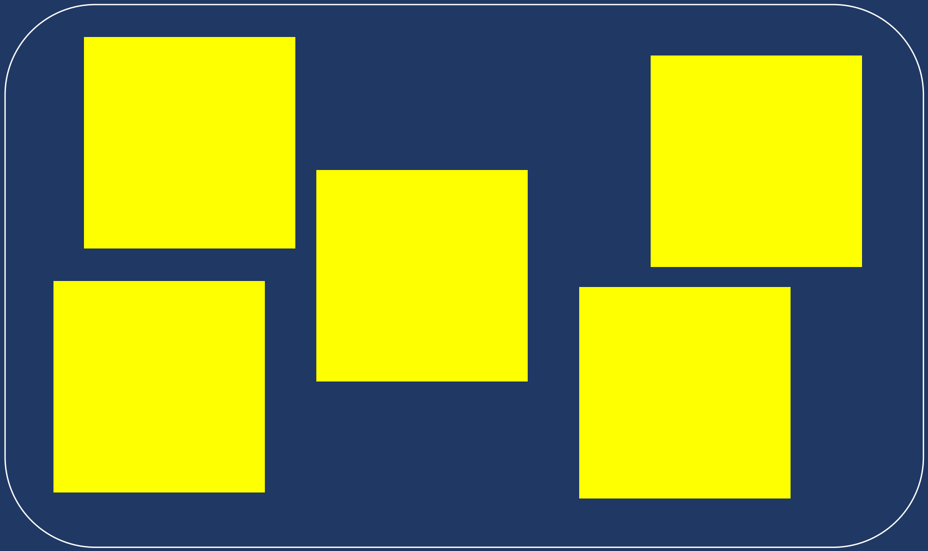
$$\begin{array}{r} 16 \\ -\quad 8 \\ \hline 8 \end{array}$$

(minuend)
(subtrahend)
(difference)

Change

Subtraction

Start with a set, take away from that set, count difference

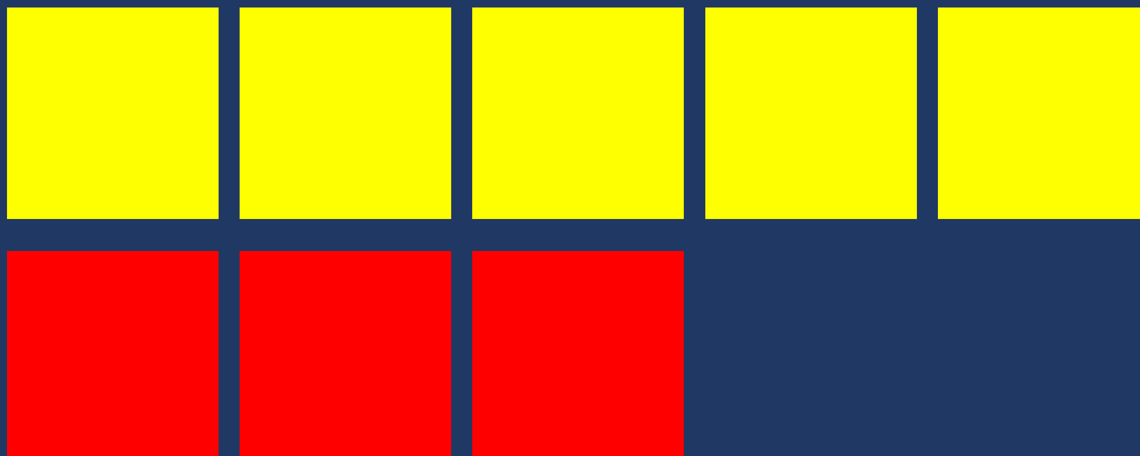


$5 - 3 = 2$

Difference

Subtraction

Compare two sets, count difference



$$5 - 3 = 2$$

Change

Subtraction

An amount that increases or **decreases**

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?

Difference

Subtraction

Greater and lesser amounts compared for a difference

Rachel has 9 apples. Jodie has 2 apples. How many more apples does Rachel have? (How many fewer does Jodie have?)

$9 - 5 = \_\_$

Subtraction

If you would chose beaches:

What's a Change/Separate story to show subtraction?

If you would chose mountains:

What's a Difference story to show subtraction?

Multiplication

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{r} 2 \\ \times\ 3 \\ \hline 6 \end{array}$$

(factor)
(factor)
(product)

Equal Groups

Multiplication

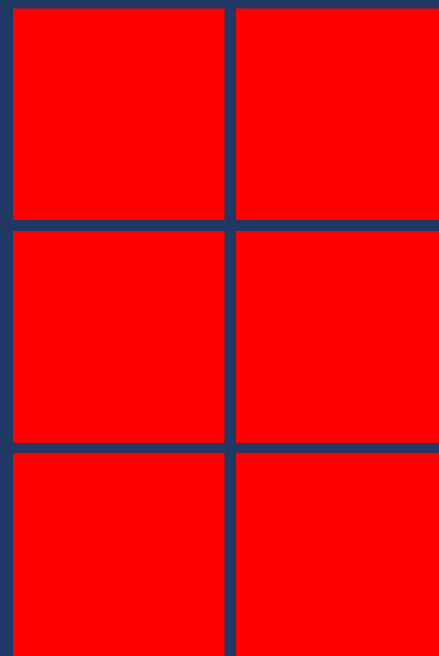
Show the groups, show the amount for each group, count product

$$3 \times 2 = 6$$

Equal Groups

Multiplication

Show the groups, show the amount for each group, count product



$3 \times 2 = 6$

Comparison

Multiplication

Show a set, then multiply the set

$3 \times 2 = 6$

Equal Groups

Multiplication

Groups multiplied by number in each group for a product

Rhiannon has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Rhiannon have altogether?

Comparison

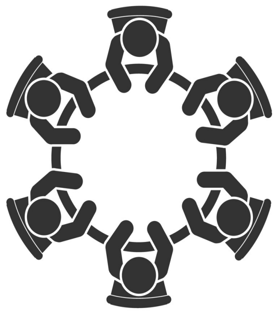
Multiplication

Set multiplied by a number of times for a product

Vivienne had 12 stickers. Jessica had 2 times as many stickers as Vivienne. How many stickers did Jessica have?

$2 \times 5 =$ __

Multiplication



If you wear glasses:

What's an Equal Groups story to show multiplication?

If you don't wear glasses:

What's a Comparison story to show multiplication?

90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

$$8 \div 4 = 2$$

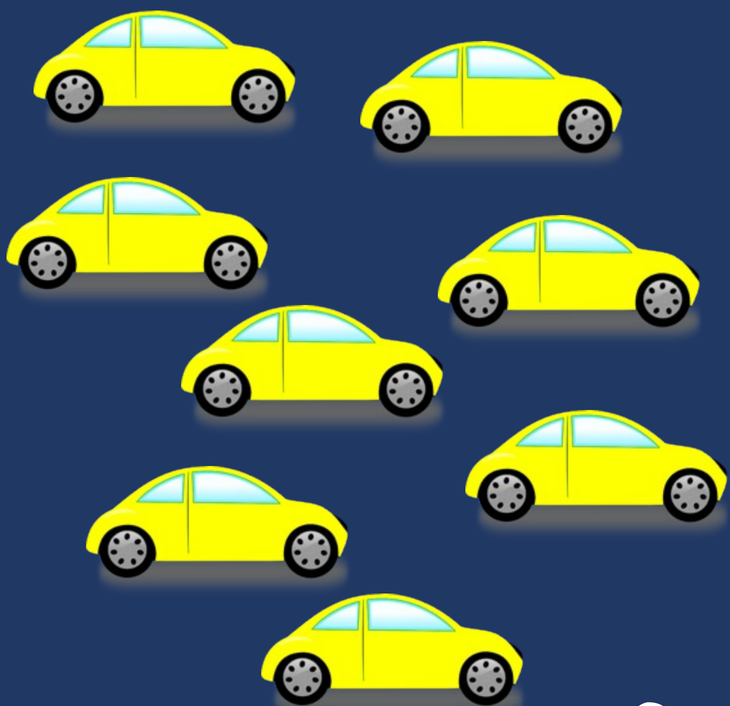
(<u>dividend</u>) (<u>divisor</u>) (<u>quotient</u>)

