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3D EM modeling of steel casings using an equivalent RL circuit network

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Summary

To avoid a massive mesh, material averaging is often used in the DC/EM modeling of a steel casing. We use a parallel beam model to quantify the behavior of a hollow casing and its solid-column approximation with averaged conductivity in terms of equivalent impedance. At high frequencies, the internal mutual inductance due to the casing's geometry can cause skin effect and make the equivalent impedance frequency-dependent. As a result, the effect of casing's geometry must be considered if a high accuracy is desired.

We propose a new multiscale approach (RLnet) that takes into account the casing's geometry without 3D mesh refinement. First, at the borehole scale, we calculate the equivalent impedance of a casing in a certain length with fast semi-analytic solutions. Second, the earth is transformed to an equivalent 3D RL network, in which the mesh edges become branches in a circuit. Then the pre-calculated equivalent impedance of casing is used to modify the existing edge impedance of the earth along the well path. Finally, the RL circuit network is solved using the circuit theory.

Keywords: forward modeling, casing, 3D electromagnetics, circuit, impedance

Introduction

Using steel casings to enhance electrical (DC) and electromagnetic (EM) imaging for deep reservoirs has gained significant traction in recent years. Both theoretical and field studies have shown that including highly conductive casings in a DC/EM survey is a viable approach in the monitoring for hydraulic fracturing, CO2 sequestration, etc. (Hoversten, Commer, Haber, & Schwarzbach, 2015). The 3D numerical modeling of those geological problems with the presence of casing is challenged by the large contrast in conductivity and spatial scale between the casing and the earth. Previously proposed algorithms fall into two categories. The most accurate approach is to use extremely small cells to capture the exact geometry of casing (Commer, Hoversten, & Um, 2015). In 3D, this can result in a significant computational cost. A common practice to avoid excessive refinement is to approximate the casing with an averaged conductivity in the casing-bearing cells that are only moderately refined (Um, Commer, Newman, & Hoversten, 2015; Weiss, Aldridge, Knox, Schramm, & Bartel, 2016; Haber, Schwarzbach, & Shekhtman, 2016) or even not refined at all (Yang, Oldenburg, & Heagy, 2016). However, how well the material averaging works in EM and whether the casing's actual geometry matters are still not well understood.

In this abstract, we first simulate a casing using a parallel beam model. This method allows us to investigate the skin effect in casings and how the current distribute on the cross section. Most importantly, it calculates the equivalent impedance of a casing segment by taking into account the internal mutual inductance that may contribute to the inaccuracy in the averaging approach. Recognizing the importance of the casing's actual geometry, we then introduce a new 3D EM algorithm based on the equivalent resistor and inductor network (RLnet). RLnet takes the pre-calculated equivalent impedance of casings as a complex and frequency-dependent "averaged property". The goal is to more accurately incorporates the effect from the casing's geometry into EM modeling without mesh refinement in 3D.

Skin effect in casing

When a half-space of the earth medium is energized by a normal plane wave, the E-field and current are parallel to the surface, and the current density is larger near the surface than in the interior. This is the skin effect well-known in EM geophysics. Similarly, when a casing is excited by an external source, the E-field and current are mostly parallel to the casing-earth interface. Based on the same physics, we should also expect larger current density near the casing's surface. Assuming a non-permeable casing of 5×10^6 S/m, the skin depth becomes a few centimeters at about 100 Hz and beyond. If the skin depth is comparable with or smaller than the casing's thickness, the current cannot be considered uniform on the cross section - currents crowd to the surface and avoid the interior. At sufficiently high frequencies, the effective cross section used for conducting current decreases with higher frequency, equivalent to an increased resistance or impedance of casing. In the following, we numerically quantify the skin effect in a casing energized by a wideband EM source.

