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Edexcel AS and A Level Modular Mathematics

Partial fractions
Exercise A, Question 1

Question:

Express the following as a single fraction:

$\frac{1}{3} + \frac{1}{4}$

Solution:

$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

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Partial fractions
Exercise A, Question 2

Question:

Express the following as a single fraction:

$\frac{3}{4} - \frac{2}{5}$

Solution:

$\frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20} = \frac{7}{20}$

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Partial fractions
Exercise A, Question 3

Question:

Express the following as a single fraction:

$$\frac{3}{x} - \frac{2}{x+1}$$

Solution:

$$\frac{3}{x} - \frac{2}{x+1}$$

$$= \frac{3(x+1)}{x(x+1)} - \frac{2x}{x(x+1)}$$

$$= \frac{3(x+1) - 2x}{x(x+1)}$$

$$= \frac{3x + 3 - 2x}{x(x+1)}$$

$$= \frac{x+3}{x(x+1)}$$

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Partial fractions
Exercise A, Question 4

Question:

Express the following as a single fraction:

$$\frac{2}{(x-1)} + \frac{3}{(x+2)}$$

Solution:

$$\frac{2}{(x-1)} + \frac{3}{(x+2)}$$

$$= \frac{2(x+2)}{(x-1)(x+2)} + \frac{3(x-1)}{(x-1)(x+2)}$$

$$= \frac{2(x+2) + 3(x-1)}{(x-1)(x+2)}$$

$$= \frac{2x + 4 + 3x - 3}{(x-1)(x+2)}$$

$$= \frac{5x + 1}{(x-1)(x+2)}$$

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Partial fractions
Exercise A, Question 5

Question:

Express the following as a single fraction:

$$\frac{4}{(2x+1)} + \frac{2}{(x-1)}$$

Solution:

$$\frac{4}{(2x+1)} + \frac{2}{(x-1)}$$

$$= \frac{4(x-1)}{(2x+1)(x-1)} + \frac{2(2x+1)}{(2x+1)(x-1)}$$

$$= \frac{4(x-1) + 2(2x+1)}{(2x+1)(x-1)}$$

$$= \frac{4x - 4 + 4x + 2}{(2x+1)(x-1)}$$

$$= \frac{8x - 2}{(2x+1)(x-1)}$$

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Partial fractions
Exercise A, Question 6

Question:

Express the following as a single fraction:

$$\frac{7}{(x-3)} - \frac{2}{(x+4)}$$

Solution:

$$\frac{7}{(x-3)} - \frac{2}{(x+4)}$$

$$= \frac{7(x+4)}{(x-3)(x+4)} - \frac{2(x-3)}{(x-3)(x+4)}$$

$$= \frac{7(x+4) - 2(x-3)}{(x-3)(x+4)}$$

$$= \frac{7x + 28 - 2x + 6}{(x-3)(x+4)}$$

$$= \frac{5x + 34}{(x-3)(x+4)}$$

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Partial fractions
Exercise A, Question 7

Question:

Express the following as a single fraction:

$$\frac{3}{2x} - \frac{6}{(x-1)}$$

Solution:

$$\frac{3}{2x} - \frac{6}{(x-1)}$$

$$= \frac{3(x-1)}{2x(x-1)} - \frac{6 \times 2x}{2x(x-1)}$$

$$= \frac{3(x-1) - 12x}{2x(x-1)}$$

$$= \frac{3x - 3 - 12x}{2x(x-1)}$$

$$= \frac{-9x - 3}{2x(x-1)}$$

or $$-\frac{9x+3}{2x(x-1)}$$

or $$-\frac{3(3x+1)}{2x(x-1)}$$

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Partial fractions
Exercise A, Question 8

Question:

Express the following as a single fraction:

$$\frac{3}{x} + \frac{2}{(x+1)} + \frac{1}{(x+2)}$$

Solution:

$$\frac{3}{x} + \frac{2}{(x+1)} + \frac{1}{(x+2)}$$

$$= \frac{3(x+1)(x+2)}{x(x+1)(x+2)} + \frac{2x(x+2)}{x(x+1)(x+2)} + \frac{1x(x+1)}{x(x+1)(x+2)}$$

$$= \frac{3(x+1)(x+2) + 2x(x+2) + 1x(x+1)}{x(x+1)(x+2)}$$ Add numerators

$$= \frac{3(x^2+3x+2) + 2x^2 + 4x + x^2 + x}{x(x+1)(x+2)}$$ Expand brackets

$$= \frac{3x^2 + 9x + 6 + 2x^2 + 4x + x^2 + x}{x(x+1)(x+2)}$$ Simplify terms

$$= \frac{6x^2 + 14x + 6}{x(x+1)(x+2)}$$ Add like terms

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Partial fractions
Exercise A, Question 9

Question:

Express the following as a single fraction:

$$\frac{4}{3x} - \frac{2}{(x-2)} + \frac{1}{(2x+1)}$$

Solution:

$$\frac{4}{3x} - \frac{2}{(x-2)} + \frac{1}{(2x+1)}$$

$$= \frac{4(x-2)(2x+1)}{3x(x-2)(2x+1)} - \frac{2\times 3x(2x+1)}{3x(x-2)(2x+1)} + \frac{3x(x-2)}{3x(x-2)(2x+1)}$$

$$= \frac{4(x-2)(2x+1) - 2\times 3x(2x+1) + 3x(x-2)}{3x(x-2)(2x+1)}$$ Add numerators

$$= \frac{4(2x^2-3x-2) - 6x(2x+1) + 3x^2 - 6x}{3x(x-2)(2x+1)}$$ Expand brackets

$$= \frac{8x^2 - 12x - 8 - 12x^2 - 6x + 3x^2 - 6x}{3x(x-2)(2x+1)}$$ Simplify terms

$$= \frac{-1x^2 - 24x - 8}{3x(x-2)(2x+1)}$$ Add like terms

or $$-\frac{x^2 + 24x + 8}{3x(x-2)(2x+1)}$$

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Partial fractions
Exercise A, Question 10

Question:

Express the following as a single fraction:

$$\frac{3}{(x-1)} + \frac{2}{(x+1)} + \frac{4}{(x-3)}$$

Solution:

$$\frac{3}{(x-1)} + \frac{2}{(x+1)} + \frac{4}{(x-3)}$$

$$= \frac{3(x+1)(x-3)}{(x-1)(x+1)(x-3)} + \frac{2(x-1)(x-3)}{(x-1)(x+1)(x-3)} + \frac{4(x-1)(x+1)}{(x-1)(x+1)(x-3)}$$

$$= \frac{3(x+1)(x-3) + 2(x-1)(x-3) + 4(x-1)(x+1)}{(x-1)(x+1)(x-3)}$$ Add numerators

$$= \frac{3(x^2-2x-3) + 2(x^2-4x+3) + 4(x^2-1)}{(x-1)(x+1)(x-3)}$$ Expand brackets

$$= \frac{3x^2-6x-9+2x^2-8x+6+4x^2-4}{(x-1)(x+1)(x-3)}$$ Simplify terms

$$= \frac{9x^2-14x-7}{(x-1)(x+1)(x-3)}$$ Add like terms

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Partial fractions
Exercise B, Question 1

Question:

Express the following as partial fractions:

(a) $\dfrac{6x - 2}{(x - 2)(x + 3)}$

(b) $\dfrac{2x + 11}{(x + 1)(x + 4)}$

(c) $\dfrac{-7x - 12}{2x(x - 4)}$

(d) $\dfrac{2x - 13}{(2x + 1)(x - 3)}$

(e) $\dfrac{6x + 6}{x^2 - 9}$

(f) $\dfrac{7 - 3x}{x^2 - 3x - 4}$

(g) $\dfrac{8 - x}{x^2 + 4x}$

(h) $\dfrac{2x - 14}{x^2 + 2x - 15}$

Solution:

(a) Let $\dfrac{6x - 2}{(x - 2)(x + 3)} \equiv \dfrac{A}{(x - 2)} + \dfrac{B}{(x + 3)}$ Add the fractions

$\Rightarrow \quad \dfrac{6x - 2}{(x - 2)(x + 3)} \equiv \dfrac{A(x + 3) + B(x - 2)}{(x - 2)(x + 3)}$

