Counting Methods

LESSON 10.1
PAGES 292–295

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Count possible outcomes
- Understand and use tree diagrams

✓ *Determine Student Readiness*

In this lesson, students apply counting methods to determine possible outcomes. To determine student readiness, ask students to solve simple multiplication problems that contain three or more factors, such as $2 \times 3 \times 4 \times 5$.

Key Concept

Counting methods can be used to determine the sample space and the number of possible outcomes in experiments.

Concept Background: In order for students to determine the probability of an event, or the results of an experiment, they must be able to find the total number of possible outcomes. There are two ways to do this. One is the use of a tree diagram. Another is the use of the Counting Principle. Tree diagrams and applying the Counting Principle summarize all of the possible events, or outcomes. Point out to students that when they go to an ice cream, yogurt, or sandwich shop, there are often many possible outcomes. For example, at an ice cream shop, they may choose 1, 2, or 3 scoops of ice cream put in either a plain cone, waffle cone, or cup, and topped with nuts, sprinkles, or nothing at all. Each unique combination of ice cream, container, and topping represents a possible outcome.

Develop Core Skills

Core Skill: Utilize Counting Techniques

Write the words *statistics* and *data* on the board. Explain to students that statistics is the study of data, or specific pieces of information. Tell students that people who study data for a living are called statisticians. These professionals collect, organize, display, interpret, and share data. Statisticians use counting techniques in their work. Those techniques include organized lists, data tables, and tree diagrams. Invite students to describe prior experiences organizing and interpreting data, such as collecting student survey data and organizing the results in a data table.

Core Practice: Reason Abstractly

Explain to students that when they use numbers and symbols to represent a relationship, they are reasoning abstractly. For example, tell students that if they read a word problem, they apply reason, or logic, to understanding the relationships that exist among the quantities described in the problem. Then they use mathematical symbols and numbers to represent the quantities. The numbers and symbols are abstractions, or non-concrete forms, of real quantities. Write a simple word problem on the board. Discuss the problem and its quantitative elements as a class. Then ask students to translate the relationships into numbers and symbols. Invite volunteers to share their abstractions. Engage students in a discussion of how they can look at both the problem and the abstraction and understand the connections between them.

Pre-Teach Vocabulary

Make Connections

Write the vocabulary words on the board. Ask students to identify familiar words. Have them call on prior knowledge to describe the contexts in which they encountered the words they recognize. Model the process by identifying a word or term you have used outside of class and explain to students the circumstance in which you used it. For example, you might say: *I was surprised by the outcome of the baseball game last night. I really hadn't expected the team to win.*

Tier 2 Words:	Tier 3 Words:
event (p. 292)	compound event (p. 292)
outcome (p. 292)	Counting Principle (p. 294)
	sample space (p. 292)
	tree diagram (p. 292)

DURING THE LESSON

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Count Possible Outcomes

Gather a coin and two number cubes or dice before the lesson. Read the first paragraph as a class. Give the coin to a volunteer and ask the student to toss it. Ask students to use the words in the paragraph to explain what they observed. Record the result on the board. Next, give a volunteer a number cube. Repeat the process. Next, give a volunteer two number cubes and repeat the process. Ask a volunteer to use his or her observations to describe the difference between simple and compound events.

Guide students through Example 1, Use a Table. You may want to give each of two volunteers a coin and ask them to confirm the possible outcomes identified in the table.

Have students locate the boldfaced term *tree diagram* in their books. Read the introductory sentence and help students complete Example 2.

MATH LINK

Have students read the text. Discuss the differences between a table and a tree diagram. Explain to students that they will learn about the Counting Principle later in the lesson.

Understand a Diagram

PAGE 293

Examine the tree diagram as a class. Ask students to describe its features, such as labels and lines. Discuss the diagram's structure, that is, how it is constructed, and whether students find the structure an effective means of identifying all of the possible outcomes at a glance. Ask students to count the total number of possible outcomes, or events.

Core Skill: Utilize Counting Techniques

Give students time to read the text and interpret the activity instructions. Ask students to explain how having possible outcomes organized in a tree diagram helps them see and interpret all of the possible outcomes at a glance. Have students re-create and complete the tree diagram. Ask: *How many possible outcomes, or events, exist with each spin? If a player spins twice, how many possible outcomes are there in all? You can use the tree diagram to count all of the possible outcomes of two spins, but is there something else you could do that would give you the same result?* Students may observe that if they multiplied the number of possible outcomes for each spin, or 3×3, the total number of possible outcomes would be the same.

The Counting Principle

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Read the opening sentence for students and then guide them through the completion of Example 3. Ask: *What do you observe about spinners A, B, and C? If the sections in each spinner weren't of equal size, how would that affect possible outcomes?* Organize students into small groups. Ask each group to design a spinner for a game, making sure that each spinner is divided into equal groups. Afterward, ask students to compare spinners. Select different numbers of spinners and ask students to use the Counting Principle to determine all possible outcomes.

Engage and Extend

ELL Instruction: Build a Menu Ask students to imagine opening a sandwich shop. Have them work in small groups to build a menu that includes several choices of breads, meats, cheeses, vegetables, and condiments. Have groups draw their menus on large sheets of paper or on the board. Then have students use tree diagrams or the Counting Principle to calculate all of the possible outcomes for each menu. Encourage students to explain their menus and their solutions for determining possible outcomes.

THINK ABOUT MATH ANSWER KEY

1. 36
2. 125

Core Practice: Reason Abstractly

Write the word *probability* on the board. Explain that probability is a quantitative, or numerical, value that describes the likely occurrence of a specific event, such as the likelihood that someone would spin the three spinners on the page and get *4, red, D*. Help students understand that when they make observations, such as where a spinner's hand lands, they observe something concrete, or real. When they use numbers and symbols to describe their observations, they are using something abstract. It's also possible to apply those abstract representations to other situations, such as with playing cards. Have students read and discuss the text in pairs. Ask volunteers to describe situations in which they applied abstract reasoning.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 428.

Extension Activity: Draw Conclusions from Diagrams

Have students work in small groups. Tell them to imagine that they work in the online catalog department for an international company that sells camping and sporting equipment. Assign each group a different camping destination, such as Maui, the Amazonian rain forest, Antarctica, and the Mojave Desert. Have students research the physical environment in their assigned location to determine what kinds of camping gear they should recommend. Have students construct tree diagrams to show all possible outcomes. For example, students might build tree diagrams that show possible combinations of tents, sleeping bags, cooking supplies, and safety gear.

Introduction to Probability

LESSON 10.2
PAGES 296–301

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Find theoretical probability
- Find experimental probability
- Make predictions

✓ Determine Student Readiness

In this lesson, students distinguish between theoretical and experimental probability. To determine student readiness, hold a number cube in your hands and ask students to identify the total number of possible outcomes. Continue adding more number cubes and asking students to find the total number of possible outcomes.

Key Concept

Understand and use concepts of probability to find probabilities and make predictions.

Concept Background: Probability is the likelihood or chance that an event will occur. A probability ratio may be expressed as a fraction or decimal between 0 and 1, or as a percent. Students can use math to determine theoretical probability, while they must conduct trials, or experiments, to determine experimental probability. Ask students to define the word *chance*, as it's used to describe the likelihood that something will happen. Explain that the chance of an outcome is the same as the probability of an outcome.

Develop Core Skills

Core Skill: Evaluate Reasoning

Write the word *reasoning* on the board. Explain that it has two meanings. One is used in mathematics to describe trying a variety of possible solutions, or processes, to find an answer. The second meaning applies to mathematics and all other topics. Explain that the word *reasoning* is a verb, or action word, that means to think, understand, and make judgments about a topic or answer. However, that analysis or those judgments are based on logic, meaning facts or proof. Write the term *Information Age* on the board. Explain to students that many people use this term to refer to modern times, because people today have access to so many forms of information, some accurate and some not. Ask students to explain why the term *Information Age* is appropriate and why a person's ability to evaluate a writer's or speaker's reasoning is so important.

Core Skill: Determine Probabilities

Write the word *probability* on the board. Next to the word, write the definition: *a quantitative description of the likelihood of a particular event.* Examine each word in the definition as a class.

Tell students that there are two kinds of probability. One is theoretical, meaning they can use math to predict the chance of an event occurring, and the other is experimental, in which they carry out trials to determine probability. Ask: *If I have a number cube with six sides, numbered 2, 4, 6, 8, 10, and 12, what is the probability that I will roll an even number?* Help students see that the probability 100 percent. Ask: *What is the probability that I will roll an odd number?* Help students see that the probability is 0 percent.

Pre-Teach Vocabulary

Word Bench

Write the vocabulary words on the board. Ask students to identify familiar words. Ask students to describe the contexts in which they have seen or heard these words before. Students may say, for example, that their favorite pizza is the combination pizza that combines different meats and vegetables. Or they may have watched trials in track and field events to determine which athletes will go on to compete in national or international games.

Tier 2 Words:	Tier 3 Words:	Test Word:
combination (p. 298)	certain event (p. 296)	support (p. 297)
trials (p. 299)	experimental probability (p. 299)	
	impossible event (p. 296)	
	permutation (p. 298)	
	probability (p. 296)	
	theoretical probability (p. 296)	

DURING THE LESSON

PAGE 296

Theoretical Probability

Write the word *ratio* on the board, and ask a volunteer to recall its meaning. To prompt discussion, you may need to explain that a ratio describes a quantitative relationship between two values. In other words, a ratio compares one value to another. Show students 2 pencils and 3 pens. Say: *The ratio of pencils to pens is two to three.* Write the different forms of the ratio on the board: 2 : 3, or $\frac{2}{3}$, or 2 to 3.