Parallel beam model

Because the casing's conductivity is many orders of magnitude greater than the surrounding, it is reasonable to assume the current only flows in its axial direction, and the current remains constant within a short distance (relative to its total length). Here we consider a 100 m long vertical hollow steel casing sample (5×10^6 S/m) with an outer radius of 12.70 and an inner radius 10.16 cm respectively. The casing's cross section is discretized by a uniform grid of 5 mm interval so that the casing is represented by a bundle of 5 mm $\times$ 5 mm $\times$ 100 m beams arranged in a ring shape (Figure 1). If a uniform voltage source $\mathbf{V}$ is applied to the two ends of the casing, the currents in the beams $\mathbf{I}$ can be solved by using the generalized Ohm's law

$$\mathbf{ZI} = \mathbf{V}, \tag{1}$$

where $\mathbf{Z}$ is a complex and symmetric impedance matrix. $\mathbf{Z}$ has the form of

$$\begin{bmatrix} R_1 + j\omega L_1 & j\omega M_{12} & \dots & j\omega M_{1n} \\ j\omega M_{21} & R_2 + j\omega L_2 & \dots & j\omega M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ j\omega M_{n1} & j\omega M_{n2} & \dots & R_n + j\omega L_n \end{bmatrix}, \tag{2}$$

where n is the number of beams, R_i and L_i are the resistance and self inductance of each beam, M_{ij} is the mutual inductance between the ith and jth beams, and ω is the angular frequency. R can be calculated using the resistivity, length and thickness of each beam. L for extremely long objects has analytic solutions (Ruehli, 1972). By treating the beams as parallel filaments, M also has a simple analytic solution only dependent on the geometric coupling (Paul, 2011). Figure 1 shows the magnitude of current on the casing's cross section when a 1 V source at 100 Hz is applied to all beams. Because the skin depth 2.2 cm is comparable with the casing's thickness, the inductive interaction between the beams is strong enough to drive significant current crowding near the surface.

Equivalent impedance

Equation 2 implies that the impedance of a casing also depends on the frequency (ω) and the geometry (M) at high frequencies. Modeling the exact geometry of casing is desireable, but mixing the mm-scale casings and km-scale geology in one modeling would result in a massive mesh. So we take the approach of equivalent impedance (EI) that describes the macroscopic behavior of a casing comprised of many parallel beams. EI is defined as the transfer function of a voltage source applied to the two ends of a casing sample and the total current in all beams. Following Paul (2011), EI can be calculated using $\mathbf{Z}$ in equation 2

$$Z_{eq} = \frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{n} Z_{ij}^{-1}} = R_{eq} + j\omega L_{eq}, \tag{3}$$

which can be split into R_{eq} as the equivalent resistance and L_{eq} as the equivalent self-inductance. Both of them are frequency-dependent.

Figure 2 shows EI of the casing sample at some common frequencies in EM geophysics. We first use the settings in Figure 1 to model the actual casing geometry and calculate EI (blue curves). Then we also examine the material averaging approach. In this case, the casing is approximated by a bundle of 16 square beams, each of which measures 25 $\times$ 25 cm on the cross section. The bundle has a cross section area of 1 m^2 and the resistivity is averaged to preserve the same resistance in DC. This is approximation is equivalent to placing E-field on face centers in a staggered-grid finite difference scheme. The difference made by calculating the actual casing's geometry is evident in the imaginary part for all frequencies. Below 1 Hz, the real part is dominant, and both approaches agree well with DC. At higher frequencies, the real parts also become frequency-dependent, but their effect may be small compared to the imaginary parts.

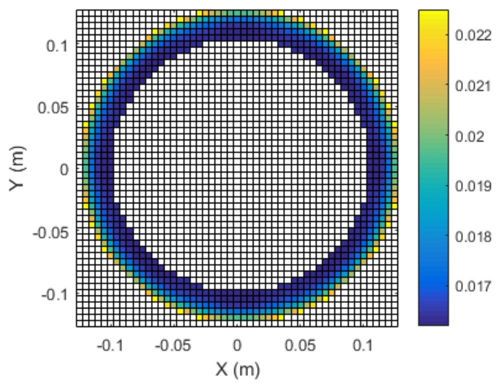

Figure 1: Current distribution over the cross section of the casing sample at 100 Hz.