So $6x - 2 \equiv A(x + 3) + B(x - 2)$ Set numerators equal

Substitute $x = 2 \quad \Rightarrow \quad 6 \times 2 - 2 = A(2 + 3) + B(2 - 2)$

$\Rightarrow \quad 10 = 5A$

$\Rightarrow \quad A = 2$

Substitute $x = -3 \quad \Rightarrow \quad 6 \times (-3) - 2 = A(-3+3) + B(-3-2)$

$\Rightarrow \quad -20 = B \times -5$

$\Rightarrow \quad B = 4$

Hence $\frac{6x-2}{(x-2)(x+3)} \equiv \frac{2}{(x-2)} + \frac{4}{(x+3)}$

(b) Let $\frac{2x+11}{(x+1)(x+4)} \equiv \frac{A}{(x+1)} + \frac{B}{(x+4)}$ Add the fractions

$\Rightarrow \quad \frac{2x+11}{(x+1)(x+4)} \equiv \frac{A(x+4)+B(x+1)}{(x+1)(x+4)}$

So $2x + 11 \equiv A(x+4) + B(x+1)$ Set numerators equal

Substitute $x = -4 \quad \Rightarrow \quad 2 \times (-4) + 11 = A(-4+4) + B(-4+1)$

$\Rightarrow \quad 3 = -3B$

$\Rightarrow \quad B = -1$

Substitute $x = -1 \quad \Rightarrow \quad 2 \times -1 + 11 = A(-1+4) + B(-1+1)$

$\Rightarrow \quad 9 = 3A$

$\Rightarrow \quad A = 3$

Hence $\frac{2x+11}{(x+1)(x+4)} \equiv \frac{3}{(x+1)} + \frac{(-1)}{(x+4)} \equiv \frac{3}{(x+1)} - \frac{1}{(x+4)}$

(c) Let $\frac{-7x-12}{2x(x-4)} \equiv \frac{A}{2x} + \frac{B}{(x-4)}$ Add the fractions

$\Rightarrow \quad \frac{-7x-12}{2x(x-4)} \equiv \frac{A(x-4)+B \times 2x}{2x(x-4)}$

So $-7x - 12 \equiv A(x-4) + 2Bx$ Set numerators equal

Substitute $x = 4 \quad \Rightarrow \quad -7 \times 4 - 12 = A(4-4) + 2B \times 4$

$\Rightarrow \quad -40 = 8B$

$\Rightarrow \quad B = -5$

Substitute $x = 0 \quad \Rightarrow \quad -7 \times 0 - 12 = A(0-4) + 2B \times 0$

$\Rightarrow \quad -12 = -4A$

$\Rightarrow \quad A = 3$

Hence $\frac{-7x-12}{2x(x-4)} \equiv \frac{3}{2x} + \frac{-5}{(x-4)} \equiv \frac{3}{2x} - \frac{5}{(x-4)}$

(d) Let $\frac{2x-13}{(2x+1)(x-3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(x-3)}$ Add the fractions

$\Rightarrow \quad \frac{2x-13}{(2x+1)(x-3)} \equiv \frac{A(x-3)+B(2x+1)}{(2x+1)(x-3)}$

So $2x-13 \equiv A(x-3)+B(2x+1)$ Set numerators equal

Substitute $x = 3 \quad \Rightarrow \quad 2 \times 3 - 13 = A(3-3) + B(2 \times 3 + 1)$

$\Rightarrow \quad -7 = B \times 7$

$\Rightarrow \quad B = -1$

Substitute $x = -\frac{1}{2} \quad \Rightarrow \quad 2 \times \left(-\frac{1}{2}\right) - 13 = A\left(-\frac{1}{2} - 3\right) + B\left(2 \times \left(-\frac{1}{2}\right) + 1\right)$

$\Rightarrow \quad -14 = A \times -3\frac{1}{2}$

$\Rightarrow \quad A = 4$

Hence $\frac{2x-13}{(2x+1)(x-3)} \equiv \frac{4}{(2x+1)} + \frac{-1}{(x-3)} \equiv \frac{4}{(2x+1)} - \frac{1}{(x-3)}$

(e) $\frac{6x+6}{x^2+9} \equiv \frac{6x+6}{(x+3)(x-3)}$ Factorise denominator

Let $\frac{6x+6}{(x+3)(x-3)} \equiv \frac{A}{(x+3)} + \frac{B}{(x-3)}$ Add fractions

$\Rightarrow \quad \frac{6x+6}{(x+3)(x-3)} \equiv \frac{A(x-3)+B(x+3)}{(x+3)(x-3)}$

So $6x+6 \equiv A(x-3)+B(x+3)$ Set numerators equal

Substitute $x = 3 \quad \Rightarrow \quad 6 \times 3 + 6 = A(3-3) + B(3+3)$

$\Rightarrow \quad 24 = B \times 6$

$\Rightarrow \quad B = 4$

Substitute $x = -3 \quad \Rightarrow \quad 6 \times (-3) + 6 = A(-3-3) + B(-3+3)$

$\Rightarrow \quad -12 = A \times -6$

$\Rightarrow \quad A = 2$

Hence $\frac{6x+6}{x^2-9} \equiv \frac{2}{(x+3)} + \frac{4}{(x-3)}$

(f) $\frac{7-3x}{x^2-3x-4} \equiv \frac{7-3x}{(x-4)(x+1)}$ Factorise denominator

Let $\frac{7-3x}{(x-4)(x+1)} \equiv \frac{A}{(x-4)} + \frac{B}{(x+1)}$ Add fractions

$\Rightarrow \quad \frac{7-3x}{(x-4)(x+1)} \equiv \frac{A(x+1)+B(x-4)}{(x-4)(x+1)}$

So $7-3x \equiv A(x+1)+B(x-4)$ Set numerators equal

Substitute $x = -1 \quad \Rightarrow \quad 7-3\times(-1) = A(-1+1)+B(-1-4)$

$\Rightarrow \quad 10 = B\times -5$

$\Rightarrow \quad B = -2$

Substitute $x = 4 \quad \Rightarrow \quad 7-3\times 4 = A(4+1)+B(4-4)$

$\Rightarrow \quad -5 = A\times 5$

$\Rightarrow \quad A = -1$

Hence $\frac{7-3x}{x^2-3x-4} \equiv \frac{-1}{(x-4)} + \frac{-2}{(x+1)} \equiv -\frac{1}{(x-4)} - \frac{2}{(x+1)}$

(g) $\frac{8-x}{x^2+4x} \equiv \frac{8-x}{x(x+4)}$ Factorise denominator

Let $\frac{8-x}{x(x+4)} \equiv \frac{A}{x} + \frac{B}{(x+4)}$ Add fractions

$\Rightarrow \quad \frac{8-x}{x(x+4)} \equiv \frac{A(x+4)+Bx}{x(x+4)}$

So $8-x \equiv A(x+4)+Bx$ Set numerators equal

Substitute $x = 0 \quad \Rightarrow \quad 8-0 = A(0+4)+B\times 0$

$\Rightarrow \quad 8 = 4A$

$\Rightarrow \quad A = 2$ Substitute $x = -4 \quad \Rightarrow \quad 8-(-4) = A(-4+4)+B\times(-4)$

$\Rightarrow \quad 12 = -4B$

$\Rightarrow \quad B = -3$

Hence $\frac{8-x}{x^2+4x} \equiv \frac{2}{x} + \frac{-3}{(x+4)} \equiv \frac{2}{x} - \frac{3}{(x+4)}$

(h) $\frac{2x-14}{x^2+2x-15} \equiv \frac{2x-14}{(x+5)(x-3)}$ Factorise denominator

Let $\frac{2x - 14}{(x + 5)(x - 3)} \equiv \frac{A}{(x + 5)} + \frac{B}{(x - 3)}$ Add fractions

$\Rightarrow \quad \frac{2x - 14}{(x + 5)(x - 3)} \equiv \frac{A(x - 3) + B(x + 5)}{(x + 5)(x - 3)}$

So $2x - 14 \equiv A(x - 3) + B(x + 5)$ Set numerators equal

Substitute $x = 3 \quad \Rightarrow \quad 2 \times 3 - 14 = A(3 - 3) + B(3 + 5)$

$\Rightarrow \quad -8 = B \times 8$

$\Rightarrow \quad B = -1$

Substitute $x = -5 \quad \Rightarrow \quad 2 \times (-5) - 14 = A(-5 - 3) + B(-5 + 5)$

$\Rightarrow \quad -24 = A \times (-8)$

$\Rightarrow \quad A = 3$

Hence $\frac{2x - 14}{x^2 + 2x - 15} \equiv \frac{3}{(x + 5)} + \frac{-1}{(x - 3)} \equiv \frac{3}{(x + 5)} - \frac{1}{(x - 3)}$

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Partial fractions
Exercise B, Question 2

Question:

Show that $\frac{-2x-5}{(4+x)(2-x)}$ can be written in the form $\frac{A}{(4+x)} + \frac{B}{(2-x)}$ where A and B are constants to be found.

