

Best Practices for Word-Problem Solving

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Say hello.

Describe the mathematics you
support.



Instructional Platform



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction



MODELING

Step-by-step
explanation

Planned examples

PRACTICE

Guided practice

Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

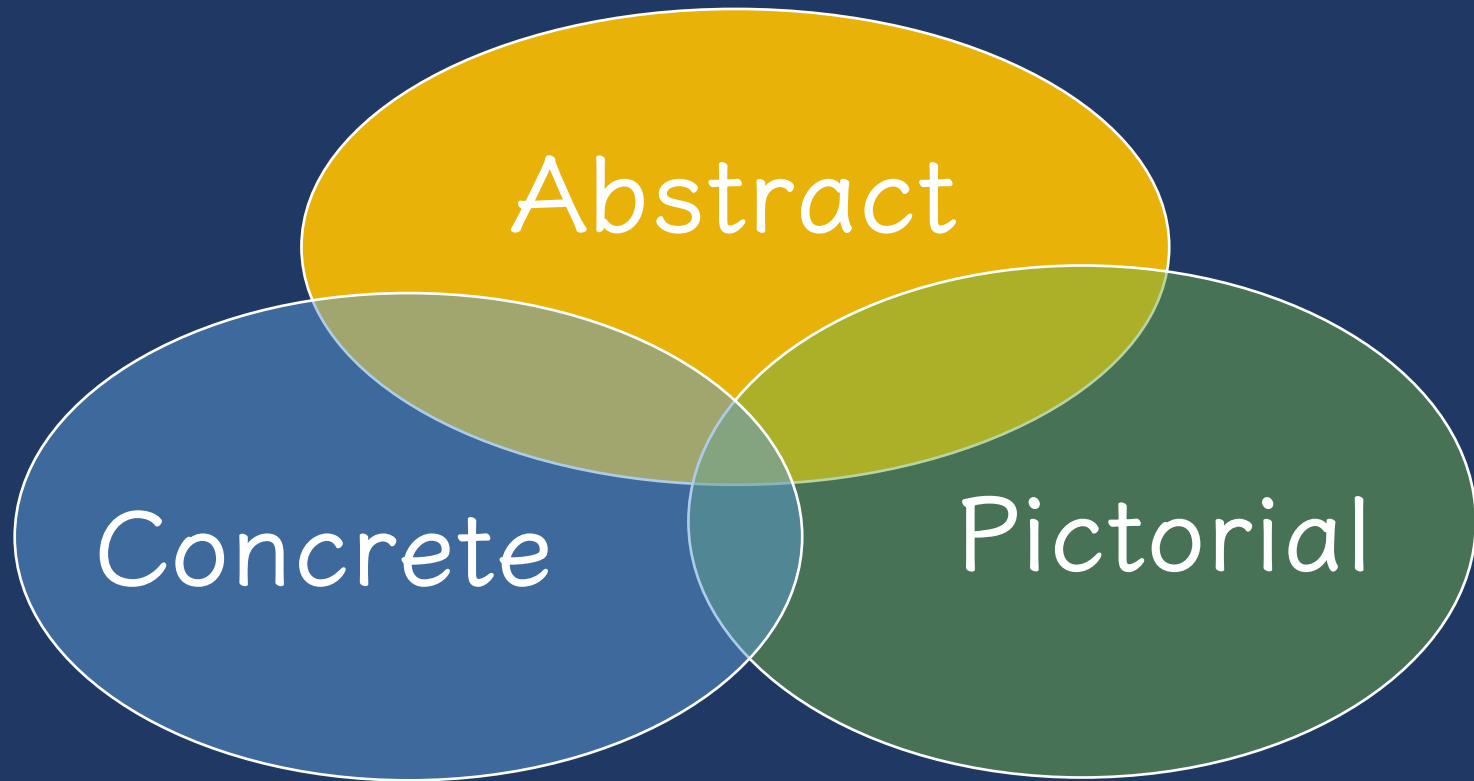
Providing affirmative and corrective feedback



Use formal math language

Use terms precisely





Addition	Subtraction
Multiplication	Division

Counting

Comparing
numbers

Counting
coins

Telling
time

Identifying
equivalent
fractions

Knowing
multiples

Identifying
shapes

Knowing
formulas



Word-Problem Solving



How Students Solve Word Problems



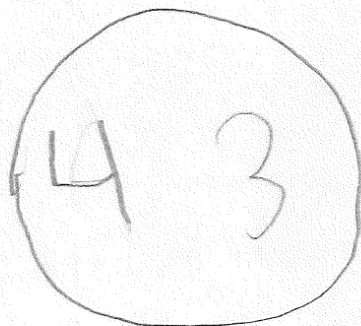
N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



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71

N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



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Donna



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

96 paper fold



- N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?



N. Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?

$$\begin{array}{r} 96 \\ + 25 \\ \hline 121 \end{array}$$



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✓

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$$\begin{array}{r} 96 \\ \times 25 \\ \hline 121 \end{array}$$

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Correct



Undetermined



Repeated
information



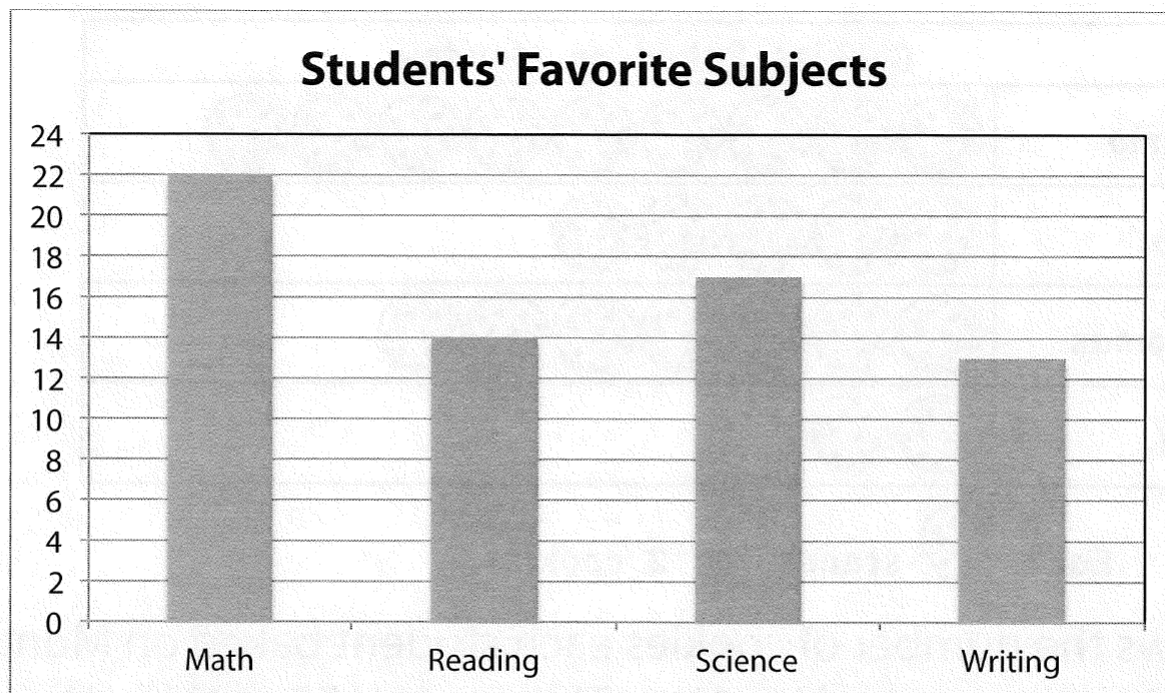
Wrong
information



Wrong
(misapplied)
operation



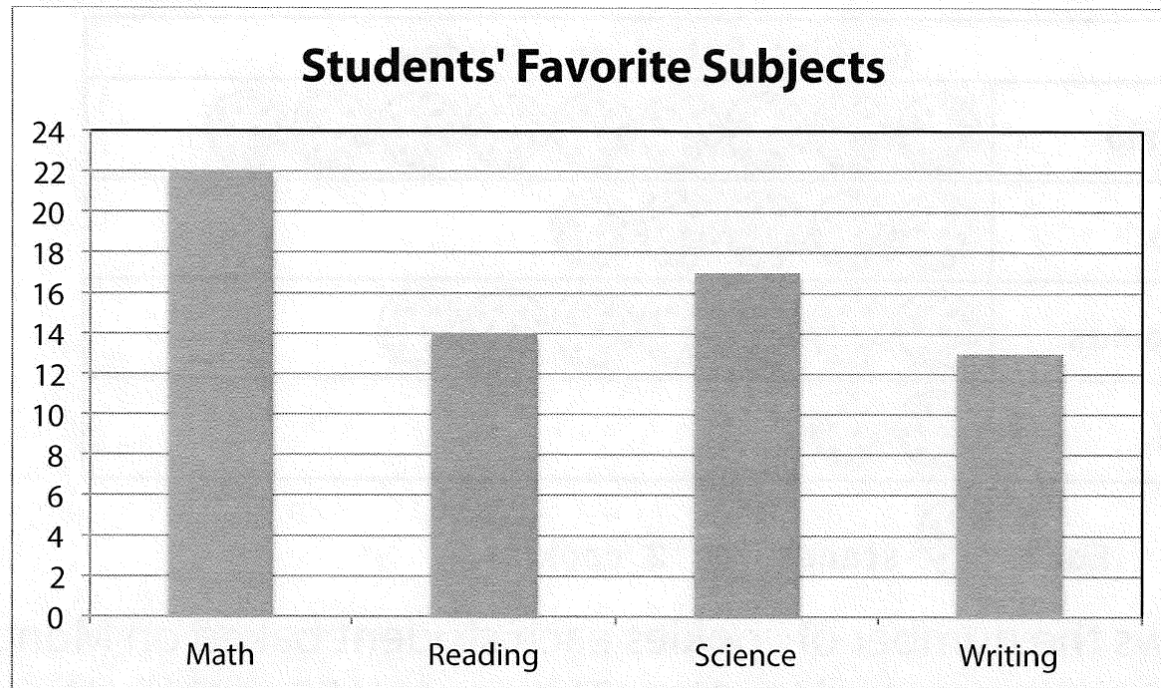
J.



The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?



J.