Equal Groups

(Partitive Division)

Division

Show the dividend, divide equally among divisor, count quotient



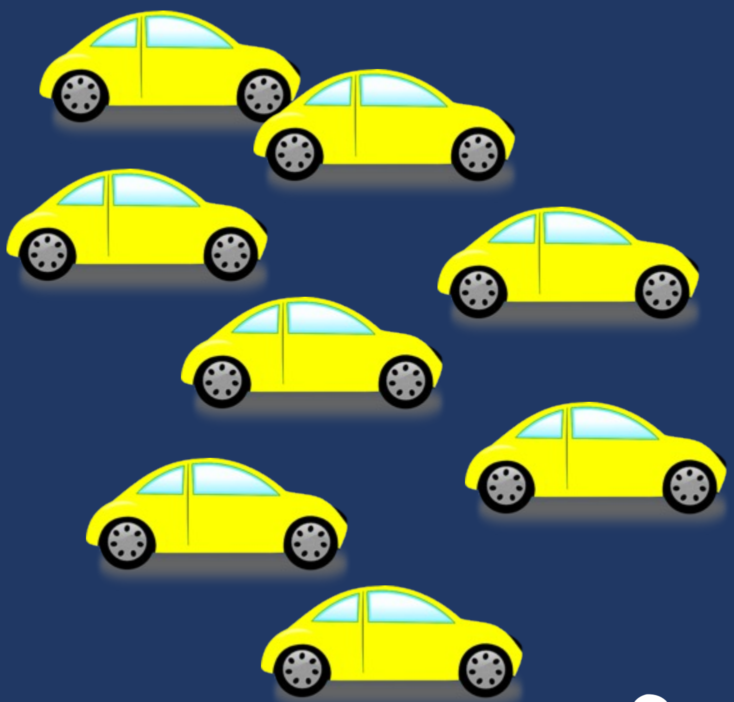
$$8 \div 2 = 4$$

Equal Groups

(Quotative Division)

Division

Show the dividend, make groups of the divisor, count groups



$8 \div 2 = 4$

Equal Groups

Division

Groups multiplied by number in each group for a product

Stefanie has 12 pencils. She wants to share them equally among her 2 friends. How many pencils will each friend receive?

Nicole has 12 pencils. She put them into pencil pockets with 6 pencils each. How many pencil pockets did Nicole use?

$12 \div 4 = \_\_$

Division

If you watch *Stranger Things*:

What's a Partitive story to show division?

If you watch *Ted Lasso*:

What's a Quotative story to show division?

Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ +\ \ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ \ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ \ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ \ 8 \\ \hline \end{array}$$

MATH

Fluency

Strategies for Building Fluency

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Cover, Copy, Compare

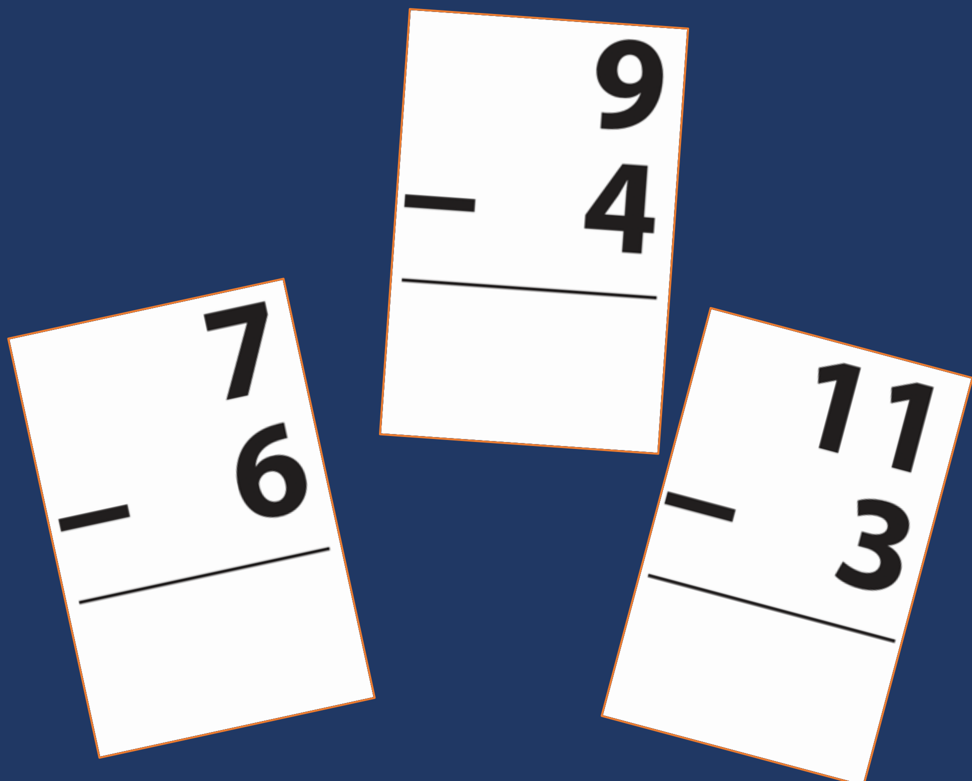
$9 \times 6 = 54$	$8 \times 6 = 48$
$7 \times 8 = 56$	$6 \times 5 = 3$
$9 \times 9 = 81$	$\times$
$6 \times 7 = 42$	
$8 \times 8 = 64$	

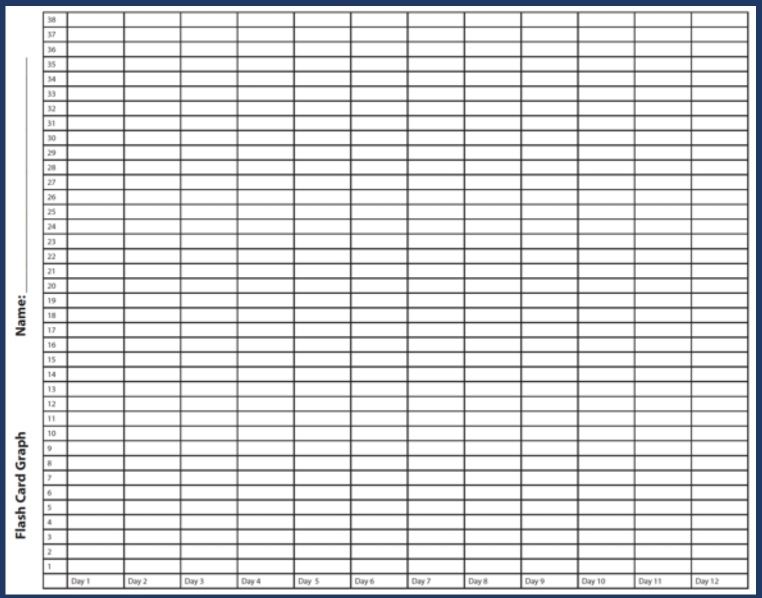
File Folder

6 + 3 =	
1 + 7 =	8
6 + 4 =	1
7 + 3 =	1
2 + 7 =	9
5 + 6 =	11
4 + 7 =	11
7 + 8 =	15
6 + 7 =	13
7 + 9 =	16
7 + 6 =	13
8 + 7 =	15
7 + 0 =	7
9 + 6 =	15
6 + 0 =	6
6 + 8 =	14