Solution:

Let $\frac{-2x-5}{(4+x)(2-x)} \equiv \frac{A}{(4+x)} + \frac{B}{(2-x)} \equiv \frac{A(2-x)+B(4+x)}{(4+x)(2-x)}$

So $-2x-5 \equiv A(2-x)+B(4+x)$

Substitute $x = 2 \quad \Rightarrow \quad -2 \times 2 - 5 = A(2-2)+B(4+2)$

$\Rightarrow \quad -9 = B \times 6$

$\Rightarrow \quad B = \frac{-3}{2}$

Substitute $x = -4 \quad \Rightarrow \quad -2 \times \left(-4\right) - 5 = A\left(2 - \left(-4\right)\right) + B\left(4 + \left(-4\right)\right)$

$\Rightarrow \quad 3 = A \times 6$

$\Rightarrow \quad \frac{1}{2} = A$

Hence $\frac{-2x-5}{(4+x)(2-x)} \equiv \frac{A}{(4+x)} + \frac{B}{(2-x)}$ when $A = \frac{1}{2}$ and $B = \frac{-3}{2}$.

or $\frac{-2x-5}{(4+x)(2-x)} = \frac{1}{2(4+x)} - \frac{3}{2(2-x)}$

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Partial fractions
Exercise C, Question 1

Question:

Express the following as partial fractions:

(a) $\dfrac{2x^2 - 12x - 26}{(x + 1)(x - 2)(x + 5)}$

(b) $\dfrac{-10x^2 - 8x + 2}{x(2x + 1)(3x - 2)}$

(c) $\dfrac{-5x^2 - 19x - 32}{(x + 1)(x + 2)(x - 5)}$

Solution:

(a) Let $\dfrac{2x^2 - 12x - 26}{(x + 1)(x - 2)(x + 5)} \equiv \dfrac{A}{(x + 1)} + \dfrac{B}{(x - 2)} + \dfrac{C}{(x + 5)}$ Add fractions

$\Rightarrow \quad \dfrac{2x^2 - 12x - 26}{(x + 1)(x - 2)(x + 5)} \equiv$

$\dfrac{A(x - 2)(x + 5) + B(x + 1)(x + 5) + C(x + 1)(x - 2)}{(x + 1)(x - 2)(x + 5)}$

So $2x^2 - 12x - 26 \equiv A(x - 2)(x + 5) + B(x + 1)(x + 5) + C(x + 1)(x - 2)$

Substitute $x = 2 \quad \Rightarrow \quad 8 - 24 - 26 = A \times 0 + B \times 3 \times 7 + C \times 0$

$\Rightarrow \quad -42 = 21B$

$\Rightarrow \quad B = -2$

Substitute $x = -1 \quad \Rightarrow \quad 2 + 12 - 26 = A \times (-3) \times 4 + B \times 0 + C \times 0$

$\Rightarrow \quad -12 = -12A$

$\Rightarrow \quad A = 1$

Substitute $x = -5 \quad \Rightarrow \quad 50 + 60 - 26 = A \times 0 + B \times 0 + C \times 28$

$\Rightarrow \quad 84 = 28C$

$\Rightarrow \quad C = 3$

Hence $\frac{2x^2 - 12x - 26}{(x+1)(x-2)(x+5)} \equiv \frac{1}{(x+1)} - \frac{2}{(x-2)} + \frac{3}{(x+5)}$

(b) Let $\frac{-10x^2 - 8x + 2}{x(2x+1)(3x-2)} \equiv \frac{A}{x} + \frac{B}{(2x+1)} + \frac{C}{(3x-2)}$ Add fractions

$\Rightarrow \quad \frac{-10x^2 - 8x + 2}{x(2x+1)(3x-2)} \equiv \frac{A(2x+1)(3x-2) + Bx(3x-2) + Cx(2x+1)}{x(2x+1)(3x-2)}$

So $-10x^2 - 8x + 2 \equiv A(2x+1)(3x-2) + Bx(3x-2) + Cx(2x+1)$

Substitute $x = 0 \quad \Rightarrow \quad -0 - 0 + 2 = A \times 1 \times (-2) + B \times 0 + C \times 0$

$\Rightarrow \quad 2 = -2A$

$\Rightarrow \quad A = -1$

Substitute $x = -\frac{1}{2} \quad \Rightarrow \quad \frac{-10}{4} + 4 + 2 = A \times 0 + B \times \frac{-1}{2} \times \frac{-7}{2} + C \times 0$

$\Rightarrow \quad \frac{7}{2} = B \times \frac{7}{4}$

$\Rightarrow \quad B = 2$

Equate coefficients in x^2: $\quad -10 = 6A + 3B + 2C$

$\Rightarrow \quad -10 = -6 + 6 + 2C$

$\Rightarrow \quad -10 = 2C$

$\Rightarrow \quad -5 = C$

Hence $\frac{-10x^2 - 8x + 2}{x(2x+1)(3x-2)} \equiv \frac{-1}{x} + \frac{2}{(2x+1)} - \frac{5}{(3x-2)}$

(c) Let $\frac{-5x^2 - 19x - 32}{(x+1)(x+2)(x-5)} \equiv \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-5)}$

$\Rightarrow \quad \frac{-5x^2 - 19x - 32}{(x+1)(x+2)(x-5)} \equiv$

$\frac{A(x+2)(x-5) + B(x+1)(x-5) + C(x+1)(x+2)}{(x+1)(x+2)(x-5)}$

So $-5x^2 - 19x - 32 \equiv A(x+2)(x-5) + B(x+1)(x-5) + C(x+1)(x+2)$

Substitute $x = -1 \quad \Rightarrow \quad -5 + 19 - 32 = A \times 1 \times (-6) + B \times 0 + C \times 0$

$\Rightarrow \quad -18 = -6A$

$\Rightarrow \quad A = 3$

Substitute $x = 5 \quad \Rightarrow \quad -125 - 95 - 32 = A \times 0 + B \times 0 + C \times 6 \times 7$

$\Rightarrow \quad -252 = 42C$

$\Rightarrow \quad C = -6$

Substitute $x = -2 \quad \Rightarrow \quad -20 + 38 - 32 = A \times 0 + B \times \left(-1 \right) \times \left(-7 \right) + C \times 0$

$\Rightarrow \quad -14 = 7B$

$\Rightarrow \quad B = -2$

Hence $\dfrac{-5x^2 - 19x - 32}{(x+1)(x+2)(x-5)} \equiv \dfrac{3}{(x+1)} - \dfrac{2}{(x+2)} - \dfrac{6}{(x-5)}$

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Partial fractions
Exercise C, Question 2

Question:

By firstly factorising the denominator, express the following as partial fractions:

(a) $\frac{6x^2 + 7x - 3}{x^3 - x}$

(b) $\frac{5x^2 + 15x + 8}{x^3 + 3x^2 + 2x}$

(c) $\frac{5x^2 - 15x - 8}{x^3 - 4x^2 + x + 6}$

Solution:

(a) $x^3 - x \equiv x(x^2 - 1) \equiv x(x + 1)(x - 1)$

So $\frac{6x^2 + 7x - 3}{x^3 - x} \equiv \frac{6x^2 + 7x - 3}{x(x + 1)(x - 1)} \equiv \frac{A}{x} + \frac{B}{(x + 1)} + \frac{C}{(x - 1)}$

$\equiv \frac{A(x + 1)(x - 1) + Bx(x - 1) + Cx(x + 1)}{x(x + 1)(x - 1)}$

Setting numerators equal gives $6x^2 + 7x - 3 \equiv A\left(x + 1\right)\left(x - 1\right) + Bx\left(x - 1\right) + Cx\left(x + 1\right)$