Read the introductory text with students, pausing often to check student understanding. After reading, say: *If I put the two pencils and three pens in a box, what is the theoretical probability that I will pull a pencil from the box?* Help students see that the probability of the favorable outcome is $\frac{2}{5}$, 0.4; or 40 percent.

Draw or project the probability line on the board (see the example at the bottom of page 296). Then guide students through Example 1. Help students identify the probability of rolling an even number on the probability line. Then ask them to identify the probability of rolling a number between 1 and 6. (1)

MATH LINK

Have students read the text. Ask them to relate what they read to the probability line you drew on the board. Ask: *What label could we write above $\frac{1}{2}$ on the probability line?* (equally likely)

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21st Century Skill: Ethics and Probability

Write the word *ethics* on the board. Ask students to skim the text to find the word's meaning. Write the definition on the board. Before students read the text, ask them to take a moment to think about a time they made a decision based on what they understood to be the right thing to do. Invite volunteers to share their decisions. Help students make the connection between their decisions and an ethical decision. Next, have students read the text. Then, as a class, discuss the questions and how probability and ethics sometimes overlap.

PAGE 298

Permutations and Computations

Write the word *permutation* on the board. Show students three colored markers. Put them in a row. Point to the linear arrangement, and say: *This is a permutation.* Now, reorder the pens. Say: *This is another permutation.* Reorder the pens, and repeat the statement again. Explain that a permutation is one of several ways objects can be arranged. In a permutation, order matters. But in a combination, it doesn't.

Guide students through Examples 2 and 3. Ask students to compare the steps in the examples. For example, point out that the first step is the same, whether working with permutations or combinations. Ask: *What is the first thing you must determine before you can calculate the probability of a particular permutation or a combination? How is Step 2 similar in both examples? How are they different?* Clarify the difference between favorable outcomes and total possible outcomes for students before asking them to calculate the probability in each example.

Core Skill: Evaluate Reasoning

Ask: *What do you do when you evaluate someone's work, ideas, conclusions, or solution to a problem?* Then, say: *Imagine I ask you to read and solve a word problem. Why do I ask you to show every step in your solution?* Explain that as you examine a solution, you're evaluating the logic in a student's thinking, the steps he or she followed, and the conclusions he or she reached.

Engage and Extend

ELL Instruction: Toss Coins Organize students into small groups. Give each group two coins. Ask students to explain how they can find both the theoretical and experimental probability of getting two heads in a coin toss. Have students walk you through the calculation for finding theoretical probability. Then have them run 10 trials to determine experimental probability. Again, have them explain the process to you.

Read the text in the sidebar as a class, and give students time to complete the task. Ask volunteers to share their explanations.

MATH LINK

Have students read the text. Invite students to draw a diagram to interpret the text. Students may, for example, draw three rows, with 5 squares in the first row, 4 squares in the second row, and 3 squares in the third row.

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THINK ABOUT MATH ANSWER KEY

1. 0
2. $\frac{1}{3}$, about 0.33. or about 33%
3. $\frac{1}{28}$, about 0.036, or about 3.6%

Experimental Probability

Ask students to describe experiments they've done, such as a building a model of a working volcano in science class or mixing paints to get a certain color. Help students recognize that an experiment often requires doing it many times, and that each time it is called a trial. Read the introductory text with students. Then guide them through Examples 4 and 5. Afterward, say: *Let's compare theoretical and experimental probability. How are they the same? How are they different?*

Core Skill: Determine Probabilities

Read the text as a class. Then organize students into small groups. Give each group a set of 25 note cards or pieces of paper. Have students label the cards or papers A through E, creating the same number of cards for each letter. Have students apply theoretical probability to predict the number of times they will draw a vowel in 100 draws, assuming that the each draw is returned to the larger set. Then have students use experimental probability and record the results in a table to predict the number of times a vowel will be drawn from the set.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 428 and 429.

Extension Activity: Explain Phenomena in Terms of Concepts Have students work in small groups to design spinners made of equal parts and attach a question to the spinner, such as *What is the experimental probability of spinning the spinner 20 times and landing on a red space?* Have groups exchange spinners and questions, and work as teams to find the answers. Afterward, have students explain their procedures and results. Have other students use each group's data to confirm the results.

Compound Events

LESSON 10.3
PAGES 302–306

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Find the probability of mutually exclusive events
- Find the probability of overlapping events
- Find the probability of independent and dependent events

✓ *Determine Student Readiness*

In this lesson, students distinguish between mutually exclusive and overlapping (or inclusive) events, as well as independent and dependent events. To determine student readiness, have students work in pairs. Give each pair a number cube or spinner. Assign probability problems related to the objects students have in hand. For example, if students have number cubes, ask them to find the probability of rolling an even number ($\frac{3}{6}$, or $\frac{1}{2}$).

Key Concept

Extend your understanding of probability to finding the probability of compound events.

Concept Background: Independent events occur independently of each other. For example, tossing a coin and a number cube simultaneously doesn't affect the outcome of either toss, so the events are independent. Dependent events *do* affect outcomes. For example, pulling a blue marble out of a bag of blue, red, green, and yellow marbles and not returning the marble to the bag afterward affects the likelihood of getting a blue marble on the next draw. The two events are dependent. At the conclusion of this lesson, have students use real-life examples to explain the differences between independent and dependent events.

Develop Core Skills

Core Skill: Represent Real-World Problems

Explain to students that they don't have to toss a number cube to find examples of probability. They can play board games, for example, that include dice. They may have observed a referee toss a coin to decide which basketball team controls the ball at the start of play. They may have listened to weather reports to determine the chance of rain before planning a picnic. Invite students to think of other real-world examples in which chance, or probability, plays a role.

Core Practice: Reason Abstractly

In the first lesson in this chapter, students learned that when they use numbers and symbols to represent mathematical relationships, they are reasoning abstractly. Ask students to model abstract reasoning using office supplies, such as finding the sum of pens and pencils in a pencil jar. Emphasize that when they assign numbers to the pens and pencils, they are using abstractions of real quantities. Write some other examples of abstractions on the board. Select formulas students have encountered in previous chapters, such as the formulas for finding: the perimeter of a square, theoretical probability, square roots, and the slope of a line. Invite students to give other examples of mathematical formulas and explain the relationship between real quantities and an abstraction.

Pre-Teach Vocabulary

Word Maps

Draw a model of a word map on the board. Begin by drawing a large square. Divide the square into four equal parts. Draw a smaller square over the space where the perpendicular lines intersect. Erase the lines within the smaller square, and write: *A vocabulary word goes here.* Then label the areas surrounding the smaller box: *Definition; Picture; Example; Sentence.* Have students copy the model, replacing the directions in the center square with any one of the words from the vocabulary list. Give students time to add information to their maps as you go through the lesson.

Tier 2 Words:	Tier 3 Words:
replacement (p. 304)	dependent events (p. 305)
	independent events (p. 304)
	mutually exclusive events (p. 302)
	overlapping events (p. 303)

DURING THE LESSON

PAGE 302

Mutually Exclusive and Overlapping Events

Write the terms *mutually exclusive* and *overlapping* on the board. Explain the following information as you write it on the board. Beneath the word *mutual*, write: from the Latin word *mutuus*, meaning "something done in exchange, or reciprocal." Beneath the word *exclusive*, write: from the Latin word *excludere*, meaning "to keep or shut out." Beneath the word *overlapping*, write: *overlap* used in the 1700s to mean "to partially extend or lay over." Invite students to predict the meaning of each term before you read the first two paragraphs on page 302. Afterward, guide students through Example 1. You may want to organize students into groups and give each group 6 notecards. Have students label the cards to match the letters in the example, and use them to model the steps in the example.

Draw two overlapping circles on the board. Explain that the overlapping section indicates that two events are not mutually exclusive. In other words, they can both occur simultaneously.

Leave the diagram on the board and read the first paragraph on page 303 to students. Use the diagram to help students visualize the steps in Example 2. For example, write the numbers 1 and 2 in the left circle because they are less than or equal to 2. Write the numbers 3 and 5 in the right circle because they are greater than 2 but odd. Write the numbers 4 and 6 in the overlapping section of the circles because they meet both requirements.

MATH LINK

Have students read the text. Write the formulas on the board, and ask students to explain what makes these formulas examples of abstractions, as discussed in the Before the Lesson Core Practice activity.

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Core Skill: Represent Real-World Problems

Read the text with students. Pause as you read to refer to the diagram you used to explain Example 2. Afterward, give students time to complete the activity. Suggest that they draw a diagram like the one on the board to visualize the problem before they write and solve an equation.

PAGE 304

Independent and Dependent Events

Gather a coin, a number cube, a spinner, and a container of marbles or paper clips before the lesson. If possible, also gather notecards and enough sets of markers to distribute among groups of students. For Example 3, each group needs 18 notecards and one red, one blue, one green, and one yellow marker. For Example 4, each group needs 5 blue markers, 4 green markers, and 3 red markers. If this is impossible, replace the notecards with colored pieces of paper, and gather one set of markers for demonstration purposes.

