Chapter 7.2. Functions and Relations

Functions

The Function, $f: A \rightarrow B$, such that f(x) = y

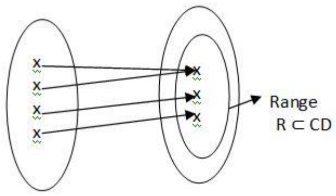
A, Domain – Set of 'x' values / pre-image of y

B, Co – Domain – Set of 'y' values

Range – Set of f(x) values (image of x values)

[Range is subset of Co - Domain]

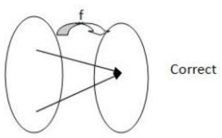
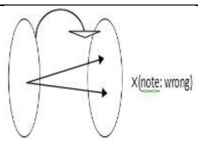
f(x) is called as the functional value

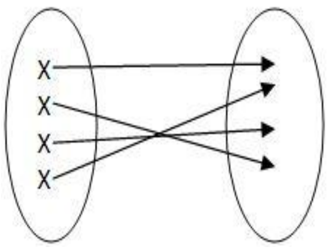


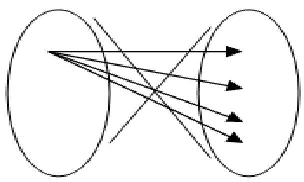
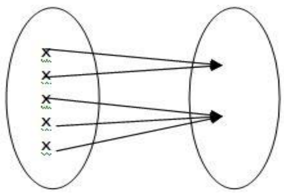
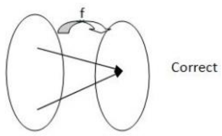
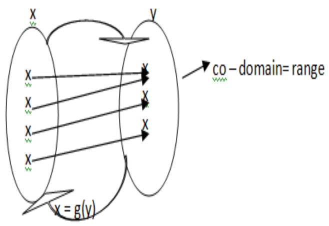
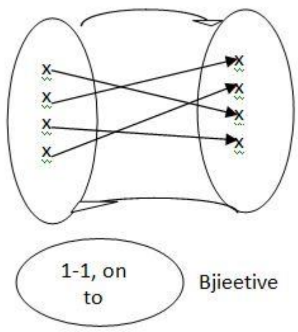
Points to Ponder:

1. All the values in the Domain should be mapped

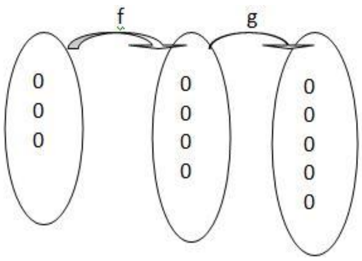
2. One value in the Domain should not have same image.

Function	Not a Function
	

Sl. No	Name of the Function	Mappings $f: A \rightarrow B$	Points to Ponder	Examples
1	Identity function (I) **I: A ⟶ *A*,** I(x) = x		1.Domain = Range 2. 1-1function 3. If, Range = Co-Domain, then Domain = Range = Co-Domain and 1-1 and onto function	**I: *N* ⟶ *W*,** I(x) = x Here, f(1) = 1 & f(2) = 2

2	One to Many		It is not a function, as one 'x' value cannot have more than one f(x) value, which means the function is not properly defined	$f: A \rightarrow E$ f(x) = x^2
3	Many to one		A properly defined Function	
4	All to one		Constant Function f(x) = k, k – constant	f: A⟶B f(x) = x^2, $1 \epsilon B$
5	Onto, function		Range = Co – Domain	**f:** $\boldsymbol{W} \longrightarrow \boldsymbol{N}$, f(x) = x + 1
6	Inverse Function		Only a function which is 1-1 and onto shall have an Inverse function	

Specific Functions				
No	**Function**	**Mapping**	**Examples**	**Remarks**
1	Single-valued function	$y = f(x), for\ all\ x$	$f(x) = x^2, x\ \in N$	Only one functional value for all x
2	Multi-valued function	$f(x) = \begin{cases} f_1(x), for\ some\ x \\ f_2(x), for\ someother\ x \end{cases}$	$f(x) = \begin{cases} x^2, for\ x\ < 0 \\ x^3, for\ x \geq 0 \end{cases}$	functional values depend on the values of 'x' falling in

				different range
3	Absolute / Modulus Function	$f(x) = \|x - a\|$ $f(x) = \begin{cases} x - a, x > a \\ a - x, x < a \\ 0, x = a \end{cases}$	$f(x) = \|x\| + \|x - 2\|$ $f(x) = \begin{cases} 2 - 2x, \quad x < 2 \\ 2, 0 \leq x \leq 2 \\ 2x - 2, x > 0 \end{cases}$	
4	Equal function	Two functions with same functional values	Let $f: N \to N, f(x) = x^2$ and $g: N \to W, g(x) = x^2$	f(x) = g(x) but co domain is different
5	Composite Function	fog= $\mathrm{f}(g(x))$ 	1. f is onto 2. In general, fog ≠ gof 3. If g = f^{-1}, then fog = I	(g.f) (x) = g (f (x)), if g(x) = f inverse
6	Even	$f(-x) = f(x)$	$f(x) = x^2$	
7	Odd	$f(-x) = -f(x)$	$f(x) = x^3$	
8	Explicit	Variables are seperable	x + y = 2	y = 2 - x
9	Implicit	Variables are not seperable	$x^2y = 0$	y = $0/x^2$, indeterminate form, is not possible