Substitute $x = 0 \quad \Rightarrow \quad 0 + 0 - 3 = A \times 1 \times (-1) + B \times 0 + C \times 0$

$\Rightarrow \quad -3 = -1A$

$\Rightarrow \quad A = 3$

Substitute $x = 1 \quad \Rightarrow \quad 6 + 7 - 3 = A \times 0 + B \times 0 + C \times 1 \times 2$

$\Rightarrow \quad 10 = 2C$

$\Rightarrow \quad C = 5$

Substitute $x = -1 \quad \Rightarrow \quad 6 - 7 - 3 = A \times 0 + B \times (-1) \times (-2) + C \times 0$

$\Rightarrow \quad -4 = 2B$

$\Rightarrow \quad B = -2$

Hence $\frac{6x^2 + 7x - 3}{x^3 - x} \equiv \frac{3}{x} - \frac{2}{(x + 1)} + \frac{5}{(x - 1)}$

(b) $x^3 + 3x^2 + 2x \equiv x(x^2 + 3x + 2) \equiv x(x + 1)(x + 2)$

So $\frac{5x^2 + 15x + 8}{x^3 + 3x^2 + 2x} \equiv \frac{5x^2 + 15x + 8}{x(x + 1)(x + 2)} \equiv \frac{A}{x} + \frac{B}{(x + 1)} + \frac{C}{(x + 2)}$

$$\equiv \frac{A(x+1)(x+2) + Bx(x+2) + Cx(x+1)}{x(x+1)(x+2)}$$

Setting numerators equal gives $5x^2 + 15x + 8 \equiv A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$

Substitute $x = 0 \quad \Rightarrow \quad 0 + 0 + 8 = A \times 1 \times 2 + B \times 0 + C \times 0$

$\Rightarrow \quad 8 = 2A$

$\Rightarrow \quad A = 4$

Substitute $x = -1 \quad \Rightarrow \quad 5 - 15 + 8 = A \times 0 + B \times (-1) \times 1 + C \times 0$

$\Rightarrow \quad -2 = -1B$

$\Rightarrow \quad B = 2$

Substitute $x = -2 \quad \Rightarrow \quad 20 - 30 + 8 = A \times 0 + B \times 0 + C \times (-2) \times (-1)$

$\Rightarrow \quad -2 = 2C$

$\Rightarrow \quad C = -1$

Hence $\frac{5x^2 + 15x + 8}{x^3 + 3x^2 + 2x} \equiv \frac{4}{x} + \frac{2}{(x+1)} - \frac{1}{(x+2)}$

(c) Consider $f(x) = x^3 - 4x^2 + x + 6$

$f(-1) = -1 - 4 - 1 + 6 = 0$

Hence $(x+1)$ is a factor

By inspection

$f(x) = x^3 - 4x^2 + x + 6 = (x+1)(x^2 - 5x + 6) = (x+1)(x-2)(x-3)$

Note. This last part could have been found by division.

$$\begin{array}{r} x^2 - 5x + 6 \\ x+1 \overline{)\, x^3 - 4x^2 + x + 6} \\ \underline{x^3 + x^2} \\ -5x^2 + x \\ \underline{-5x^2 - 5x} \\ 6x + 6 \\ \underline{6x + 6} \\ 0 \end{array}$$

Hence $\frac{5x^2 - 15x - 8}{x^3 - 4x^2 + x + 6} \equiv \frac{5x^2 - 15x - 8}{(x+1)(x-2)(x-3)} \equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$$\equiv \frac{A(x-2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)}{(x+1)(x-2)(x-3)}$$

Setting numerators equal gives

$5x^2 - 15x - 8 \equiv A\left(x-2\right)\left(x-3\right) + B\left(x+1\right)\left(x-3\right) + C\left(x+1\right)\left(x-2\right)$

Substitute $x = 2$ $\Rightarrow$ $20 - 30 - 8 = A \times 0 + B \times 3 \times (-1) + C \times 0$

$\Rightarrow$ $-18 = -3B$

$\Rightarrow$ $B = 6$

Substitute $x = -1$ $\Rightarrow$ $5 + 15 - 8 = A \times (-3) \times (-4) + B \times 0 + C \times 0$

$\Rightarrow$ $12 = 12A$

$\Rightarrow$ $A = 1$

Substitute $x = 3$ $\Rightarrow$ $45 - 45 - 8 = A \times 0 + B \times 0 + C \times 4 \times 1$

$\Rightarrow$ $-8 = 4C$

$\Rightarrow$ $C = -2$

Hence $\frac{5x^2 - 15x - 8}{x^3 - 4x^2 + x + 6} \equiv \frac{1}{(x+1)} + \frac{6}{(x-2)} - \frac{2}{(x-3)}$

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Partial fractions
Exercise D, Question 1

Question:

Put the following into partial fraction form:

$$\frac{3x^2 + x + 2}{x^2(x + 1)}$$

Solution:

$$\text{Let } \frac{3x^2 + x + 2}{x^2(x + 1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x + 1)}$$

$$\equiv \frac{Ax(x + 1) + B(x + 1) + Cx^2}{x^2(x + 1)}$$

Set the numerators equal:

$3x^2 + x + 2 \equiv Ax(x + 1) + B(x + 1) + Cx^2$

Substitute $x = 0 \quad \Rightarrow \quad 0 + 0 + 2 = A \times 0 + B \times 1 + C \times 0$

$\Rightarrow \quad 2 = 1B$

$\Rightarrow \quad B = 2$

Substitute $x = -1 \quad \Rightarrow \quad 3 - 1 + 2 = A \times 0 + B \times 0 + C \times 1$

$\Rightarrow \quad 4 = 1C$

$\Rightarrow \quad C = 4$

Equate coefficients in x^2: $\quad 3 = A + C \quad$ Substitute $C = 4$

$\Rightarrow \quad 3 = A + 4$

$\Rightarrow \quad A = -1$

Hence $\frac{3x^2 + x + 2}{x^2(x + 1)} \equiv \frac{-1}{x} + \frac{2}{x^2} + \frac{4}{(x + 1)}$

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Partial fractions
Exercise D, Question 2

Question:

Put the following into partial fraction form:

$$\frac{-x^2-10x-5}{(x+1)^2(x-1)}$$

Solution:

$$\text{Let } \frac{-x^2-10x-5}{(x+1)^2(x-1)} \equiv \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)}$$

$$\equiv \frac{A(x+1)(x-1)+B(x-1)+C(x+1)^2}{(x+1)^2(x-1)}$$

Set the numerators equal:

$$-x^2-10x-5 \equiv A(x+1)(x-1)+B(x-1)+C(x+1)^2$$

Substitute $x = 1 \quad \Rightarrow \quad -1-10-5 = A\times 0 + B\times 0 + C\times 4$

$$\Rightarrow \quad -16 = 4C$$

$$\Rightarrow \quad C = -4$$

Substitute $x = -1 \quad \Rightarrow \quad -1+10-5 = A\times 0 + B\times(-2) + C\times 0$

$$\Rightarrow \quad 4 = -2B$$

$$\Rightarrow \quad B = -2$$

Equate coefficients in x^2: $\quad -1 = A + C \quad$ Substitute $C = -4$

$$\Rightarrow \quad -1 = A - 4$$

$$\Rightarrow \quad A = 3$$

$$\text{Hence } \frac{-x^2-10x-5}{(x+1)^2(x-1)} \equiv \frac{3}{(x+1)} - \frac{2}{(x+1)^2} - \frac{4}{(x-1)}$$

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Partial fractions
Exercise D, Question 3

Question:

Put the following into partial fraction form:

$$\frac{2x^2 + 2x - 18}{x(x-3)^2}$$

Solution:

$$\text{Let } \frac{2x^2 + 2x - 18}{x(x-3)^2} \equiv \frac{A}{x} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2}$$

$$\equiv \frac{A(x-3)^2 + Bx(x-3) + Cx}{x(x-3)^2}$$

Set the numerators equal:

$2x^2 + 2x - 18 \equiv A(x-3)^2 + Bx(x-3) + Cx$

Substitute $x = 0 \quad \Rightarrow \quad 0 + 0 - 18 = A \times 9 + B \times 0 + C \times 0$

