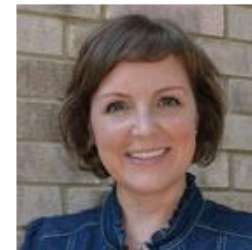


# Five Essential Components of Mathematics Intervention

# sarahpowellphd.com

Evidence-based mathematics resources for educators



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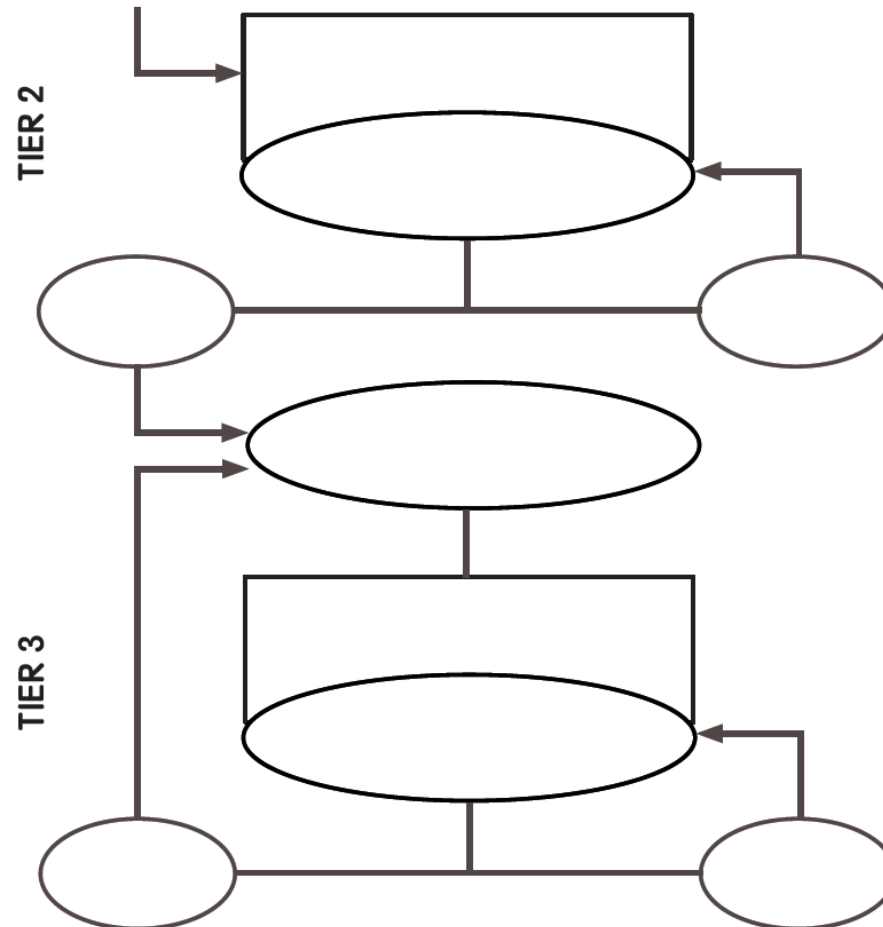
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@sarahpowellphd

# Five Essential Components of Mathematics Intervention

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www.sarahpowellphd.com

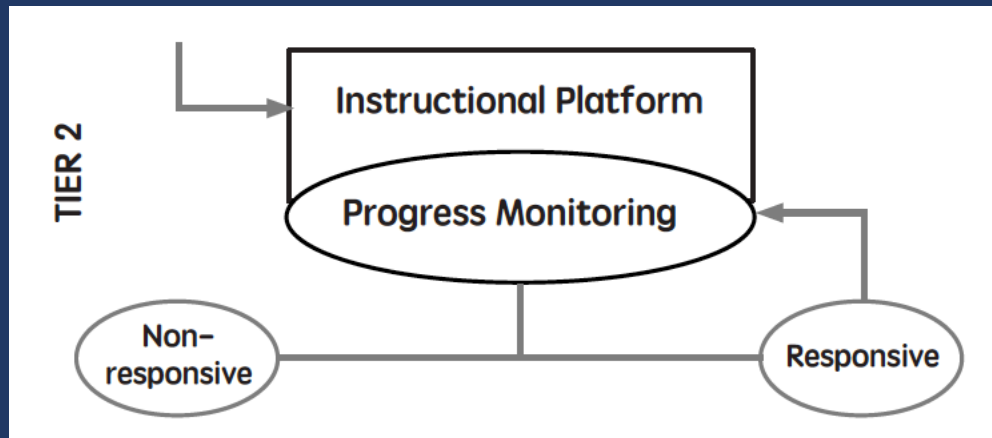


## TIER 1

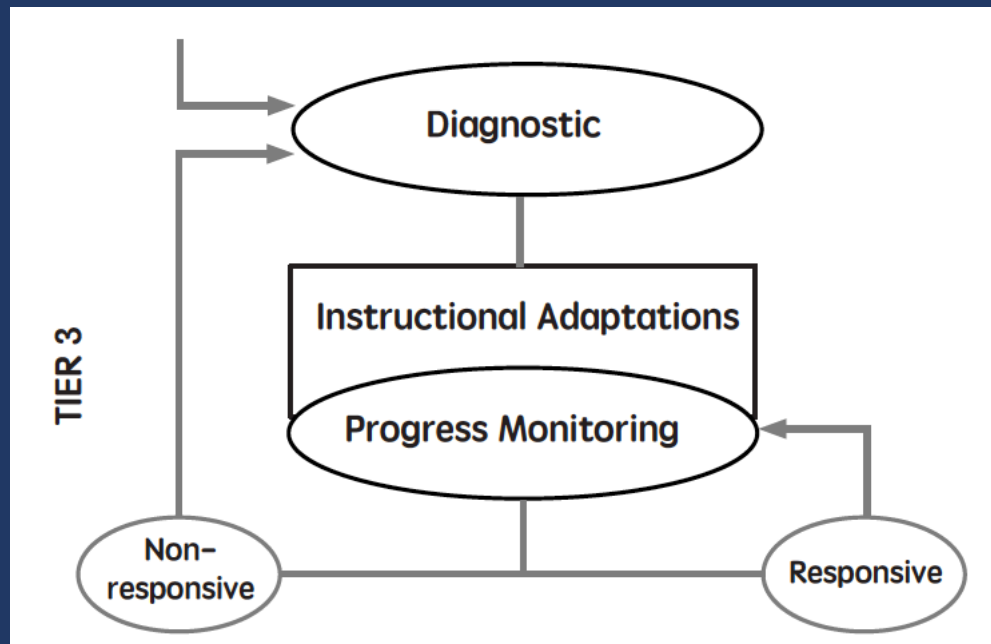


- Core instruction utilizes **evidence-based practices**
- All students **screened** (universal screener)
- Students scoring below a cut-score are suspected **at risk** for math difficulties
- Suspected **at-risk students** monitored for 6 to 10 weeks during primary prevention using **progress monitoring**

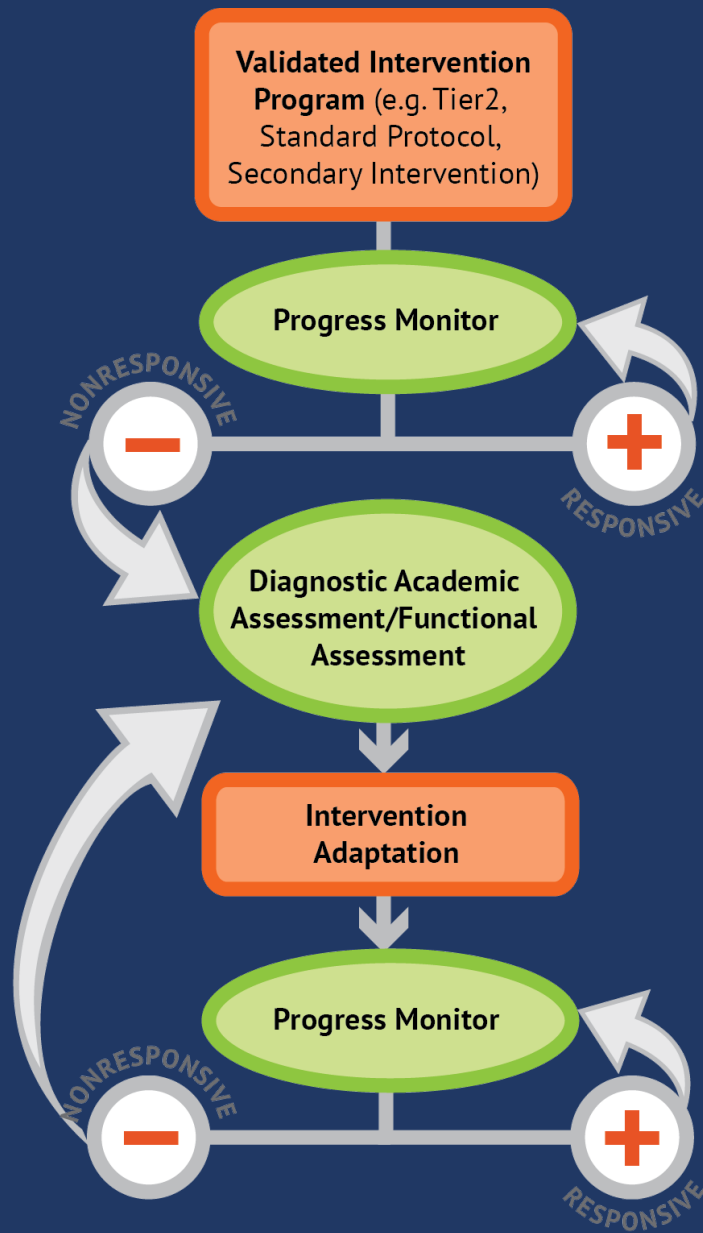


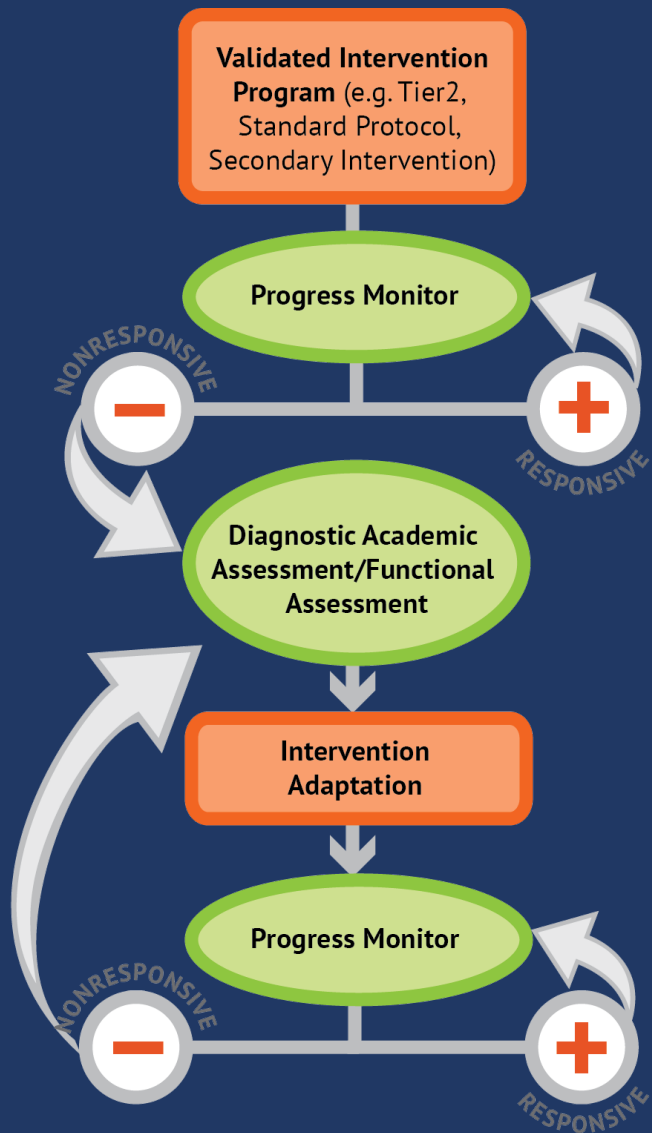


- Students are tutored in small groups using **evidence-based practices**
- Tutoring takes place three or four times a week
- Each tutoring session lasts 30 to 60 minutes
- Tutoring lasts 10 to 20 weeks
- **Progress monitoring** continues weekly