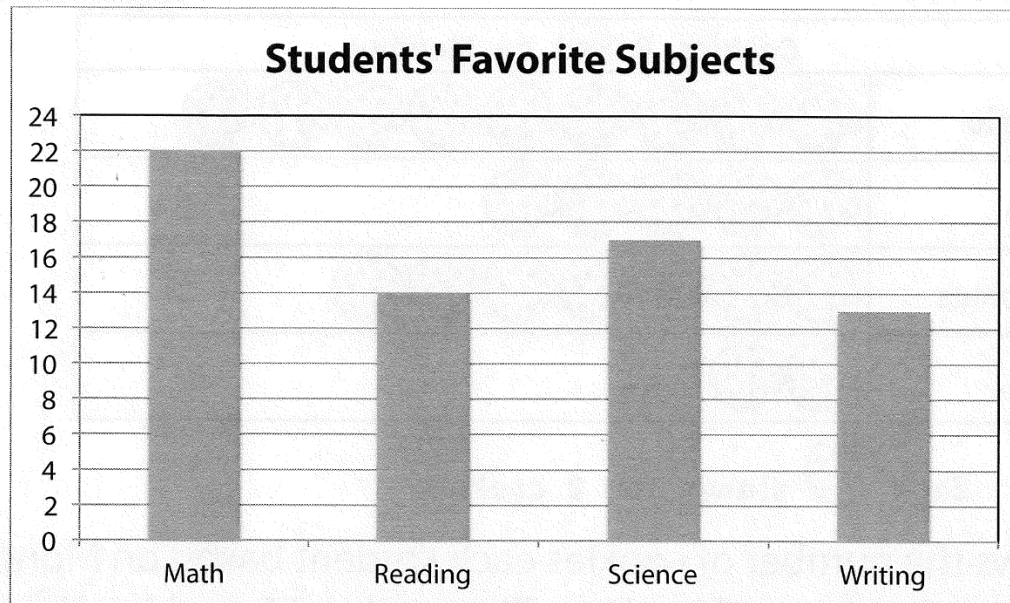


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

60

|||||

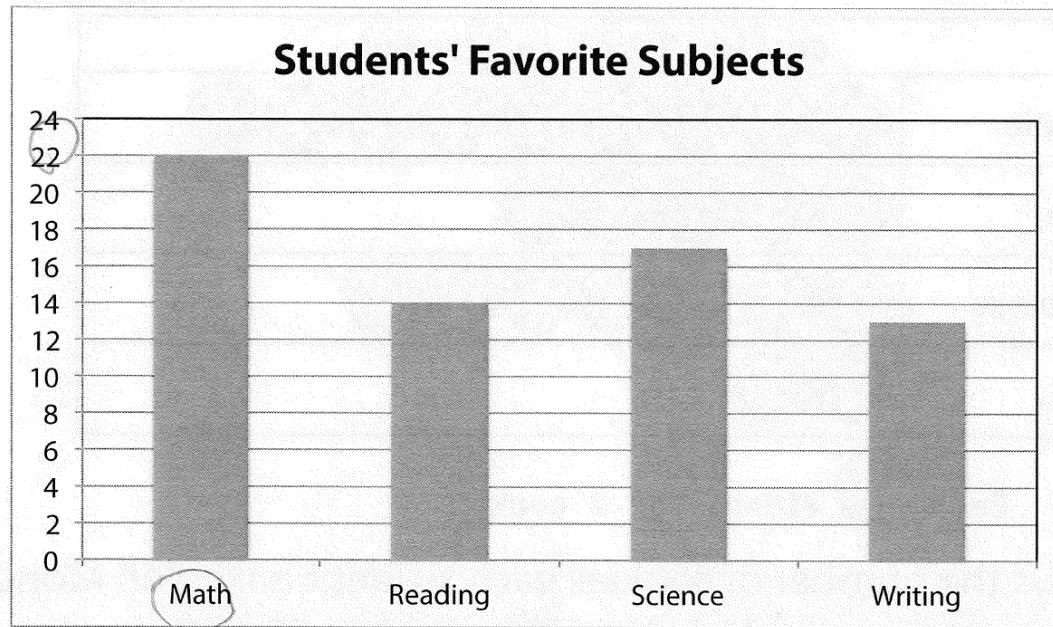
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The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

They choose 8% more than
reading

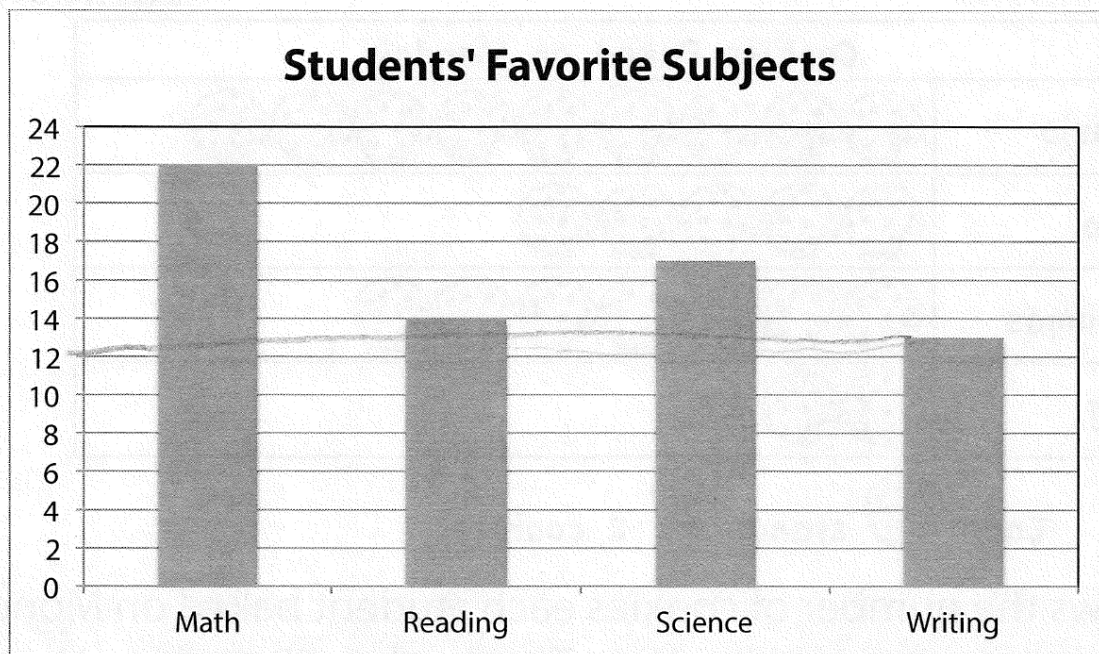
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The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

22
Math

J.



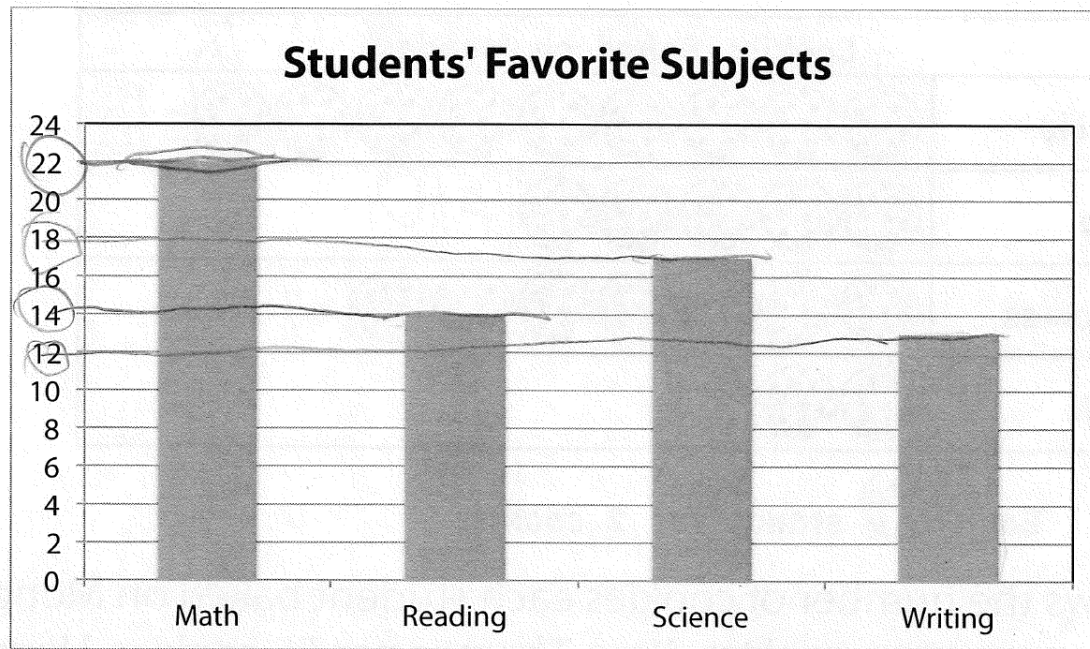
The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

(22) math

(13) writing



J.

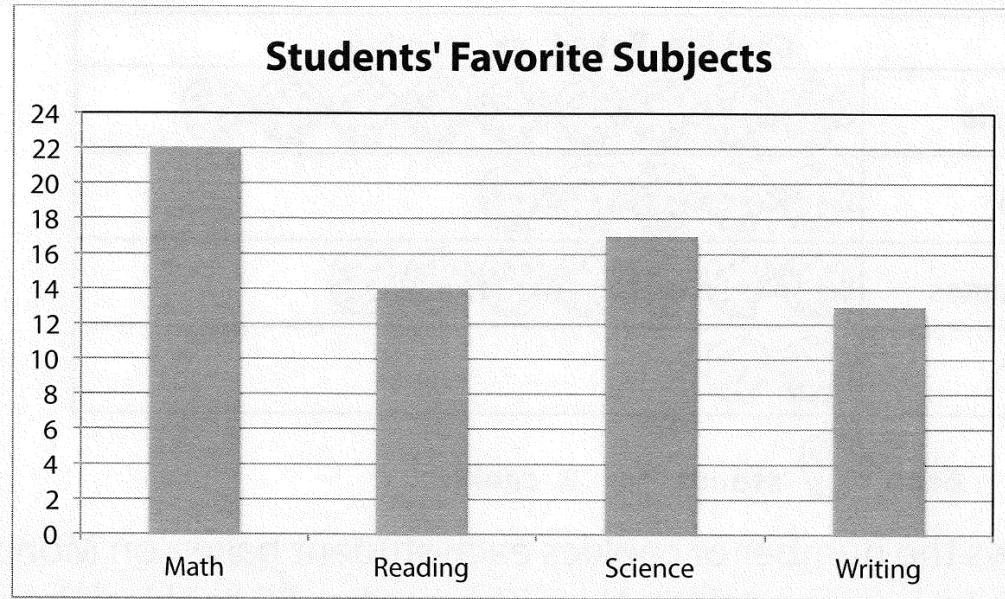


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

22
Math

14
Writing

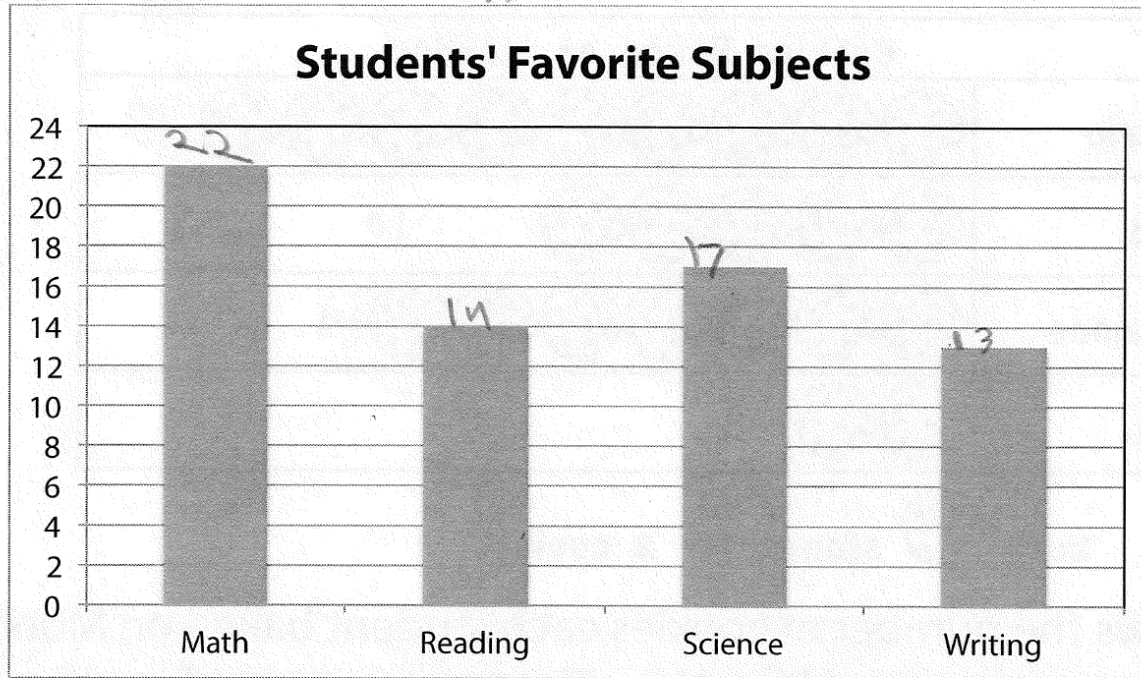
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the students chooses they
love more math d'can es
so 22 math