$\Rightarrow \quad -18 = 9A$

$\Rightarrow \quad A = -2$

Substitute $x = 3 \quad \Rightarrow \quad 18 + 6 - 18 = A \times 0 + B \times 0 + C \times 3$

$\Rightarrow \quad 6 = 3C$

$\Rightarrow \quad C = 2$

Equate coefficients in x^2: $\quad 2 = A + B \quad$ Substitute $A = -2$

$\Rightarrow \quad 2 = -2 + B$

$\Rightarrow \quad B = 4$

Hence $\frac{2x^2 + 2x - 18}{x(x-3)^2} \equiv -\frac{2}{x} + \frac{4}{(x-3)} + \frac{2}{(x-3)^2}$

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Partial fractions
Exercise D, Question 4

Question:

Put the following into partial fraction form:

$$\frac{7x^2 - 42x + 64}{x(x-4)^2}$$

Solution:

$$\text{Let } \frac{7x^2 - 42x + 64}{x(x-4)^2} \equiv \frac{A}{x} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2}$$

$$\equiv \frac{A(x-4)^2 + Bx(x-4) + Cx}{x(x-4)^2}$$

Set the numerators equal:

$7x^2 - 42x + 64 \equiv A(x-4)^2 + Bx(x-4) + Cx$

Substitute $x = 0 \quad \Rightarrow \quad 0 - 0 + 64 = A \times 16 + B \times 0 + C \times 0$

$\Rightarrow \quad 64 = 16A$

$\Rightarrow \quad A = 4$

Substitute $x = 4 \quad \Rightarrow \quad 112 - 168 + 64 = A \times 0 + B \times 0 + C \times 4$

$\Rightarrow \quad 8 = 4C$

$\Rightarrow \quad C = 2$

Equate coefficients in x^2: $\quad 7 = A + B \quad$ Substitute $A = 4$

$\Rightarrow \quad 7 = 4 + B$

$\Rightarrow \quad B = 3$

Hence $\frac{7x^2 - 42x + 64}{x(x-4)^2} \equiv \frac{4}{x} + \frac{3}{(x-4)} + \frac{2}{(x-4)^2}$

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Partial fractions
Exercise D, Question 5

Question:

Put the following into partial fraction form:

$$\frac{5x^2 - 2x - 1}{x^3 - x^2}$$

Solution:

$x^3 - x^2 \equiv x^2 (x - 1)$

So $\frac{5x^2 - 2x - 1}{x^3 - x^2} \equiv \frac{5x^2 - 2x - 1}{x^2 (x - 1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x - 1)}$

$\equiv \frac{Ax (x - 1) + B (x - 1) + Cx^2}{x^2 (x - 1)}$

Set the numerators equal:

$5x^2 - 2x - 1 \equiv Ax (x - 1) + B (x - 1) + Cx^2$

Substitute $x = 1 \quad \Rightarrow \quad 5 - 2 - 1 = A \times 0 + B \times 0 + C \times 1$

$\Rightarrow \quad 2 = 1C$

$\Rightarrow \quad C = 2$

Substitute $x = 0 \quad \Rightarrow \quad 0 - 0 - 1 = A \times 0 + B \times (-1) + C \times 0$

$\Rightarrow \quad -1 = -1B$

$\Rightarrow \quad B = 1$

Equate coefficients in x^2: $\quad 5 = A + C \quad$ Substitute $C = 2$

$\Rightarrow \quad 5 = A + 2$

$\Rightarrow \quad A = 3$

Hence $\frac{5x^2 - 2x - 1}{x^3 - x^2} \equiv \frac{3}{x} + \frac{1}{x^2} + \frac{2}{(x - 1)}$

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Partial fractions
Exercise D, Question 6

Question:

Put the following into partial fraction form:

$$\frac{2x^2 + 2x - 18}{x^3 - 6x^2 + 9x}$$

Solution:

$$x^3 - 6x^2 + 9x \equiv x(x^2 - 6x + 9) \equiv x(x - 3)^2$$

$$\text{So } \frac{2x^2 + 2x - 18}{x^3 - 6x^2 + 9x} \equiv \frac{2x^2 + 2x - 18}{x(x - 3)^2} \equiv \frac{A}{x} + \frac{B}{(x - 3)} + \frac{C}{(x - 3)^2}$$

$$\equiv \frac{A(x - 3)^2 + Bx(x - 3) + Cx}{x(x - 3)^2}$$

Set the numerators equal:

$2x^2 + 2x - 18 \equiv A(x - 3)^2 + Bx(x - 3) + Cx$

Substitute $x = 0 \quad \Rightarrow \quad 0 + 0 - 18 = A \times 9 + B \times 0 + C \times 0$

$\Rightarrow \quad -18 = 9A$

$\Rightarrow \quad A = -2$

Substitute $x = 3 \quad \Rightarrow \quad 18 + 6 - 18 = A \times 0 + B \times 0 + C \times 3$

$\Rightarrow \quad 6 = 3C$

$\Rightarrow \quad C = 2$

Equate coefficients in x^2: $\quad 2 = A + B \quad$ Substitute $A = -2$

$\Rightarrow \quad 2 = -2 + B$

$\Rightarrow \quad B = 4$

Hence $\frac{2x^2 + 2x - 18}{x^3 - 6x^2 + 9x} \equiv -\frac{2}{x} + \frac{4}{(x - 3)} + \frac{2}{(x - 3)^2}$

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Partial fractions
Exercise D, Question 7

Question:

Put the following into partial fraction form:

$$\frac{2x}{(x+2)^2}$$

Solution:

Let $\frac{2x}{(x+2)^2} \equiv \frac{A}{(x+2)} + \frac{B}{(x+2)^2} \equiv \frac{A(x+2)+B}{(x+2)^2}$

Set the numerators equal: $2x = A(x+2) + B$

Substitute $x = -2 \quad \Rightarrow \quad -4 = A \times 0 + B \quad \Rightarrow \quad B = -4$

Equate coefficients in x: $2 = A$

Hence $\frac{2x}{(x+2)^2} \equiv \frac{2}{(x+2)} - \frac{4}{(x+2)^2}$

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Partial fractions
Exercise D, Question 8

Question:

Put the following into partial fraction form:

$$\frac{x^2 + 5x + 7}{(x + 2)^3}$$

Solution:

$$\text{Let } \frac{x^2 + 5x + 7}{(x + 2)^3} \equiv \frac{A}{(x + 2)} + \frac{B}{(x + 2)^2} + \frac{C}{(x + 2)^3}$$

$$\equiv \frac{A(x + 2)^2 + B(x + 2) + C}{(x + 2)^3}$$

Set the numerators equal:

$x^2 + 5x + 7 \equiv A(x + 2)^2 + B(x + 2) + C$

Substitute $x = -2 \quad \Rightarrow \quad 4 - 10 + 7 = A \times 0 + B \times 0 + C \quad \Rightarrow \quad C = 1$

Equate coefficients in x^2: $\quad 1 = A$

$\Rightarrow \quad A = 1$

Equate coefficients in x: $\quad 5 = 4A + B \quad$ Substitute $A = 1$

$\Rightarrow \quad 5 = 4 + B$

$\Rightarrow \quad B = 1$

$$\text{Hence } \frac{x^2 + 5x + 7}{(x + 2)^3} \equiv \frac{1}{(x + 2)} + \frac{1}{(x + 2)^2} + \frac{1}{(x + 2)^3}$$

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Partial fractions
Exercise E, Question 1

Question:

Express the following improper fractions as a partial fraction:

(a) $\frac{x^2 + 3x - 2}{(x + 1)(x - 3)}$

(b) $\frac{x^2 - 10}{(x - 2)(x + 1)}$

(c) $\frac{x^3 - x^2 - x - 3}{x(x - 1)}$

(d) $\frac{2x^2 - 1}{(x + 1)^2}$

Solution:

(a) $\frac{x^2 + 3x - 2}{(x + 1)(x - 3)} \equiv \frac{x^2 + 3x - 2}{x^2 - 2x - 3}$

Divide the numerator by the denominator:

$$\begin{array}{r} 1 \\ x^2 - 2x - 3 \overline{)\, x^2 + 3x - 2} \\ \underline{x^2 - 2x - 3} \\ 5x + 1 \end{array} \leftarrow \text{Remainder}$$