- **Diagnostics** are conducted
- **Adaptations** are made to the student's intervention
- Student progress is monitored weekly
  - With adequate slopes or end levels, students return to Tier 1 or 2





# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

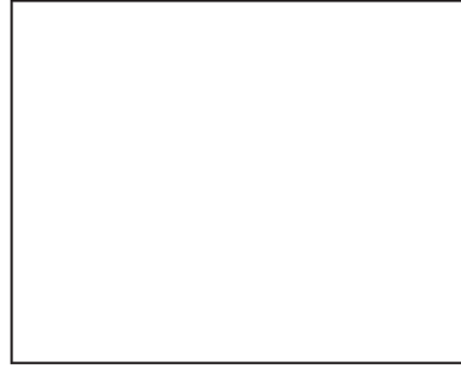
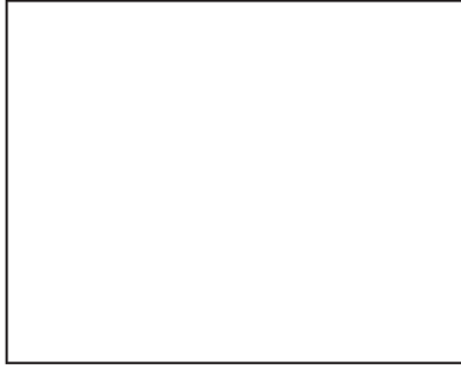
Multiple representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

## Explicit Instruction



## Modeling

Clear  
Explanation

Planned  
Examples

## Practice

Guided  
Practice

Independent  
Practice

## Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

## Goal and importance

### Modeling



Clear  
Explanation

Planned  
Examples

“Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.”

“Let’s continue working with our three-dimensional shapes and volume. Understanding volume and calculating volume helps with measuring capacity.”

## Goal and importance

## Model steps

### Modeling



Clear  
Explanation

Planned  
Examples

“To solve 26 plus 79, I first decide about the operation. Do I add, subtract, multiply or divide?”

“The plus sign tells me to add. So, I’ll add 26 plus 79. I’ll use the partial sums strategy. First, I add 20 plus 70. What’s 20 plus 70?”

“20 plus 70 is 90. I write 90 below the equal line. Where?”

“Then I add 6 plus 9. What’s 6 plus 9?”

“6 plus 9 is 15. So, I write 15 here.”

“Finally, we add the partial sums: 90 and 15. 90 plus 15 is 105. So, 26 plus 79 equals 105. What’s 26 plus 79?”



Goal and importance

Model steps

With examples

## Modeling

Clear  
Explanation

Planned  
Examples

“Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.”

$$24 / 6$$

$$28 \div 7$$

$$35 \overline{) 5}$$

## Modeling

Clear  
Explanation



Planned  
Examples

Goal and importance

Model steps

With examples

With non-examples

“Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.”

$$32 \div 8$$

$$42 \div 7$$

$$25 - 5$$

## Modeling

Clear  
Explanation

Planned  
Examples

## Practice

Guided  
Practice

Independent  
Practice

## Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

## Practice

Guided  
Practice

Independent  
Practice



Teacher and student  
practice together



Student practices with teacher support

## Modeling

Clear  
Explanation

Planned  
Examples

## Practice

Guided  
Practice

Independent  
Practice

## Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

## Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace



## Low-level and high-level

“What is 7 times 9?”

“Which shape has 6 sides?”

“What do you do when you see a word problem?”

“Why do you have to regroup?”

“How would you solve this problem?”

“Why do you have to use zero pairs?”



### Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual,  
partner, write on paper,  
write on whiteboard,  
thumbs up, etc.

“Turn and discuss the formula for perimeter with your partner.”

“Write the multiplication problem on your whiteboard.”

“In your math journal, draw a picture to help you remember to term *parallelogram*.”

### Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

Low-level and high-level

Classwide, individual,  
partner, write on paper,  
write on whiteboard,  
thumbs up, etc.

Affirmative and corrective

### Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

“Good work using your word-problem attack strategy.”

“Let’s look at that again. Tell me how you added in the hundreds column.”

Low-level and high-level

Classwide, individual,  
partner, write on paper,  
write on whiteboard,  
thumbs up, etc.

Affirmative and corrective

Planned and organized

### Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace

## Modeling

Clear  
Explanation

Planned  
Examples

## Practice

Guided  
Practice

Independent  
Practice

## Supports

- Asking the right questions
- Eliciting frequent responses
- Providing immediate specific feedback
- Maintaining a brisk pace



Introduction of material



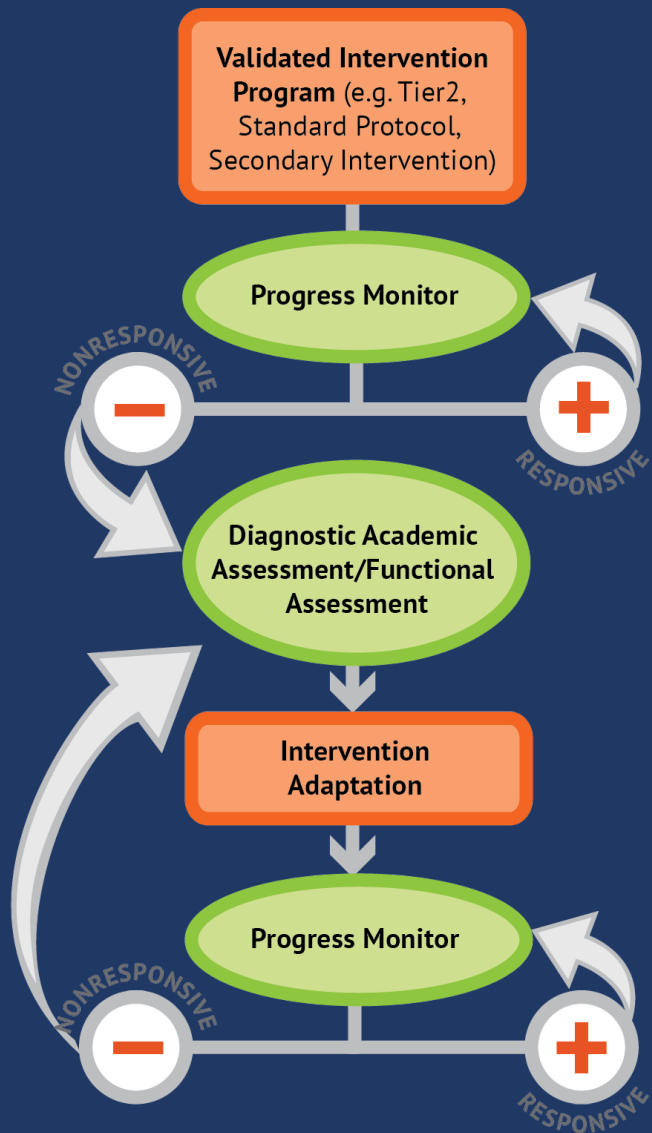
Review of material



# How do you use explicit instruction?



- ☐ Model steps using concise language
- ☐ Provide guided practice opportunities
- ☐ Provide independent practice opportunities
- ☐ Use supporting practices during modeling and practice
  - ☐ Ask the right questions
  - ☐ Elicit frequent responses
  - ☐ Provide feedback
  - ☐ Be planned and organized



# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

## INSTRUCTIONAL STRATEGIES

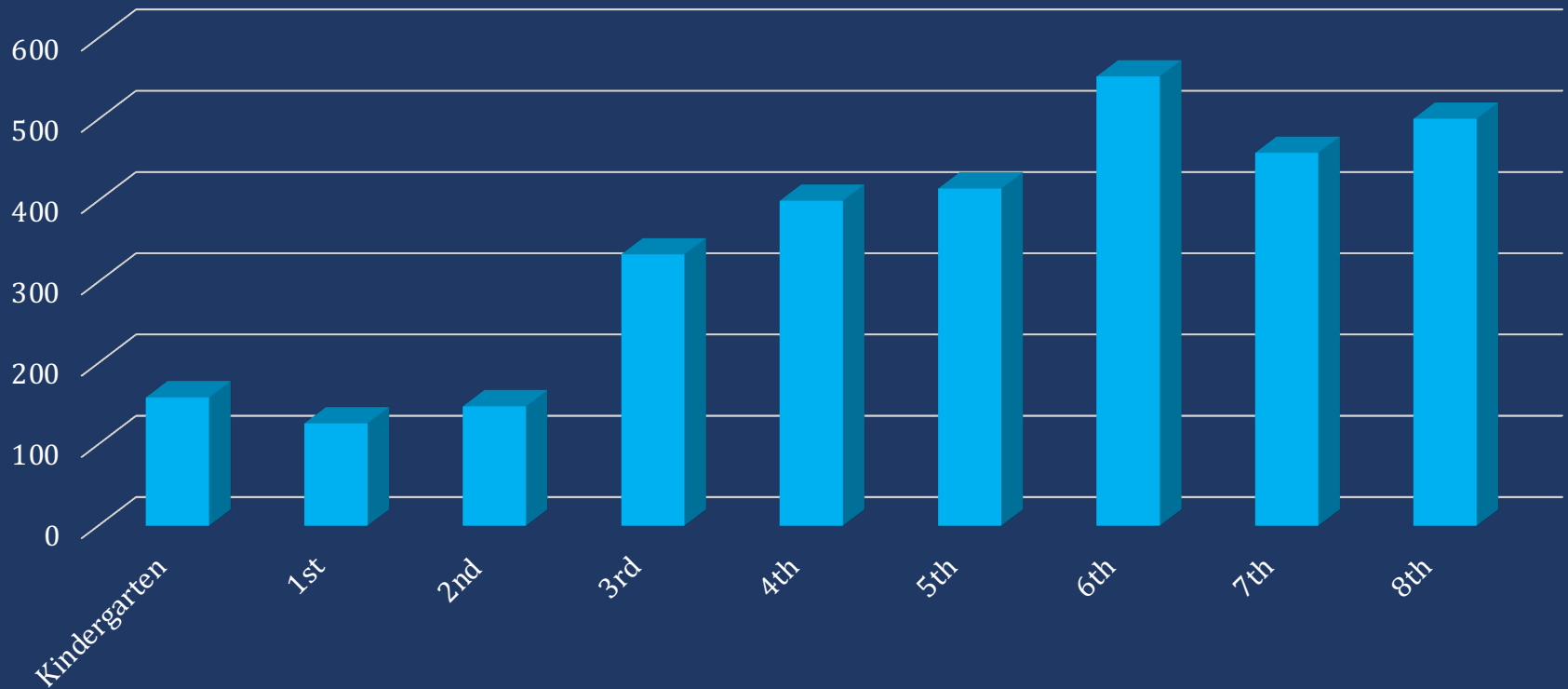


## Mathematical Language

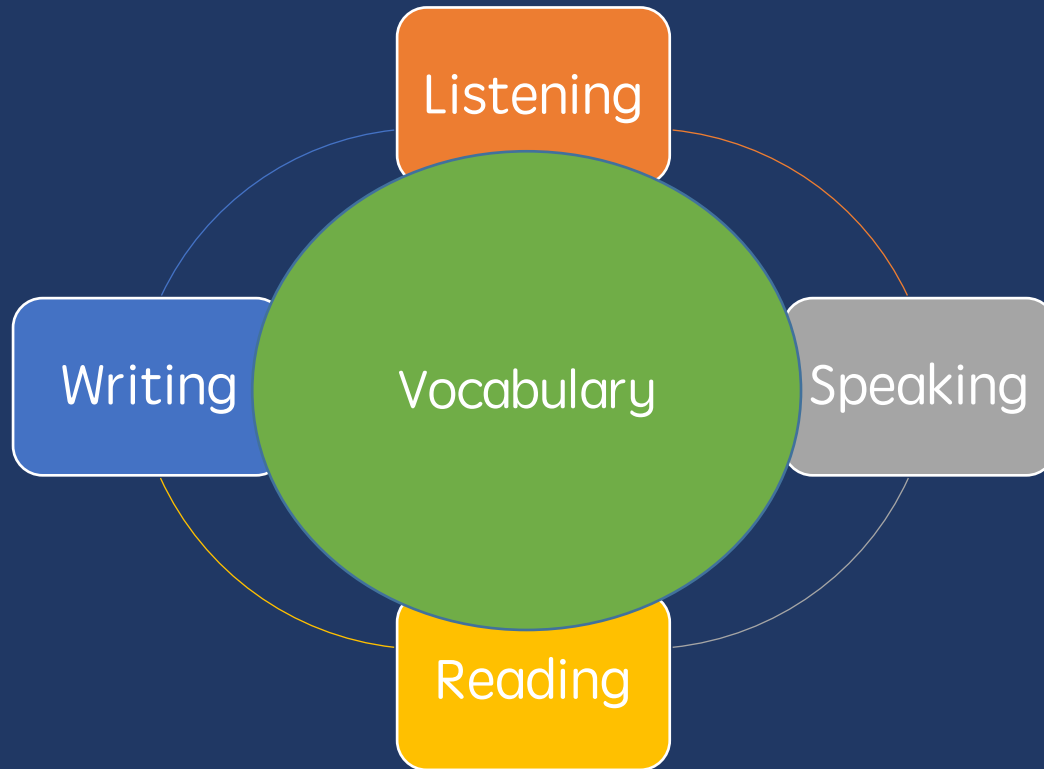
Instead of that...	Say this...

# degree

# Vocabulary Across Grades



# The Language of Mathematics



## 1. Some math terms are shared with English but have different meanings

**base**

**right**

**degree**

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)

**difference**

**even**

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)
3. Some math terms are only used in math

**trapezoid**

**numerator**

**parallelogram**

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

**round**

**square**

**second**

**base**

Rubenstein & Thompson (2002)



1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings

**divide vs.  
Continental  
Divide**

**variable vs.  
variably cloudy**

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs

**eight vs. ate**

**sum vs. some**

**rows vs. rose**

**base vs. bass**

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings

**factor vs.  
multiple**

**hundreds vs.  
hundredths**

**numerators vs.  
denominator**

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings

**mesa vs. tabla**

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities

**four vs. forty**

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
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6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way

**skip count vs.  
multiples**

**one-fourth vs.  
one quarter**

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings  
(but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way
11. Informal terms may be used for formal math terms

**rhombus vs.  
diamond**

**vertex vs.  
corner**


Use formal math language

Use terms precisely




## Mathematical Language

Instead of that...	Say this...



What number is in  
the tens place?




What digit is in the tens place?  
What is the value of the digit in  
the tens place?

135


**Why this is important...**

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.



The alligator eats the  
bigger number

is less than  
OR  
is greater than



**Why this is important...**

- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because  $<$  and  $>$  are two distinct symbols.



carry OR borrow

regroup OR  
trade OR  
exchange

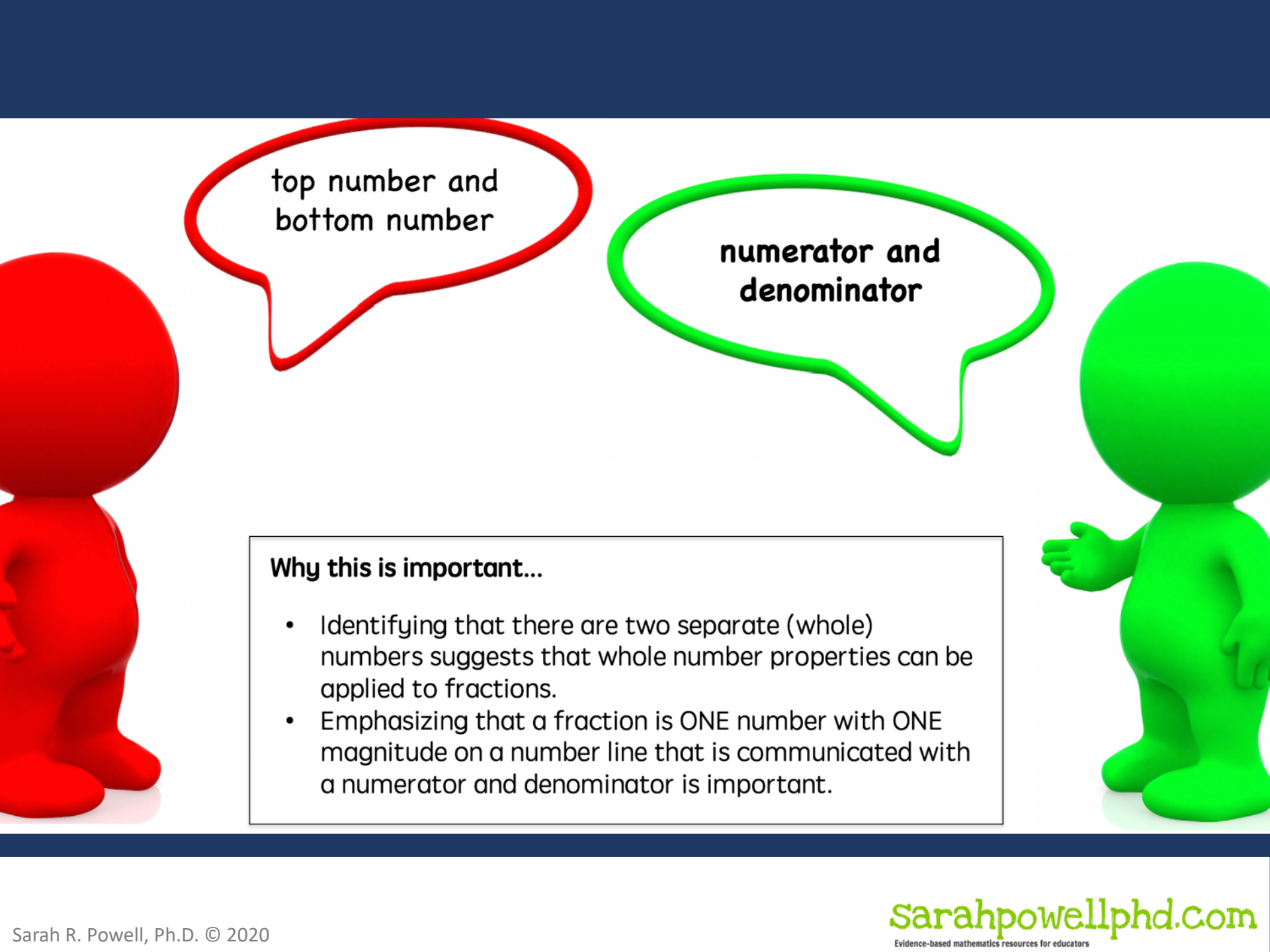


$$\begin{array}{r} 167 \\ + 294 \\ \hline \end{array}$$

**Why this is important...**

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.





top number and  
bottom number

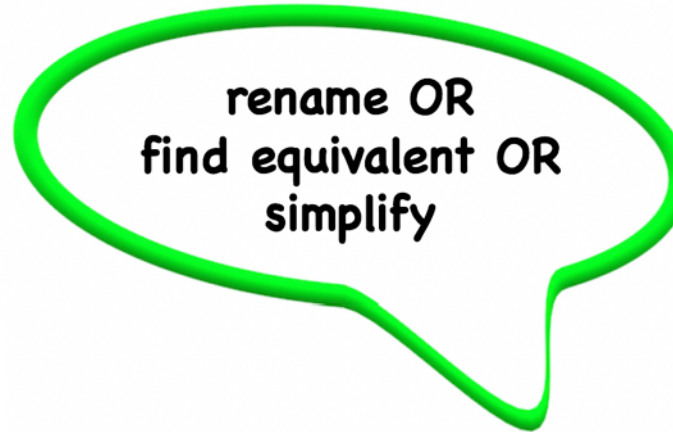

numerator and  
denominator

**Why this is important...**

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.



reduce the fraction



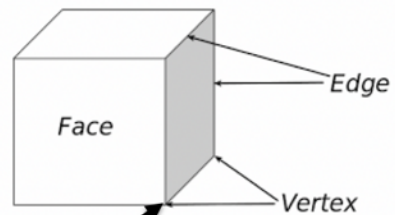
rename OR  
find equivalent OR  
simplify

**Why this is important...**

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

point

vertex



### Why this is important...

- This is the endpoint where two or more line segments or rays meet.



flips, slides, turns

**reflections,  
translations,  
rotations**

**Why this is important...**

- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.