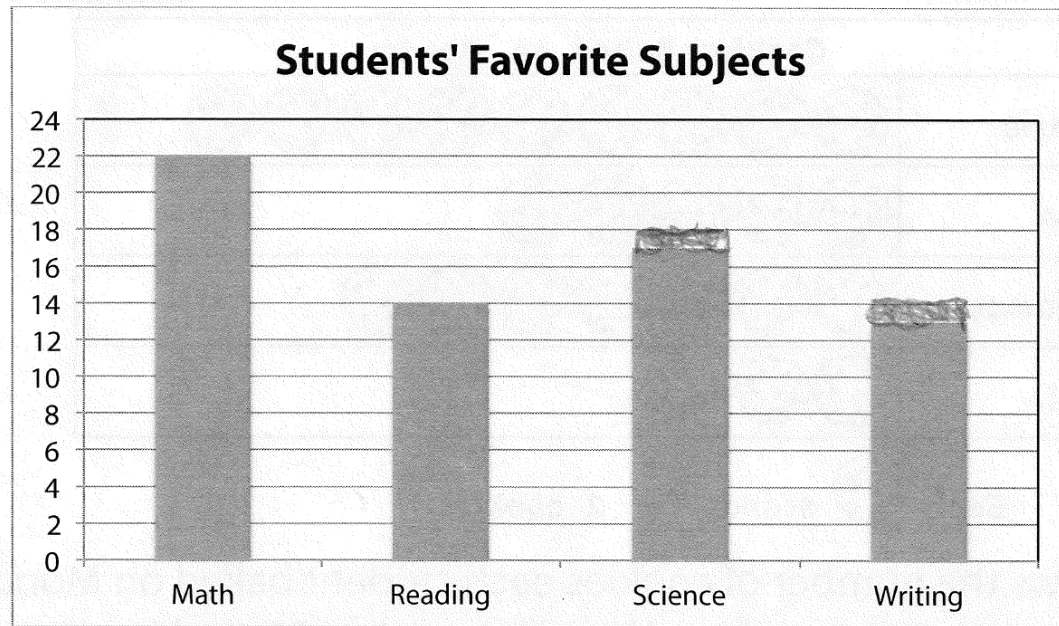
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The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ - 13 \\ \hline 9 \end{array}$$

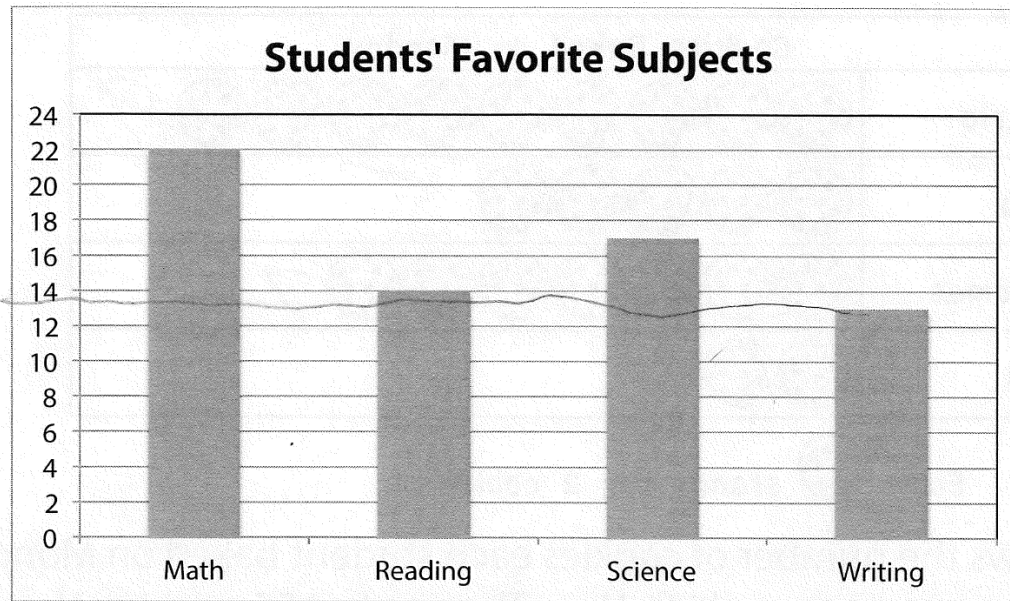
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The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 24 \\ + 14 \\ \hline 36 \end{array}$$

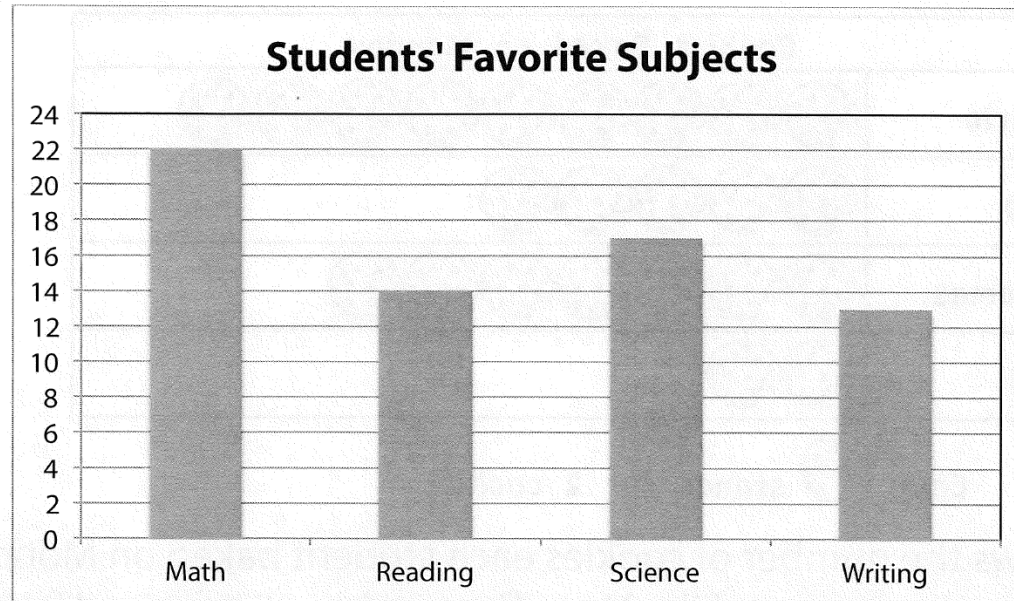
J.



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$$\begin{array}{r} 24 \\ - 13 \\ \hline 11 \end{array}$$

J.

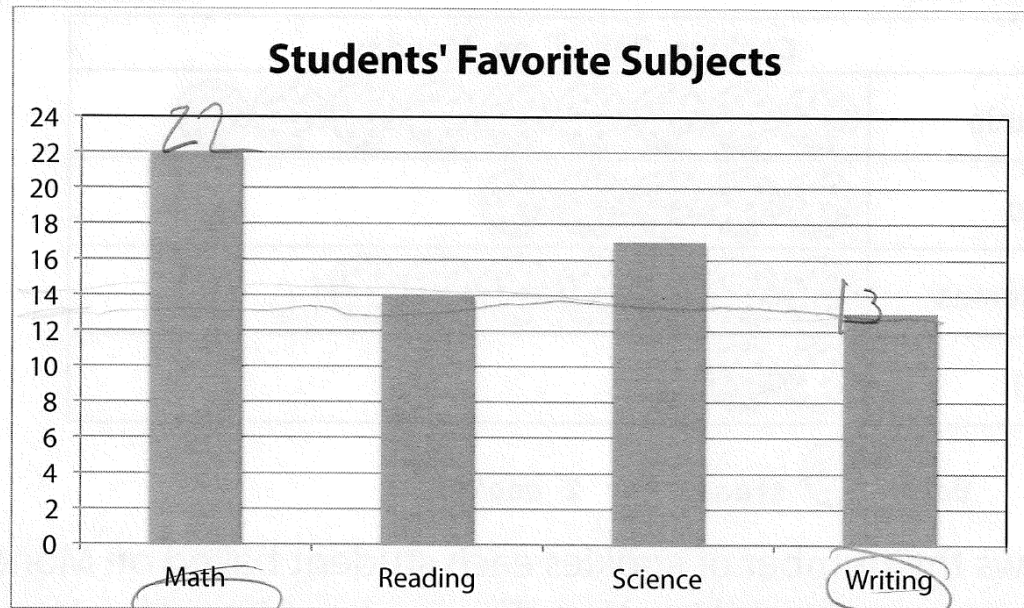


The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?

$$\begin{array}{r} 22 \\ + 12 \\ \hline 34 \end{array}$$

34

J.

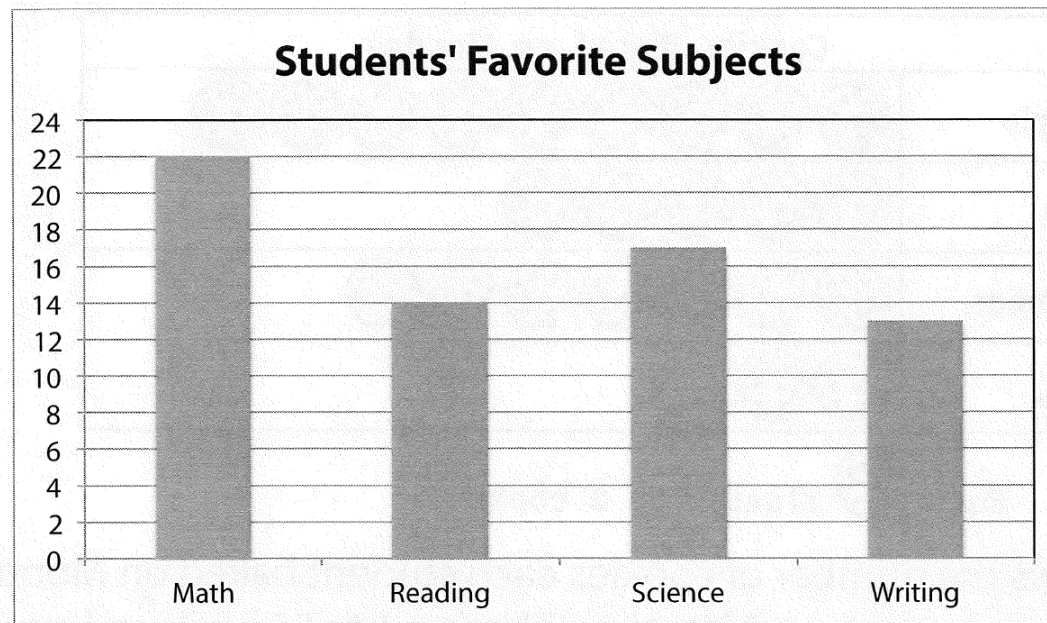


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$$\begin{array}{r} 22 \\ + 13 \\ \hline 35 \end{array}$$

14

J.

