Edexcel GCE

Core Mathematics C3

Differentiation

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (i) Differentiate with respect to x

(*a*) $x^2 \cos 3x$,

(3)

(*b*) $\dfrac{\ln(x^2+1)}{x^2+1}$.

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\frac{1}{4}, \quad y > 0.$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

2. (*a*) Find the value of $\dfrac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2\sqrt{(5x-1)}.$$

(6)

(*b*) Differentiate $\dfrac{\sin 2x}{x^2}$ with respect to x.

(4)

3. (*a*) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x+2)$.

(3)

Given that $y = \dfrac{3x^2+6x-7}{(x+1)^2}$, $x \neq -1$,

(*b*) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(*c*) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)

4. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(*a*) Show that the turning points on C occur where $\tan x = -1$.

(6)

(*b*) Find an equation of the tangent to C at the point where $x = 0$.

(2)

5. A curve C has equation $y = x^2 e^x$.

(*a*) Find $\frac{dy}{dx}$, using the product rule for differentiation.

(3)

(*b*) Hence find the coordinates of the turning points of C.

(3)

(*c*) Find $\frac{d^2y}{dx^2}$.

(2)

(*d*) Determine the nature of each turning point of the curve C.

(2)

6. The curve C has equation $x = 2 \sin y$.

(*a*) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(*b*) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(*c*) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)

7. (i) The curve C has equation $y = \frac{x}{9 + x^2}$.

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that $y = (1 + e^{2x})^{\frac{3}{2}}$, find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2} \ln 3$.

(5)

8. Differentiate, with respect to x,

(a) $e^{3x} + \ln 2x$,

(3)

(*b*) $(5 + x^2)^{\frac{3}{2}}$.

(3)

9. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(5)

10. (*a*) Differentiate with respect to x

(i) $x^2 e^{3x+2}$,

(4)

(ii) $\dfrac{\cos(2x^3)}{3x}$.

(4)

(*b*) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x.

(5)

11. (*a*) Differentiate with respect to x

(i) $3\sin^2 x + \sec 2x$,

(3)

(ii) $\{x + \ln(2x)\}^3$.

(3)

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq 1$,

(*b*) show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$.

(6)

12. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$$

(*a*) Show that $f(x) = \dfrac{x-3}{x-2}$.

(5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2.$$

(*b*) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$.

(3)

(*c*) Find the exact values of x for which $g'(x) = 1$

(4)

13.

$$f(x) = \frac{2x+2}{x^2 - 2x - 3} - \frac{x+1}{x-3}.$$

(*a*) Express $f(x)$ as a single fraction in its simplest form.

(4)

(*b*) Hence show that $f'(x) = \dfrac{2}{(x-3)^2}$.

(3)

14.

$$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2 + 3x - 2}, \quad x > \frac{1}{2}.$$

(*a*) Show that $f(x) = \dfrac{4x-6}{2x-1}$.

(7)

(*b*) Hence, or otherwise, find $f'(x)$ in its simplest form.

(3)