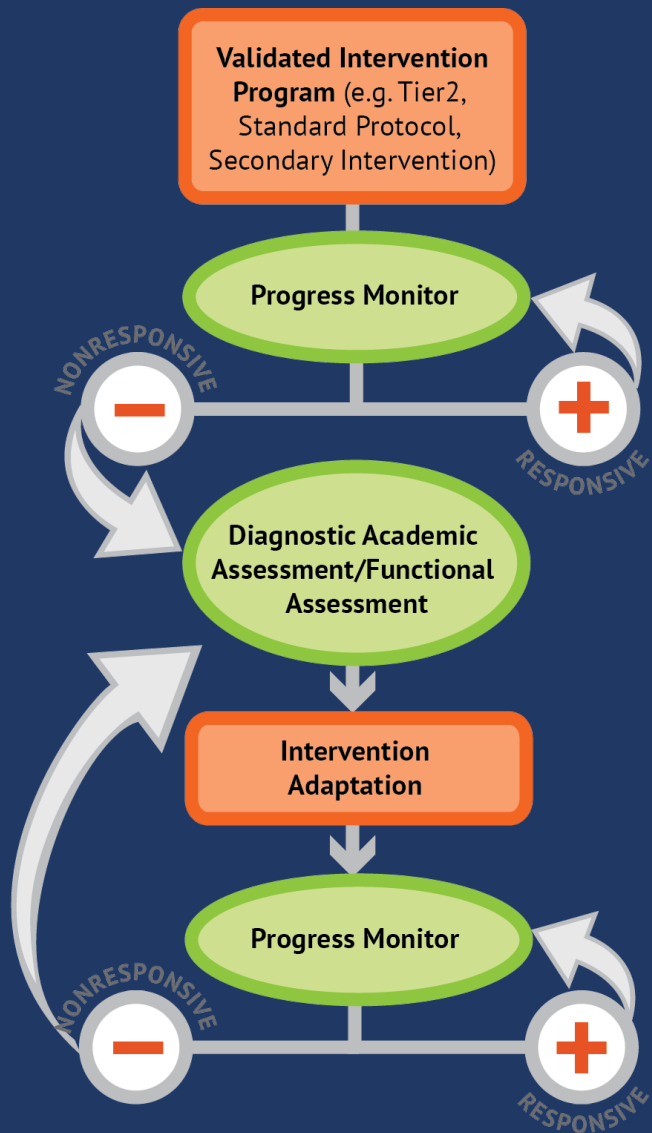
## Mathematical Language

Instead of that...	Say this...

# How do you use mathematical language?

- ☐ Use formal mathematical language
- ☐ Use precise terms





# Instructional Platform

## INSTRUCTIONAL DELIVERY

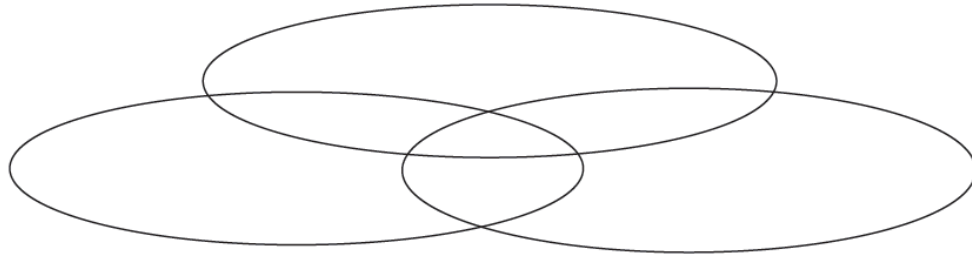
Explicit instruction

Precise language

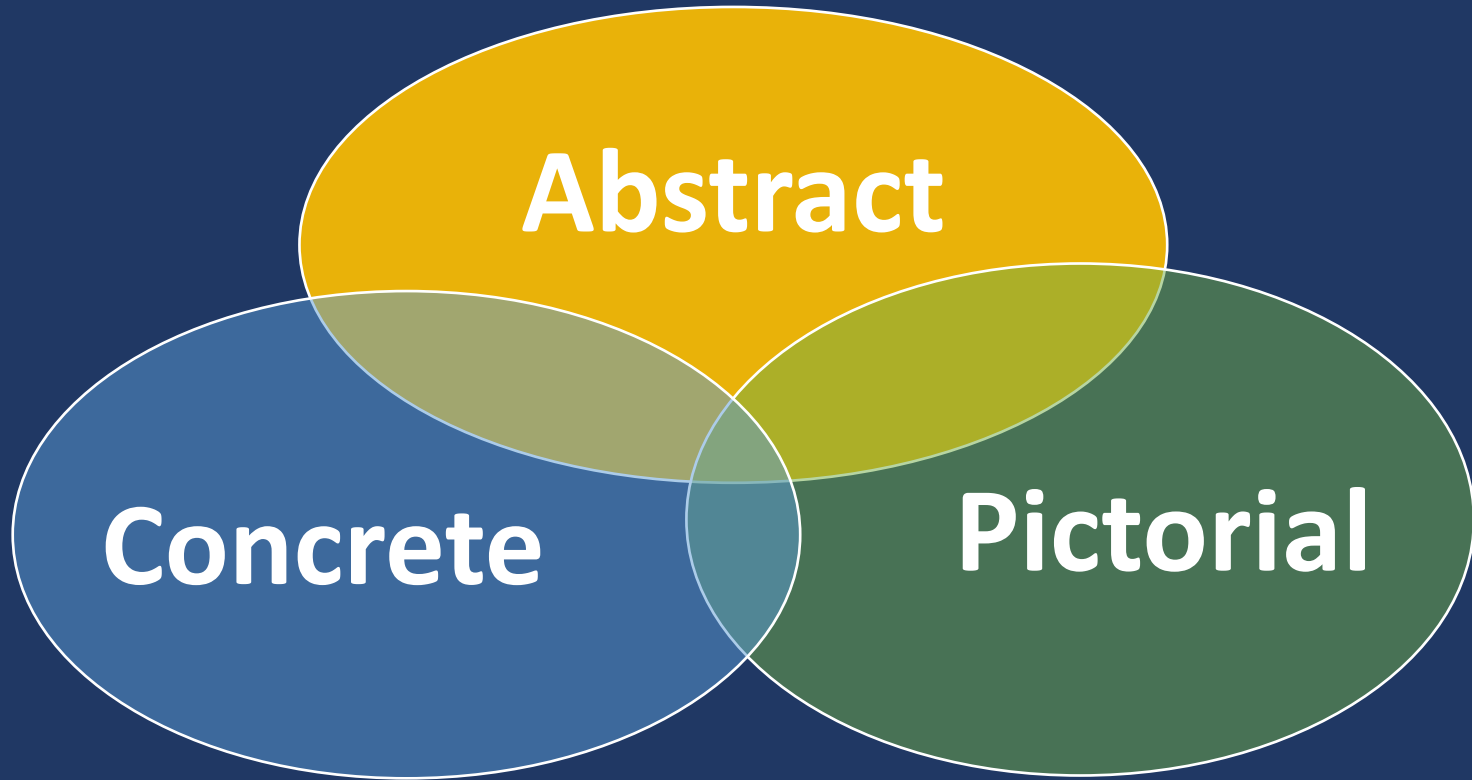
Multiple  
representations

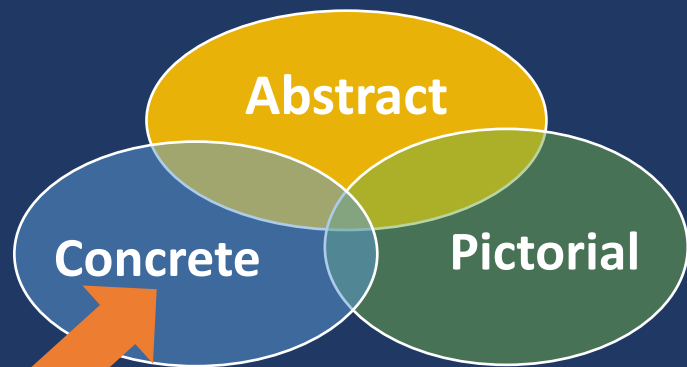
## INSTRUCTIONAL STRATEGIES

## Multiple Representations

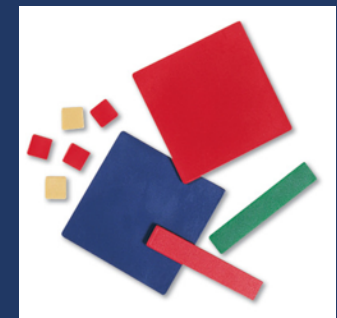
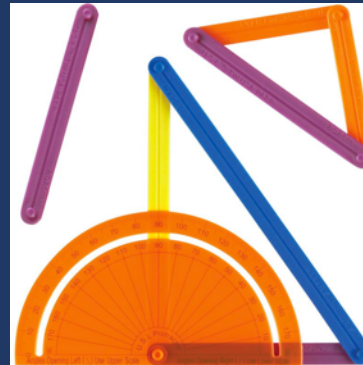
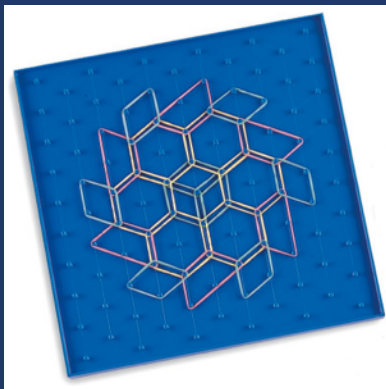


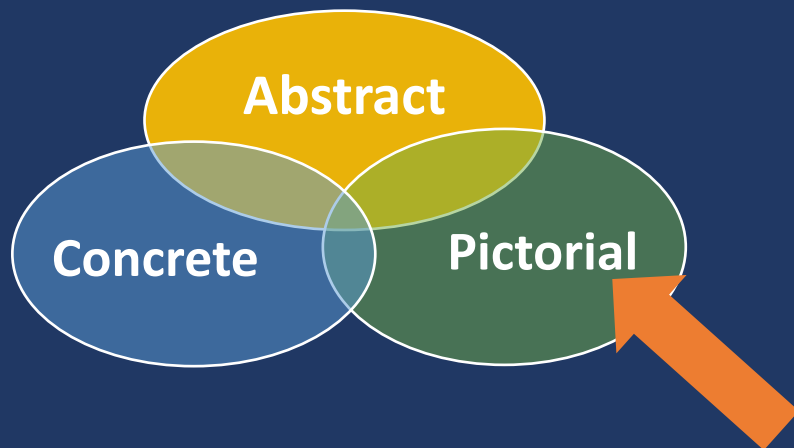
# Multiple Representations



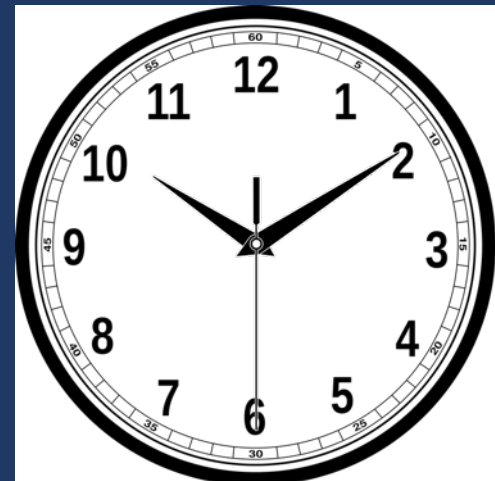
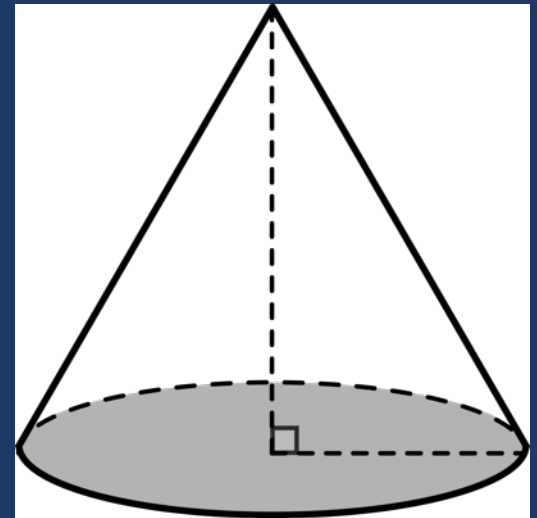
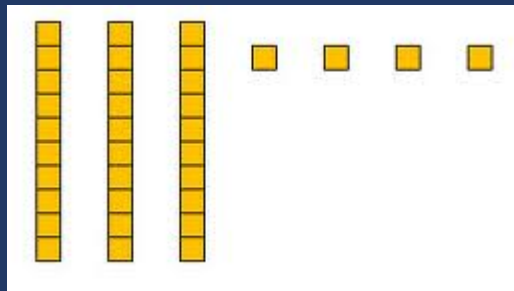