Taped Problems

6×5	8×6	7×9	6×8
9×8	8×5	7×8	6×6
7×7	6×9	5×9	8×4
9×4	6×9	9×5	8×7
6×7	8×8	4×8	5×7


7
− 6
9
− 4
11
− 3


Flash Card Graph
Name:
38
37
36
35
34
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21
20
19
18
17
16
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14
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12
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8
7
6
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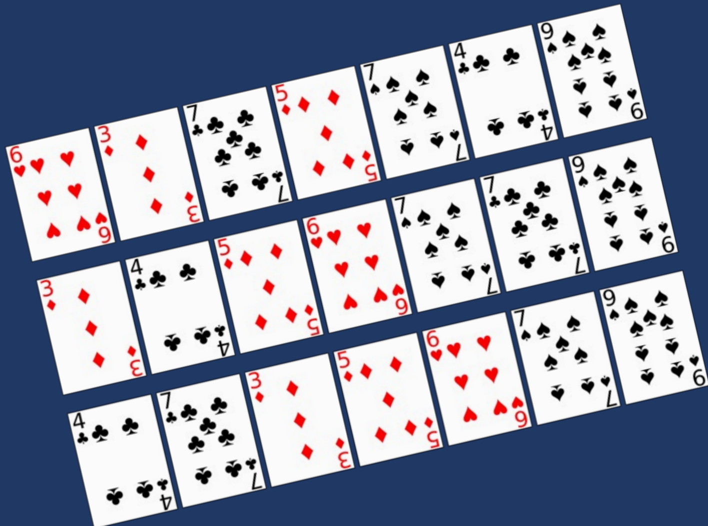
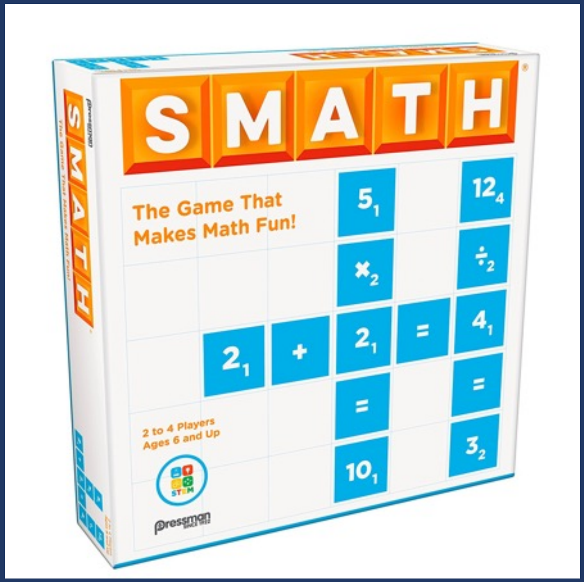
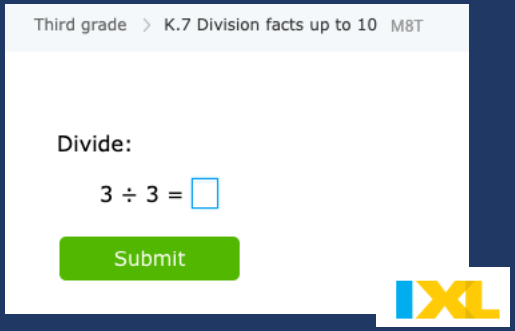
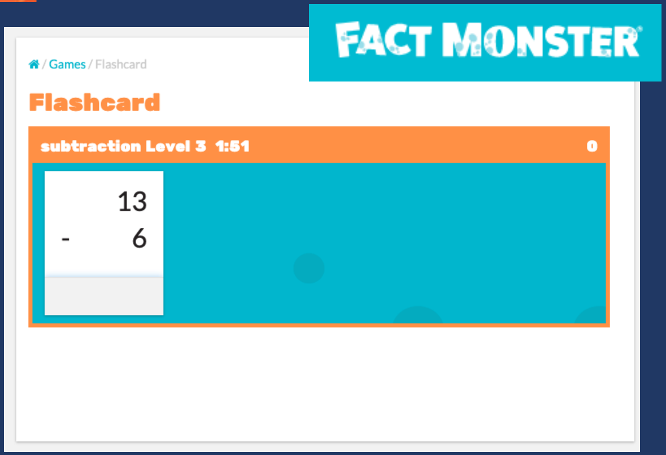
Day 1
Day 2
Day 3
Day 4
Day 5
Day 6
Day 7
Day 8
Day 9
Day 10
Day 11
Day 12




SMATH
The Game That Makes Math Fun!
2 to 4 Players
Ages 6 and Up
Pressman

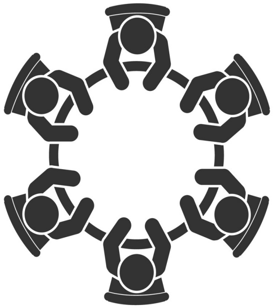

möbi





DAILY and BRIEF

Addition	Subtraction
Multiplication	Division



What are five ways you help students build fact fluency?

Fluency with Computation

Addition

Subtraction

Multiplication

Division

Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ +\ \ 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ \ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ -\ \ 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div\ \ 15 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ +\quad 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div\quad 0.14 \\ \hline \end{array}$$

MATH

Addition	Subtraction
Multiplication	Division

Build fluency with integer computation

$$\begin{array}{r} 1.4 \\ +\quad -3.9 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times\quad -12 \\ \hline \end{array}$$

$-14 - (-7) =$

$-135 \div 2 =$

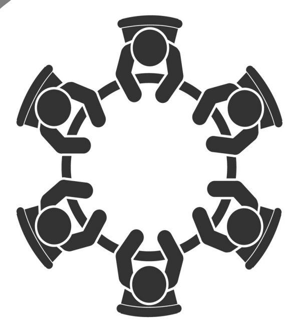
Partial Sums

A.

$$\begin{array}{r} 74 \\ +\ 18 \\ \hline 80 \\ +12 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ +\ 365 \\ \hline 1{,}000 \\ 80 \\ +\ 10 \\ \hline 1{,}090 \end{array}$$



$$\begin{array}{r} 227 \\ +\ \ 185 \\ \hline \end{array}$$

Opposite Change

A.

$$\begin{array}{r} 74 \\ +\ 18 \\ \hline \end{array} \quad \begin{array}{c} \xrightarrow{-4} \\ \xrightarrow{+4} \end{array} \quad \begin{array}{r} 70 \\ +\ 22 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 725 \\ +\ 365 \\ \hline \end{array} \quad \begin{array}{c} \xrightarrow{+5} \\ \xrightarrow{-5} \end{array} \quad \begin{array}{r} 730 \\ +\ 360 \\ \hline 1{,}090 \end{array}$$

$$\begin{array}{r} 227 \\ +\ \ 185 \\ \hline \end{array}$$

Partial Differences

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline +50 \\ -\ 5 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline +300 \\ -90 \\ -1 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

Same Change

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline \end{array} \quad \begin{array}{c} \xrightarrow{+3} \\ \xrightarrow{+3} \end{array} \quad \begin{array}{r} 65 \\ -20 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline \end{array} \quad \begin{array}{c} \xrightarrow{+4} \\ \xrightarrow{+4} \end{array} \quad \begin{array}{r} 309 \\ -100 \\ \hline 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

Add Up

A.

$$\begin{array}{r} 62 \\ -\ 17 \\ \hline \end{array}$$

$$\begin{array}{rr} 17 & \\ 20 & 3 \\ 60 & 40 \\ 62 & +\ 2 \\ \hline & 45 \end{array}$$

B.

$$\begin{array}{r} 305 \\ -\ 96 \\ \hline \end{array}$$

$$\begin{array}{rr} 96 & \\ 100 & 4 \\ 300 & 200 \\ 305 & +\ 5 \\ \hline & 209 \end{array}$$

$$\begin{array}{r} 232 \\ -\ 164 \\ \hline \end{array}$$

Partial Products

A.