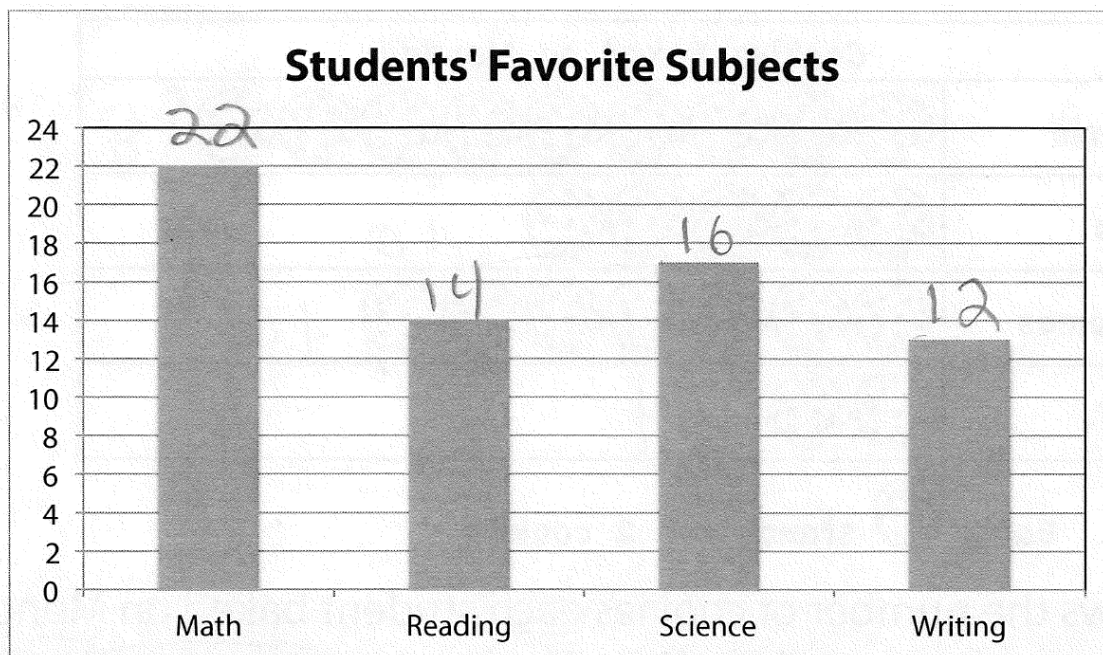


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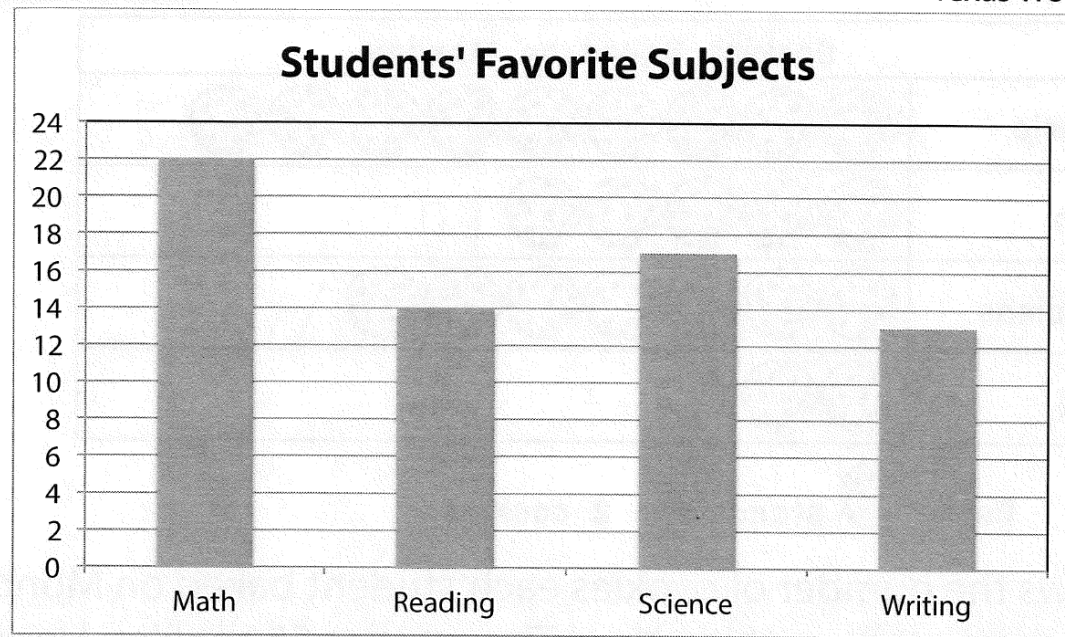
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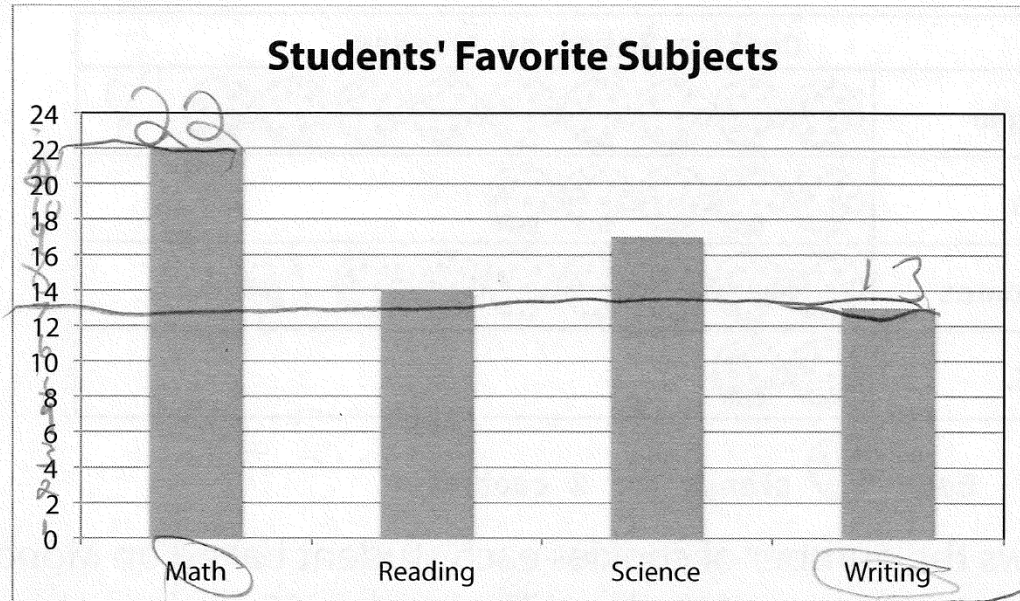
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$$\begin{array}{r}
 22 \\
 - 13 \\
 \hline
 9
 \end{array}$$

Correct



Undetermined



Didn't provide
answer



Wrong
(misapplied)
operation



Wrong
information



Subtraction error



COMMON

Undetermined

Repeated
information from
problem

Didn't provide
answer

Wrong
information

Wrong
(misapplied)
operation

UNCOMMON

Addition error

Subtraction error



Word-Problem Solving

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20

Solve the problem

What skills are necessary to solve this problem?



Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
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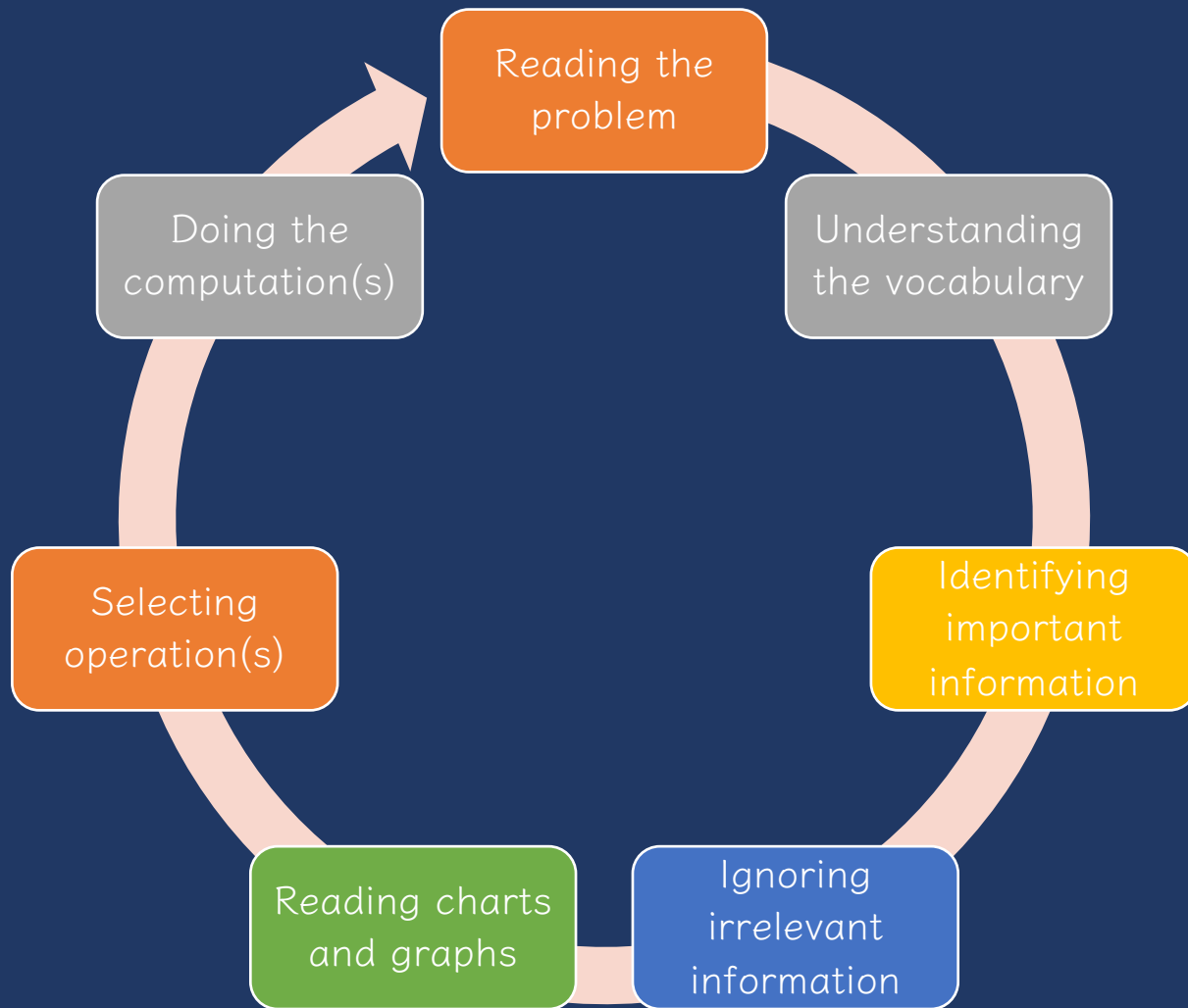
How would you solve this problem?
What skills are necessary to solve this problem?