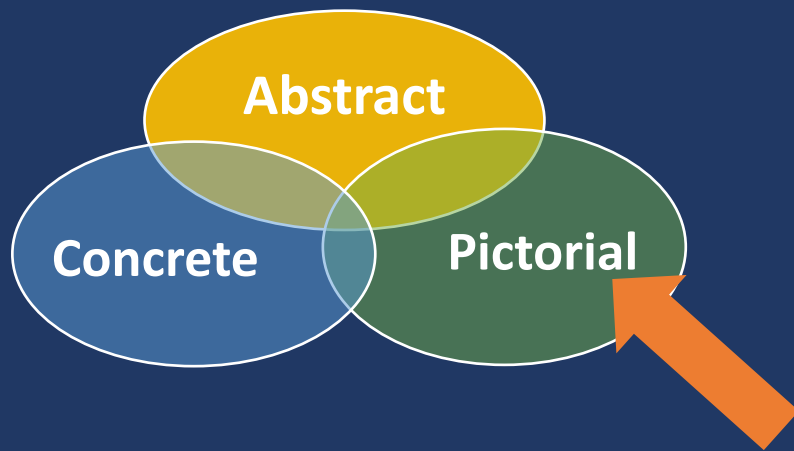
Three-dimensional objects



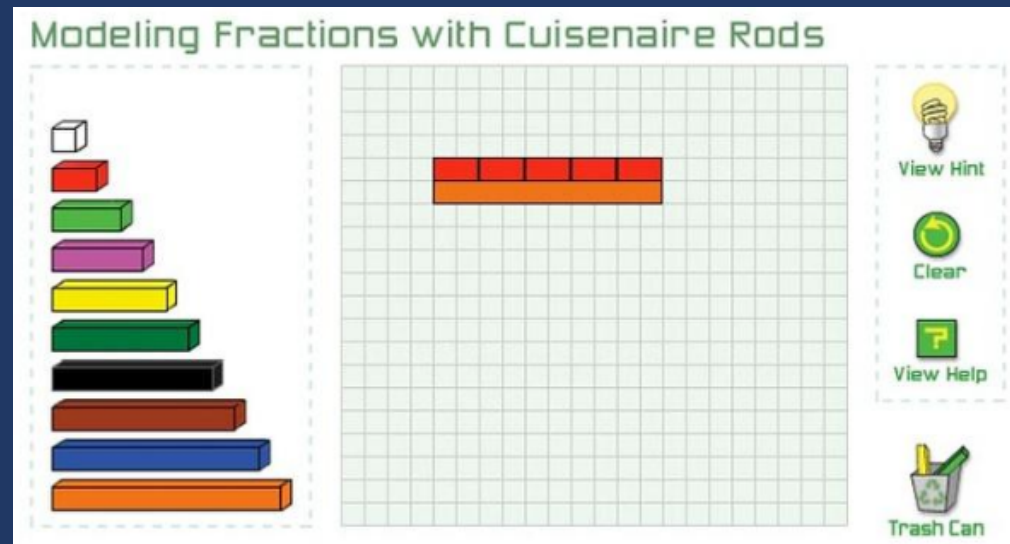
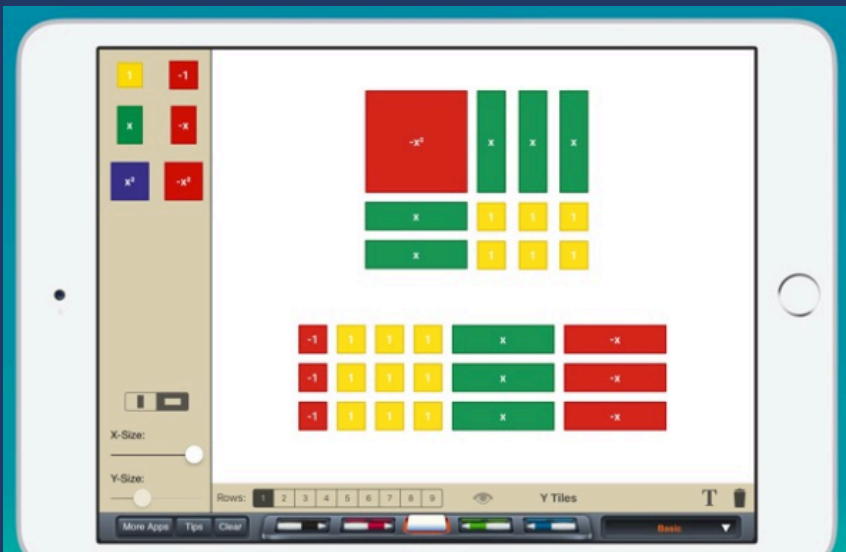
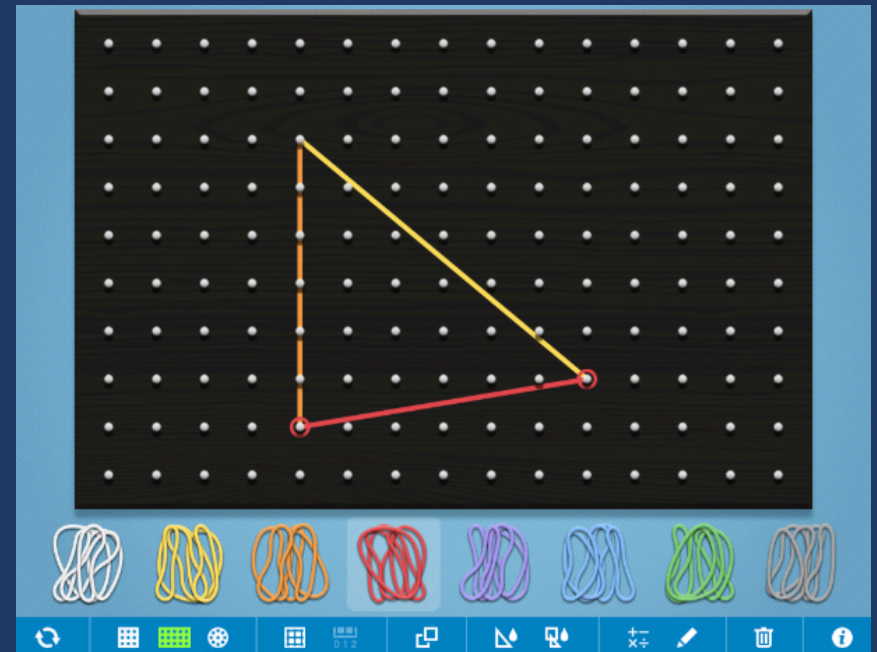


Two-dimensional images

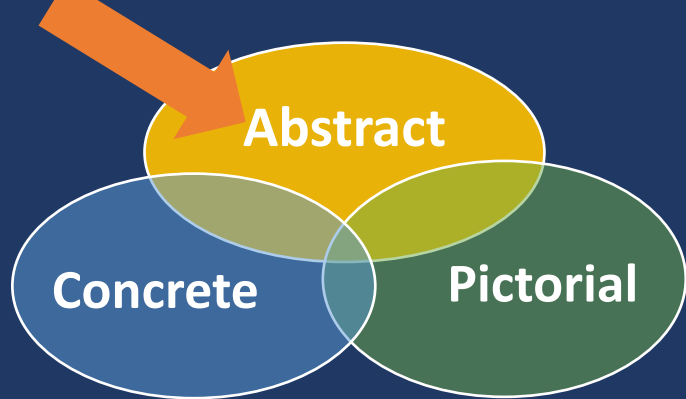




Two-dimensional images







Numerals and symbols

$$2 + 8 = 10$$

$$34 = 3 \text{ tens and } 4 \text{ ones}$$

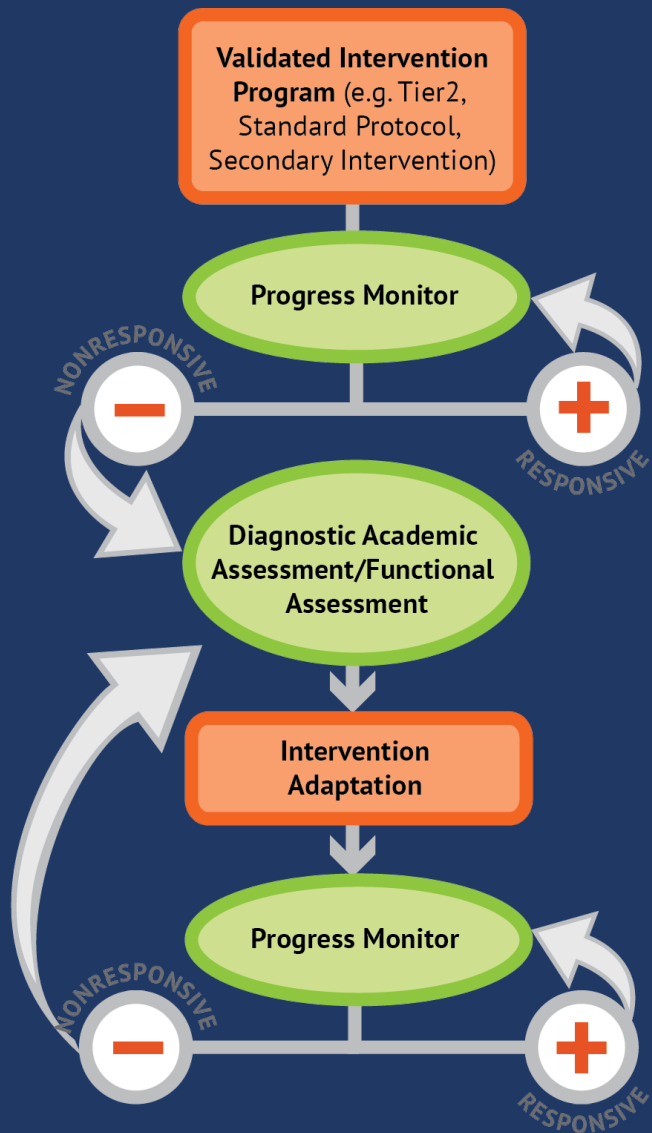
$$x - 6 = 8$$

$$\begin{array}{r} 4,179 \\ + \quad 569 \\ \hline \end{array}$$

# How do you use multiple representations?



- ❑ Use concrete materials
- ❑ Use pictorial representations
- ❑ Ensure students understand mathematics with numbers and symbols and words (i.e., the abstract)



# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple  
representations

## INSTRUCTIONAL STRATEGIES

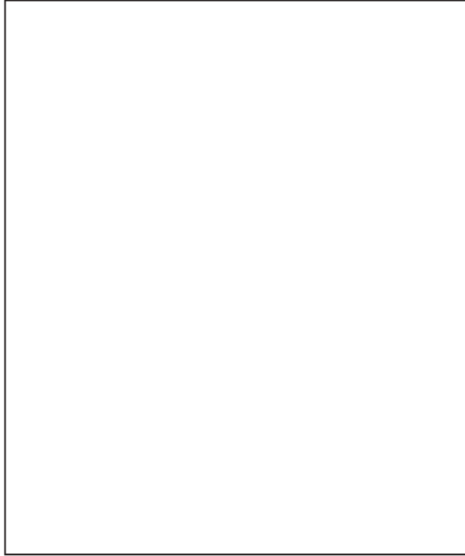
Fluency building

## Fluency

Addition



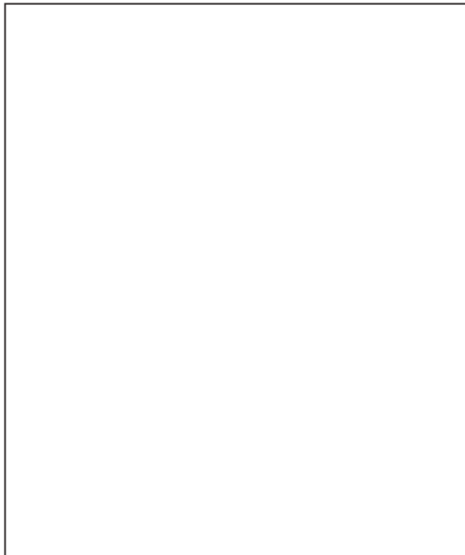
Subtraction



Multiplication



Division



# Addition Facts

## 100 addition basic facts

- Single-digit addends sum to a single- or double-digit number

$$\begin{array}{r} 5 \\ + 4 \\ \hline 9 \end{array}$$

(addend)  
(addend)  
(sum)

# Addition: Total (Part-Part-Whole, Combine)

Count one set, count another set, put sets together, count sum



$$2 + 3 = 5$$

# Addition: Join (Change Increase)

Start with a set, add the other set, count sum



$$2 + 3 = 5$$

Why is it important to understand addition in two separate ways?



# Total

**Parts** put together into a **total**

- Angie saw **4** cardinals and **5** blue jays. How many birds did Angie see?

# Change

An amount that **increases** or decreases

- Pam had \$4. Then she earned \$3 for cleaning her room. How much money does Pam have now?

# Subtraction Facts

## 100 subtraction basic facts

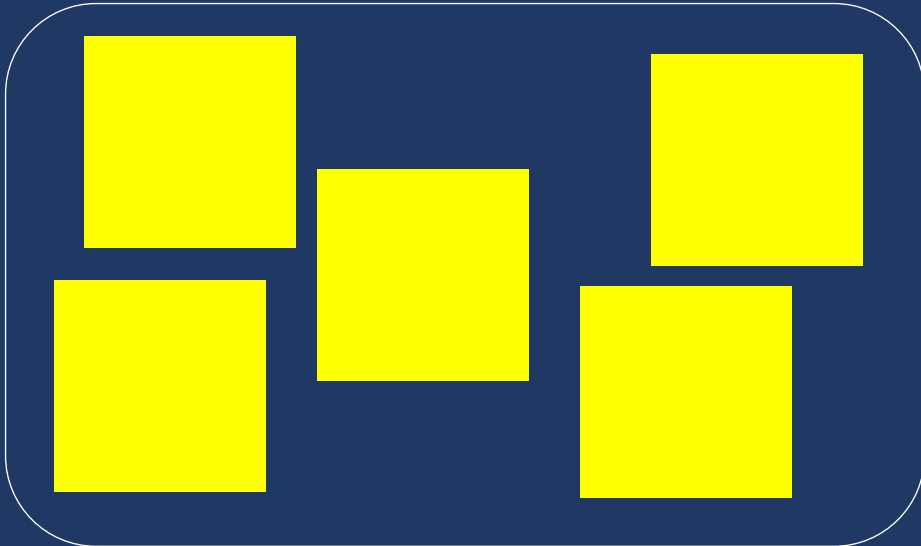
- Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{r} 16 \\ - \quad 8 \\ \hline 8 \end{array}$$

(minuend)  
(subtrahend)  
(difference)

# Subtraction: Separate (Change Decrease)

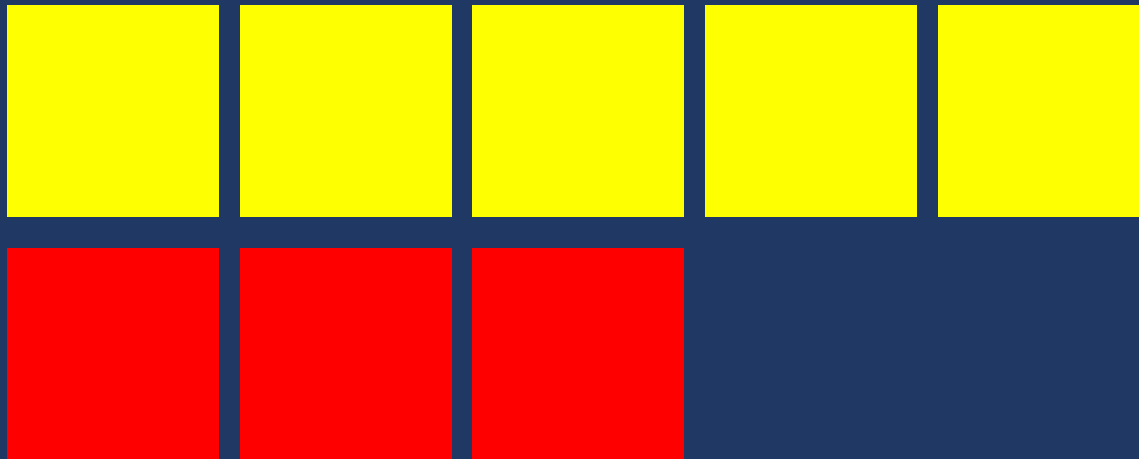
Start with a set, take away from that set, count difference



$$5 - 3 = 2$$

# Subtraction: Difference (Compare)

Compare two sets, count difference



$$5 - 3 = 2$$

Why is it important to understand subtraction in two separate ways?

# Change

An amount that increases or **decreases**

- Amanda had **9** cookies. Then she ate **2** of the cookies. How many cookies does Amanda have now?

# Difference

Greater and less amounts compared for a difference

- Scott has 9 apples. Cathy has 4 apples. How many more apples does Scott have? (How many fewer does Cathy have?)



# Multiplication Facts

## 100 multiplication basic facts

- Multiplication of single-digit factors results in a single- or double-digit product

	2	( <u>factor</u> )
×	3	( <u>factor</u> )
<hr/>		
	6	( <u>product</u> )

# Multiplication: Equal Groups

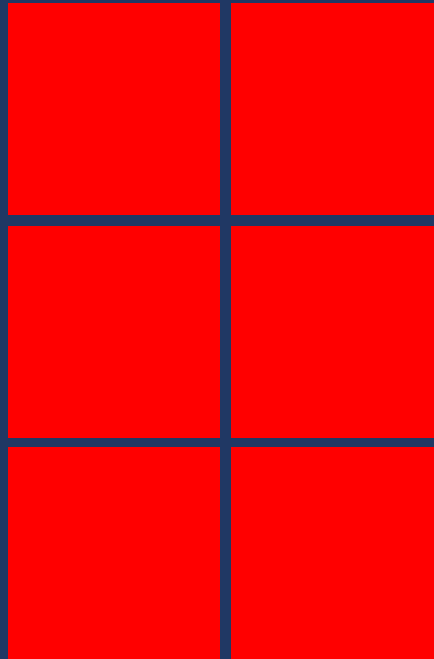
Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

# Multiplication: Equal Groups

Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

# Multiplication: Comparison

Show a set, then multiply the set



$$3 \times 2 = 6$$

Why is it important to understand multiplication in different ways?

# Equal Groups

**Groups** multiplied by **number in each group** for a **product**

- Carlos has **2** bags of apples. There are **6** apples in each bag. How many apples does Carlos have altogether?

# Comparison

**Set** multiplied by a number of **times** for a **product**

- Beth picked **6** apples. Amy picked **2** times as many apples as Beth. How many apples did Amy pick?

# Division Facts

## 90 division basic facts

- Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

$$8 \div 4 = 2$$

(dividend)    (divisor)    (quotient)



# Multiplication: Equal Groups

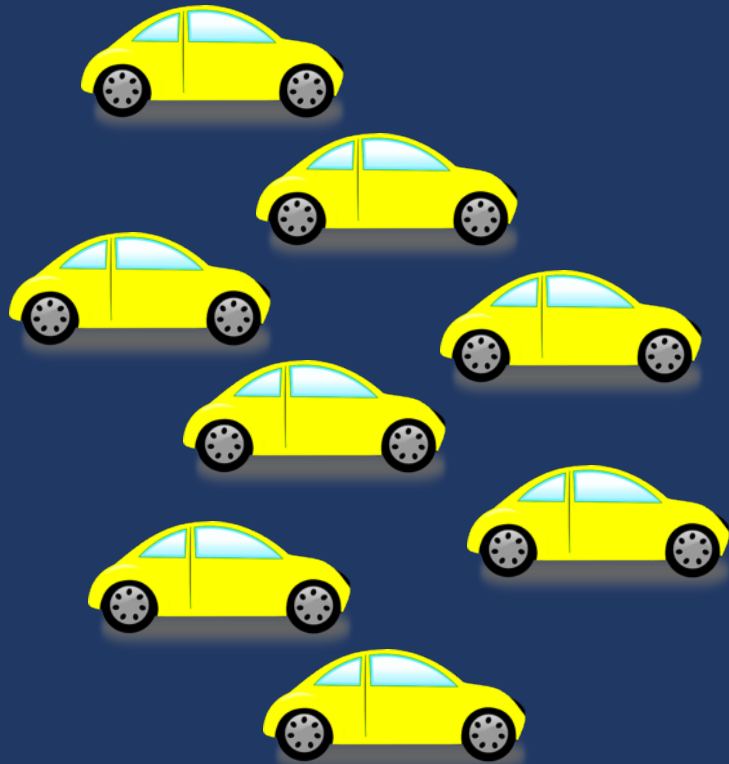
Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

# Division: Equal Groups (Partitive Division)

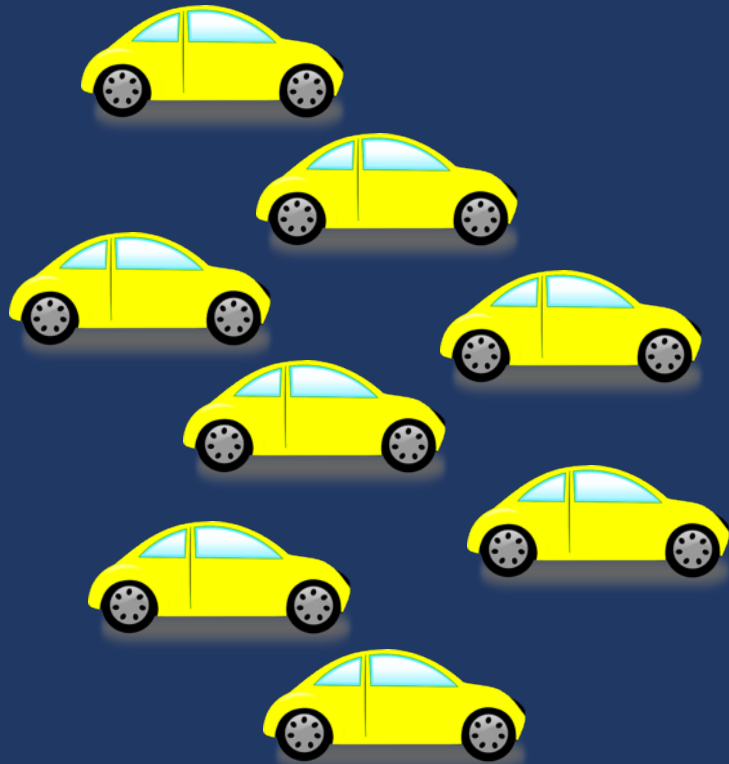
Show the dividend, divide equally among divisor, count quotient



$$8 \div 2 = 4$$

# Division: Equal Groups (Measurement Division)

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

Why is it important to understand division in different ways?