Organize students into small groups. Give each group 18 notecards or pieces of paper and red, blue, green, and yellow markers. Have students read the first sentence in Example 3 and then color some part of each card to match what they read.

Read the introduction to Example 3 on page 304, and guide students through the example. Ask students to use the cards to model each step. Ask a volunteer to explain the process of determining the probability of drawing a red tile first, followed by a yellow tile.

Have students remain in their small groups as you read the introduction to Example 4 on page 305, and work through the example. Invite volunteers to summarize the steps they followed to solve the problem.

THINK ABOUT MATH **ANSWER KEY**

1. overlapping; $\frac{7}{12} + \frac{6}{12} - \frac{3}{12} = \frac{10}{12} = \frac{5}{6}$
2. mutually exclusive; $\frac{1}{12} + \frac{6}{12} = \frac{7}{12}$
3. mutually exclusive; $\frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$
4. overlapping; $\frac{6}{12} + \frac{5}{12} - \frac{2}{12} = \frac{9}{12} = \frac{3}{4}$

Core Practice: Reason Abstractly

Remind students that a formula is an abstraction for reasoning, or working logically through the steps of a problem. Read the first paragraph together. Then ask students to read the second paragraph and think about the usefulness of using a tree diagram to solve the problem in Example 4. Ask: *What are some advantages of using a formula instead of building a tree diagram?* Discuss students' responses as a class.

PAGE 305

MATH LINK

Have students read the text. Ask students to explain the symbols and steps in each formula and then compare the formulas to determine and explain the differences.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 429 and 430.

Engage and Extend

ELL Instruction: Talk to a Partner Organize students into pair. Tell one student in each pair to name two events. Have the partner use objects or formulas to tell whether the events are overlapping, mutually exclusive, independent, or dependent. Walk among students as they challenge each other. Intervene when necessary, to clarify understandings.

Extension Activity: Formulate a Demonstration of Probability Explain to students that many games are based on dependent events, such as a memory games like *Concentration*, in which matched pairs are removed. Have students work as partners or in small groups to design a card game that involves dependent events, such as removing matching pairs from play. Then have them write an explanation for determining the probability of dependent events in their game. Have students play their games to test their explanations.

Customary Units

LESSON 11.1
PAGES 312–315

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Change from one customary unit to another
- Change from mixed units
- Change to mixed units

✓ *Determine Student Readiness*

In this lesson, students convert units of customary measures, a task that requires the use of two mathematical operations—multiplication and division. To determine student readiness for the lesson, ask students to solve simple multiplication and division problems that you write on the board. Observe students as they work, intervening whenever necessary to correct students' solutions.

Key Concept

Understand how to use division and multiplication to change from one customary unit of length, capacity, weight, or time to another, and how to change to and from mixed units.

Concept Background: In the United States, people generally use customary units to measure length, weight, and capacity. Inches, feet, yards, and miles are examples of customary units of length. Ounces, pounds, and tons are examples of customary units of weight. Cups, pints, quarts, and gallons are customary units of capacity. Have students tell which units they would use to measure the width of the room, the weight of a piece of furniture, and the amount of lemonade in a glass.

Develop Core Skills

Core Skill: Evaluate Expressions

Invite students to share some of their favorite expressions. To prompt student thought, suggest some of your own favorite expressions, such as *a dime a dozen, a drop in the bucket,* and *saved by the bell.* Explain to students that like language, mathematics has expressions, too. In mathematics, an expression is any combination of numbers and symbols. Write the expression $4x + 3$ on the board as an example. Then invite students to come to the board to write other mathematical expressions.

Core Skill: Represent Real-World Problems

Invite students to discuss a recent experience in which they measured something. It might be a measurement they made while working on a hobby, cooking, or buying groceries. Ask volunteers to explain how often they use or refer to measurements in their daily activities.

Pre-Teach Vocabulary

Make Connections

Write the vocabulary words on the board. Invite students to share what they know to help you define or give examples of each term. Supply additional information as needed. Ask students to describe their most recent experience related to measurement. Ask: *What did you measure? What kinds of tools did you use?*

Tier 2 Words:	Tier 3 Words:	Test Word:
capacity (p. 312)	unit (p. 312)	abbreviation (p. 312)
length (p. 312)		
time (p. 312)		
weight (p. 312)		

DURING THE LESSON

Change Units of Customary Measure

PAGE 312

Read the introduction to the table to the class. Examine the table as a class, and ask students to identify which units they use most and least often.

Write the word *convert* on the board. Next to the word, write: from the Latin word *convertere*, meaning "to turn around, to transform." Ask volunteers to find examples in the table of measurements that have been converted, such as feet to inches.

Guide students through Examples 1 and 2 on page 313. Afterward, ask volunteers to explain when it is necessary to divide to change units, and when it is necessary to multiply.

MATH LINK

Write the words *United States* on the board. Ask students how they would abbreviate the words. Explain that an abbreviation is a short form of a word. Have students read the Math Link and share examples of the abbreviations they recognize.

Change *To* and *From* Mixed Units

PAGE 313

Read the introduction to this section with students. Walk through the steps in Examples 3 and 4 together. Next, ask volunteers to tell you their heights in feet and inches. Record their measurements on the board. Have students work with partners to convert each measurement into a single unit. Check students' answers as a class. Then write examples of single units of measurements on the board, such as 480 minutes. Ask students to work with their partners to convert the single units into mixed units. Leave students' work on the board for use in discussion of the Core Skill activity.

Core Skill: Evaluate Expressions
Ask volunteers to point to examples of expressions on the board. For example, students might refer to the expression *480 minutes ÷ 60*. Ask students to identify the elements in the expression, which are two numbers, a division symbol, and a unit of measurement. Have students read the expressions and work with partners to simplify and evaluate them.

PAGE 314

THINK ABOUT MATH **ANSWER KEY**

1. 48
2. 10
3. 5
4. 10,000
5. 74
6. 455
7. 2, 3
8. 3, 2

MATH LINK

Have students read the Math Link and share a few more examples. Then discuss why these statements are conceptually true. It takes multiple units of a smaller unit to equal a greater unit.

Core Skill: Represent Real-World Problems
Organize students into pairs. Before they read the text, explain that they are going to be presented with a real-world problem related to measuring the length of a fence. Invite students to share similar experiences related to measuring spaces in and around their homes. Then give students time to read the text and complete the problem. Afterward, discuss students' solutions.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 431.

Engage and Extend

ELL Instruction: Fill in the Blanks Write the following Cloze passage on the board. Ask students to help you fill in the blanks.
I use inches to measure _____,
but I use pounds to measure _____.
_____ is measured in minutes and hours,
and _____ is measured in gallons and cups.

Extension Activity: Solve Measurement Problems Tell students that 1 pint of water weighs about 1 pound. Have students determine the weight of 1 quart, 1 cup, and 1 gallon of water. Then have them determine how many gallons are in 1 ton of water, and how many cups are in 5 pounds of water.

Metric Units

LESSON 11.2
PAGES 316-321

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Change from one metric unit to another
- Understand the basic metric unit for length
- Understand the basic metric unit for capacity
- Understand the basic metric unit for mass

✓ Determine Student Readiness

In this lesson, students convert units of metric measures, a task that, like converting customary units of measure, requires the use of two mathematical operations — multiplication and division. To determine student readiness for the lesson, ask students to solve simple standard unit conversion problems, such as 24 inches = _____ feet, that you write on the board. Observe students as they work, intervening whenever necessary to correct students' solutions.

Key Concept

Understand how to change from one metric unit of length, capacity, or mass to another.

Concept Background: The metric system of measurement, based on multiples of 10, is used internationally and by scientists in the United States. It is different from the customary system in that it uses a measure of mass rather than weight. Weight is a measure of mass affected by gravity, so weight and mass are related. Units of mass are based on the gram. Units of length are based on meters. Units of capacity are based on the liter. Have students identify objects that can be measured in grams, meters, and liters.

Develop Core Skills

Core Skill: Build Solution Pathways

Remind students that the word *solution* has more than one meaning. Ask students to define the term as they understand it. Help them recognize that a solution can refer to an answer to a problem. It can also refer to any process they use to find an answer. Write the word *pathways* on the board. Explain that a pathway is a course, a trail, a track, or simply, a path. Ask students to apply this understanding to explain the meaning of the term *solution pathway*. Remind students that there are often multiple paths a person can follow to find the answer to a problem. Ask students to share some of the strategies that work best for them, for example, how they converted customary measurements in the previous lesson. Explain that these strategies will likely work in this lesson, too.

Core Skill: Use Ratio Reasoning

Write the ratio "12 inches to 1 foot" on the board. Point to the example, and remind students that a ratio describes a comparison of two values. Ask: *What values are compared in this ratio?* Next, write the word *reasoning* on the board. Ask students to recall what they learned about reasoning in the previous lesson. To prompt recall, ask: *When you convert measurements from one unit to another unit or to mixed units, how do you apply reasoning?*

Pre-Teach Vocabulary

Background Knowledge

Have students recall circumstances where they have read, heard, or used the words *gram, liter,* and *meter*. Record students' responses on the board and review the list before asking students to explain how these words are related to measurement.