Word-Problem Solving

Teaching Problem Solving





Ineffective Strategies





1. Keywords tied to operations



Lincoln had 8 pencils **fewer** than Roscoe. If Roscoe had 18 pencils, how many pencils did Lincoln have?

Lincoln had 8 pencils **fewer** than Roscoe. If Lincoln had 18 pencils, how many pencils did Roscoe have?

Key Words Used In Math Word Problems

Addition Words	Subtraction Words
<ul style="list-style-type: none"> + all together or altogether + and + both + combined + how many in all + how much + in all + increased by + plus + sum + together + total 	<ul style="list-style-type: none"> - change - decreased by - difference - fewer or fewer than - how many are left (or have left) - how many did not have - how many - how much - taller, heavier, less or less than - lost - minus - need to - reduce - remain - subtract - take away

Multiplication Words

- x by (dimension)
- x double
- x each group
- x every
- x factor of
- x increased by
- x multiplied by
- x of
- x product
- x times
- x triple

Division Words

- ÷ as much
- ÷ cut up
- ÷ each group
- ÷ equal share
- ÷ half (or at)
- ÷ how many
- ÷ parts
- ÷ per
- ÷ percent
- ÷ quotient
- ÷ ratio of
- ÷ separated
- ÷ share some

OPERATION clue words

ADDITION	SUBTRACTION
<ul style="list-style-type: none"> and total join more than in all altogether sum increased 	<ul style="list-style-type: none"> less than decreased remaining change left fewer take away difference minus

MULTIPLICATION	DIVISION
<ul style="list-style-type: none"> product times twice as many as by of equal groups 	<ul style="list-style-type: none"> quotient each broken into per distribute evenly parts

KEY Words

ADDITION	MULTIPLICATION
<ul style="list-style-type: none"> -sum -total -more than -plus 	<ul style="list-style-type: none"> -both -combined -increased by -perimeter

SUBTRACTION	DIVISION
<ul style="list-style-type: none"> -difference -remain -left -less than -minus -how many more 	<ul style="list-style-type: none"> -fewer -decrease -give away -reduce -discount -how many more

Division

Taking a whole and sharing it into equal parts

Addition

Putting two or more things/amounts together.

Keywords

- Total
- Altogether
- In all
- Sum
- more than
- combined

Problem Solving Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add together 	<ul style="list-style-type: none"> are not decrease difference fewer, larger, shorter left less than minus remain take away

key words

Addition

- combined
- addition
- sum
- both
- in all
- together
- total
- plus
- perimeter
- add
- more than

Subtraction

- triple
- factor
- product
- multiply
- each
- per
- in all
- multiple
- area
- double
- times

Division

- average
- division
- split
- quotient
- equal groups
- divide
- half
- shared equally
- each
- distribute

Math Operation - Key Words

Addition	Subtraction
<ul style="list-style-type: none"> add altogether and both in all sum total increase 	<ul style="list-style-type: none"> difference fewer than gave/take away decreased by how many more show much longer/smaller/shorter minus remaining

Multiplication	Division
<ul style="list-style-type: none"> area product Each by - of - per Times double, twice, triple total increase 	<ul style="list-style-type: none"> quotient divide into equal parts/share equally per amount of each

Math Key Words

Addition	Subtraction	Multiplication	Division
<ul style="list-style-type: none"> plus sum add total all together increase more combine 	<ul style="list-style-type: none"> subtract minus difference left left over decrease take away fewer 	<ul style="list-style-type: none"> times product factor double groups each area rows 	<ul style="list-style-type: none"> quotient split share divide separate each average equal groups







Description of Single-Step Word Problems (n = 132)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords ^a		Multiple keywords ^a		Keyword(s) led to correct solution ^a	
	n	%	n	%	n	%	n	%	n	%
Total	27	20.5	26	96.3	23	88.5	5	19.2	21	80.8
Difference	17	12.9	17	100.0	14	82.4	2	11.8	12	70.6
Change	11	8.3	7	63.6	5	71.4	5	71.4	2	28.6
Equal groups	29	22.0	26	89.7	22	84.6	18	69.2	8	30.8
Comparison	10	7.6	9	90.0	9	100.0	4	44.4	5	55.6
Ratios or proportions	29	22.0	23	79.3	9	39.1	9	39.1	6	26.1
Product of measures	9	6.8	9	100.0	8	88.9	1	11.1	5	55.6

^aWhen a problem featured a keyword.



Description of Multi-Step Word Problems (n = 84)

Schema	Occurrence of schema ^a		Any keyword		Keyword(s) led to correct solution ^b	
	n	%	n	%	n	%
Total	40	47.6	39	97.5	3	7.7
Difference	11	13.1	11	100.0	1	9.1
Change	21	23.8	19	95.0	1	5.3
Equal groups	49	58.3	48	98.0	1	2.1
Comparison	7	8.3	7	100.0	0	0.0
Ratios or proportions	22	25.0	16	76.2	1	6.3
Product of measures	7	8.3	7	100.0	2	28.6

^aSum across schemas does not equal 100 because each word problem featured more than one schema.

^bWhen a problem featured a keyword.



Mr. Rivera's taxable income is \$20 each hour before taxes are taken out. Mr. Rivera worked a total of 40 hours each week for 50 weeks.

What is the dollar amount, to the nearest dollar, taken out for taxes based on Mr. Rivera's taxable income?

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount, in dollars, did Jessica pay to rent each movie?

The temperature of a substance decreased by 24°C per minute for 3 minutes. What was the overall change of the temperature of the substance?



Keywords are important to identify and understand

Keywords are the mathematical vocabulary that help an students understand what the story is about and what they need to do

Talk about keywords
("What does *more than* tell you about?")



But, *do not* tie a keyword to a specific operation!





2. Presenting problems by operation



Name: _____

Date: _____

Addition Word Problems

Solve the word problems. Show your work.

1. Noah had 12 books. He got 5 more books. How many books did Noah have in all?
2. Bonnie found 8 rocks on her front yard and 7 rocks in her backyard. How many rocks did she find in all?
3. Edward had 5 toy cars. He got 3 more toy cars. How many toy cars did Edward have in all?
4. Mariela collected 11 feathers. She found 3 more feathers. How many feathers did she have in all?
5. LaMonte made 14 cookies. He made 7 more cookies. How many cookies did LaMonte have in all?

Division Word Problems

1. Zookeeper Al has 567 bananas. He has 9 monkeys at the zoo and an equal number of bananas for each monkey. How many bananas does each monkey get? And how many are left over?
2. Betty has 427 oranges. She wants to pack them up equally in 23 boxes. How many oranges will she have in each box and how much does she have left over?
3. Mr. King has 1376 pages of scrap paper. He wants to divide the paper equally among 32 students. How many pages will each student get? How many extra pages will there be?
4. Mr. King has 1376 pages of scrap paper. He instead wants to divide the paper equally among 32 students. Will there be enough paper for each student? If not, how much more scrap paper does he need?

Effective Strategies



Teach an attack strategy

Teach about schemas



Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20



What was your process for working through this problem?



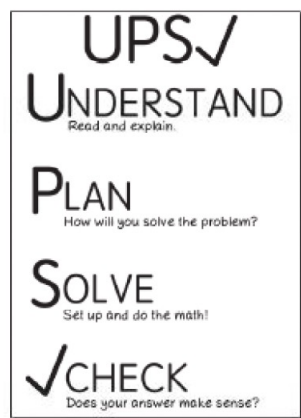
Attack Strategy

SOLVE

Study the problem.
Organize the facts.
Line up the plan.
Verify the plan with computation.
Examine the answer.

R-CUBES

Read the problem.
Circle key numbers.
Underline the question.
Box action words.
Evaluate steps.
Solve and check.



RIDE

Read the problem.

Identify the relevant information.

Determine the operation and unit for the answer.

Enter the correct numbers and calculate, then check the answer.

RIDGES

Read the problem.

I know statement.

Draw a picture.

Goal statement.

Equation development.

Solve the equation.



STAR

Stop and read the problem carefully.

Think about your plan and the strategy you will use.

Act. Follow your plan and solve the problem.

Review your answer.

RICE

Read and record the problem.

Illustrate your thinking.

Compute.

Explain your thinking.



SUPER

Slowly read the story problem twice.
Underline the question and circle the numbers you need.
Picture it. Draw the scenario to show what is happening.
Explain the problem with a number sentence.
Rewrite the answer in a sentence.

SHINES

Slowly and carefully read the problem.
Highlight or underline key information.
Identify the question by drawing a circle around it.
Now solve the problem. Show your work.
Examine your work for precision, accuracy, and clarity.
Share your answer by writing a sentence.



SOLVE

Study the problem.

Organize the facts.

Line up the plan.

Verify the plan with computation.

Examine the answer.

R-CUBES

Read the problem.

Circle key numbers.

Underline the question.

Box action words.

Evaluate steps.

Solve and check.



UPS✓
UNDERSTAND
Read and explain.

PLAN
How will you solve the problem?

SOLVE
Set up and do the math!

✓CHECK
Does your answer make sense?

Created by: Sarah Powell (srpowell@austin.utexas.edu)





Share your favorite attack strategy.



Teach an attack strategy

Teach about schemas



Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Additive Word Problems

Meanings of Addition

Meanings of Subtraction



Total

Count one set, count another set, put sets together, count sum



$$2 + 3 = 5$$



Total

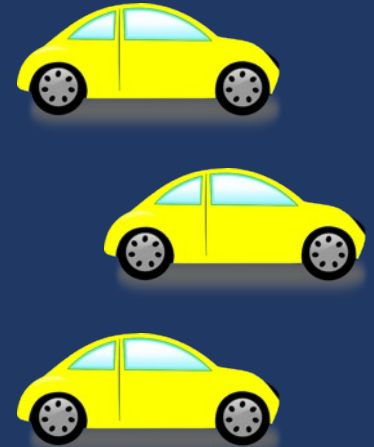
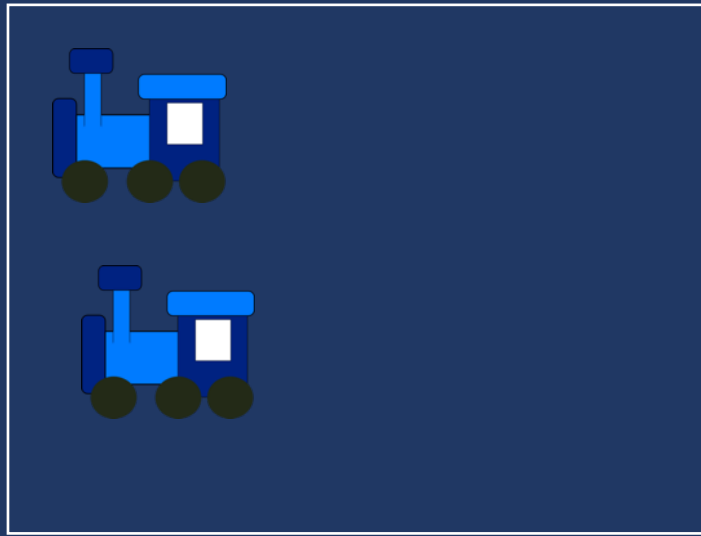
Parts put together into a **total**

Karly saw **4** cardinals and **5** blue jays. How many birds did Karly see?