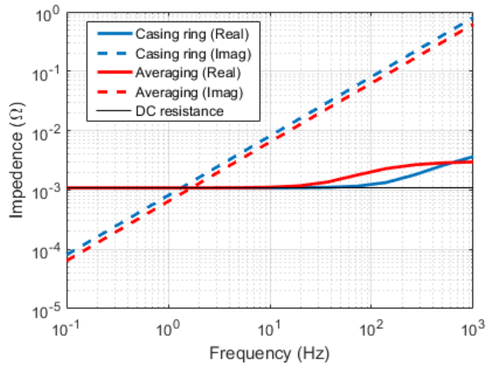

Figure 2: Equivalent impedance of the casing sample when modelled using the actual ring shape and using an averaged resistivity.

Figure 3 shows how the two approaches differ in terms of the total current in the casing sample. Below 1 Hz, when EM induction is small, the difference can be negligible. At higher frequencies, the error introduced by not considering the casing's geometry is mostly seen in the imaginary part. The relative error increases quickly from DC to 1 Hz, but afterward the error is bounded within 30% up to 1 kHz.

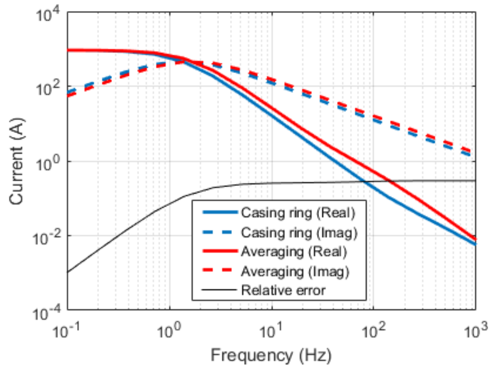

Figure 3: Equivalent total current in the casing sample when modelled using the actual ring shape and using a solid rod with an averaged conductivity.

RLnet: an equivalent circuit approach

The calculation above shows that equivalent impedance calculated using the parallel beam model can be a better choice of upscaled casing property since it takes into account the internal mutual inductance using the casing's actual geometry. EI can be easily utilized in the equivalent circuit modeling approach. An equivalent circuit represents the earth medium with a large number of straight wires connected in 3D. Each wire is a branch in the 3D circuit network. At DC, each branch has a resistance determined by averaging the materials around the corresponding wire, a procedure very similar to a finite difference scheme defining E-field on mesh edges. When a steel casing collocates with the branches, the casing is treated as a high-conductance parallel circuit added to the existing network. The DC code using a resistor network is referred to as RESnet (Yang et al., 2016). Here we generalize the circuit approach to EM by adding (self and mutual) inductors.

Formulation

For the network, the constitutive relation in equation 1 still holds with $\mathbf{I}$ being the currents on branches and $\mathbf{V}$ the voltages across branches. The diagonal of $\mathbf{Z}$ represents the self-impedance of branches. When a casing collocates with a branch, the self-impedance of that branch is modified using the pre-calculated EI of a casing. The off-diagonals are from the mutual inductance between any two branches using the Neumann formula (Ruehli, 1972)

$$M_{ij} = \frac{\mu}{4\pi} \sum_{i=1}^{n} \sum_{j=1}^{k} \frac{\delta\ell_i \cdot \delta\ell_j}{r_{ij}}, \tag{4}$$

where the two branches are discretized into n and k segments depending on how their distance compares to their own lengths.