Tier 2 Words:	Tier 3 Words	Test Word:
gram (p. 316)	power of 10 (p. 318)	prefix (p. 316)
liter (p. 316)		
meter (p. 316)		

DURING THE LESSON

PAGE 316

The Metric System

Engage students in a discussion of international sporting events, such as Olympic events. Ask students to give examples of some of the units of distance used in sprints and marathons. Ask about the units of length used to determine the height of a pole bar in a pole-vaulting event. Ask about the units of time used to measure how quickly swimmers cover a given distance, and the units of mass that athletes lift in weightlifting events. Explain that all of these units of measure belong to the metric system of measurement.

Read the text with students, helping them understand the meaning and value of the prefixes *milli–*, *centi–*, and *kilo–*. Review the table of metric unit measures as a class. Call out a unit of measure, and ask students to use the table to find an equivalent value.

Have students turn to page 318. Examine the diagram as a class, and invite volunteers to explain it. Ask them to explain how they know when they should multiply and when they should divide during the conversion process.

Complete Examples 1 through 4 on pages 318 and 319 as a class, or organize students into groups, and have groups work together to read and analyze the solution to each problem. In either case, ask volunteers to summarize the task in each example. Provide additional problems, if necessary, to help students apply their learning before they complete the Think about Math activities.

MATH LINK

Write the expression *powers of 10* on the board. Explain to students that a power of ten is any integer power of the number ten. As an example, write the expression 10×10 on the board. Ask a volunteer to use the list in the Math Link to simplify the expression. Write more expressions on the board, and ask students to use powers of ten to simplify them.

Use Prefixes

PAGE 317

Write the word *prefix* on the board. Explain that the word comes from the combination of the Latin word *prae*, meaning "before," and *figere*, meaning "to fix or fasten." Ask students to use the roots to define the term *prefix*.

Read the introduction to this section with students and review the table. Invite volunteers to give examples of words they often see, hear, or speak that include these prefixes. Next, have students use the table to complete the exercise. Discuss the answers as a class.

Core Skill: Build Solution Pathways

Ask students to read the text and complete the task. Afterward, ask volunteers to summarize the solution pathway they can use to convert metric units of measure from one unit to another. Challenge students to test their solution pathways by asking them to convert simple units of measurement, such as 100 meters into centimeters.

PAGE 318

THINK ABOUT MATH ANSWER KEY

1. 0.001
2. 100
3. 1,000
4. 1,000
5. 100,000
6. 1,000,000
7. 0.00001
8. 0.01
9. 10
10. 0.000001
11. 0.001
12. 0.1

Core Skill: Use Ratio Reasoning

Read the first paragraph as a class. Write the directions "Convert 1 mile to inches" on the board. Challenge students to apply reasoning to suggest a strategy for solving the problem. Say: *As we think about how we might solve this problem, let's consider how we can use ratios in the solution. For example, let's compare miles to feet. Write the ratio* $\frac{1 \text{ mile}}{5{,}280 \text{ feet}}$ *on the board. We want to know how many inches are in one mile, but our ratio compares miles to feet. So, we need to consider another ratio.* Write the ratio $\frac{1 \text{ foot}}{12 \text{ inches}}$ on the board. Point out the same ratios in the Core Skill activity, and ask students why the arrangement of each ratio is important, emphasizing the need to cancel all units but those that students are being asked to find. Discuss the answer as a class.

PAGE 319

THINK ABOUT MATH ANSWER KEY

1. 3.2
2. 400
3. 5.5
4. 8.9
5. 1,400
6. 2
7. 0.24
8. 342
9. 6,500

MATH LINK

Read the Math Link to the class. Ask volunteers to write some of the problems they solved in Think about Math on the board and demonstrate how they moved the decimal point to find the answer.

AFTER THE LESSON

Read through with students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 431.

Engage and Extend

ELL Instruction: Pronunce Metric Words Have students practice reading units of measurement and their prefixes. Pay special attention to the word *kilometer*, because the pronunciation varies from the pronunciation of other metric-measurement words. Ask students to give examples of when each kind of unit is used. Write abbreviations for units of measure, and have students read the words they represent.

Extension Activity: Relate the Conversion Process Have students work individually or with partners to explain the process of metric conversions through the use of a chart. The chart should indicate when to multiply or divide, what power of 10 to use, and the decimal point's direction of movement. Have students compare their charts and combine the best elements across the charts to create a single chart that someone unfamiliar with metric conversions could use successfully.

Geometric Figures

LESSON 12.1
PAGES 326–331

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Identify geometric figures
- Identify and classify angles
- Identify and classify triangles and quadrilaterals
- Identify the parts of circles

✓ *Determine Student Readiness*

Draw triangles, circles, squares, and rectangles on the board. Have students identify real-world examples of items that are in these shapes. Have volunteers identify each shape by name.

Key Concept

Learn how to identify and classify two-dimensional shapes by their properties.

Concept Background: By studying geometry, students develop spatial thinking and awareness of the similarities and differences between real-world shapes and solids. Have students identify several objects in the room as parallel, perpendicular, square, or any other geometric terms they already know.

Develop Core Skills

Core Skill: Analyze Events and Ideas

Explain to students that modern mathematics was influenced by many mathematicians in the past. Ask students if they know the story of a mathematician or to tell about someone who has influenced their own math knowledge.

Core Skill: Evaluate Reasoning

Remind students of the importance of checking answers for reasonableness. Explain to them that they will also check their answers to geometry problems for reasonableness. Ask students if they can think of ways this might be different or the same as how they have checked for reasonableness in the past, considering that most geometry problems can be drawn.

Pre-Teach Vocabulary

Study Cards

Have students make study cards for the vocabulary in this lesson. Provide definitions of the words. Have students add one term, its definition, pronunciation, and a sentence using the word to each card. Encourage students to use drawings to illustrate their words. Have students make cards as complete as possible and then add to or revise them during the lesson.

Tier 2 Words:	Tier 3 Words:	Test Words:
circle (p. 330)	angle (p. 326)	classify (p. 328)
rectangle (p. 329)	parallel lines (p. 326)	
segment (p. 326)	perpendicular lines (p. 326)	
square (p. 329)	quadrilateral (p. 329)	

DURING THE LESSON

PAGE 326

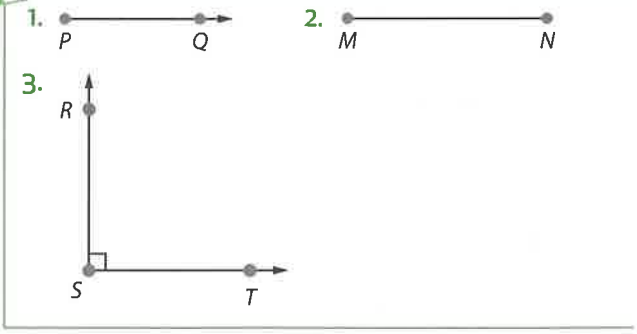
Basic Geometric Terms

Allow students time to read the section. Then have students close the book and draw each figure: line, segment, ray, angle, right angle, parallel lines, and perpendicular lines as you say it. Then have students compare their drawings to the figures in the book. Discuss with students how each type of figure is named using letters, making sure they can identify the vertex of an angle.

You may wish to have students start a geometry notebook or designate part of their notebook just for geometry. As they complete this chapter, have them record geometric terms; properties of each type of shape and solid; formulas for perimeter, circumference, area, and volume; and the Pythagorean theorem.

PAGE 327

THINK ABOUT MATH **ANSWER KEY**



Contributions of Ancient Civilizations

Have students read the text. If time allows, you may want to encourage them to research the Babylonia clay tablets or Moscow Papyrus online.

Core Skill: Analyze Events and Ideas

Discuss with students what pi represents. Some students may know that it is used in formulas for calculations with circles. Pi is equal to the ratio of a circle's circumference to its diameter. It is not a whole number or even a rational number, which is why only approximations can be found. Have students search for an interactive representation of Archimedes' calculations to help them visualize it.

MATH LINK

Draw a few polygons and have students practice naming them, making sure they list the vertices in order.

PAGE 328

Angles

Have students read the text and then draw each type of angle. Have students compare their angles. Lead students to discover that all right angles and straight angles have the same degree measure, but they can be drawn in different orientations. Acute angles and obtuse angles have a range of possible degree measures and can also be drawn in different orientations.

Triangles and Quadrilaterals

After students read the section, have them draw each type of triangle. Have them use tick marks to indicate equal sides and miniature squares to indicate right angles. Then have them exchange papers with a partner and classify each triangle by angle and side. Write the types of angles on the board so students can practice using the correct spelling.

Guide students through the steps of Example 1, making sure students are comfortable classifying each triangle into two categories: angles and sides.

Then have students read the section on quadrilaterals. Have students show why all three figures on the page are parallelograms (opposite sides are equal in length and parallel). Explain that rectangles are a special type of parallelogram, where all of the angles are right angles. A square is a special type of rectangle, where all the sides are the same length.

Guide students through the steps in Example 2, making sure they understand the informal proof.

THINK ABOUT MATH ANSWER KEY

1. acute equilateral triangle
2. obtuse scalene triangle

Core Skill: Evaluate Reasoning

Have students read the sidebar. Point out that sometimes in the lesson, details are given in the problem, and sometimes they are included on the diagram. Then have them flip through the lesson, identifying the given information in diagrams.