Examples		
Sl. No	**Function**	**Type**
	$\boldsymbol{f: A \to B, f(x) = x^2}$	
1	A = W & B = W	1 -1 & Onto
2	A = IN & B = W	1 -1
3	A = E & B = E	Not a function
4	A = _ & B= W	many -1 on to
5	A = _ & B = IN	Not a function
6	$f: N \to N, f(x) = x$	Identity function / 1 – 1 / Onto
7	$f: N \to W, f(x) = x$	Identity function / 1 – 1 , but not onto
8	$f: W \to N, f(x) = x$	Not a function

Relations

Let S = $\{a, b, c\}$ and Relation R – Subset of the set $S \times S$

Sl.No.	Relation	Explanation	Examples		
			"Is equal to"	"Is parallel to"	"Is perpendicular to"
1	Reflexive 'a is relate ϕ to it self'	(a,a) ϵ R	a = a	a∥a	a⊥a is not true → Not Reflexive
2	Symmetric if (a,b) ϵ R then	(b,a) ϵ R	a =b ⇒ b=a	a∥b ⇒ b∥a	a⊥b b⊥a → Bymetric
3	Transitive	if (a,b) ϵR & (b,c) ϵ R ⇒ (a,c) ϵR	a=b, b=c ⇒ a=c	a∥b and b∥c ⇒ a∥c	a⊥b, b⊥ c a⊥c → not transitive
4	Equivalence	Reflexive + symmetry + Transitive	Equivalence	Equivalence	Not Equivalence

Cartesian Product (ordered pairs)

Let A = {1,2,3} and B = {2,4,6}

Then A × B = {(1,2), (1,4), (1,6), (2,2), (2,4), (2,6), (3,2), (3,4), (3,6)}

Points to Ponder:

By definition, Every subset of A × B is a relation from A to B.

Consider the product set X × Y = {(1,3), (2,3), (4,3), (2,2), (3,2), (4,2), (1,1), (2,1), (3,1), (4,1)}

Subset	Relation
{(1,1), (2,2), (3,3)}	'Is equal to'
{(1,3), (2,3), (1,2)}	'Is less than'
$\{(\mathbf{4},\mathbf{3}), (\mathbf{3},\mathbf{2}), (\mathbf{4},\mathbf{2}), (\mathbf{2},\mathbf{1}), (\mathbf{3},\mathbf{1}), (\mathbf{4},\mathbf{1})\}$	'Is greater than'
$\{(\mathbf{4},\mathbf{3}), (\mathbf{3},\mathbf{2}), (\mathbf{4},\mathbf{2}), (\mathbf{2},\mathbf{1})(\mathbf{3},\mathbf{1}), (\mathbf{4},\mathbf{1}), (\mathbf{1},\mathbf{1}), (\mathbf{2},\mathbf{2}), (\mathbf{3},\mathbf{3}))\}$	'Is greater than or equal'

Identity and Inverse Relation

1. Identity Relation: The relation I = {(a, a): a∈ A} is called the identity relation on A.

Illustration: Let A = {1, 2, 3} then I = {(1,1),(2,2),(3,1)}

2. Inverse Relation: Let R be a relation on A, then the relation R^{-1} on A, defined by

R^{-1} = {(b,a): (a,b) ∈R} is called an iverse relation on A.

Here, Dom (R^{-1}) = Range (R) & Range(R^{-1}) = Dom (R)

Then R being a subset of A× A, it is a relation on A. Dom (R) = {1, 2, 3} and Range (R) = {2,1}

Now, (R^{-1}) = {(2,1),(2,2),(1,3),(2,3)} Here, Dom (R^{-1}) = {2,1} = Range (R) and

Range (R^{-1}) = {1, 2, 3} = Dom (R).

Illustration: Let A = {1, 2, 3}, then

R1 = {(1,1),(2,2),(3,3),(1,2)}	Reflexive and transitive but not symmetric, since (1,2) ∈ R_1 but (2,1) does not belongs to R_1.
R2 = {(1,1), (2,2),(1,2), (2,1)	Symmetric and transitive but not reflexive, since (3,3) does not belong to R_2.
R3 = {(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)}	Reflexive and symmetric but not transitive, since (1,2)∈ R3&(2,3) ∈ R3 but (1,3) does not belong to R3.