Change

Start with a set, add the other set, count sum



$$2 + 3 = 5$$

Change

An amount that **increases** or **decreases**

Silas had \$4. Then they earned \$5 for cleaning their room. How much money does Silas have now?



$$6 + 7 = \underline{\quad}$$



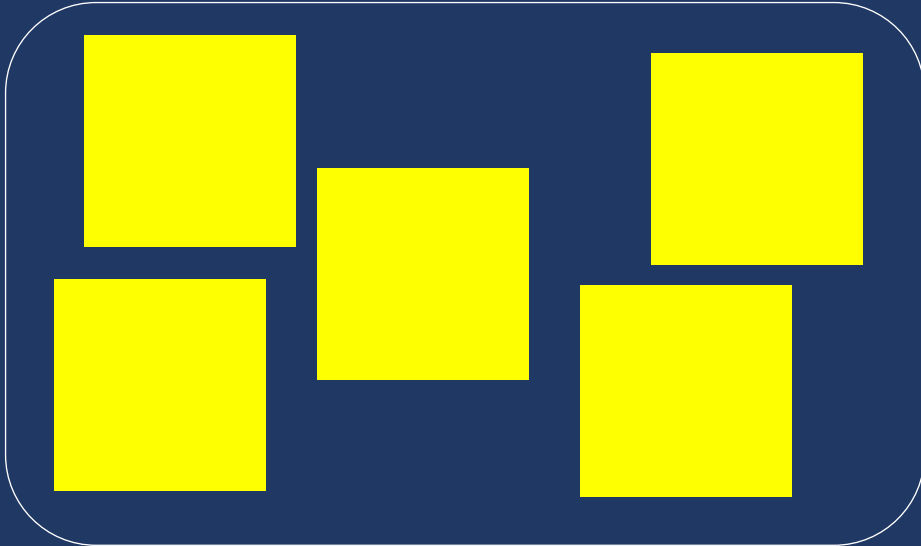
Share a Total story.

Share a Change/Join story.



Change

Start with a set, take away from that set, count difference



$$5 - 3 = 2$$

Change

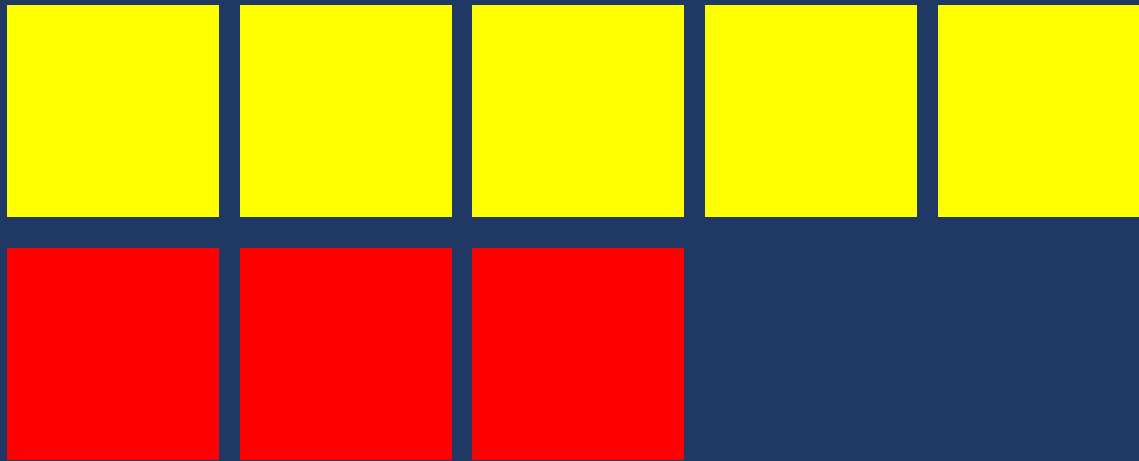
An amount that increases or decreases

Bronwyn had 9 cookies. Then they ate 2 of the cookies. How many cookies does Bronwyn have now?



Difference

Compare two sets, count difference



$$5 - 3 = 2$$



Difference

Greater and less amounts compared for a difference

Rachel has 9 pencils. Jodie has 4 pencils. How many more pencils does Rachel have? (How many fewer does Jodie have? What's the difference between Rachel's and Jodie's pencils?)



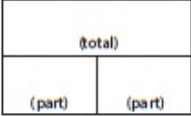

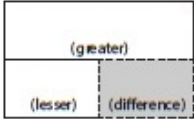
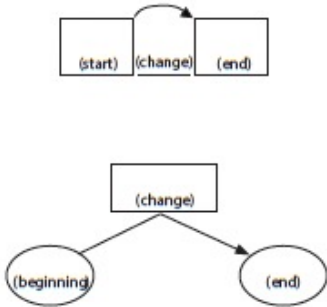
$$14 - 8 = \underline{\quad}$$



Share a Change/Separate story.

Share a Difference story.



Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	$P1 + P2 = T$ (part + part = total) 	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Difference (Compare) Sets compared for a difference	$B - s = D$ (bigger - smaller = difference)  $G - L = D$ (greater - less = difference) 	Difference unknown: Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	Bigger/greater unknown: Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	Smaller/lesser unknown: Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	$ST +/ - C = E$ (start +/- change = end) 	End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now? End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?	Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting? Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?	Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with? Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?	Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?

Total

Additive Word Problems

A.

Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.

Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

NOTES ABOUT TOTAL PROBLEMS:



Total

Part-part-whole
Combine

Parts put together into a **total**

Daniela saw **3** canoes and **8** kayaks.
How many boats did Daniela see?

Total

Daniela saw **11** boats. If **3** of the boats
were canoes, how many were kayaks?

Part

Daniela saw **11** boats. **8** of the boats
were kayaks, how many were canoes?

Part



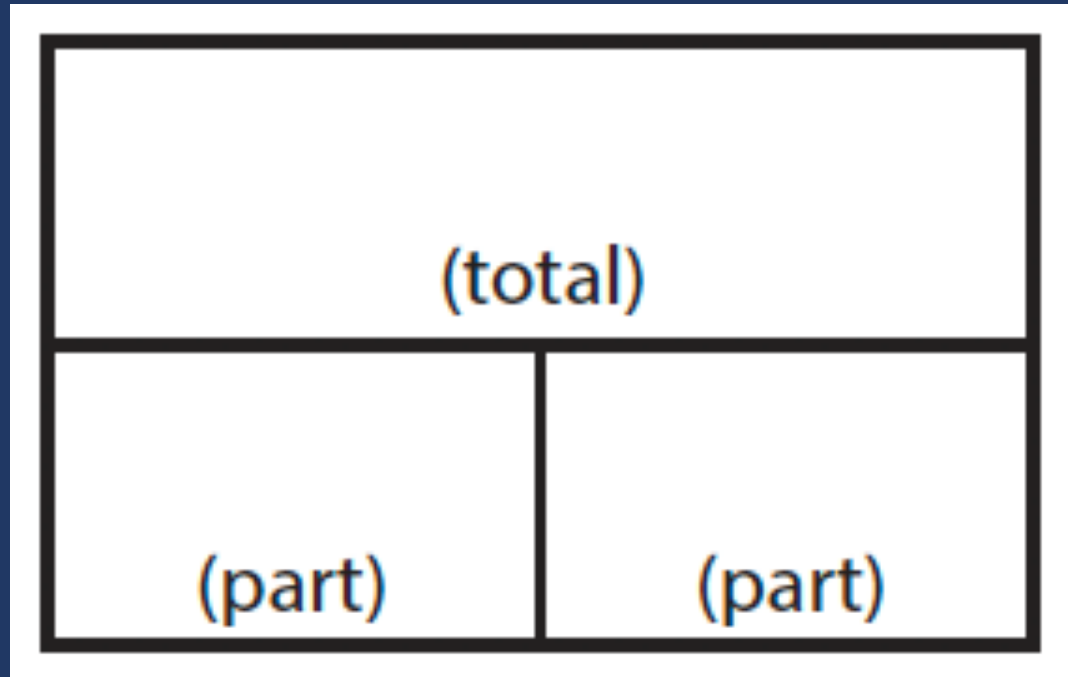
Total

“Are parts put together for a total?”



Total

$$P1 + P2 = T$$



Total

Additive Word Problems

A.

Ali delivered 12 boxes of cookies on Friday and 25 boxes of cookies on Saturday. How many boxes of cookies did Ali deliver?

B.

In March and April, it rained a total of 11.4 inches. If it rained 3.9 inches in March, how many inches did it rain in April?

C.

Sam mows lawns and made \$560 last week. She made \$95 on Monday, \$135 on Tuesday, and \$70 on Wednesday. How much did Sam make on Thursday and Friday?

NOTES ABOUT TOTAL PROBLEMS:



Total



Write a Total problem.



Difference

Compare

Greater and **lesser** amounts compared for a **difference**

Adrianna has **10** pencils. Tracy has **4** pencils.
How many more pencils does Adrianna have?

Difference

Adrianna has **6** more pencils than Tracy. If Tracy has **4** pencils, how many does Adrianna have?

Greater
amount

Tracy has **6** fewer pencils than Adrianna.
Adrianna has **10** pencils. How many pencils does Tracy have?

Lesser
amount



Total

“Are parts put together for a total?”