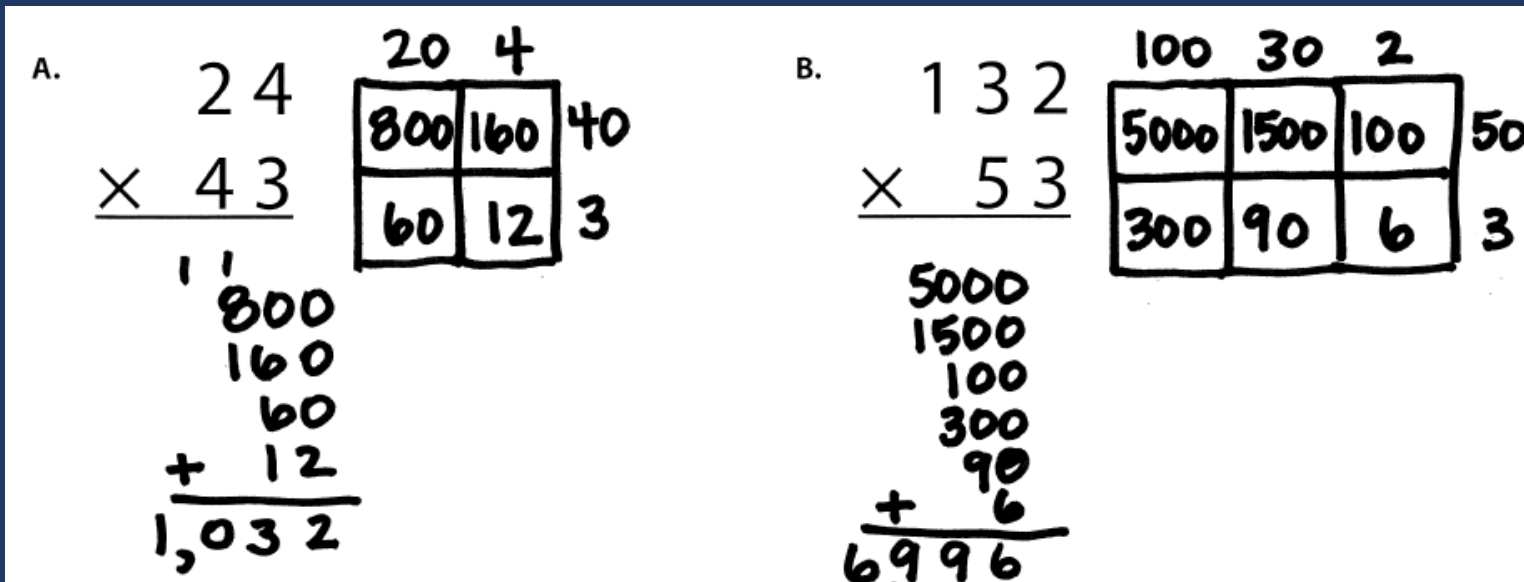
$$\begin{array}{r} 24 \\ \times\ 43 \\ \hline 800 \\ 160 \\ 60 \\ +\ 12 \\ \hline 1{,}032 \end{array}$$

B.

$$\begin{array}{r} 132 \\ \times\ 53 \\ \hline 5000 \\ 1500 \\ 100 \\ 300 \\ 90 \\ +\ 6 \\ \hline 6996 \end{array}$$

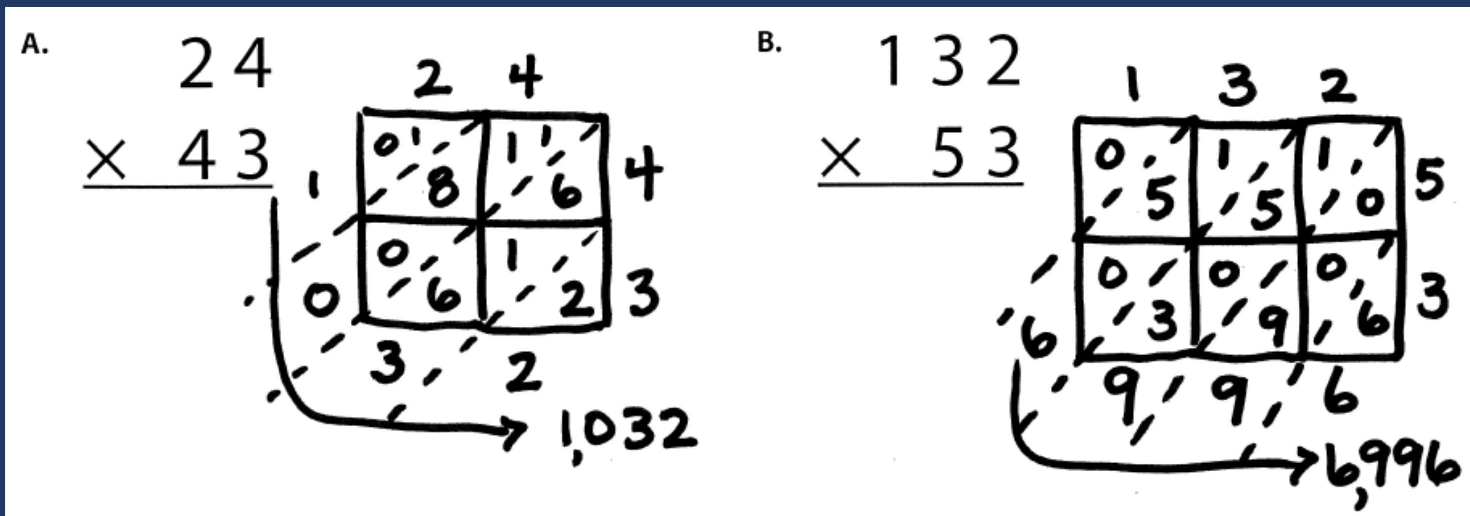
$$\begin{array}{r} 13 \\ \times\ 47 \\ \hline \end{array}$$

Area (Array)



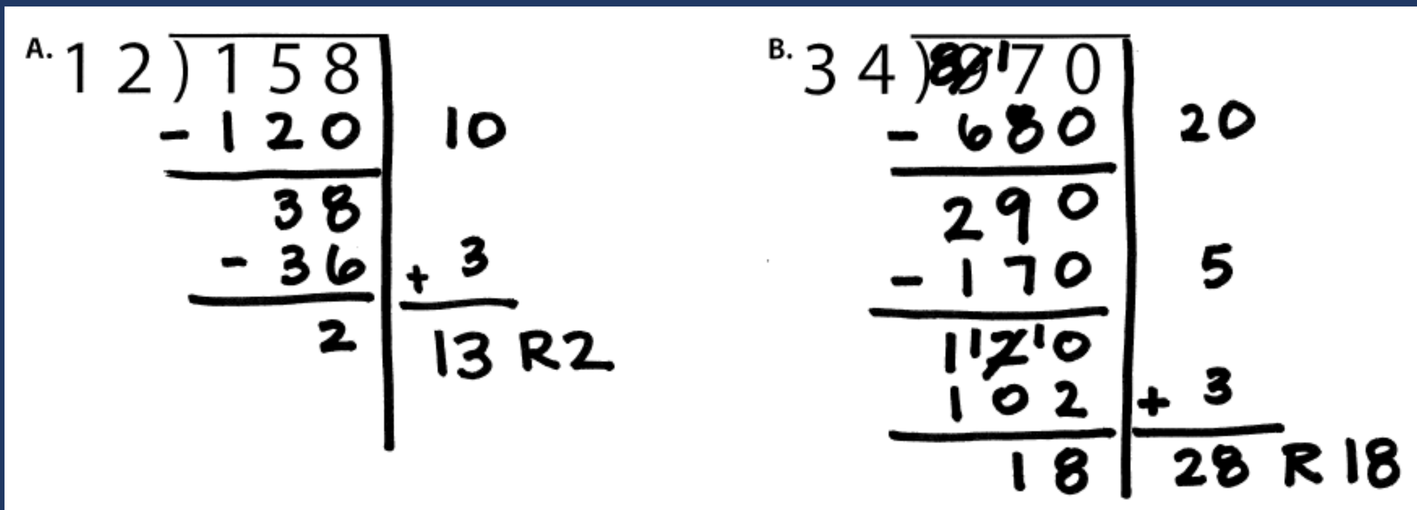
$$\begin{array}{r} 13 \\ \times \quad 47 \\ \hline \end{array}$$