The RL circuit is governed by Kirchhoff's current law

$$-\mathbf{D}^\top \left[\mathbf{Z}^{-1}\left(\mathbf{D}\phi - \mathbf{Z}_0\mathbf{I}_0\right)\right] = \mathbf{I}_s. \tag{5}$$

Eventually, we solve a complex system

$$\mathbf{D}^\top\mathbf{Z}^{-1}\mathbf{D}\phi = \mathbf{D}^\top\mathbf{Z}^{-1}\mathbf{Z}_0\mathbf{I}_0 - \mathbf{I}_s, \tag{6}$$

where ϕ is the potential at the circuit junctions, $\mathbf{D}$ is a differential operator calculating the potential difference, and $\mathbf{Z}^{-1}$ is the admittance matrix. The first term on the right-hand side represents the inductive coupling between the source wires and the earth, as $\mathbf{I}_0$ is the current in the sources and $\mathbf{Z}_0$ is the one-way mutual inductance between the source wires and the earth's branches. $\mathbf{I}_s$ is the galvanic current injection from the source to the earth. $\mathbf{I}_s$ does not exist for an inductive source.

Numerical example

We run a simple example to demonstrate the modeling of casing using RLnet. The earth is a 100 Ω half-space. A casing is buried vertically from the surface to 1 km depth. One end of the source is attached to the top of casing, and the other end is grounded at about 2.1 km north (+Y) of the well. The E-field receivers are placed along a line perpendicular to the source wire on the surface (Figure 4). The top 1 km of the earth is discretized to 10 layers in the 3D mesh, so the casing is approximated by ten 100-m segments. The horizontal cell size at the well head is 50 m and it expends exponentially outwards. A rectlinear mesh is preferred because the orthogonality of grid simplifies the calculation of mutual inductance.

Two modelings are shown in Figure 5. The first one only models the half-space with no casing. For the entire receiver line, the real part dominates and quickly decays from the well head to large offset. Then the equivalent impedances of *casing ring* at 10 Hz in Figure 2 are populated to the edges representing the well path. The result shows that the casing decreases the overall intensity of field around the well head, but at a long offset, the effect of casing gradually vanishes. This example takes about 8 minutes in serial mode on a desktop computer, and the efficiency can be further improved if a semi-structured mesh that merges skinny cells is used.

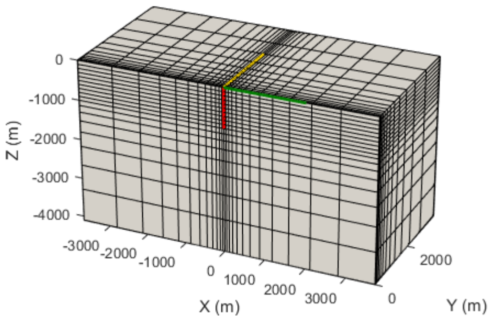

Figure 4: 3D mesh used in the demonstrative example. Red line: casing; yellow line: source wire; green line: receiver line.

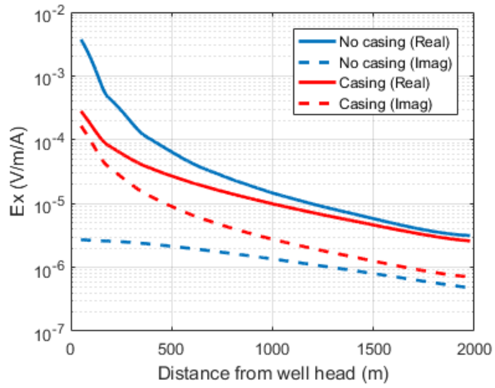

Figure 5: Simulated surface E-field data at 10 Hz with and without casing.

Conclusions

We use a semi-analytic parallel beam model to examine the behavior of a casing energized by an EM source. At high frequencies, the equivalent impedance of a casing depends on frequency as well as the casing's actual geometry, because of the internal mutual inductance. Although the material averaging may work reasonably well at certain frequency bands, we propose a new EM algorithm (RLnet) to include the effect of casing's geometry using the concept of equivalent RL circuit network. Without 3D mesh refinement, RLnet uses the pre-calculated equivalent impedance as a more accurate upscaled property to modify the branches (edges) that represent a casing.

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