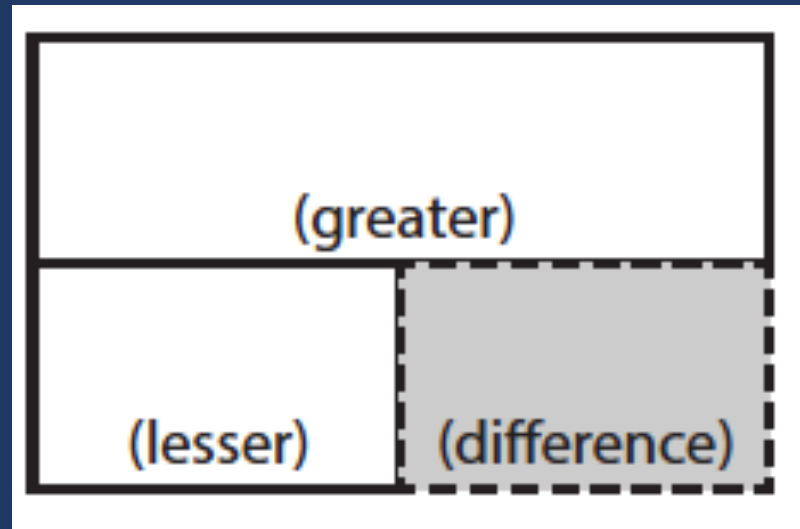
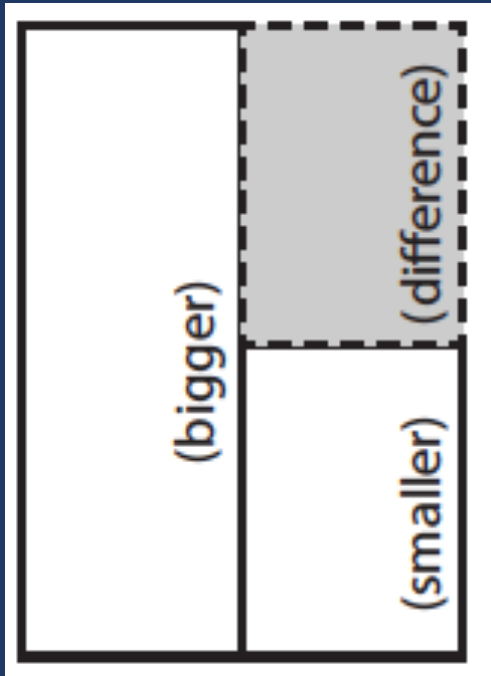
Difference

“Are amounts compared for a difference?”



Difference

$$G - L = D$$



Difference

Additive Word Problems

D.

Audrey has 162 wooden beads and 95 glass beads. What is the difference between Audrey's wooden beads and glass beads?

E.

Damian's dog eats $5\frac{1}{2}$ cups of dog food each week. Monte's dog eats $4\frac{1}{2}$ cups more each week than Damian's dog. How much does Monte's dog eat in a week?

F.

The temperature in Norfolk was 12 degrees warmer than in Roanoke where the temperature was 79 degrees. It was 86 degrees in Marion. What was the temperature in Norfolk?

NOTES ABOUT DIFFERENCE PROBLEMS:



Difference



Write a Difference problem.



Change

Join

An amount that **increases** or decreases

Nickole had 6 notebooks. Then, she bought 3 notebooks. How many notebooks does Nickole have now?

End amount

Nickole had 6 notebooks. Then, she bought a few more notebooks. Now, Nickole has 9 notebooks. How many notebooks did she buy?

Change
amount

Nickole had some notebooks. Then, she bought 3 notebooks. Now, Nickole has 9 notebooks. How many notebooks did she have to start with?

Start
amount



Change

Separate

An amount that increases or **decreases**

Samantha baked **20** cookies. Then, she ate **3** of the cookies. How many cookies does Samantha have now?

End amount

Samantha baked **20** cookies. Then, she ate some of the cookies. Now, she has **17** cookies. How many cookies did Samantha eat?

Change
amount

Samantha baked some cookies. She ate **3** of the cookies and has **17** cookies left. How many cookies did Samantha bake?

Start
amount



Total

“Are parts put together for a total?”

Difference

“Are amounts compared for a difference?”

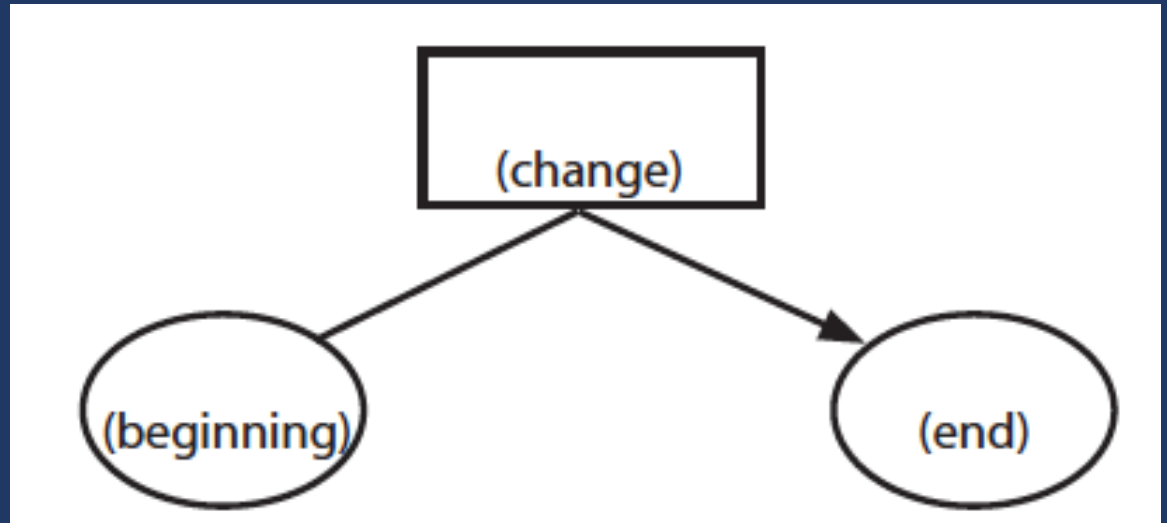
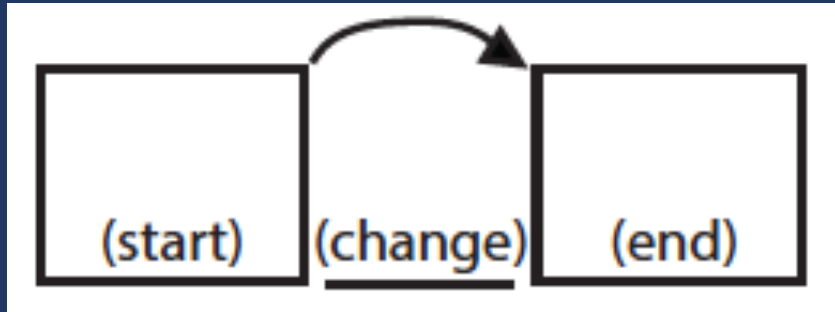
Change

“Does an amount increase or decrease?”



Change

$$ST + / - C = E$$



Change

Additive Word Problems

G.

A plant was $3\frac{3}{4}$ inches tall at the beginning of June. By the end of July, the plant was $9\frac{1}{8}$ inches tall. How many inches did the plant grow in 2 months?

H.

Martina has some money in her bank account. Then, she spent \$135.69 and has a balance of -\$24.80. How much money did Martina have to begin with?

I.

Hui saved \$70 in January. In February, she spent \$64 of the money she saved. She saved \$92 more in March. How much has Hui saved by the end of March?

NOTES ABOUT CHANGE PROBLEMS:



Change



Write a Change problem.





Schema Check!



Change

Pablo goes to a stamp show where he can share, buy, and sell stamps.

26. Part A

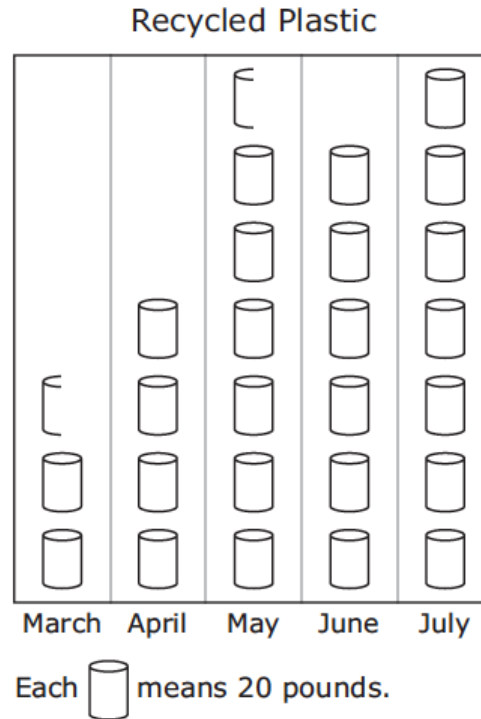
The first day, Pablo starts with 744 stamps. He buys 27 stamps from his friend. He then sells 139 stamps.

What is the total number of stamps that Pablo has after the first day of the stamp show?



Difference

The graph below shows the number of pounds of plastic the Keller family recycled for five months.



Based on the graph, how many more pounds of plastic did the family recycle in July than in April?

Total

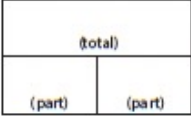

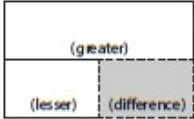

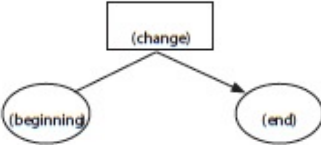
Mr. Conley delivers packages. The bar graph shows the total number of packages he delivered on five days last week.



10. Part A

What is the total number of packages Mr. Conley delivered on Monday and Tuesday?

- (A) 300
- (B) 340
- (C) 350
- (D) 360

Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Total (Combine; Part-part-whole) Parts combined for a sum	$P1 + P2 = T$ (part + part = total) 	Sum unknown: Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	Part unknown: Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		More than two parts: Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
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Change (Join; Separate) An amount that increases or decreases	$ST +/ - C = E$ (start +/- change = end)  	End (increase) unknown: Jorge had \$52. Then, he earned \$16 babysitting. How much money does Jorge have now? End (decrease) unknown: Jorge had \$52. Then, he spent \$29 at the ballpark. How much money does Jorge have now?	Change (increase) unknown: Jorge had \$52. Then, he earned some money babysitting. Now, Jorge has \$68. How much did Jorge earn babysitting? Change (decrease) unknown: Jorge had \$52 but spent some money when he went to the ballpark. Now, Jorge has \$23. How much did Jorge spend at the ballpark?	Start (increase) unknown: Jorge has some money, and then he earned \$16 for babysitting. Now, Jorge has \$68. How much money did he have to start with? Start (decrease) unknown: Jorge had some money. Then, he spent \$29 at the ballpark and has \$23 left. How much money did Jorge have before going to the ballpark?	Multiple changes: Jorge had \$78. He stopped and bought a pair of shoes for \$42 and then he spent \$12 at the grocery. How much money does Jorge have now?