# Equal Groups

**Groups** multiplied by **number in each group** for a **product**

- Carlos has **12** apples. He wants to share them equally among his **2** friends. How many apples will each friend receive?
- Carlos has **12** apples. He put them into bags containing **6** apples each. How many bags did Carlos use?

# Partitive versus Measurement

$$10 \div 5 = \underline{\quad}$$

$$12 \div 4 = \underline{\quad}$$

Addition

Subtraction

Multiplication

Division

BRIEF  
(1-2 min)

DAILY  
(everyday)

# Other Types of Fluency



A.

$$\begin{array}{r} 1 \\ 74 \\ + 18 \\ \hline 92 \end{array}$$

B.

$$\begin{array}{r} 1 \quad 1 \\ 725 \\ + 365 \\ \hline 1,090 \end{array}$$

A.

$$\begin{array}{r} 5 \\ \cancel{6}12 \\ - 17 \\ \hline 45 \end{array}$$

B.

$$\begin{array}{r} 29 \\ \cancel{30}15 \\ - 96 \\ \hline 209 \end{array}$$

A.

$$\begin{array}{r}
 24 \\
 \times 43 \\
 \hline
 11800 \\
 160 \\
 60 \\
 + 12 \\
 \hline
 1,032
 \end{array}$$

20	4
800	160
60	12

40  
3

B.

$$\begin{array}{r}
 132 \\
 \times 53 \\
 \hline
 5000 \\
 1500 \\
 100 \\
 300 \\
 90 \\
 + 6 \\
 \hline
 6996
 \end{array}$$

100	30	2
5000	1500	100
300	90	6

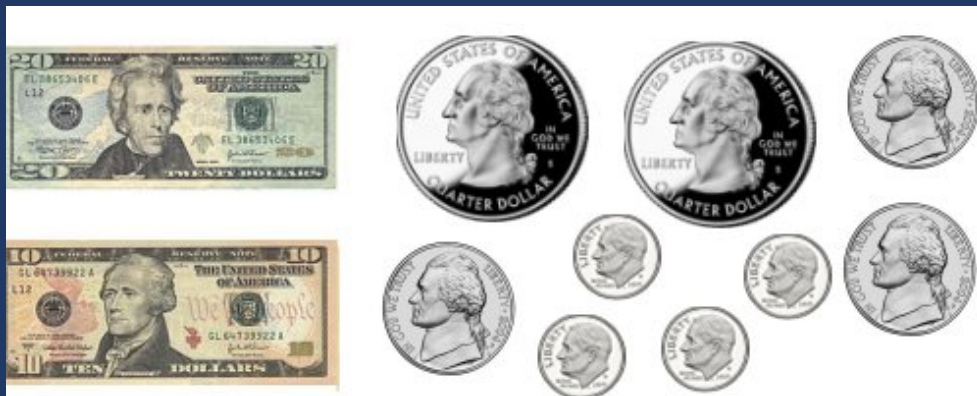
50  
3

A.  $12 \overline{)158}$

158	10
- 120	
38	
- 36	+ 3
2	13 R2

B.  $34 \overline{)170}$

170	20
- 680	
290	
- 170	5
120	
102	+ 3
18	28 R18



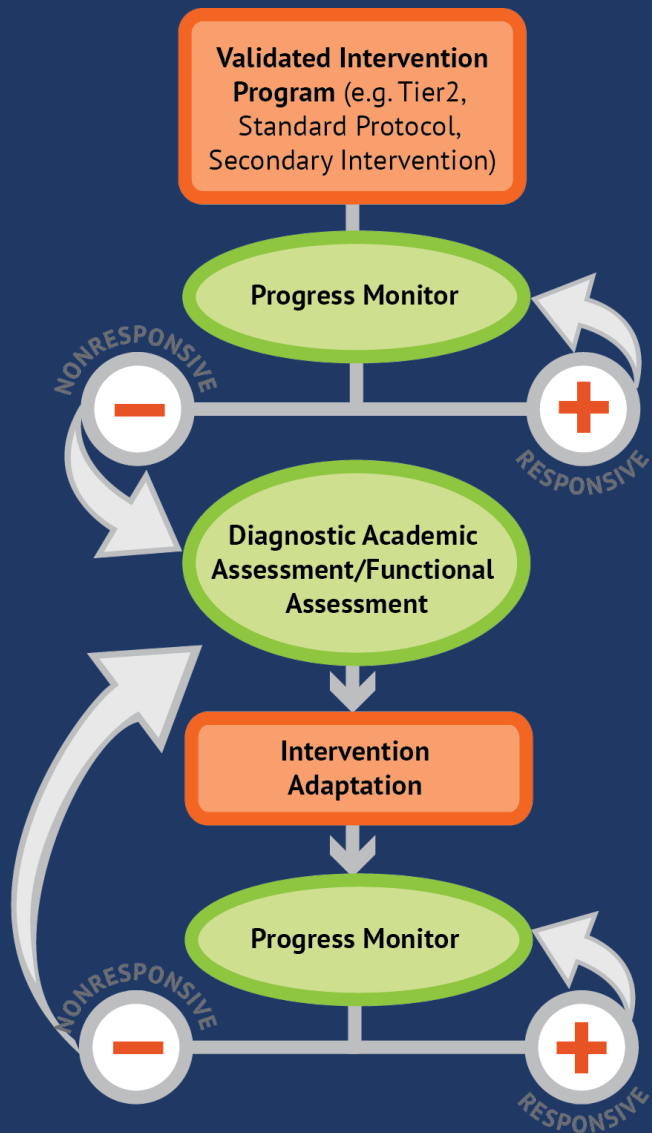
X	1	2	3	4	5

$$-3 + (-4) = \underline{\hspace{2cm}} \qquad 5 - (-6) = \underline{\hspace{2cm}}$$

# How to build fluency?

- ❑ Teach the *concepts* of the operations
- ❑ Teach *strategies* to understand how facts fit together
- ❑ Practice building *fluency* with a variety of activities and games





# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

## Problem Solving

Three Things to Remember


Attack Strategies

--



**Don't tie key words to operations**



**Do have an attack strategy**



**Do teach word-problem schemas**



**Addition**

total, plus, in all, join, sum, together

What it looks like:

$1 + 3 = 4$

$1 + 2 = 3$

$1 + 1 = 2$

**Subtraction**

Part, minus, left, take away, fewer

What it looks like:

$4 - 1 = 3$

$5 - 2 = 3$

$3 - 1 = 2$

## Key Words Used in Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> <li>+ add</li> <li>+ all together or altogether</li> <li>+ and</li> <li>+ both</li> <li>+ combined</li> <li>+ how many in all</li> <li>+ how much</li> <li>+ in all</li> <li>+ increased by</li> <li>+ plus</li> <li>+ sum</li> <li>+ together</li> <li>+ total</li> </ul>	<ul style="list-style-type: none"> <li>- change</li> <li>- decreased by</li> <li>- difference</li> <li>- fewer or fewer than</li> <li>- how many are left (or have left)</li> <li>- how many did not have</li> <li>- how many (or much) more</li> <li>- how much longer (shorter, taller, heavier, etc.)</li> <li>- less or less than</li> <li>- lost</li> <li>- minus</li> <li>- need to</li> <li>- reduce</li> <li>- remain</li> <li>- subtract</li> <li>- take away</li> </ul>
Multiplication Words	Division Words
<ul style="list-style-type: none"> <li>x by (dimension)</li> <li>x double</li> <li>x each group</li> <li>x every</li> <li>x factor of</li> <li>x increased by</li> <li>x multiplied by</li> <li>x of</li> <li>x product</li> <li>x times</li> <li>x triple</li> </ul>	<ul style="list-style-type: none"> <li>+ as much</li> <li>+ cut up</li> <li>+ each group</li> <li>+ equal share</li> <li>+ half (or)</li> <li>+ how many</li> <li>+ parts</li> <li>+ per</li> <li>+ percent</li> <li>+ quotient</li> <li>+ rate of</li> <li>+ separate</li> <li>+ share</li> </ul>

**key words**

combined, addition, in all, total, sum, together, plus, both, perimeter, add, more than

**key words**

triple, factor, product, multiply, each, product, in all, multiple, times

**key words**

subtract, decrease, fewer, remain, take away, minus, less than, how many more

**key words**

division, split, quotient, equal groups, divide, half, shared equally, each, distribute

**Addition**

Sum  
Total  
Altogether  
Ex: How much did it cost altogether?  
In all  
Added to  
Increase / Increased by

**Subtraction**

minus  
Less / Less than  
Difference / Comparing  
Ex: How much more...  
How much taller...  
How much further...  
Decrease / Decreased by  
How much is Left?  
L must know how much you've used first!

**Multiplication**

Product  
times  
multiplied  
Of  
Ex: He ate  $\frac{2}{3}$  of the cookies of these  
Each  
Per  
Ex: when increasing  
Ex: It cost \$1 per soda, how much for 10 sodas?

**Division**

Quotient  
Average  
Divided by  
Each  
Per > Just One!  
Ex: It cost \$2.40 for 12 apples. How much per apple?  
How much will fit into a certain thing?  
Ex: Each shirt cost \$8, how many can you buy for \$40?

**Key Words**

Students need to understand *key words*. But,  
key words should not be directly tied to  
*operations*.

## Word Problems: Subtraction

Name \_\_\_\_\_ Date \_\_\_\_\_

Subtraction

Read each problem. Write a number sentence and solve.

1. Mrs. Smith has 33 poodles and 18 boxers. How many more poodles does Mrs. Smith have?



\_\_\_\_\_

2. The kennel holds 91 dogs. Mr. Glass has 67 dogs in the kennel now. How many spaces does he have left?



\_\_\_\_\_

3. Mr. Kelly has 44 beagles. 26 of them are puppies. How many adult beagles does Mr. Kelly have?



\_\_\_\_\_

4. Mrs. Green has 60 terriers. 25 of them are boys. How many terriers are girls?



\_\_\_\_\_

5. There were 58 kittens at the pet shop on Friday. 29 of them were sold on Saturday. How many kittens were left?



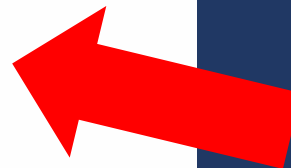
\_\_\_\_\_

6. Pat counted 22 lizards in the tank at the pet shop. 8 were sold later that day. How many lizards were left in the tank?



\_\_\_\_\_

# LONG DIVISION WORD PROBLEMS



1. Zookeeper Al wants to give each monkey at the zoo an equal number of bananas. There are 37 monkeys in the zoo and 567 bananas. How many bananas does each monkey get? And How many are left over for him to eat himself?