Lattice



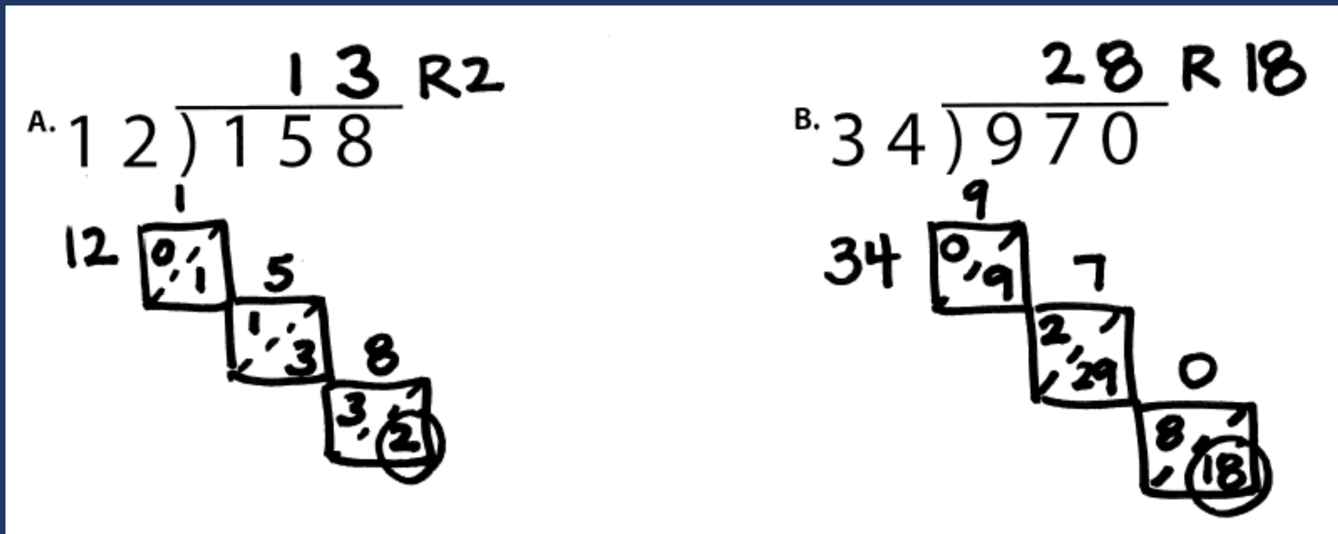
$$\begin{array}{r} 13 \\ \times\ \ 47 \\ \hline \end{array}$$

Partial Quotients



$$\begin{array}{r} 804 \\ \div \quad 12 \\ \hline \end{array}$$

Lattice



$$\begin{array}{r} 804 \\ \div \quad 12 \\ \hline \end{array}$$

Fluency

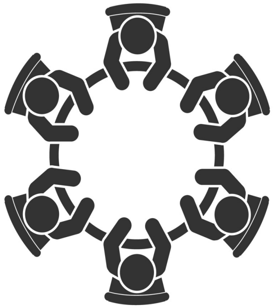
Strategies for Building Fluency

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Teachers should use fluency building activities to build counting fluency and fluency with the operations.



What are your strengths with building fluency?

What are the opportunities for growth?

What are your immediate next steps?

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Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

Solve Word Problems

Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit
- Vocabulary
- Representations

INSTRUCTIONAL STRATEGIES

- Fluency
- Word Problems

Word Problems

Research and Information

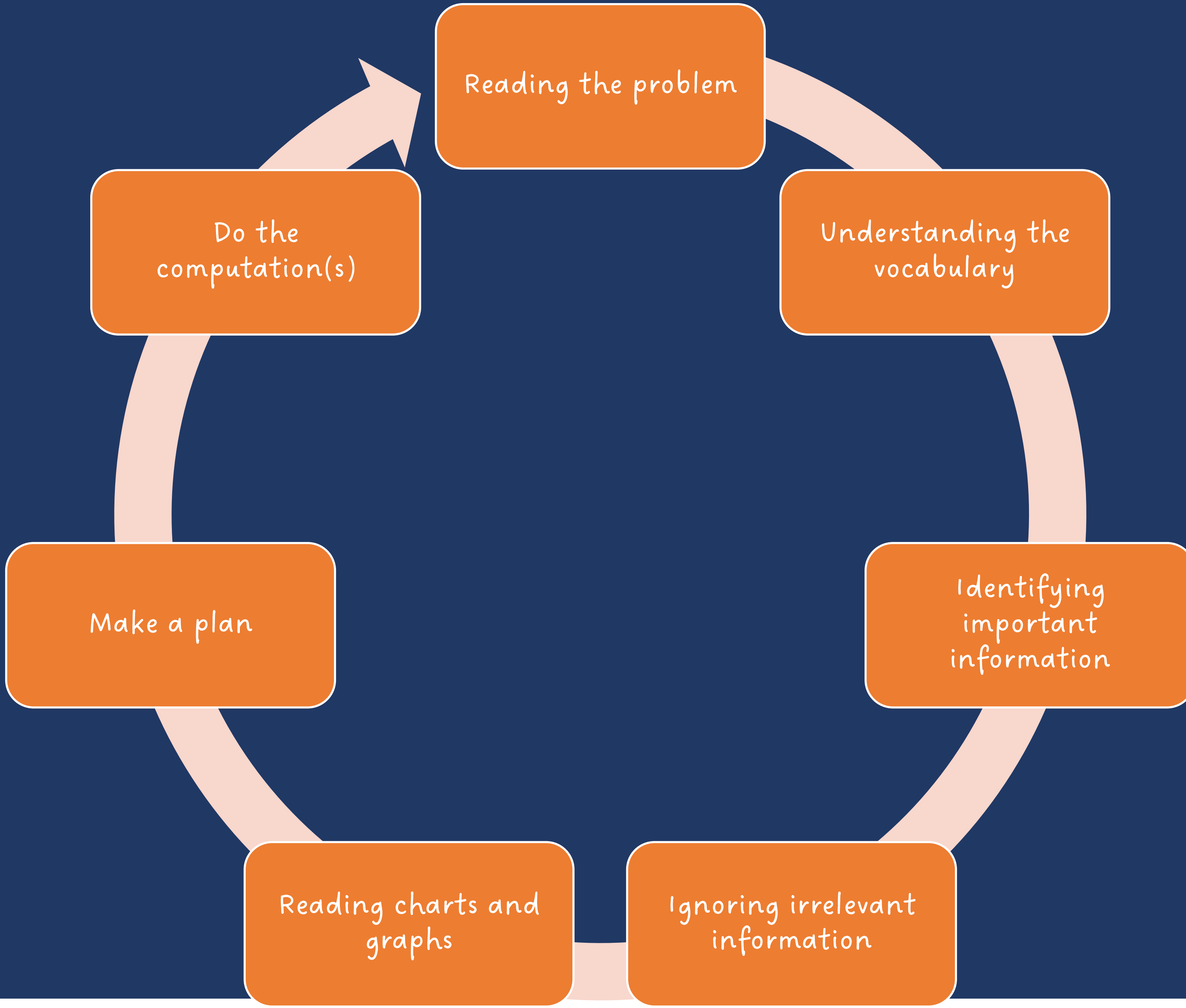
Key words tied to operations is an ineffective word-problem strategy.
(Karp et al., 2019; Powell et al., 2022)

Using a meta-cognitive strategy improves word-problem performance.
(Freeman-Green et al., 2015; Krawec et al., 2012; Montague et al., 2011; Swanson et al., 2014)

A focus on schemas improves word-problem performance.
(Alghamdi et al., 2020; Cook et al., 2020; Flores et al., 2016; Fuchs et al., 2021; Griffin et al., 2019; Jitendra et al., 2013; Lein et al., 2020; Peltier et al., 2020; Powell et al., 2022; Xin & Xhang, 2009; Zheng et al., 2013)

