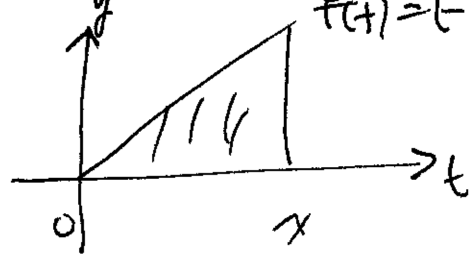
Math 1496 – Calc 1

Fundamental Th^m – Part 2



Consider $F(x) = \int_0^x t\,dt$

$= \frac{t^2}{2}\Big|_0^x$

$= \frac{x^2}{2}$

then $F'(x) = \frac{2x}{2} = x$

so $\frac{d}{dx}\int_0^x t\,dt = x$

Consider $F(x) = \int_1^x \frac{dt}{t} = \ln|t|\Big|_1^x = \ln x$

$F'(x) = \frac{d}{dx}\ln x = \frac{1}{x}$

so $\frac{d}{dx}\int_1^x \frac{dt}{t} = \frac{1}{x}$

So in general

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

This is the second fundamental Th^m of Calculus

<u>Ex 1</u>

$$\frac{d}{dx}\int_1^x \sqrt{1+t^2}\,dt = \sqrt{1+x^2}$$

<u>Ex 2</u>

$$\frac{d}{dx}\int_0^x \sin(t^2)\,dt = \sin(x^2)$$

<u>Ex 3</u> $\frac{d}{dx}\int_x^1 e^t\,dt$

(1) $\frac{d}{dx}\left(e^t \Big|_x^1 \right) = \frac{d}{dx}\left(e^1 - e^x\right) = -e^x$

(2) $\frac{d}{dx}\int_x^1 e^t\,dt = -\frac{d}{dx}\int_1^x e^t\,dt$

$= -e^x$ easier

so in general

$$\frac{d}{dx}\int_x^a f(t)\,dt = -\frac{d}{dx}\int_a^x f(t)\,dt = -f(x)$$

Consider

$$\frac{d}{dx}\int_a^{x^2} f(t)\,dt$$

we first consider and example

$$\frac{d}{dx}\left(\int_1^{x^2} \cos(t)\,dt\right)$$

so $\displaystyle \frac{d}{dx}\left(\sin(t)\Big|_1^{x^2}\right) = \frac{d}{dx}\left(\sin(x^2) - \sin(1)\right)$

$= \cos(x^2)\cdot 2x$ (chain Rule)

so if we let $u = x^2$

$$\frac{d}{dx}\int_1^u \cos(t)\,dt = \frac{d}{du}\int_1^u \cos(t)\,dt \cdot \frac{du}{dx}$$

From the fundamental Th^m

$$\cos(u)\frac{du}{dx} = \cos(x^2)\cdot 2x$$

In general

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$$

consider

$$\frac{d}{dx}\int_x^{2x} f(t)\,dt\ ?$$

Again, we'll consider an example

$$\frac{d}{dx}\int_x^{2x} e^t\,dt$$

$$\frac{d}{dx}\ e^t\Big|_x^{2x} = \frac{d}{dt}\left(e^{2x} - e^x\right)$$

$$= e^{2x}\cdot 2 - e^x$$

2 pieces

lets split up the integral

$$\int_x^{2x} e^t dt = \int_x^0 e^t dt + \int_0^{2x} e^t dt$$

$$= \int_0^{2x} e^t dt - \int_0^x e^t dt$$

$$\frac{d}{dx}\int_x^{2x} e^t dt = e^{2x}\cdot 2 \; - \; e^x$$

chain rule   fund th^m

In general

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)dt = \frac{d}{dx}\int_c^{b(x)} f(t)dt - \frac{d}{dx}\int_c^{a(x)} f(t)dt$$

$$= f(b(x))\cdot b'(x) - f(a(x))\cdot a'(x)$$