Engage and Extend

ELL Instruction: Ask Questions Have students ask each other questions about angles, triangles, and quadrilaterals: *How many triangle types are there? How can you tell if an angle is a right angle? What is the difference between a square and a rectangle?*

For example, tick marks show equal lengths, and little squares show right angles. To answer the questions at the end of the sidebar, have students draw the triangle as described. They will need to recall that a right angle is 90° and the sum of the angles of a triangle is 180°. Have them label the angles "90°?" Then have them consider: *If the sum of all three angles of a triangle is 180°, can two of the angles be 90°?*

MATH LINK

Tell students that congruent means "equal." If two figures are congruent, their corresponding angles and sides have the same measure. Have students draw two congruent triangles, labeling the vertices and indicating the congruent angles and sides. Have them write statements of congruence by using the congruence symbol, $\cong$: for example, $\angle A \cong \angle J$.

PAGE 329

MATH LINK

Make sure students are able to make these marks on their own diagrams.

PAGE 330

Circles

Draw a circle on the board. Label its center. Explain to students that the point is the center because it is the same distance from any point on the circle. Draw a few lines from the center to the outside of the circle, explaining that they are called radii, and are all the same length. Then draw a line through the center from one side of the circle to the other. Tell students that this line is a diameter, and that any line through the center connecting both sides of the circle is a diameter. Point out that a diameter is made up of two radii. Guide students to understand that any diameter is twice as long as any radius. Then have them read the section and solve Example 3 with a partner.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 433.

Extension Activity: Classify Lines Place and label four points in a plane. No three points can lie in a single line. Ask students: *How many lines can be formed?* Have students sketch intersecting, perpendicular, and parallel lines to the line segments. They should label the points and use the correct symbols and vocabulary to name each figure. Then have them classify each line by type in a list.

Perimeter and Circumference

LESSON 12.2
PAGES 332-337

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Find the perimeter of polygons
- Find the circumference of circles

✓ *Determine Student Readiness*

Draw a square and a rectangle on the board. Label the square "square" and the rectangle "rectangle." Then have volunteers label the sides of the figures with tick marks to show congruency. Note that students will need to use a single tick mark on one pair of congruent sides of the rectangle, and double tick marks on the other pair of congruent sides. Single tick marks can be used on the square because all the sides are the same length. Explain to students that knowing these properties of a square and rectangle will help them find missing lengths.

Key Concept

Understand and apply concepts of perimeter and circumference.

Concept Background: Explain to students that the perimeter is the distance around a polygon. The circumference is the distance around a circle. The perimeter can be found simply by adding the lengths of the sides or by using a formula. The circumference of a circle is found by using a formula.

Develop Core Skills

Core Practice: Model with Mathematics

Explain to students that math can be used to represent real-life situations and answer questions about those situations. Ask students to think of a time when they have used math or seen math used to model a situation. One example is a model town; city planners use math to scale down roads and buildings and use the model to help plan.

Core Skill: Build Lines of Reasoning

Point out to students that throughout the lessons, they have solved problems step-by-step. Ask students to recall an example of solving a multi-step problem. Explain that they can break down geometry problems into steps to make them easier to solve, too. Ask them to pay attention to how this is done when they read the lesson.

Pre-Teach Vocabulary

Study Cards

Have students write *circumference* and *perimeter* on separate index cards. Have them write down anything they know about these measurements, including any formulas they already know. Have them update the cards as they complete the lesson.

Tier 2 Words:
perimeter (p. 332)

Tier 3 Words:
circumference (p. 335)
diameter (p. 335)
pi (p. 335)
radius (p. 335)

DURING THE LESSON

PAGE 332

Perimeter

Give students rulers or tape measures and have them measure the distance around an object, such as a book or desk. Explain to students that this distance is called perimeter. If they know the lengths of the sides of the object, they can add them up to find the perimeter.

Read the first paragraph as a class, relating it to the activity they just completed. Guide students through the steps of Examples 1 and 2.

For Example 2, ask students which property of squares can be used to find the lengths of the missing sides (all sides are the same length). Have students try solving the problem by adding all the sides and compare the answer to the answer found using the formula.

Have students read Example 3 on page 334 independently. Tell them that the property of rectangles can be used to find the lengths of the missing sides (opposite sides are equal). As students work through the problem, make sure they understand how the formula works (two of the sides are equal the length of the rectangle and two of the sides are equal the width). A conceptual understanding of perimeter will help prevent students confusing perimeter, area, and volume in later lessons.

Take time to work through Example 4 carefully. Students may not immediately understand how x and y are calculated. Drawing dotted lines to show the two lengths that make up the missing side x and the difference between the lengths that make up y may help.

Mathematical Practices

PAGE 333

Tell students that a common mistake when finding perimeter is to just add up all the numbers shown in the diagram. Explain to student that like other word problems, it is important to think through what you are looking for and the steps to find it. Have students read the text. Then ask students to apply the formula to solve the problem a different way.

Core Practice: Model with Mathematics

Have students read the passage and work in pairs to complete the activity at the end. It may be tricky because it is abstract—the actual dimensions of the garden are not given. Students can choose an actual value for the length and width to help them identify the steps needed. A possible solution is: 1) Measure the length and width of the plot. 2) Use the formula to find the perimeter of the plot. This is the amount of fencing needed. 3) Divide the perimeter by the number of families, 25, to find the amount of fencing each family gets to paint.

PAGE 334

THINK ABOUT MATH ANSWER KEY

1. 100 yd
2. 40 cm
3. 46 m

Core Skill: Build Lines of Reasoning

Have the students read the text and then complete the activity in pairs. Make sure students identify the end goal (perimeter of the square) first, and then show logical steps toward the solution.

Circumference

PAGE 335

Explain to students that the distance around a circle has a special name—circumference. Conceptually, circumference is the same as perimeter. Have students read the text and then review converting from radius to diameter. Guide students through the steps in Example 5, pointing out the formula for circumference in Step 2. As a class, complete Examples 6 and 7.

MATH LINK

Tell students that this rule is given because it simplifies calculations. The results will be approximately the same. No answer is exact, though, because both $\frac{22}{7}$ and 3.14 are approximations for pi.

Have students examine this list of circle measures: $r = 21$ cm, $d = 10$ m, $d = 35$ in., $r = 15$ ft. Ask them to identify when it would be easier to use $\frac{22}{7}$ for pi and when it would be easier to use 3.14.

PAGE 336

THINK ABOUT MATH ANSWER KEY

1. 62.8 m
2. 43.96 m or 44 m

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 434.

Engage and Extend

ELL Instruction: Rephrase Finding Circumference Review the section on finding circumference, paying special attention to the relationship between the radius and the diameter. Have students explain the relationship to partners in their own words.

Extension Activity: Revise a Formula Tell students that there is another formula for circumference that uses radius instead of diameter. Explain that they can revise the diameter formula to find it. See if students can figure out what it is ($C = 2\pi r$) and then develop a logical argument explaining why it is true.

Scale Drawing and Measurement

LESSON 12.3
PAGES 338–345

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Compute actual lengths from a scale drawing
- Draw geometric shapes with given conditions
- Reproduce a scale drawing at a different scale

✓ *Determine Student Readiness*

Draw sketches of one dog and two cats on the board. Ask students to write the ratios of dogs to cats, cats to dogs, cats to animals, and dogs to animals, both with a colon and as a fraction.

Key Concept

You can use scale drawings to discover information about the actual objects those drawings represent.

Concept Background: This lesson covers scale drawings and how these are used to model real objects that are too large to represent on paper. Tell students that they will learn how to interpret and produce scale drawings using mathematical concepts that were developed in earlier lessons. Explain that a scale drawing of an object can be constructed to represent the object on a convenient (usually smaller) scale. The scale factor of a scale drawing is the key to interpreting and creating scale drawings. The scale factor is expressed as a ratio, but may be written in various formats, e.g., $\frac{1 \text{ in}}{100 \text{ miles}}$, 1 in = 100 miles, or it may be displayed graphically.

Develop Core Skills

Core Practice: Use Appropriate Math Tools

Ask students what types of tools they use in daily life or at work. Then discuss with students why those particular tools are important for getting the desired results. Explain that the same reasoning applies to solving math problems.

Core Skill: Solve Problems Involving Scale Drawings of Geometric Figures

Ask students to perform an image search on the Internet to find the different ways that scale factors are indicated in various types of scale drawings, such as maps, architectural plans, manufacturing specification drawings, etc. Provide examples before students research independently.

Pre-Teach Vocabulary

Study Cards

Have students make study cards for the vocabulary in this lesson. Provide students with definitions. Then have them add one term, its definition, and pronunciation, and a sentence using the word to each card. Encourage students to use drawings to illustrate their words. Have students make cards as complete as possible and then add to them during the lesson.

Tier 2 Words:
proportion (p. 338)
scale drawing (p. 339)

Tier 3 Words:
scale factor (p. 338)

DURING THE LESSON

Scale Factor

PAGE 338

This section introduces the concept of a scale factor, which is essentially a ratio of lengths. It then covers how to use a scale factor to convert between the measurements of two different objects.

Have students read the first three paragraphs. Then review how to use a proportion to solve a problem with equal ratios. Remind students that if they have one ratio, they can find an equivalent ratio with a specific value by setting up a proportion and cross-multiplying. Walk through the steps with the class.

Scale Drawings

PAGE 339

This section defines what a scale drawing is and has students compute distances on a rudimentary scale drawing of a nature park.

Have students read the first paragraph and then study the drawing.