Word Problems

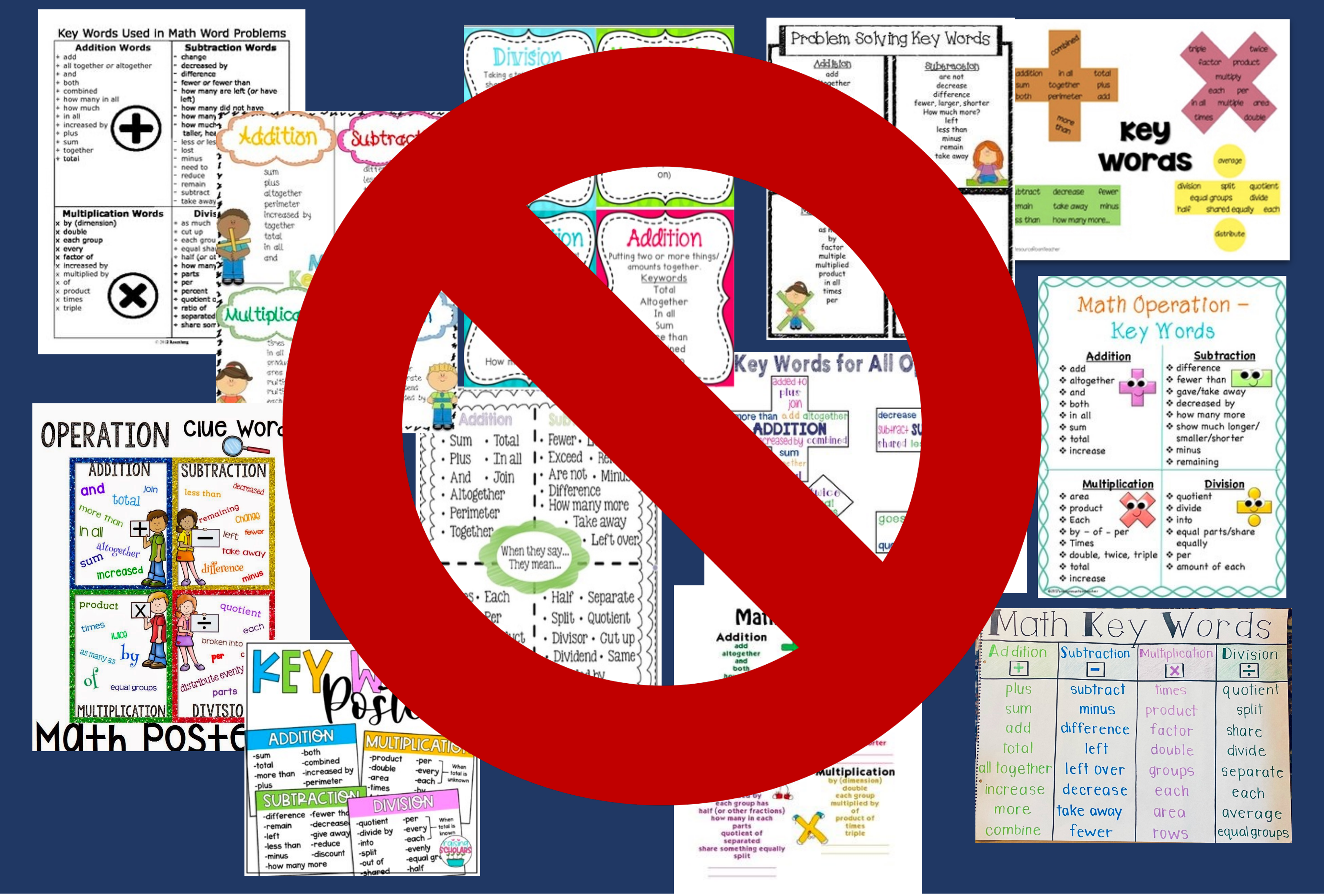
Research and Information


Reading the problem
Understanding the vocabulary
Identifying important information
Ignoring irrelevant information
Reading charts and graphs
Make a plan
Do the computation(s)

1. Keywords tied to operations

Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?


Key Words Used in Math Word Problems
Addition Words
Subtraction Words
Multiplication Words
Problem Solving Key Words
Key words
Math Operation – Key Words
Addition
add
altogether
and
both
in all
sum
total
increase
Subtraction
difference
fewer than
gave/take away
decreased by
how many more
show much longer/smaller/shorter
minus
remaining
Multiplication
area
product
Each
by – of – per
Times
double, twice, triple
total
increase
Division
quotient
divide
into
equal parts/share equally
per
amount of each
OPERATION Clue Words
ADDITION
SUBTRACTION
MULTIPLICATION
Math Poster
When they say... They mean...
Key Words for All Operations
Math Key Words
Addition
plus
sum
add
total
all together
increase
more
combine
Subtraction
subtract
minus
difference
left
left over
decrease
take away
fewer
Multiplication
times
product
factor
double
groups
each
area
rows
Division
quotient
split
share
divide
separate
each
average
equal groups


m Words Poster Set
(4.1)
75%
7 Reviews
Write a Review
pondents would
nend this to a friend
do you want to know about
oduct?
wer in about 48 hours
th 3 monthly payments of
just $4.33. No fees.
Learn more
1 post
Quantity
1
Add to Wish list
total
join
both
increase
Addition
combined
plus
in all
add
how many more
remain
minus
decrease
Subtraction
left
subtract
less than
difference
split equally
times
product
total
by

Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords[a]		Multiple keywords[a]		Keyword(s) led to correct solution[a]	
	n	%	*n*	%	*n*	%	*n*	%	*n*	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

[a]When a problem featured a keyword.

Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema[a]		Any keyword		Keyword(s) led to correct solution[b]	
	n	%	*n*	%	*n*	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.

[b]When a problem featured a keyword.

Mr. Rivera's taxable income is $20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?

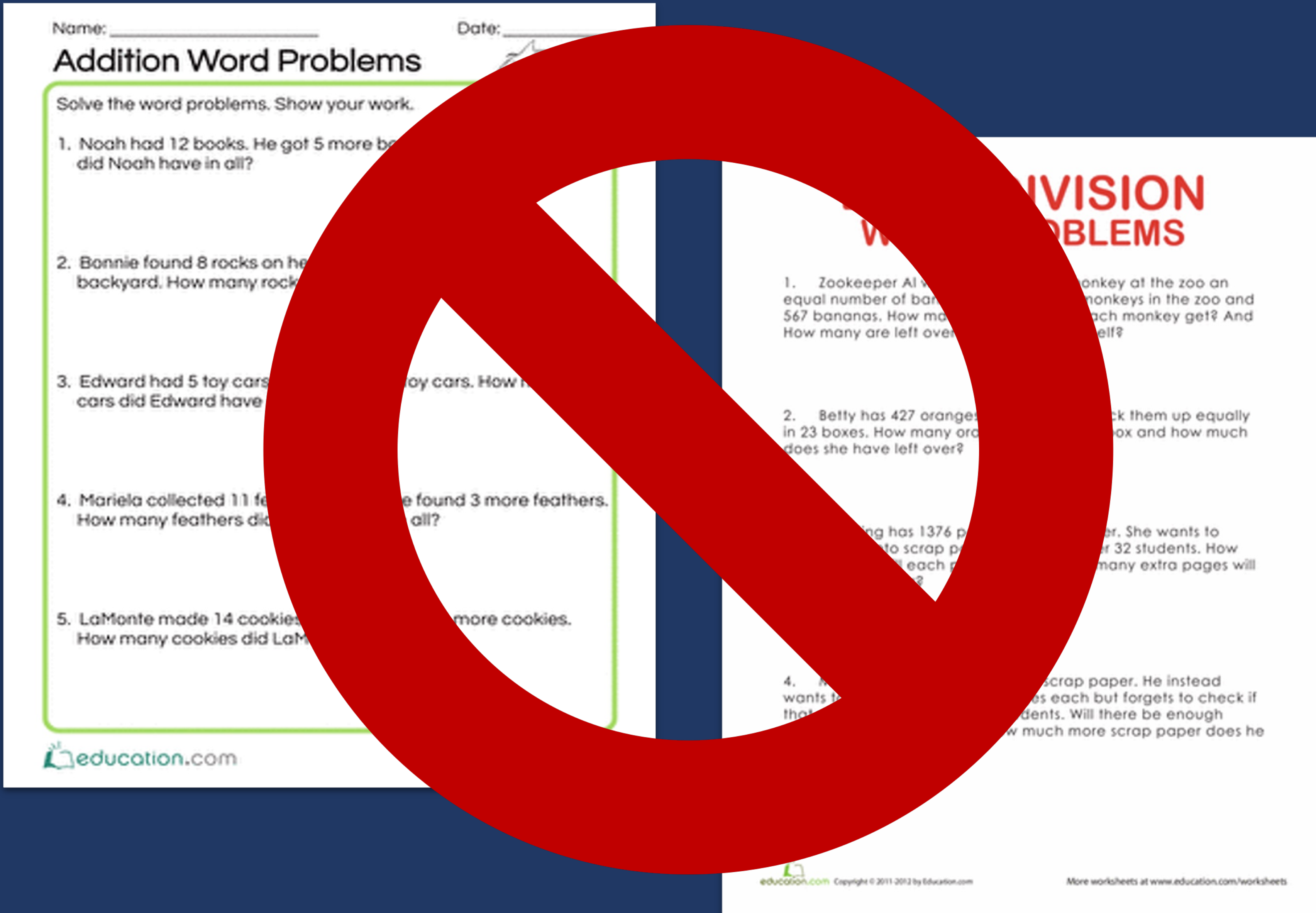
Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords ("What does *more than* tell you about?")

