# Elmwood Press

# Core Mathematics C4

# Paper G

# (Question Paper)

# Core Mathematics C4      For Edexcel
# Advanced Level

## Paper G
## Time: 1 hour 30 minutes

**1.** The parametric equations of a curve are

$$x = \cos t, \ y = 2 \sin t, \text{ for } 0 \le t \le 2\pi.$$

(a) Find the gradient of the curve at the point where $t = \dfrac{\pi}{2}$.

(2)

(b) Find the cartesian equation of the curve.

(3)

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**2.** (a) Express

$$\frac{7x^2 + 2x - 12}{(x - 2)(2x + 1)(x + 2)}$$

in partial fractions

(4)

(b) Hence show that $\displaystyle\int_{3}^{4} \frac{7x^2 + 2x - 12}{(x - 2)(2x + 1)(x + 2)}\,dx = \ln\left(\frac{12}{5}\right) + \frac{3}{2}\ln\left(\frac{9}{7}\right)$

(5)

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**3.** (a) Expand $(1 + ax)^6$ in ascending powers of $x$ up to and including the term in $x^2$.

(3)

(b) The coefficients of $x$ and $x^2$ in the expansion of

$$(1 + bx)(1 + ax)^6$$

are $-9$ and $24$ respectively.

Find the values of $a$ and $b$, given that $a$ is an integer.

(7)

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**4.** (a) Work out $\displaystyle\int x \sec^2 x \ dx$.

(4)

(b) Given that

$$\frac{dy}{dx} = (x \sec^2 x) y^{\frac{1}{2}},$$

and that $y = 4$ when $x = 0$, show that when $x = \dfrac{\pi}{4}$ the value of $y$ is

$$\left(\frac{\pi}{8} + 2 - \frac{1}{4}\ln 2\right)^2.$$

(6)

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**5.** (*a*) Use the trapezium rule, with the number of trapeziums given below, to find approximate values for

$$\int_1^3 \frac{12}{x}\,dx,$$

    (i) using two trapeziums,

**(2)**

    (ii) using four trapeziums.

**(2)**

  (*b*) Work out the exact value for this integral. For each case in part (a) work out the error when using the trapezium rule as a percentage of the exact value, giving your answers correct to one decimal place.

**(5)**

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**6.** Relative to an origin $O$, the position vectors of the points $A$ and $B$ are $\mathbf{i} - 2\mathbf{j} + 8\mathbf{k}$ and $3\mathbf{i} + 3\mathbf{j} + m\mathbf{k}$ respectively, where $m$ is a constant.

The point $C$ is such that $OABC$ is a rectangle.

(*a*) Find the value of $m$.

**(4)**

(*b*) Write down the position vector of $C$.

**(1)**

(*c*) Find a vector equation of the line $AC$.

**(2)**

(*d*) Calculate the acute angle between the diagonals of $OABC$, giving your answer to the nearest degree.

**(3)**

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**7.** A curve has the equation

$$y = 12x - e^{\frac{1}{2}x}.$$

(*a*) Show that the $x$-coordinate of the stationary point on the curve is $2\ln 24$.

**(3)**

(*b*) Find the corresponding $y$-coordinate in the form $a\ln 24 + b$, where $a$ and $b$ are integers to be determined.

**(1)**

(*c*) Find an expression for $\dfrac{d^2 y}{dx^2}$ and hence determine the nature of the stationary point.

**(1)**

(*d*) Show that the area of the region enclosed by the curve, the $x$-axis and lines $x = 2$ and $x = 4$ is $72 + 2e(1 - e)$.

**(5)**

**8.** The equation of a curve is $2x^2 - xy + y^2 = 56$.

(*a*) Find $\dfrac{dy}{dx}$ in terms of $x$ and $y$.

**(4)**

(*b*) Find the equation of the normal to the curve at the point $(2, -6)$.

**(3)**

(*c*) Find the coordinates of the two points on the curve at which the tangents to the curve are parallel to the $x$-axis.

**(5)**

**END**                **TOTAL 75 MARKS**