As a class, work through the example that starts on the bottom of page 339. This section demonstrates a systematic process of computing actual distance from a scale drawing. The steps are: (1) write a ratio using the scale factor, (2) write a proportional equation that features the unknown distance, and (3) solve the proportional equation for the unknown distance. Have students record this process in their notebooks.

PAGE 340

THINK ABOUT MATH **ANSWER KEY**

No, the answer does not make sense. In the drawing, the table is 1.5 in. long. The scale is 1 in. = 4 ft. A dining room table would likely be longer than 0.375 feet. Thus, the proportion was written incorrectly. 1.5 inches should be in the numerator and not in the denominator.

Draw Geometric Shapes with Given Conditions

This section covers the converse of the earlier section "Computing Distances on a Scale Drawing." A systematic process, similar to the one presented in this earlier section is used to create a scale drawing from a description of an object, in this case an office building.

Guide students through the first two steps. Complete the first half of Step 3 as a class, and then allow students time to find the width of the door in pairs or independently. Check students' proportions to make sure the ratios are equivalent.

Finally, distribute rulers and protractors so students can draw the doors. Have students compare their results and explain how they drew their doors.

PAGE 341

Core Practice: Use Appropriate Math Tools

This section discusses various types of tools for measuring length. The type of tool used is highly dependent on the size of the object to be measured, e.g., a micrometer or calipers would be used to make very small measurements (on the order of $\frac{1}{100}$ of $\frac{1}{1000}$ of an inch), whereas a yardstick could be used to make medium-sized measurements (anywhere from 1 to 36 inches.)

Have students read the section. Then ask them to perform an Internet search to learn about different measurement tools for measuring length. Ask them to categorize them according to the size of measurements that need to be made, e.g., for measuring the perimeter of a room, it would be more practical to use a tape measure or a yardstick as opposed to a 12-inch ruler.

PAGE 342

Reproduce a Scale Drawing of a Different Scale

This section covers the process of converting the scale of a drawing from one scale to another. The process first involves determining the measurements of the actual object from the scale drawing and then converting these measurements using the scale for the new drawing.

Guide students through the steps in the example. Make sure students understand the many steps involved in finding the solution. To help students understand, draw and label a 3 inch by 5 inch rectangle on the board with the scale 1 in. = 3 ft before beginning the problem. Then draw a 6 inch by 10 inch rectangle, labeling the 6 inch side "6 in." and the 10 inch side "? in." Write the scale "1 in. = ? ft." Replace the question marks with numbers as you complete the solution process.

Core Skill: Solve Problems Involving Scale Drawings of Geometric Figures

This section covers times in which students needs to use both geometry and algebra to obtain measurements of actual objects from a scale drawing.

Have students read the text. Allow students to work in pairs or groups to solve the problem. Make sure students understand that the ratio they are working with is length to width.

PAGE 343

MATH LINK

This Math Link provides a tip to students for using mental math for readily recognizable patterns in measurements, which can enable them to solve proportion problems more quickly. If students find the missing number by calculating $\frac{12 \times 5}{3} = 20$, this still results in the correct answer, although it may take longer.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 434 and 435.

Engage and Extend

ELL Instruction: Find Multiple Meanings Have students look up the word "scale" and examine its various meanings and usages (it can be used as a noun and a verb). Ask students to use these meanings and usages in different sentences to develop a better understanding of the concept of scale. Support student understanding with pictures, if necessary.

Extension Activity: Investigate Angles and Scale Drawings Have students investigate whether angles remain the same when converting from the measurements of a real object to that of a scale drawing. Using a scale drawing of simple geometric shapes, such as triangles, trapezoids, etc., have students measure the angles associated with these shapes using a protractor. Then have them draw these shapes to a specified scale and measure the angles again. The angles should remain the same.

Area

LESSON 12.4
PAGES 346-351

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Find the area of squares, rectangles, and triangles
- Find the area of circles
- Find the area of complex shapes

✓ *Determine Student Readiness*

Draw the four shapes from the top of the student lesson on the board. Label the measurements. Have students identify the length and width of the square and rectangle, the base and height of the triangle, and the radius and diameter of the circle.

Key Concept

Develop and apply the concept of area to find the areas of simple and complex shapes.

Concept Background: Explain to students that area is the amount of space a two-dimensional surface covers. It has many real-life applications. Applications include building and container construction. Have students describe situations in which they would need to find the amount of space something covers (area).

Develop Core Skills

Core Skill: Build Lines of Reasoning

Have students draw a rectangle. Then have them draw a diagonal line through the rectangle dividing it into two triangles. Ask students to think about how the size (area) of one of the triangles relates to the size of the rectangle. Explain that they will apply this reasoning when finding the areas of triangles.

Core Skill: Build Solution Pathways

Have students write a list of steps detailing the process for finding the perimeter of a shape. Explain that they will follow a similar process in the lesson for finding the area of shapes.

Pre-Teach Vocabulary

Word Web

Tell students that the original meaning of the word *area* was "a vacant piece of level ground." We now use the term area of town to mean "a section of town." Have students make a word web including terms related to area.

Tier 2 Words:
area (p. 346)
complex shape (p. 349)
height (p. 347)
length (p. 346)
width (p. 346)

Tier 3 Words:
base (p. 347)

DURING THE LESSON

PAGE 346

Area of Rectangles, Squares, and Triangles

Distribute index cards. Have students place them on their desks, noting how much of the desk it covers. Explain that the area of the index card is the amount of surface it covers. Have them place their notebooks on the desk beside the index card. Guide students to understand that the area of the notebook is greater than the area of the index card because it covers more space.

Have students read the first paragraph. Ask a volunteer to rephrase the main point of the paragraph.

Work through Example 1 as a class. Make sure students understand that they need to identify the length and width, and not multiply all of the numbers on the diagram. Point out the units in the answer and explain that area is always given in square units.

Then guide students through Example 2. Have a student explain to the class the property of squares that allows them to find the length and width—all sides of a square are equal in length.

PAGE 347

Area of Triangles

Allow students time to read the first paragraph. Point out the fourth triangle, where a line had to be drawn outside the triangle to show the height. Draw a few triangles on the board and have students label the heights and bases, making sure they identify the height as being perpendicular to the base.

Then as a class, complete Example 3. Have students identify the extra information in the diagram: the label "6 m" is not needed to solve the problem. If necessary, remind students that multiplying by $\frac{1}{2}$ is the same as dividing by 2.

Have students use a calculator to complete Example 4. Point out that the fraction $\frac{1}{2}$ has been converted to the equivalent decimal, 0.5, for ease of use with the calculator.

MATH LINK

Have a student show why this is true. Since the length and width both equal the same amount, s, $l \times w = s \times s = s^2$.

Core Skill: Build Lines of Reasoning
Give students time to read the sidebar. Have them discuss the activity in small groups. They should come to the conclusion that the area of a triangle is equal to half the area of a rectangle with the length and width equal to the base and height. Tell students that they can remember this if they forget the formula for the area of a triangle, and it will help them to remember to multiply by $\frac{1}{2}$.

PAGE 348

Area of Circles

Have students read the first paragraph and write down the formula for the area of a circle. Have them also write down the formula for the circumference of a circle, $2\pi r$, and compare. Explain that the formulas look similar and may be easy to confuse. Guide students through the steps of Example 5. Point out that just like for finding the circumference, it is easier to use the decimal representation of pi unless the radius is a multiple of 7.

Then have students use calculators to solve Example 6.

Core Skill: Build Solution Pathways
Have students read the sidebar and compare the list of steps to the list they made for perimeter.

Have students work in groups to solve the problem at the end. It may be tricky because they have not solved this type of problem before. If students need help, lead them to understand that the first step is to use the circumference to find the radius. Then they can use the radius to calculate the area. Because the circumference equals two times pi times radius, divide 25.12 by 2π, or set up an equation to solve for r. Since the radius is 4, the area is approximately 50.24 cm^2.

PAGE 349

THINK ABOUT MATH **ANSWER KEY**

1. 60 cm^2
2. 154 ft^2
3. 153.9 $in.^2$
4. 64 mm^2

Area of Complex Shapes

Explain to students that they know how to find the area of rectangles, squares, triangles, and circles. When they are faced with a different shape, they can find its area by dividing it up into shapes they know, find the area of each shape separately, and then add the areas. Draw a few complex shapes on the board and have students draw lines to divide them into squares, rectangles, triangles, or circles.

Have students read the first paragraph. Then guide students through Example 7, modeling each step.

Then work through Example 8 as a class. Explain that although they do not have a formula for a semicircle, it is half of a circle, so they can use the formula for a circle and then divide it by two.

Have a student identify the diameter of the circle (6 ft) and explain the answer.

MATH LINK

Have students refer to the complex shapes on the board when reading this text. Suggest that they solve Skills Practice problem 3 by dividing the complex figure two different ways and showing that the result is the same.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 435.

Engage and Extend

ELL Instruction: Use a Graphic Organizer Have students draw or complete copies of a Venn diagram (see the Graphic Organizer section of the Instructor Resource Binder for a blackline master) for circles with one section labeled "Area" and the other section "Circumference." Have students fill in as much information as they can in the diagram.

Extension Activity: Investigate the Area of Parallelograms Have students develop a formula for finding the area of a parallelogram. Have them research online or investigate on their own with drawings or graph paper. They can apply what they know about the area of a rectangle and the area of triangles. Have them develop a logical argument for their formula. (The area of a parallelogram is found by multiplying the base by the height.)