But, *do not* tie a keyword to a specific operation!

2. Presenting problems by operation


Name:
Date:
Addition Word Problems
Solve the word problems. Show your work.
1. Noah had 12 books. He got 5 more
did Noah have in all?
2. Bonnie found 8 rocks on
3. Edward had 5 toy cars
4. Mariela collected 11
How many feathers
5. LaMonte made 14 cookies
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Word Problems

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Teach an attack strategy

Teach about schemas

RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.

SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

STAR

Stop and read the problem carefully.
Think about your plan and the strategy you will use.
Act. Follow your plan and solve
the problem.
Review your answer.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.

SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

Evaluate steps.

Solve and check.

UPS✓

UNDERSTAND

Read and explain.

PLAN

How will you solve the problem?

SOLVE

Set up and do the math!

✓CHECK

Does your answer make sense?

Created by: Sarah Powell (srpowell@austin.utexas.edu)

Share your favorite attack strategy.

Teach an attack strategy

Teach about schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Total

Part-part-whole
Combine

Parts put together into a **total**

Daniela saw 3 canoes and 8 kayaks. How many boats did Daniela see?

Daniela saw 11 boats. If 3 of the boats were canoes, how many were kayaks?

Daniela saw 11 boats. 8 of the boats were kayaks, how many were canoes?

Total

Part

Part

Difference

Compare

Greater and lesser amounts compared for a difference

Adrianna has 10 pencils. Tracy has 4 pencils. How many more pencils does Adrianna have?

Difference

Adrianna has 6 more pencils than Tracy. If Tracy has 4 pencils, how many does Adrianna have?

Greater amount

Tracy has 6 fewer pencils than Adrianna. Adrianna has 10 pencils. How many pencils does Tracy have?

Lesser amount

Change

Join

An amount that increases or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

End amount

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Change amount

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

Start amount

Change

Separate

An amount that increases or **decreases**

Samantha baked 20 cookies. Then, she ate 3 of the cookies. How many cookies does Samantha have now?

Samantha baked 20 cookies. Then, she ate some of the cookies. Now, she has 17 cookies. How many cookies did Samantha eat?

Samantha baked some cookies. She ate 3 of the cookies and has 17 cookies left. How many cookies did Samantha bake?

End amount

Change amount

Start amount

Equal Groups

Array

Vary

Groups multiplied by number in each group for a product

Toni has 2 boxes of crayons. There are 12 crayons in each box. How many crayons does Toni have altogether?

Product

Toni has 24 crayons. They want to place them equally into 2 boxes. How many crayons will Toni place in each box?

Number in each group

Toni has 24 crayons. They put them into boxes with 12 crayons each. How many boxes did Toni use?

Groups

Comparison

Set multiplied by a number of times for a product

Brooke ran 6 minutes. Shaleeni ran 4 times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of times

Product

Ratios/Proportions

Description of relationships among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

Emma typed 56 words in 2 minutes. At this rate, how many words could Emma type in 7 minutes?

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

Teach an attack strategy

Teach about schemas

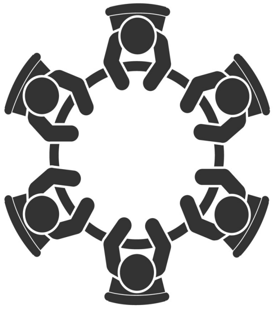
Word Problems

What are your strengths?

What are your opportunities for growth?

What are your immediate next steps?

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.



What are your strengths with word-problem solving?

What are the opportunities for growth?

What's one thing you can start doing next week?

Teachers should use systematic and explicit instruction to help students develop a strong foundation for specific mathematics skills.

Students require modeling and practice on how to use the language of mathematics.

udents should use hands-on tools, virtual manipulatives, drawings, and other visuals to understand mathematics concepts and procedures.

Teachers should use fluency building activities to build counting fluency and fluency with the operations.

Students should learn how to set up and solve word problems by combining an attack strategy with a focus on word-problem schemas.

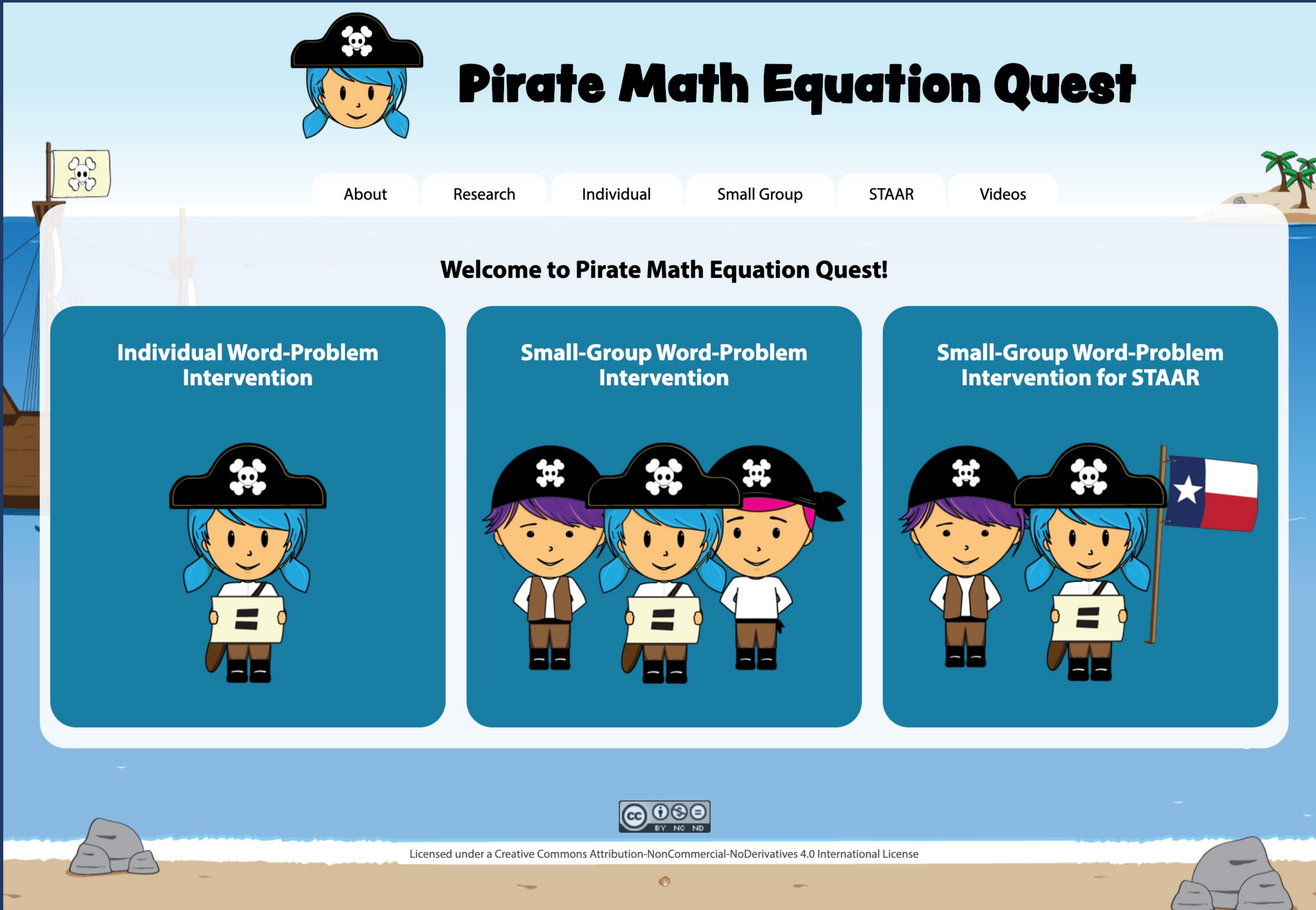
Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit
- Vocabulary
- Representations