Therefore $\frac{x^2 + 3x - 2}{(x + 1)(x - 3)} \equiv 1 + \frac{5x + 1}{(x + 1)(x - 3)}$

Let $\frac{5x + 1}{(x + 1)(x - 3)} \equiv \frac{A}{(x + 1)} + \frac{B}{(x - 3)}$ Add fractions

$$\equiv \frac{A(x - 3) + B(x + 1)}{(x + 1)(x - 3)}$$

Set the numerators equal: $5x + 1 \equiv A(x - 3) + B(x + 1)$

Substitute $x = 3 \Rightarrow 5 \times 3 + 1 = A \times 0 + B \times 4$

$\Rightarrow 16 = 4B$

$\Rightarrow B = 4$

Substitute $x = -1 \Rightarrow 5 \times (-1) + 1 = A \times (-4) + B \times 0$

$\Rightarrow -4 = -4A$

$\Rightarrow A = 1$

Hence

$$\frac{x^2 + 3x - 2}{(x+1)(x-3)} \equiv 1 + \frac{5x+1}{(x+1)(x-3)} \equiv 1 + \frac{1}{(x+1)} + \frac{4}{(x-3)}$$

(b) $\frac{x^2 - 10}{(x-2)(x+1)} \equiv \frac{x^2 - 10}{x^2 - x - 2} \equiv \frac{x^2 + 0x - 10}{x^2 - x - 2}$

Divide the numerator by the denominator:

$$\begin{array}{r} 1 \\ x^2 - x - 2 \overline{)\, x^2 + 0x - 10} \\ \underline{x^2 - x - 2} \\ x - 8 \end{array} \leftarrow \text{Remainder}$$

Therefore $\frac{x^2 - 10}{(x-2)(x+1)} \equiv 1 + \frac{x-8}{(x-2)(x+1)}$

Let $\frac{x-8}{(x-2)(x+1)} \equiv \frac{A}{(x-2)} + \frac{B}{(x+1)}$ Add fractions

$$\equiv \frac{A(x+1) + B(x-2)}{(x-2)(x+1)}$$

Set the numerators equal: $x - 8 \equiv A(x+1) + B(x-2)$

Substitute $x = 2 \Rightarrow 2 - 8 = A \times 3 + B \times 0$

$\Rightarrow -6 = 3A$

$\Rightarrow A = -2$

Substitute $x = -1 \Rightarrow -1 - 8 = A \times 0 + B \times (-3)$

$\Rightarrow -9 = -3B$

$\Rightarrow B = 3$

Hence

$$\frac{x^2 - 10}{(x-2)(x+1)} \equiv 1 + \frac{x-8}{(x-2)(x+1)} \equiv 1 + \frac{-2}{(x-2)} + \frac{3}{(x+1)}$$

$$\equiv 1 - \frac{2}{(x-2)} + \frac{3}{(x+1)}$$

(c) $\frac{x^3 - x^2 - x - 3}{x(x-1)} \equiv \frac{x^3 - x^2 - x - 3}{x^2 - x}$

Divide the numerator by the denominator:

$$\begin{array}{r} -3x + 2 \\ x^2 + 2x - 3 \overline{)\, -3x^3 + 4x^2 - 19x + 8} \\ \underline{-3x^3 - 6x^2 - 9x} \\ 2x^2 + 10x + 8 \\ \underline{2x^2 + 4x - 6} \\ 6x + 14 \end{array} \leftarrow \text{Remainder}$$

Therefore $\frac{x^3 - x^2 - x - 3}{x(x-1)} \equiv x + \frac{-x-3}{x(x-1)}$

Let $\frac{-x-3}{x(x-1)} \equiv \frac{A}{x} + \frac{B}{(x-1)}$ Add fractions

$\equiv \frac{A(x-1)+Bx}{x(x-1)}$

Set the numerators equal: $-x-3 \equiv A(x-1)+Bx$

Substitute $x = 1$ $\Rightarrow$ $-1-3 = A \times 0 + B \times 1$

$\Rightarrow$ $-4 = 1B$

$\Rightarrow$ $B = -4$

Substitute $x = 0$ $\Rightarrow$ $-3 = A \times (-1) + B \times 0$

$\Rightarrow$ $A = 3$

Hence $\frac{x^3-x^2-x-3}{x(x-1)} \equiv x + \frac{-x-3}{x(x-1)} \equiv x + \frac{3}{x} - \frac{4}{(x-1)}$

(d) $\frac{2x^2-1}{(x+1)^2} \equiv \frac{2x^2-1}{(x+1)(x+1)} \equiv \frac{2x^2-1}{x^2+2x+1} \equiv \frac{2x^2+0x-1}{x^2+2x+1}$

Divide the numerator by the denominator:

$$\begin{array}{r} 2 \\ x^2+2x+1 \overline{)\,2x^2+0x-1} \\ \underline{2x^2+4x+2} \\ -4x-3 \end{array} \leftarrow \text{Remainder}$$

Therefore $\frac{2x^2-1}{(x+1)^2} \equiv 2 + \frac{-4x-3}{(x+1)^2}$

Let $\frac{-4x-3}{(x+1)^2} \equiv \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$ Add fractions

$\equiv \frac{A(x+1)+B}{(x+1)^2}$

Set the numerators equal: $-4x-3 \equiv A(x+1)+B$

Substitute $x = -1$ $\Rightarrow$ $-4 \times (-1) - 3 = A \times 0 + B$

$\Rightarrow$ $1 = B$

$\Rightarrow$ $B = 1$

Equate coefficients in x: $-4 = A$

$\Rightarrow$ $A = -4$

Hence $\frac{2x^2-1}{(x+1)^2} \equiv 2 + \frac{-4x-3}{(x+1)^2} \equiv 2 - \frac{4}{(x+1)} + \frac{1}{(x+1)^2}$

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Partial fractions
Exercise E, Question 2

Question:

By factorising the denominator, express the following as partial fraction:

(a) $\dfrac{4x^2 + 17x - 11}{x^2 + 3x - 4}$

(b) $\dfrac{x^4 - 4x^3 + 9x^2 - 17x + 12}{x^3 - 4x^2 + 4x}$

Solution:

(a) Divide the numerator by the denominator:

$$\begin{array}{r} 4 \\ x^2+3x-4\overline{)\,4x^2+17x-11} \\ \underline{4x^2+12x-16} \\ 5x+5 \end{array} \leftarrow \text{Remainder}$$

Therefore $\dfrac{4x^2 + 17x - 11}{x^2 + 3x - 4} \equiv 4 + \dfrac{5x + 5}{x^2 + 3x - 4}$ Factorise denominator

$$\equiv 4 + \frac{5x + 5}{(x + 4)(x - 1)}$$

Let $\dfrac{5x + 5}{(x + 4)(x - 1)} \equiv \dfrac{A}{(x + 4)} + \dfrac{B}{(x - 1)}$ Add fractions

$$\equiv \frac{A(x - 1) + B(x + 4)}{(x + 4)(x - 1)}$$

Set the numerators equal: $5x + 5 \equiv A(x - 1) + B(x + 4)$

Substitute $x = 1 \quad \Rightarrow \quad 5 \times 1 + 5 = A \times 0 + B \times 5$

$\Rightarrow \quad 10 = 5B$

$\Rightarrow \quad B = 2$

Substitute $x = -4 \quad \Rightarrow \quad 5 \times (-4) + 5 = A \times (-5) + B \times 0$

$\Rightarrow \quad -15 = -5A$

$\Rightarrow \quad A = 3$

Hence

$$\frac{4x^2 + 17x - 11}{x^2 + 3x - 4} \equiv 4 + \frac{5x + 5}{(x + 4)(x - 1)} \equiv 4 + \frac{3}{(x + 4)} + \frac{2}{(x - 1)}$$

(b) Divide the numerator by the denominator:

$$4x^2+4x+0\overline{)\,x^4-4x^3+9x^2-17x+12}$$

$$\begin{array}{r} x \\ \hline x^4-4x^3+4x^2+0x \\ \hline 5x^2-17x+12 \end{array}$$

← Remainder

Therefore $\dfrac{x^4-4x^3+9x^2-17x+12}{x^3-4x^2+4x}$

$\equiv x+\dfrac{5x^2-17x+12}{x^3-4x^2+4x}$ Take out a factor of x in the denominator