Pythagorean Theorem

LESSON 12.5
PAGES 352-359

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Explain the Pythagorean theorem
- Apply the Pythagorean theorem to solve problems

✓ *Determine Student Readiness*

To determine student readiness, have students classify triangles by their angles (acute, right, obtuse) and have them practice computing squares and square roots of numbers.

Key Concept

The Pythagorean theorem shows a special relationship between the sides of a right triangle.

Concept Background: This lesson covers the Pythagorean theorem, its proof, and several of its applications in mathematics. It begins with the properties of right triangles and the relationships between its sides, which are governed by the Pythagorean theorem.

Tell students that right triangles are a special class of triangles, in which one of its angles measures 90 degrees, i.e., it is a right angle. The side of the triangle opposite the right angle is the hypotenuse. The remaining sides are called legs, which are adjacent to the right angle in the triangle.

Discuss with students that the terms legs and hypotenuse only refer to right triangles, and no other types of triangles.

Develop Core Skills

Core Skill: Analyze Events and Ideas

Ask students to perform an Internet search on Pythagoras, the man credited with the Pythagorean theorem, and have volunteers give a brief report. See if any of them find that it is not known whether Pythagoras actually proved the theorem or whether one of his colleagues did.

Core Skill: Solve Quadratic Equations

Prepare students for solving quadratic equations by reviewing and practicing the rules associated with manipulation of square roots, e.g., $\sqrt{xy} = \sqrt{x}\sqrt{y}$; $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$; $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$; $\sqrt{x-y} \neq \sqrt{x} - \sqrt{y}$. Ask students to state the rules in their own words.

Pre-Teach Vocabulary

Respond to Questions

Organize students into groups. Give each group 7 notecards with a different vocabulary word printed on each card and its definition on the back. Have students choose among themselves which word card to take. If there are extra cards, invite students to collaborate to complete them.

Have students study the words and their definitions. Then have them exchange cards and have students test each other on the definitions and create sentences for the words.

Tier 2 Words:	Tier 3 Words:	Test Words:
congruent (p. 353)	Pythagorean theorem (p. 353)	hypotenuse (p. 352)
proof (p. 353)	quadratic equations (p. 354)	leg (p. 352)
	theorem (p. 353)	

DURING THE LESSON

PAGE 352

Right Triangles

This section covers the definition of a right triangle, along with its components, the hypotenuse and legs. Have students read the section. Then draw a few right triangles on the board and have students practice identifying the hypotenuse and legs.

Pythagorean Theorem

This section covers the pre-history of the Pythagorean theorem by noting that ancient peoples and early mathematicians recognized the special properties of right triangles. However, nobody had put forward a theorem to prove this relationship.

Have students read the first two paragraphs. Then given them pieces of string or thin strips of paper measuring 3 inches, 4 inches, and 5 inches and allow students to form and investigate the triangle formed by the lengths of the string or paper.

Walk through the problem at the top of page 353 as a class, and then have students read the last two paragraphs of the section.

PAGE 353

The Proof of the Pythagorean Theorem

This section leads students through the proof of the Pythagorean theorem by the rearrangement of four identical right triangles and performing area calculations that use the lengths of the legs and hypotenuse of the right triangles.

Work through the proof as a class. Have students recall the formulas they learned for the area of a square and a triangle.

Core Skill: Analyze Events and Ideas

This sidebar introduces the philosopher Pythagoras and his work in mathematics. Have students read the text. Then allow them to work in groups to research the life of Pythagoras and how his theories had a profound effect on science and mathematics that has lasted into modern times. Have them summarize their findings and share them with the class.

Identifying Right Triangles

PAGE 354

Explain to students that they can use the Pythagorean theorem to determine whether a triangle is a right triangle. Substitute the lengths of the sides, simplify, and see whether the equation is true. Point out that if they only have side lengths, they can determine which one is the hypotenuse (c) because it is always the longest side.

Guide students through the steps in the example.

Recognizing Pythagorean Triples

This section introduces the notion of a Pythagorean triple, which is a set of three numbers, (a, b, c) that satisfy the Pythagorean theorem, $a^2 + b^2 = c^2$. If the elements of a Pythagorean triple are multiplied by a constant, the result set of numbers is also a Pythagorean triple.

Have students read the text. Then have pairs identify their own Pythagorean triples by choosing a multiple. Have them exchange triples and check that they are the sides of a right triangle.

Core Skill: Solve Quadratic Equations

This section guides students through the solution of a simple quadratic equation that has the form $x^2 = c$, where c is a constant. This solution method can be used to determine the length of the hypotenuse of a right triangle, given the lengths of its legs.

Have students read the text. Then have a volunteer explain why the value of x is 5.

Missing Side Lengths

PAGE 355

This section provides students with a systematic approach using the Pythagorean theorem for solving for the length of one leg of a right triangle, given the length of the other leg and the hypotenuse. The approach consists of three steps: (1) write the equation for the Pythagorean theorem, (2) substitute the known information into the equation, and (3) solve the resulting quadratic equation for the positive root.

Guide students through the steps of the first example. Remind students how the quadratic equation was solved in the Core Skill sidebar. Be sure to show students that the final answer is obtained by taking the square root of both sides of the equation.

Then have students complete the second example as independently as possible. Have them examine the diagram to help them understand the problem.

Engage and Extend

ELL Instruction: Make Word Comparisons Provide definitions, and then have students compare the words "theorem" and "theory." While these words seem similar, they have different meanings.

THINK ABOUT MATH ANSWER KEY

$3^2 + 7^2 \neq 8^2$

$9 + 49 \neq 64$

No, the triangle is not a right triangle.

The Distance between Two Points on a Coordinate Graph

PAGE 356

This section provides students with a systematic approach for computing the distance between two points on the coordinate plane.

As you complete the example as a class, have students make a list of the steps. For example:

(1) Plot each point on the coordinate plane.

(2) Draw a right triangle for which the hypotenuse is the line that connects the two points.

(3) Determine the length of the legs by counting increments along the horizontal and vertical axes.

(4) Use the Pythagorean theorem to compute the hypotenuse.

Make sure that students do not try to count the spaces along the hypotenuse but instead use the theorem to calculate the distance. Remind students of the need for precision that they have learned throughout the lessons.

THINK ABOUT MATH ANSWER KEY

5.0

MATH LINK

This Math Link addresses a common error when using the Pythagorean theorem: substituting the wrong values of the sides into the formula $a^2 + b^2 = c^2$. The lengths of the legs are represented by a and b. It does not matter which leg is represented by the variables a or b, but it is critical that the length of the hypotenuse is represented by the variable c. Have students determine whether a triangle with sides 5, 13, and 12 inches is a right triangle.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 436.

Extension Activity: Apply the Pythagorean Theorem

Have students research on the Internet the generalization of the Pythagorean theorem to the areas of squares and other similar figures that have one boundary coincident with the side of a right triangle. Students should find that the same relationship is still true of the area of the similar shapes touching the corresponding sides.

Geometric Solids and Volume

LESSON 12.6
PAGES 360–365

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Identify three-dimensional figures
- Identify rectangular solids and cubes
- Find the volume of rectangular solids and cubes

✓ *Determine Student Readiness*

Write several multiplication problems with three factors on the board and have students solve them. For example: $10 \times 8 \times 6$; $13 \times 7 \times 4$; $10 \times 10 \times 10$. Encourage students to use any strategies they have learned in the lessons.

Key Concept

Extend understanding of geometric figures to include solids and the concept of volume.

Concept Background: Geometric solids are three-dimensional geometric figures. They have length, width, and height. The geometric solids in this lesson are also known as right solid figures because all sides that intersect are perpendicular. The measurement most associated with solids is volume: the number of cubic units that fit into the solid. Have students identify rectangular solids in the room and describe the edges, faces, and vertices. Define any of these terms, if necessary.

Develop Core Skills

Core Skill: Solve Real-World Problems

Ask students to think about some real-world problems that can be solved using geometry. Examples include finding the perimeter and area of a piece of land before planting a garden. Discuss with students how geometry word problems are similar to other word problems they have solved. For example, you can usually draw a picture to illustrate the problem, and you may need to apply a formula.

Core Skill: Calculate Volume

Explain to students that volume is the amount of space that would fill a shape if it were hollow. Discuss with students how they use volume at work or in daily life and why it is important to be able to calculate volume. For example, the amount of water that will fill a 10 inch by 5 inch by 8 inch fish tank is $10 \times 5 \times 8 = 400$ cubic inches.

Pre-Teach Vocabulary

Connect to Life Experience

Ask students to use any meaning of the tier 2 words they know in a sentence about themselves (for example, *face* as part of the body or *volume* as a level of noise). Then tell them to try to relate those meanings of the words to the mathematical meaning of the word as they complete the lesson.

Tier 2 Words:	Tier 3 Words:
cube (p. 362)	rectangular prism (p. 360)
edge (p. 360)	rectangular solid (p. 360)
face (p. 360)	three-dimensional figure (p. 360)
volume (p. 361)	vertex (p. 360)

DURING THE LESSON

PAGE 360

Solids

If available, pass out three-dimensional solids to the class, including cubes and rectangular prisms. You can use number cubes or household items such as small boxes and sugar cubes.

Have students read the text. Then have students identify the faces, edges, and vertices on their solids, and identify which ones are rectangular solids.