2. Betty has 427 oranges and needs to pack them up equally in 23 boxes. How many oranges go in each box and how much does she have left over?

3. Miss King has 1376 pages of scrap paper. She wants to make them into scrap paper packets for her 32 students. How many pages will each packet have? How many extra pages will she have left over?

4. Mr. Chong has 1,440 pages of scrap paper. He instead wants to make packets of 40 pages each but forgets to check if that will be enough for his 37 students. Will there be enough packets per student? If not how much more scrap paper does he need?



**Don't tie key words to operations**



**Do have an attack strategy**



**Do teach word-problem schemas**



# RIDGES

**R**ead the problem.

**I** know statement.

**D**raw a picture.

**G**oal statement.

**E**quation development.

**S**olve the equation.



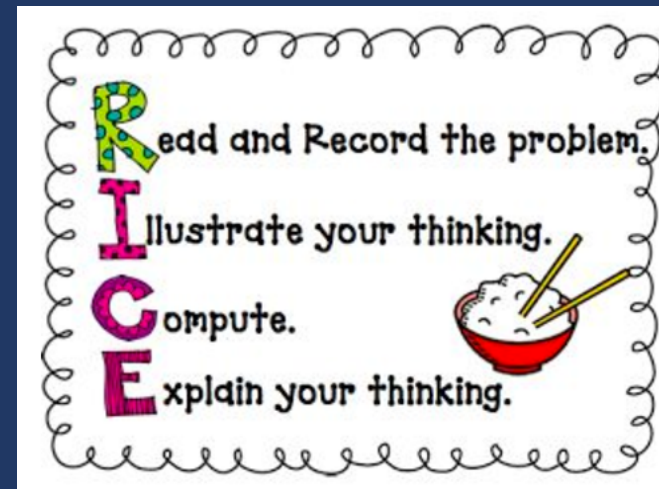
# RIDE

**R**ead the problem.

**I**dentify the relevant information.

**D**etermine the operation and unit for the answer.

**E**nter the correct numbers and calculate, then check the answer.

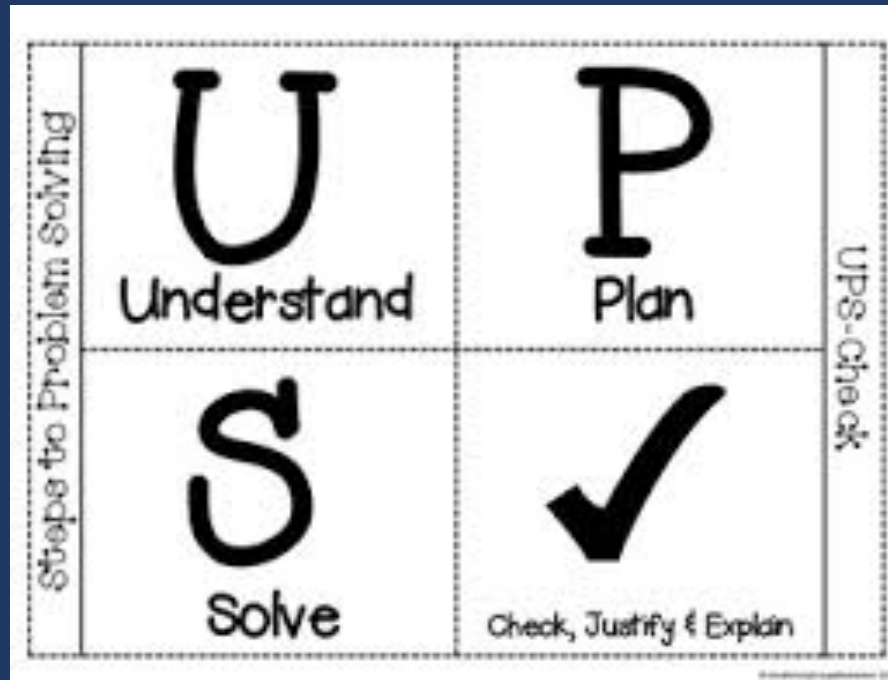


# R-CUBES

Read the problem.  
Circle key numbers.  
Underline the question.  
Box action words.  
Evaluate steps.  
Solve and check.

## Problem-Solving Model

Step	Description of Step
1	<b>Analyze the given information.</b> <ul style="list-style-type: none"><li>• Summarize the problem in your own words.</li><li>• Describe the main idea of the problem.</li><li>• Identify information needed to solve the problem.</li></ul>
2	<b>Formulate a plan or strategy.</b> <ul style="list-style-type: none"><li>• Draw a picture or a diagram.</li><li>• Find a pattern.</li><li>• Guess and check.</li><li>• Act it out.</li><li>• Create or use a chart or a table.</li><li>• Work a simpler problem.</li><li>• Work backwards.</li><li>• Make an organized list.</li><li>• Use logical reasoning.</li><li>• Brainstorm.</li><li>• Write a number sentence or an equation.</li></ul>
3	<b>Determine a solution.</b> <ul style="list-style-type: none"><li>• Estimate the solution to the problem.</li><li>• Solve the problem.</li></ul>
4	<b>Justify the solution.</b> <ul style="list-style-type: none"><li>• Explain why your solution solves the problem.</li></ul>
5	<b>Evaluate the process and the reasonableness of your solution.</b> <ul style="list-style-type: none"><li>• Make sure the solution matches the problem.</li><li>• Solve the problem in a different way.</li></ul>



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**Don't tie key words to operations**



**Do have an attack strategy**



**Do teach word-problem schemas**

# Schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions

# Total

**Parts** put together into a **total**

- Autumn saw **4** cardinals and **5** blue jays. How many birds did Autumn see?
  - $4 + 5 = ?$
- Autumn saw **9** birds. If **4** of the birds were cardinals, how many were blue jays?
  - $4 + ? = 9$
- Autumn r saw **9** birds. **5** of the birds were blue jays, how many were cardinals?
  - $5 + ? = 9$

# Difference

**Greater** and **less** amounts compared for a **difference**

- Lydia has **9** apples. Carol has **4** apples. How many more apples does Lydia have? (How many fewer?)
  - $9 - 4 = ?$
- Lydia has **5** more apples than Carol. If Carol has **4** apples, how many does Lydia have?
  - $? - 4 = 5$
- Carol has **5** fewer apples than Lydia. Lydia has **9** apples. How many apples does Carol have?
  - $9 - ? = 5$

# Change

An amount that **increases** or decreases

- Victoria had \$**4**. Then she earned \$**3** for cleaning her room. How much money does Victoria have now?
  - **$4 + 3 = ?$**
- Victoria has \$**4**. Then she earned money for cleaning her room. Now Victoria has \$**7**. How much money did she earn?
  - **$4 + ? = 7$**
- Victoria had some money. Then she made \$**3** for cleaning her room. Now she has \$**7**. How much money did Victoria start with?
  - **$? + 3 = 7$**

# Change

An amount that increases or **decreases**

- Julie baked **9** cookies. Then, she ate **2** of the cookies. How many cookies does Julie have now?
  - **$9 - 2 = ?$**
- Julie baked **9** cookies. Then, she ate some of the cookies. Now, she has **7** cookies. How many cookies did Julie eat?
  - **$9 - ? = 7$**
- Julie baked some cookies. She ate **2** of the cookies and has **7** cookies left. How many cookies did Julie bake?
  - **$? - 2 = 7$**

# Equal Groups

**Groups** multiplied by **number in each group** for a **product**

- Scott has **2** bags of apples. There are **6** apples in each bag. How many apples does Scott have altogether?
  - $2 \times 6 = ?$
- Scott has **12** apples. He wants to share them equally among his **2** friends. How many apples will each friend receive?
  - $2 \times ? = 12$
- Scott has **12** apples. He put them into bags containing **6** apples each. How many bags did Scott use?
  - $? \times 6 = 12$

# Comparison

**Set** multiplied by a number of **times** for a **product**

- Julie picked **6** apples. Amy picked **2** times as many apples as Marcie. How many apples did Lisa pick?
  - $6 \times 2 = ?$
- Amy picked **12** apples. She picked **2** times as many apples as Julie. How many apples did Julie pick?
  - $? \times 2 = 12$
- Amy picked **12** apples, and Julie picked **6** apples. How many times as many apples did Amy pick?
  - $6 \times ? = 12$



# Schemas

Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



**Don't tie key words to operations**



**Do have an attack strategy**

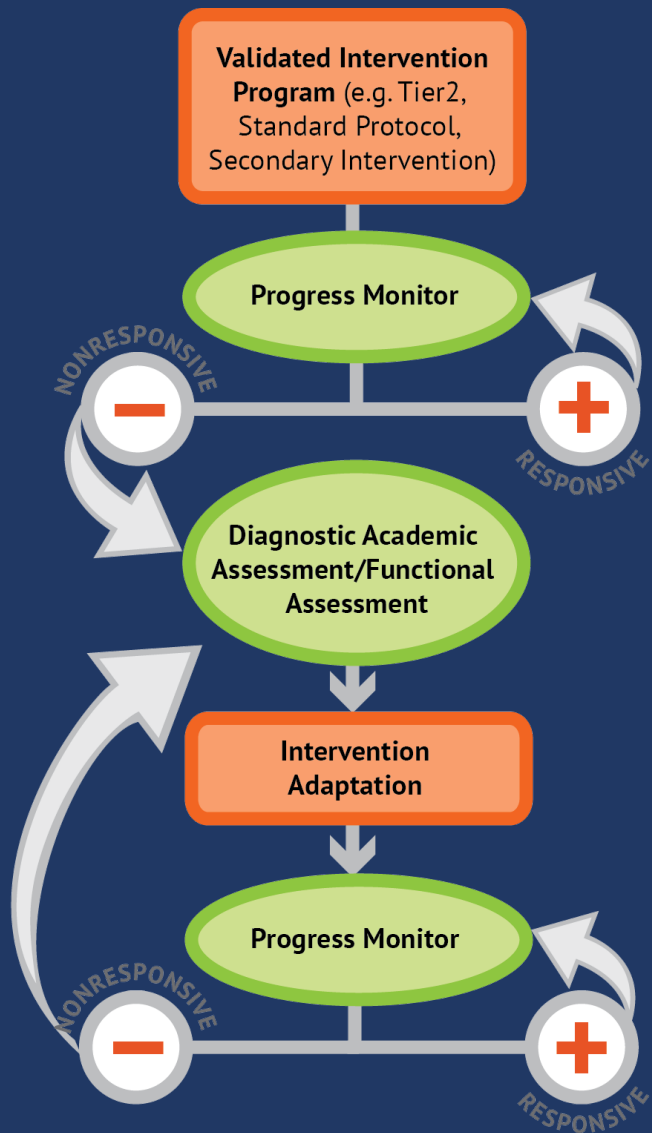


**Do teach word-problem schemas**

# How do you teach problem solving?

- ❑ **Don't** use key words tied to operations
- ❑ **Do** teach students an *attack strategy*
- ❑ **Do** teach students *schemas*





# Instructional Platform

## INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

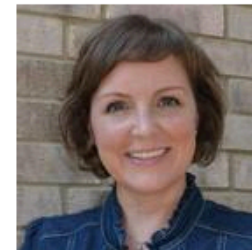
## INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving instruction

# sarahpowellphd.com

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