An Alternative Approach to Solving a Quadratic Equation and an Extension to Approximating Higher Degree Equations with Even Powers

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Abstract - This paper deals with the mathematical process of finding the roots of a quadratic equation using the minimum point on the functional parabola which it has been taken from. This idea can then be extended to finding an approximation to any equation that has constant coefficients and even powers in the variable (x).

Keywords: *Quadratic Equation, Polynomial, Equation, Roots, Solution*

I. INTRODUCTION

"The universe is governed by science. But science tells us that we can't solve the equations, directly in the abstract." S. Hawking

Dating back to as early as 2000 BC humans have always been interested in solving ideas relating to the sides of rectangles. Notions of solutions to primitive equations which were quadratic in nature (as termed much later) have inspired mathematicians for centuries. This paper is in true sense a revisit to the idea of solving a quadratic equation using something as basic as high school calculus. This idea has then been extended further to find a suitable approximation for equations with even powers on the variable (x) which has pretty much been a rather puzzling area of mathematics in general for centuries.

Objectives

1. To find the roots of a quadratic equation of the form $ax^2+bx+c=0$ using the minimum point (x_0, y_0) on the functional parabola which it has been taken from.
2. To extend this idea to ensure a suitable approximation for higher order equations in x with even powers of the form $a_{0+}a_1x^2 + a_2x^4 + \cdots + a_nx^{2n} = 0$ where a_i such that i = 0,1,2,...,n is an arbitrary real coefficient of an even power of x

Pre-requisites

The reader would have to have an understanding of the following terms and results in order to understand the true essence of the paper:-

1. Quadratic Equation: An equation of the form $ax^2+bx+c=0$ where a, b and c represent arbitrary constants and $a\neq0$.
2. Functional Form of a Quadratic Equation: the second degree polynomial $f(x) = ax^2+bx+c$ that represents a parabolic curve.
3. Roots / Solutions of a Quadratic Equation: The values of x for which $f(x) = ax^2+bx+c$ takes the value 0 is called a root of the Quadratic Equation i.e.

If $f(\alpha) = 0$ and $f(\beta) = 0$ then, α and β are called the roots / solutions to the Quadratic Equation.

4. A higher order equations in x with even powers of the form $a_{0+}a_1x^2 + a_2x^4 + \cdots + a_nx^{2n} = 0$ where a_i such that i = 0,1,2,...,n is an arbitrary real coefficient of an even power of x
5. Fundamental Theorem of Algebra: Every polynomial equation of degree n with constant complex number coefficients has n roots.

Corollary: A quadratic equation is of degree 2 and hence it has two roots.

6. A quadratic equation of the form $ax^2+bx+c=0$ can be reduced to $x^2+\frac{b}{a}x + \frac{c}{a} = 0$ by dividing through by the arbitrary constant a (as $a\neq0$). Further, taking the parabolic curve of the form $f(x) = x^2+\frac{b}{a}x + \frac{c}{a}$ we deduce that its only extremal point happens to be a minimum point.

a) $f(x) = x^2+\frac{b}{a}x + \frac{c}{a}$

$\frac{d(f(x))}{dx} = 2x+ \frac{b}{a} = 0$ which implies $x_0 = \frac{-b}{2a}$ where x_0 represents the abscissa of the extremum

Also,

$\frac{d^2(f(x))}{dx^2} = 2$ which is greater than 0 and is suggestive that the extremul point is a minimal point.

b) Notationally, let us define y_0 (the ordinate of the minimum point) $= f(x_0) = x_0^2+\frac{b}{a}x_0 + \frac{c}{a}$

Analysis

Consider, a Quadratic Equation of the form:-

$ax^2+bx+c=0$, where a, b and c are arbitrary constants where $a\neq0$.

Dividing through by a (since $a\neq0$), we arrive at $x^2+\frac{b}{a}x + \frac{c}{a} = 0$.

The claimed solution to such an equation is given by $x = x_0 \pm \sqrt{-y_0}$ i.e. $x = x_0 \pm \sqrt{-f(x_0)}$

Proof

x_0 as demonstrated earlier was given by $x_0 = \frac{-b}{2a}$ and $y_0 = f(x_0) = x_0^2+\frac{b}{a}x_0 + \frac{c}{a}$

i.e. $y_0=\frac{-b^2+4ac}{4a^2}$. On substitution and further simplification, $x= x_0 \pm \sqrt{-y_0} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ which corresponds to the Quadratic Formula. *(Hence the proof)*

Corollary: *This idea can be extended to A higher order equations in x with even powers of the form* $a_{0+}a_1x^2 + a_2x^4 + \cdots + a_nx^{2n} = 0$ *where* a_i *such that i = 0,1,2,...,n is an arbitrary real coefficient of an even power of x with an approximation of the form*
$x=((x_0)^n\pm \sqrt{-y_0})^{(1/n)}$

1. When n = 2, it forms a fourth order equation in even powers, it has been observed that the solution obtained for x is an accurate enumeration of the accurate value of x
2. When n = 3 , it forms a sixth order equation with even powers, assuming that the obtained equation in fifth order is solvable, it results in a solution which is within 5% of the true solution to the problem. However a slight numerical analysis shows that $x=((x_0)^{3.1}\pm \sqrt{-y_0})^{(1/n)}$ can reduce the error to within 2% of the true solution.
3. Any higher order equation of the form $a_{0+}a_1x^2 + a_2x^4 + \cdots + a_nx^{2n} = 0$ where a_i such that i = 0,1,2,...,n is an arbitrary real coefficient of an even power of x can be approximated using $x=((x_0)^n\pm \sqrt{-y_0})^{(1/n)}$ *where* x_0 represents the abscissa of the minimum point and y_0 represents the ordinate of the minimum point.

II. CONCLUSION

A quadratic equation of the form $ax^2+bx+c=0$ can be reduced to $x^2+\frac{b}{a}x +\frac{c}{a} = 0$ by dividing through by the arbitrary constant a (as $a\neq0$). The solution to such an equation is given by $x = x_0 \pm \sqrt{-y_0}$ i.e. $x = x_0 \pm \sqrt{-f(x_0)}$ where (x_0 , y_0) are the co-ordinates of the minimum point of the parabolic curve $f(x) = x^2+\frac{b}{a}x +\frac{c}{a}$.

On further extension, an equation of the form $a_{0+}a_1x^2 + a_2x^4 + \cdots + a_nx^{2n} = 0$ where a_i such that i = 0,1,2,...,n is an arbitrary real coefficient of an even power of x can be approximated using $x=((x_0)^n\pm \sqrt{-y_0})^{(1/n)}$ *where* x_0 represents the abscissa of the minimum point and y_0 represents the ordinate of the minimum point.

III. REFERENCE

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