PAGE 361

Volume of Rectangular Solids

Have students read the first paragraph. To help them conceptualize volume, have them examine the rectangular solids in front of them. Explain that the larger boxes, the ones that can hold more, have greater volume than the smaller boxes. Then have students practice identifying the length, width, and height of their boxes.

Guide students through the steps of Example 1. Have student confirm that the correct dimensions have been recorded. Point out the units in the answer: cubic inches. Explain that the units for three-dimensional objects will always be cubic units, or units^3.

Have the students complete Example 2 independently.

Then walk through Example 3 as a class. Students may be tempted to just multiply the three numbers together since this is a volume problem. Make sure they understand and apply the formula correctly, resulting in an equation. Lead students to understand that the volume formula can be used to find any missing dimension or volume.

Core Skill: Solve Real-World Problems
Have students read the text. Then have them work in small groups to answer the question at the end. To determine whether there is enough concrete, students will need to find how many pounds one cubic foot of concrete weighs, and then divide the pounds of concrete by that number and see if it is at least the number of square feet needed for the cube.

Volume of Cubes

PAGE 362

Have students examine a cube and identify its properties. They should notice that all the edges are the same length. Have them read the first paragraph and explain why the formula is true.

Guide students through the steps of Example 4. Then have students use a calculator to solve Example 5. Make sure students understand that the formula can be used to find either volume or the side length.

MATH LINK

After students read about outliers, ask them to explain why 11 is an outlier in the data set presented in Example 2.

PAGE 363

THINK ABOUT MATH **ANSWER KEY**

1. 288 m^3 2. 512 cm^3

Core Skill: Calculate Volume
Have students read the text. Then have them work in small groups to solve the problem. They will need to know that 3 feet equal 1 yard. Encourage them to draw pictures and provide them with small cubes to model the problem. One way to think about it is to imagine slicing the large cube into three slices that are 1 foot high. Each slice is 1 yard by 1 yard, which equals 3 feet by 3 feet. So each slice can make nine 1 $ft.^3$ cubes, and the entire large cube could be made into 3 times 9 or 27 cubic foot cubes.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson pages 436 and 437.

Engage and Extend

ELL Instruction: Visualize Solids Most students will have seen many examples of rectangular solids in their lives. Have them visualize the solids they already know as they learn the geometric terms related to them. Have them make a list of the rectangular solids they think of and verbally explain the use of the solids.

Extension Activity: Construct Prisms Ask students: *What are the dimensions of two different rectangular prisms you could make using 24 cubes?* Then ask: *How many different rectangular prisms can you make using 30 cubes?* Have students describe the prisms in terms of dimensions and volume.

Volume of Cones, Cylinders, and Spheres

LESSON 12.7
PAGES 366-371

BEFORE THE LESSON

Objectives

After completing the lesson, students will be able to

- Calculate the volumes of cones, cylinders, and spheres
- Calculate the volumes of complex 3-D objects

✓ *Determine Student Readiness*

Review with students the method for calculating the areas of composite and complex objects. This will set the stage for calculating the volumes of composite and complex 3-D shapes. For example, a washer is an object placed on a bolt, between the end of the bolt, and a nut. It is used to help distribute the weight of the bolt. The area of the flat side of a washer with an outer radius 1 inch and an inner radius 0.5 inches is $\pi \times 1^2 - \pi \times 0.52 = 0.75\pi$ square inches.

Key Concept

The volume of a cone, cylinder, or sphere is the amount of measurable space inside the object. These objects are three-dimensional, meaning they have length, width, and height. So, their units of measurement are cubed, such as $in.^3$, $ft.^3$, and m^3.

Concept Background: This lessons covers volume calculations for common 3-D shapes, applications of these calculations, and the manipulation of volume calculations to determine the volume of composite and more complex 3-D shapes. For instance, a cube with sides 1 inch by 1 inch by 1 inch has a volume of 1 cubic inch, just as a square with sides 1 inch by 1 inch has an area of 1 square inch.

Develop Core Skills

Core Practice: Model with Mathematics

Discuss with students the methods for calculating the areas of composite and complex objects. Explain that sometimes an area can be found by dividing a complex shape into smaller shapes. Other times an area can be found by finding the difference between two shapes. For example, when they found the area of a semicircle, they first found the area of the full circle and then divided it by two to get the area of just the semicircle. Have students find the area of a square with side lengths 6 inches that has its center removed in the shape of a smaller square that connects to the midpoints of the larger square. (The answer is 18 square inches.)

Core Practice: Make Use of a Structure

Review with students the formulas for the area and circumference of a circle, $A = \pi r^2$ and $C = 2\pi r$. Ask them to identify the common factor between these formulas. Tell students that this will help them recognize common factors among the volume formulas for a sphere, cylinder, and cone.

Pre-Teach Vocabulary

Study Cards

Provide students with the definition of the vocabulary words. Have them make study cards, including the definition and a drawing. Allow them to add to the cards as they work through the lesson.

Tier 2 Words:
apex (p. 368)
base (p. 368)
cone (p. 368)
cylinder (p. 367)
sphere (p. 369)

Tier 3 Words:
frustum (p. 368)

DURING THE LESSON

Volume and 3-D Shapes

PAGE 366

This section helps students transition from working with 2-D shapes (and the measurement of area) to working with 3-D shapes (and the measurement of volume).

Provide students with solid shapes, including cones, cylinders, and spheres. Have them feel the difference between flat surfaces and curved surfaces. Allow students time to read the text.

Volume of a Cylinder

PAGE 367

This section describes the 3-D features of a cylinder, along with its volume formula. Allow students time to read the first two paragraphs. Explain that the volume formula comes from the general formula they learned in the previous lesson: volume equals area of the base times height. In this case, the base is a circle, so the area of the base is πr^2.

Guide students through the first example in the third paragraph. Make sure students understand that the formula requires the radius, and since the diameter is given, they must first divide it by 2 to find the radius before applying the formula.

Then complete the second example measuring a birthday candle as a class. Have students explain whether the width of the candle is the diameter or radius (diameter, since width is all the way across).

THINK ABOUT MATH **ANSWER KEY**

The package is a cylinder. $V = \pi r^2 h$

$r = \frac{d}{2} = \frac{6}{2} = 3$

$V = (3.14)(3^2)(12)$

The cylinder's volume is ≈ 339 $in.^3$

Volume of a Cone

PAGE 368

This section describes the 3-D features of a cone, along with its volume formula.

Allow students time to read the first three paragraphs. Ask: *How does the formula compare to the formula for the volume of a cylinder?* (It has $\frac{1}{3}$ in front.) *How will the volumes of a cylinder and cone with the same height and radius compare?* (The volume of the cone will be $\frac{1}{3}$ the volume of the cylinder.)

Guide students through the first example in the fourth paragraph. Make sure they remember to find the radius first before applying the formula. Then work through the second example measuring the party hat as a class.

THINK ABOUT MATH **ANSWER KEY**

The disk is shaped like a cone.

$V = \frac{1}{3}\pi r^2 h$

$r = \frac{d}{2} = \frac{7.5}{2} = 3.75$

$V = \frac{1}{3}(3.14)(3.75^2)(2.25)$

The disk's volume is ≈ 33 in.^{3}

Core Practice: Model with Mathematics

Have students read the text. Then have students work in small groups to find the volume of the frustum. Volumes of complex 3-D shapes can be calculated by subtracting volumes. In this case, cutting off the top of a cone results in a smaller cone and a frustum. Thus, the volume of a frustum is the difference in volume between the two cones.

Volume of a Sphere

PAGE 369

This section describes the 3-D features of a sphere, along with its volume formula.

Have students read the first three paragraphs. Have them compare the formula to the formulas for a cylinder and a cone. Students may notice the different fraction, the exponent on r is 3, and there is no variable for height.

Guide students through the first example in paragraph 4. Point out that they only need one measurement to find the volume of a sphere—the radius.

Then allow students time to complete the second example measuring Eartha's volume independently. Make sure students find the radius before applying the formula.

MATH LINK

This exercise guides students in the application of the subtraction-of-volume concept from an earlier section to compute the volume of a complex 3-D object: a tube. Explain to students that the tube is formed by subtracting an inner cylinder from the whole cylinder.

Core Practice: Make Use of a Structure

This section guides students to recognize that the area of a circle, πr^2, is a factor that is common to the formulas for a cone, sphere, and cylinder. It then reinforces this notion geometrically by guiding students to recognize that a circle is common to each of these 3-D shapes.

Have students read the text. Then have small groups see how many different ways they can relate the three formulas. Have them complete the activity independently in their notebooks.

AFTER THE LESSON

Read through with the students the answers to the vocabulary and skill reviews and the skill practice items located on student lesson page 437.

Engage and Extend

ELL Instruction: Find Multiple Meanings Have students look up the various definitions of the word *volume* and ask them to use the word in separate sentences illustrating the different meanings. Provide support if they have trouble understanding any of the definitions. Ask them where they see similarities in meanings among the various definitions, particularly those that directly pertain to mathematics.

Extension Activity: Compare Formulas Have students compare the similarities and differences among the volume formulas covered in this lesson with those of 3-D objects that have straight edges, such as a cube, a pyramid, and a triangular prism. Have them create a Venn diagram comparing these two types of solids (see the Graphic Organizer section of the Instructor Resource Binder for a blackline master).