$\equiv x+\dfrac{5x^2-17x+12}{x(x^2-4x+4)}$ Factorise the denominator fully

$\equiv x+\dfrac{5x^2-17x+12}{x(x-2)^2}$

Let $\dfrac{5x^2-17x+12}{x(x-2)^2}\equiv\dfrac{A}{x}+\dfrac{B}{(x-2)}+\dfrac{C}{(x-2)^2}$ Add fractions

$\equiv\dfrac{A(x-2)^2+Bx(x-2)+Cx}{x(x-2)^2}$

Set the numerators equal: $5x^2-17x+12\equiv A(x-2)^2+Bx(x-2)+Cx$

Substitute $x=0$ $\Rightarrow$ $0-0+12=A\times4+B\times0+C\times0$

$\Rightarrow$ $12=4A$

$\Rightarrow$ $A=3$

Substitute $x=2$ $\Rightarrow$ $20-34+12=A\times0+B\times0+C\times2$

$\Rightarrow$ $-2=2C$

$\Rightarrow$ $C=-1$

Equate coefficients in x^2: $5=A+B$

$\Rightarrow$ $5=3+B$

$\Rightarrow$ $B=2$

Hence

$$\frac{x^4-4x^3+9x^2-17x+12}{x^3-4x^2+4x}\equiv x+\frac{5x^2-17x+12}{x(x-2)^2}$$

$$\equiv x+\frac{3}{x}+\frac{2}{(x-2)}-\frac{1}{(x-2)^2}$$

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Partial fractions
Exercise E, Question 3

Question:

Show that $\frac{-3x^3 - 4x^2 + 19x + 8}{x^2 + 2x - 3}$ can be expressed in the form

$A + Bx + \frac{C}{(x - 1)} + \frac{D}{(x + 3)}$, where A, B, C and D are constants to be found.

Solution:

Divide the numerator by the denominator:

$$
\begin{array}{r}
-3x + 2 \\
x^2 + 2x - 3 \overline{)\,-3x^3 - 4x^2 + 19x + 8} \\
\underline{-3x^3 - 6x^2 + 9x} \\
2x^2 + 10x + 8 \\
\underline{2x^2 + 4x - 6} \\
6x + 14 \quad \leftarrow \text{Remainder}
\end{array}
$$

Therefore $\frac{-3x^3 - 4x^2 + 19x + 8}{x^2 + 2x - 3} \equiv -3x + 2 + \frac{6x + 14}{x^2 + 2x - 3}$ Factorise denominator

$$\equiv -3x + 2 + \frac{6x + 14}{(x - 1)(x + 3)}$$

Let $\frac{6x + 14}{(x - 1)(x + 3)} \equiv \frac{C}{(x - 1)} + \frac{D}{(x + 3)} \equiv \frac{C(x + 3) + D(x - 1)}{(x - 1)(x + 3)}$

Set the numerators equal: $6x + 14 \equiv C(x + 3) + D(x - 1)$

Substitute $x = 1$ $\Rightarrow$ $6 + 14 = C \times 4 + D \times 0$

$\Rightarrow$ $20 = 4C$

$\Rightarrow$ $5 = C$

Substitute $x = -3$ $\Rightarrow$ $6 \times (-3) + 14 = C \times 0 + D \times (-4)$

$\Rightarrow$ $-4 = -4D$

$\Rightarrow$ $D = 1$

Hence

$$\frac{-3x^3 - 4x^2 + 19x + 8}{x^2 + 2x - 3} \equiv -3x + 2 + \frac{6x + 14}{(x - 1)(x + 3)}$$

$$\equiv -3x + 2 + \frac{5}{(x - 1)} + \frac{1}{(x + 3)}$$

So $A = 2$, $B = -3$, $C = 5$ and $D = 1$

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Partial fractions
Exercise F, Question 1

Question:

Express the following as a partial fraction:

(a) $\frac{x-3}{x(x-1)}$

(b) $\frac{7x^2+2x-2}{x^2(x+1)}$

(c) $\frac{-15x+21}{(x-2)(x+1)(x-5)}$

(d) $\frac{x^2+1}{x(x-2)}$

Solution:

(a) Let $\frac{x-3}{x(x-1)} \equiv \frac{A}{x} + \frac{B}{(x-1)}$ Add the fractions

$\equiv \frac{A(x-1)+Bx}{x(x-1)}$

Set the numerators equal: $x-3 \equiv A(x-1)+Bx$

Substitute $x=1$ $\Rightarrow$ $1-3 = A\times 0 + B\times 1$

$\Rightarrow$ $B = -2$

Substitute $x=0$ $\Rightarrow$ $0-3 = A\times(-1) + B\times 0$

$\Rightarrow$ $-3 = -1A$

$\Rightarrow$ $A = 3$

Hence $\frac{x-3}{x(x-1)} \equiv \frac{3}{x} - \frac{2}{(x-1)}$

(b) Let $\frac{7x^2+2x-2}{x^2(x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)}$ Add the fractions

$\equiv \frac{Ax(x+1)+B(x+1)+Cx^2}{x^2(x+1)}$

Set the numerators equal:

$7x^2+2x-2 \equiv Ax(x+1)+B(x+1)+Cx^2$

Substitute $x=0$ $\Rightarrow$ $0+0-2 = A\times 0 + B\times 1 + C\times 0$

$\Rightarrow$ $-2 = 1B$

$\Rightarrow$ $B = -2$

Substitute $x=-1$ $\Rightarrow$ $7-2-2 = A\times 0 + B\times 0 + C\times 1$

$\Rightarrow \quad 3 = 1C$

$\Rightarrow \quad C = 3$

Equate coefficients in x^2: $\quad 7 = A + C \quad$ Substitute $C = 3$

$\Rightarrow \quad 7 = A + 3$

$\Rightarrow \quad A = 4$

Hence $\dfrac{7x^2 + 2x - 2}{x^2(x+1)} \equiv \dfrac{4}{x} - \dfrac{2}{x^2} + \dfrac{3}{(x+1)}$

(c) Let $\dfrac{-15x + 21}{(x-2)(x+1)(x-5)} \equiv \dfrac{A}{(x-2)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x-5)}$ Add the fractions

$$\equiv \frac{A(x+1)(x-5) + B(x-2)(x-5) + C(x-2)(x+1)}{(x-2)(x+1)(x-5)}$$

Set the numerators equal:

$-15x + 21 \equiv A(x+1)(x-5) + B(x-2)(x-5) + C(x-2)(x+1)$

Substitute $x = -1 \quad \Rightarrow \quad 15 + 21 = A \times 0 + B \times (-3) \times (-6) + C \times 0$

$\Rightarrow \quad 36 = 18B$

$\Rightarrow \quad B = 2$

Substitute $x = 5 \quad \Rightarrow \quad -75 + 21 = A \times 0 + B \times 0 + C \times 3 \times 6$

$\Rightarrow \quad -54 = 18C$

$\Rightarrow \quad C = -3$

Substitute $x = 2 \quad \Rightarrow \quad -30 + 21 = A \times 3 \times (-3) + B \times 0 + C \times 0$

$\Rightarrow \quad -9 = -9A$

$\Rightarrow \quad A = 1$

Hence $\dfrac{-15x + 21}{(x-2)(x+1)(x-5)} \equiv \dfrac{1}{(x-2)} + \dfrac{2}{(x+1)} - \dfrac{3}{(x-5)}$

(d) $\dfrac{x^2 + 1}{x(x-2)} \equiv \dfrac{x^2 + 1}{x^2 - 2x} \equiv \dfrac{x^2 + 0x + 1}{x^2 - 2x + 0}$

Divide the numerator by the denominator:

$$\begin{array}{r} 1 \\ x^2 - 2x + 0 \overline{)\, x^2 + 0x + 1} \\ \underline{x^2 - 2x + 0} \\ 2x + 1 \end{array} \leftarrow \text{Remainder}$$

Therefore $\dfrac{x^2 + 1}{x(x-2)} \equiv 1 + \dfrac{2x + 1}{x(x-2)}$

Let $\dfrac{2x + 1}{x(x-2)} \equiv \dfrac{A}{x} + \dfrac{B}{(x-2)} \equiv \dfrac{A(x-2) + Bx}{x(x-2)}$