INSTRUCTIONAL STRATEGIES

- Fluency
- Word Problems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https://intensiveintervention.org/intensive-intervention-math-course

National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

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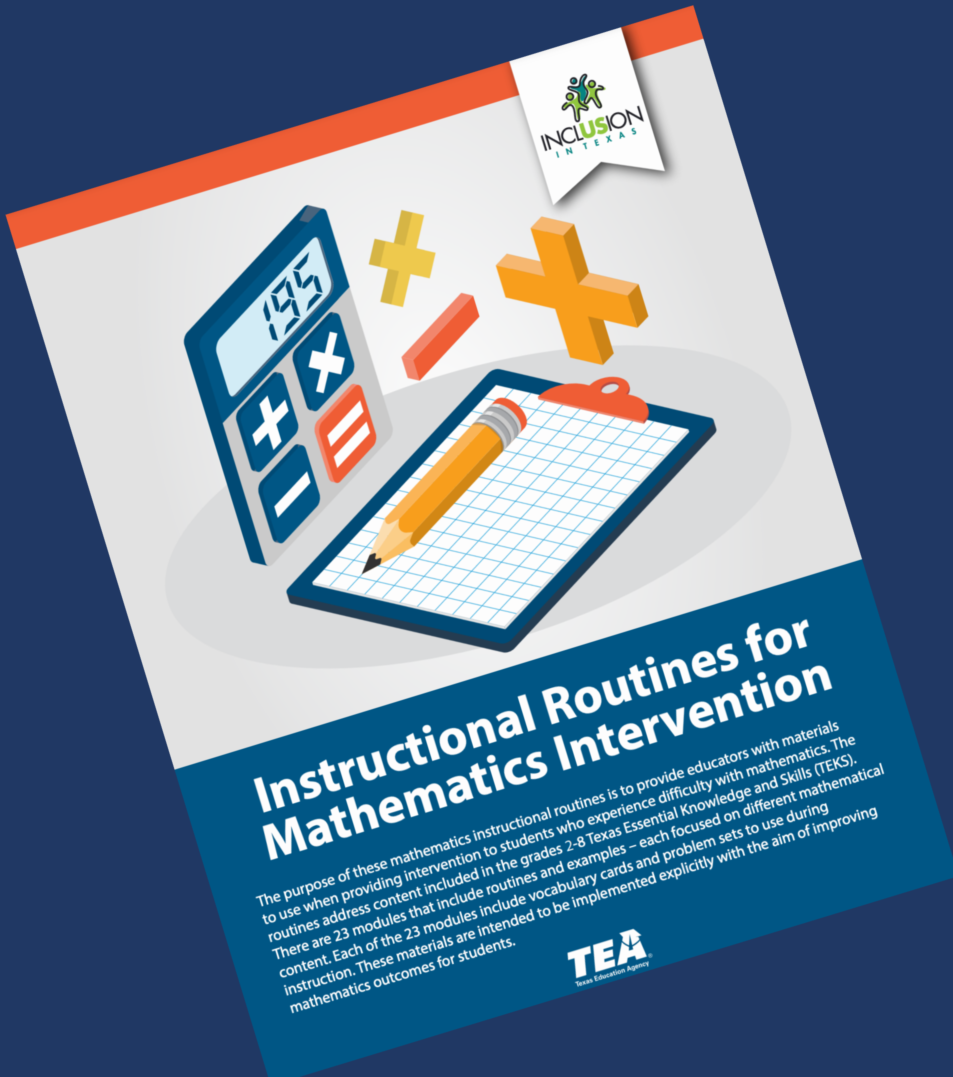
Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a high-leverage practice in special education, and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center, developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.

MODULE 5: INTENSIVE MATHEMATICS INTERVENTION: INSTRUCTIONAL STRATEGIES



https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859

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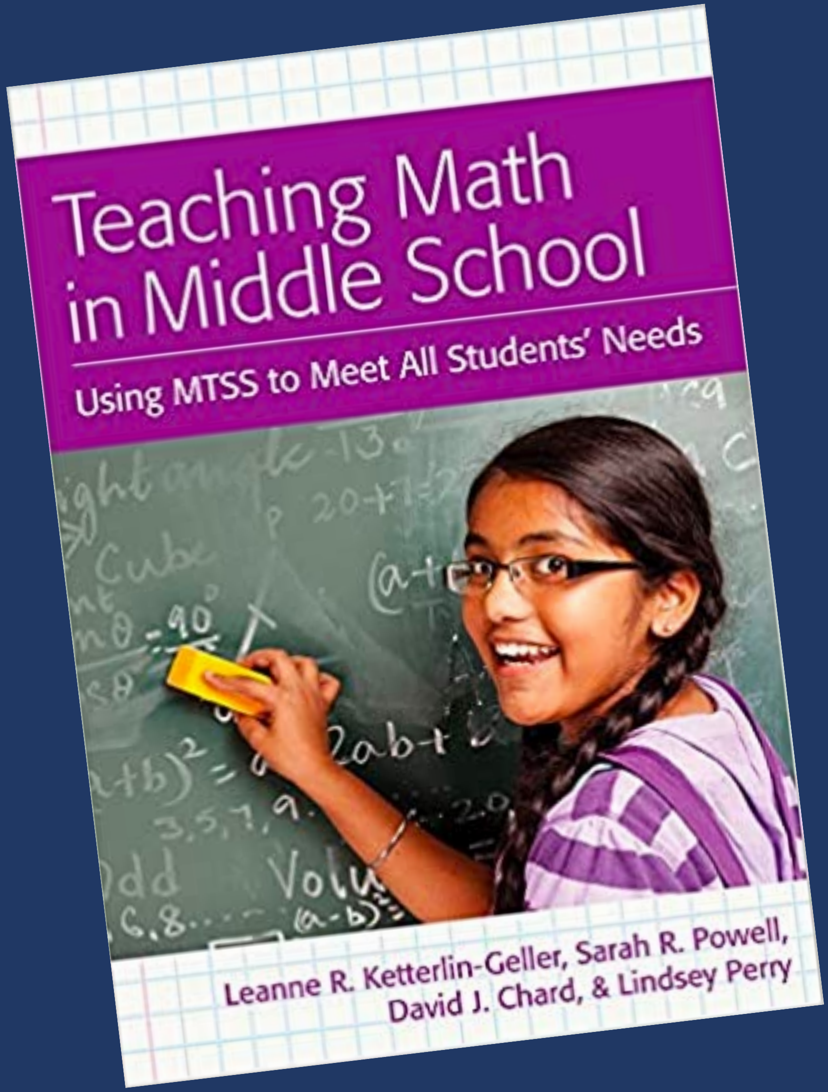

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