Set the numerators equal: $\quad 2x + 1 \equiv A(x-2) + Bx$

Substitute $x = 0 \quad \Rightarrow \quad 1 = A \times (-2) + B \times 0$

$\Rightarrow \quad 1 = -2A$

$\Rightarrow \quad A = -\frac{1}{2}$

Substitute $x = 2 \quad \Rightarrow \quad 2 \times 2 + 1 = A \times 0 + B \times 2$

$\Rightarrow \quad 5 = 2B$

$\Rightarrow \quad B = \frac{5}{2}$

Hence

$$\frac{x^2 + 1}{x(x - 2)} \equiv 1 + \frac{2x + 1}{x(x - 2)} \equiv 1 + \frac{-\frac{1}{2}}{x} + \frac{\frac{5}{2}}{(x - 2)}$$

$$\equiv 1 - \frac{1}{2x} + \frac{5}{2(x - 2)}$$

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Partial fractions
Exercise F, Question 2

Question:

Write the following algebraic fractions as a partial fraction:

(a) $\frac{3x + 1}{x^2 + 2x + 1}$

(b) $\frac{2x^2 + 2x - 8}{x^2 + 2x - 3}$

(c) $\frac{3x^2 + 12x + 8}{(x + 2)^3}$

(d) $\frac{x^4}{x^2 - 2x + 1}$

Solution:

(a) $\frac{3x + 1}{x^2 + 2x + 1} \equiv \frac{3x + 1}{(x + 1)^2}$ Repeated factor in denominator

Let $\frac{3x + 1}{(x + 1)^2} \equiv \frac{A}{(x + 1)} + \frac{B}{(x + 1)^2} \equiv \frac{A(x + 1) + B}{(x + 1)^2}$

Set the numerators equal: $3x + 1 \equiv A(x + 1) + B$

Substitute $x = -1$ $\Rightarrow$ $-3 + 1 = A \times 0 + B$

$\Rightarrow$ $B = -2$

Equate coefficients of x: $3 = A$

$\Rightarrow$ $A = 3$

Hence $\frac{3x + 1}{x^2 + 2x + 1} \equiv \frac{3x + 1}{(x + 1)^2} \equiv \frac{3}{(x + 1)} - \frac{2}{(x + 1)^2}$

(b) $\frac{2x^2 + 2x - 8}{x^2 + 2x - 3}$ is an **improper fraction**

Dividing gives

$$
\begin{array}{r}
2 \\
x^2+2x-3\overline{)2x^2+2x-8} \\
\underline{2x^2+4x-6} \\
-2x-2
\end{array}
\quad \leftarrow \text{Remainder}
$$

Therefore $\frac{2x^2 + 2x - 8}{x^2 + 2x - 3} \equiv 2 + \frac{-2x - 2}{x^2 + 2x - 3}$ Factorise the denominator

$$\equiv 2 + \frac{-2x-2}{(x+3)(x-1)}$$

Let $\frac{-2x-2}{(x+3)(x-1)} \equiv \frac{A}{(x+3)} + \frac{B}{(x-1)} \equiv \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$

Set the numerators equal: $-2x-2 \equiv A(x-1)+B(x+3)$

Substitute $x = 1$ $\Rightarrow$ $-2-2 = A \times 0 + B \times 4$

$\Rightarrow$ $-4 = 4B$

$\Rightarrow$ $B = -1$

Substitute $x = -3$ $\Rightarrow$ $6-2 = A \times (-4) + B \times 0$

$\Rightarrow$ $4 = -4A$

$\Rightarrow$ $A = -1$

Hence

$$\frac{2x^2+2x-8}{x^2+2x-3} \equiv 2 + \frac{-2x-2}{(x+3)(x-1)}$$

$$\equiv 2 - \frac{1}{(x+3)} - \frac{1}{(x-1)}$$

(c) Let $\frac{3x^2+12x+8}{(x+2)^3} \equiv \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}$

$$\equiv \frac{A(x+2)^2 + B(x+2) + C}{(x+2)^3}$$

Set the numerators equal:

$3x^2+12x+8 \equiv A(x+2)^2 + B(x+2) + C$

Substitute $x = -2$ $\Rightarrow$ $12-24+8 = A \times 0 + B \times 0 + C$

$\Rightarrow$ $C = -4$

Equate coefficients in x^2: $3 = A$

$\Rightarrow$ $A = 3$

Equate coefficients in x: $12 = 4A + B$ Substitute $A = 3$

$\Rightarrow$ $12 = 12 + B$

$\Rightarrow$ $B = 0$

Hence $\frac{3x^2+12x+8}{(x+2)^3} \equiv \frac{3}{(x+2)} - \frac{4}{(x+2)^3}$

(d) $\frac{x^4}{x^2-2x+1} \equiv \frac{x^4+0x^3+0x^2+0x+0}{x^2-2x+1}$

Divide the numerator by the denominator:

$$
\begin{array}{r}
x^2+2x+3 \\
x^2-2x+1\overline{)\,x^4+0x^3+0x^2+0x+0} \\
\underline{x^4-2x^3+x^2} \\
2x^3-x^2+0x \\
\underline{2x^3-4x^2+2x} \\
3x^2-2x+0 \\
\underline{3x^2-6x+3} \\
4x-3
\end{array}
\quad \leftarrow \text{Remainder}
$$

Therefore

$$\frac{x^4}{x^2-2x+1} \equiv x^2+2x+3+\frac{4x-3}{x^2-2x+1}$$ Factorise the denominator

$$\equiv x^2+2x+3+\frac{4x-3}{(x-1)^2}$$

Let $\frac{4x-3}{(x-1)^2} \equiv \frac{A}{(x-1)} + \frac{B}{(x-1)^2} \equiv \frac{A(x-1)+B}{(x-1)^2}$

Set the numerators equal: $4x-3 \equiv A(x-1)+B$

Substitute $x = 1 \quad \Rightarrow \quad 4-3 = B \quad \Rightarrow \quad B = 1$

Equate coefficients in x: $4 = A$

Hence

$$\frac{x^4}{x^2-2x+1} \equiv x^2+2x+3+\frac{4x-3}{(x-1)^2} \equiv x^2+2x+3+\frac{4}{(x-1)}+\frac{1}{(x-1)^2}$$

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Partial fractions
Exercise F, Question 3

Question:

Given that $f(x) = 2x^3 + 9x^2 + 10x + 3$:

(a) Show that -3 is a root of $f(x)$.

(b) Express $\frac{10}{f(x)}$ as partial fractions.

E

Solution:

(a) $f(-3) = 2 \times (-27) + 9 \times 9 + 10 \times (-3) + 3 = -54 + 81 - 30 + 3 = 0$

Therefore -3 is a root $\Rightarrow$ $(x + 3)$ is a factor

(b) $f(x) = 2x^3 + 9x^2 + 10x + 3$ $(x + 3)$ is a factor

$= (x + 3)(2x^2 + 3x + 1)$ By inspection

$= (x + 3)(2x + 1)(x + 1)$

$$\frac{10}{f(x)} \equiv \frac{10}{(x+3)(2x+1)(x+1)} \equiv \frac{A}{(x+3)} + \frac{B}{(2x+1)} + \frac{C}{(x+1)}$$

$$\equiv \frac{A(2x+1)(x+1) + B(x+3)(x+1) + C(x+3)(2x+1)}{(x+3)(2x+1)(x+1)}$$

Set the numerators equal:

$10 \equiv A(2x + 1)(x + 1) + B(x + 3)(x + 1) + C(x + 3)(2x + 1)$

Substitute $x = -1$ $\Rightarrow$ $10 = A \times 0 + B \times 0 + C \times 2 \times (-1)$

$\Rightarrow$ $10 = -2C$

$\Rightarrow$ $C = -5$

Substitute $x = -3$ $\Rightarrow$ $10 = A \times (-5) \times (-2) + B \times 0 + C \times 0$

$\Rightarrow$ $10 = 10A$

$\Rightarrow$ $A = 1$

Substitute $x = -\frac{1}{2}$ $\Rightarrow$ $10 = A \times 0 + B \times \left(2\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + C \times 0$

$\Rightarrow \quad 10 = 1.25B$

$\Rightarrow \quad B = 8$

Hence $\dfrac{10}{\mathrm{f}(x)} \equiv \dfrac{1}{(x+3)} + \dfrac{8}{(2x+1)} - \dfrac{5}{(x+1)}$

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