Multiplicative Word Problems

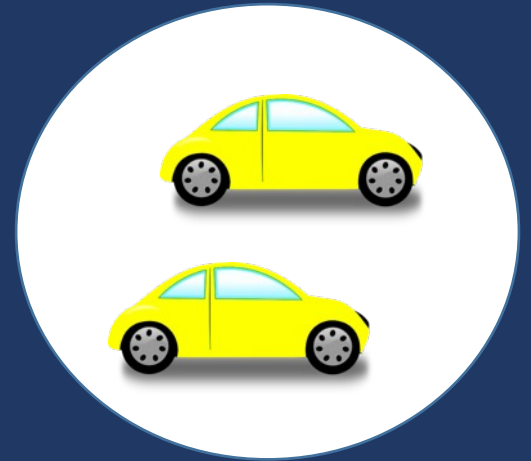
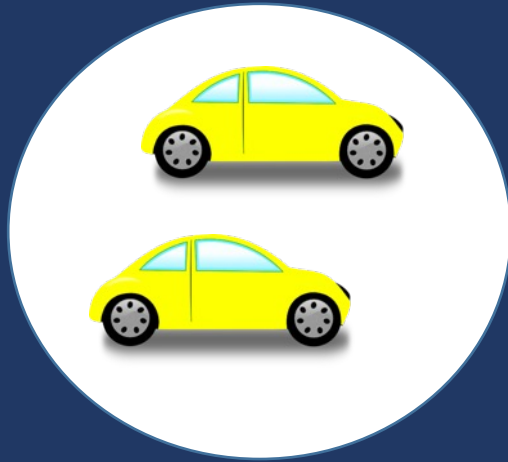
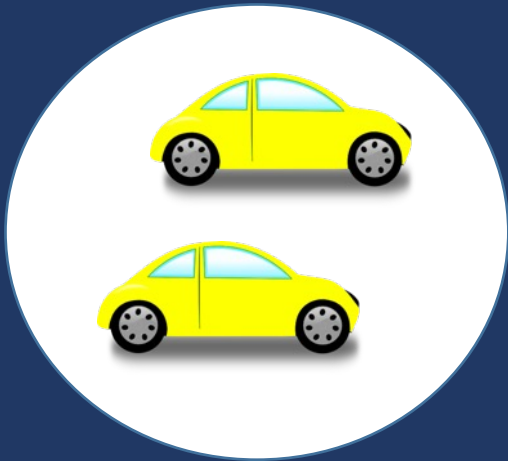
Meanings of Multiplication

Meanings of Division



Equal Groups

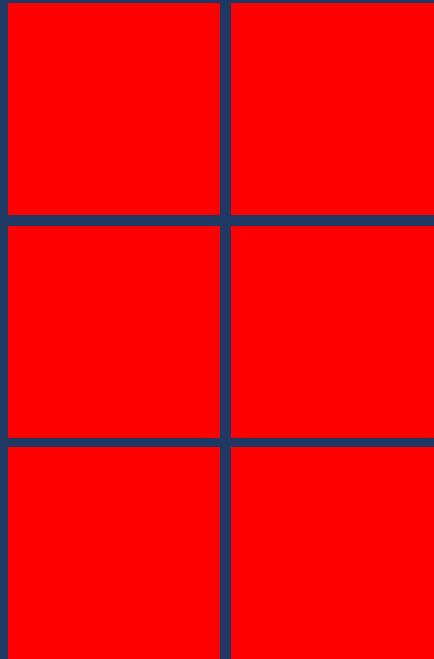
Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$

Equal Groups

Show the groups, show the amount for each group, count product



$$3 \times 2 = 6$$



Equal Groups

Groups multiplied by **number in each group** for a **product**

Rhiannon has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Rhiannon have altogether?



Comparison

Show a set, then multiply the set



$$3 \times 2 = 6$$

Comparison

Set multiplied by a number of **times** for a **product**

Vivienne picked **6** apples. Jessica picked **2** times as many apples as Vivienne. How many apples did Jessica pick?



$$4 \times 3 = \underline{\quad}$$



Share an Equal Groups story.

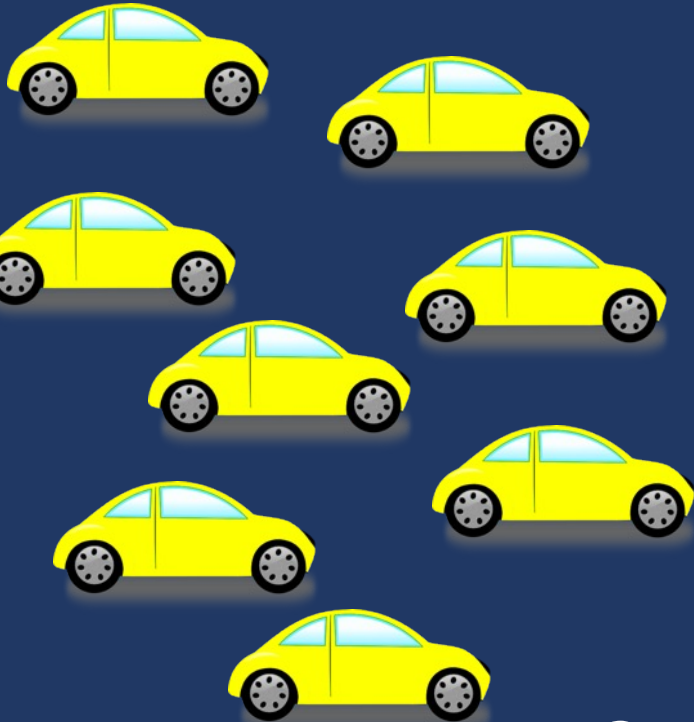
Share a Comparison story.



Equal Groups

(Partitive Division)

Show the dividend, divide equally among divisor, count quotient

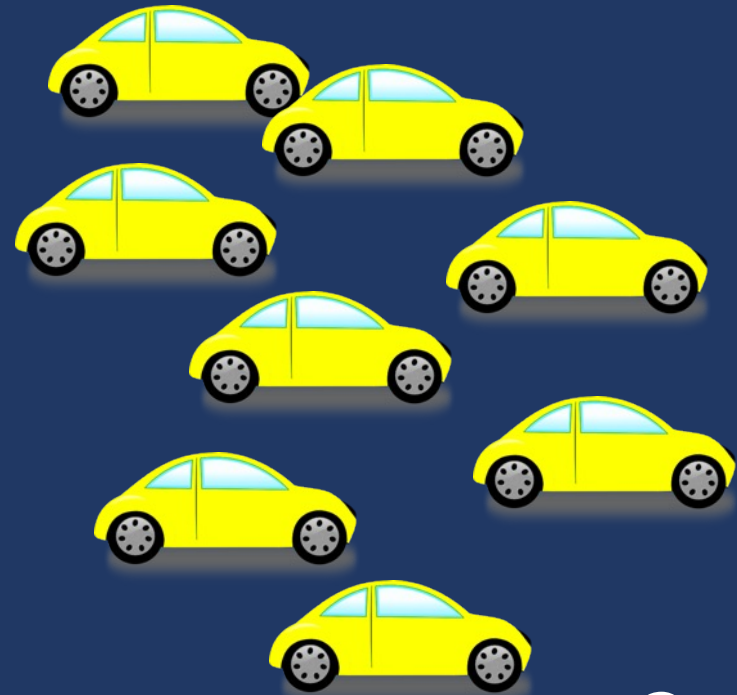


$$8 \div 2 = 4$$

Equal Groups

(Quotative Division)

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

Equal Groups

Groups multiplied by **number in each group** for a **product**

Stefanie has **12** apples. She wants to share them equally among her **2** friends. How many apples will each friend receive?

Nicole has **12** apples. She put them into bags containing **6** apples each. How many bags did Nicole use?



$$15 \div 5 = \underline{\quad}$$



Share a Partitive story.

Share a Quotative story.



Total

Difference

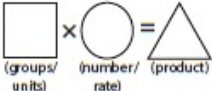
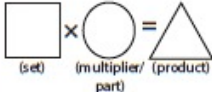
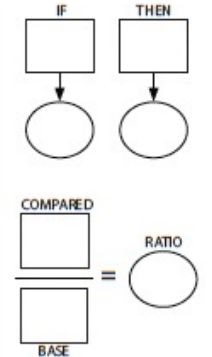
Change

Equal Groups

Comparison

Ratios/Proportions



Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		Product unknown: Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	Set unknown: Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	Times unknown: Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	With fraction: Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions		Subject unknown: Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	Object unknown: Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?	Ratio unknown: Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	With percentage: Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly? With unit rate: Paula bought 5 boxes of markers. She spent \$9.75. What is the price of one box of markers?



Equal Groups

Array
Vary

Groups multiplied by **number in each group**
for a **product**

Toni has **2** boxes of crayons. There are **12** crayons in each box. How many crayons does Toni have altogether?

Product

Toni has **24** crayons. They want to place them equally into **2** boxes. How many crayons will Toni place in each box?

Number in
each group

Toni has **24** crayons. They put them into boxes with **12** crayons each. How many boxes did Toni use?

Groups



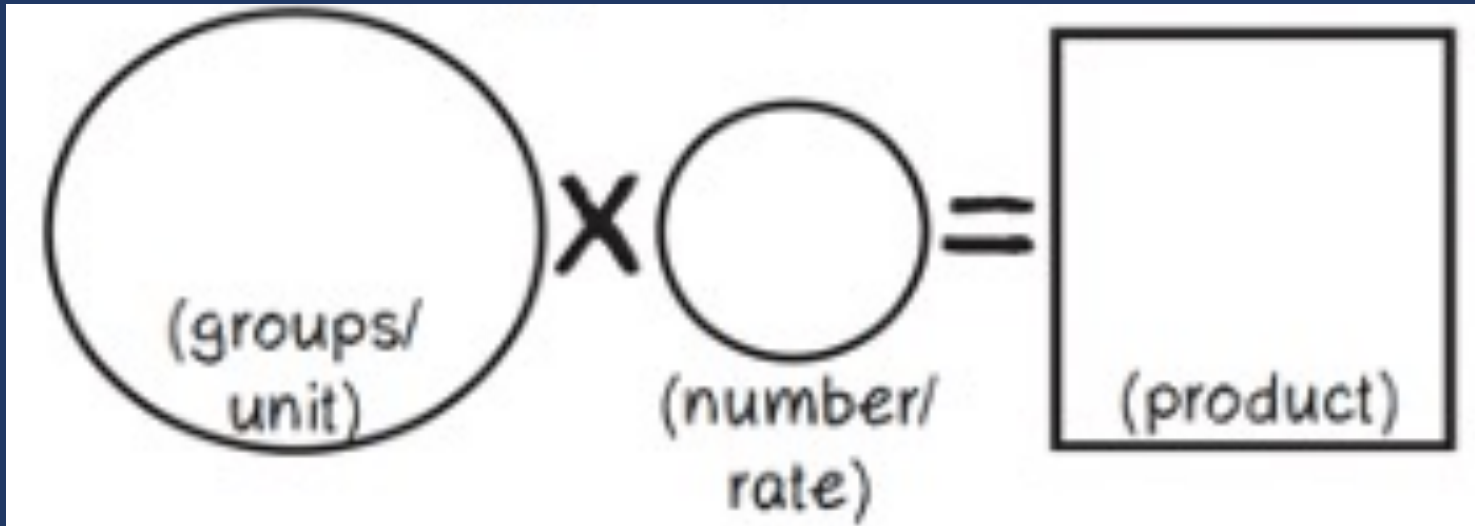
Equal Groups

“Are there groups with an equal number in each group?”



Equal Groups

$$GR \times N(E) = P$$



Equal Groups

Multiplicative Word Problems

A.

Lola baked 6 pies. For each pie, Lola used 5 apples. How many apples did Lola use?

B.

Jane bought 112 light bulbs. The light bulbs come in packs of 4. How many packs of light bulbs did Jane buy?

C.

Zachary has 3 feet of string. He makes bracelets, and each bracelet needs $5 \frac{1}{4}$ inches of string. How many bracelets could Zachary make?

NOTES ABOUT EQUAL GROUPS PROBLEMS:



Equal Groups



Write an Equal Groups problem.



Comparison

Set multiplied by a number of **times**
for a **product**

Brooke ran **6** minutes. Shaleeni ran **4** times longer than Brooke. How many minutes did Shaleeni run?

Set

Number of
times

Product



Equal Groups

“Are there groups with an equal number in each group?”

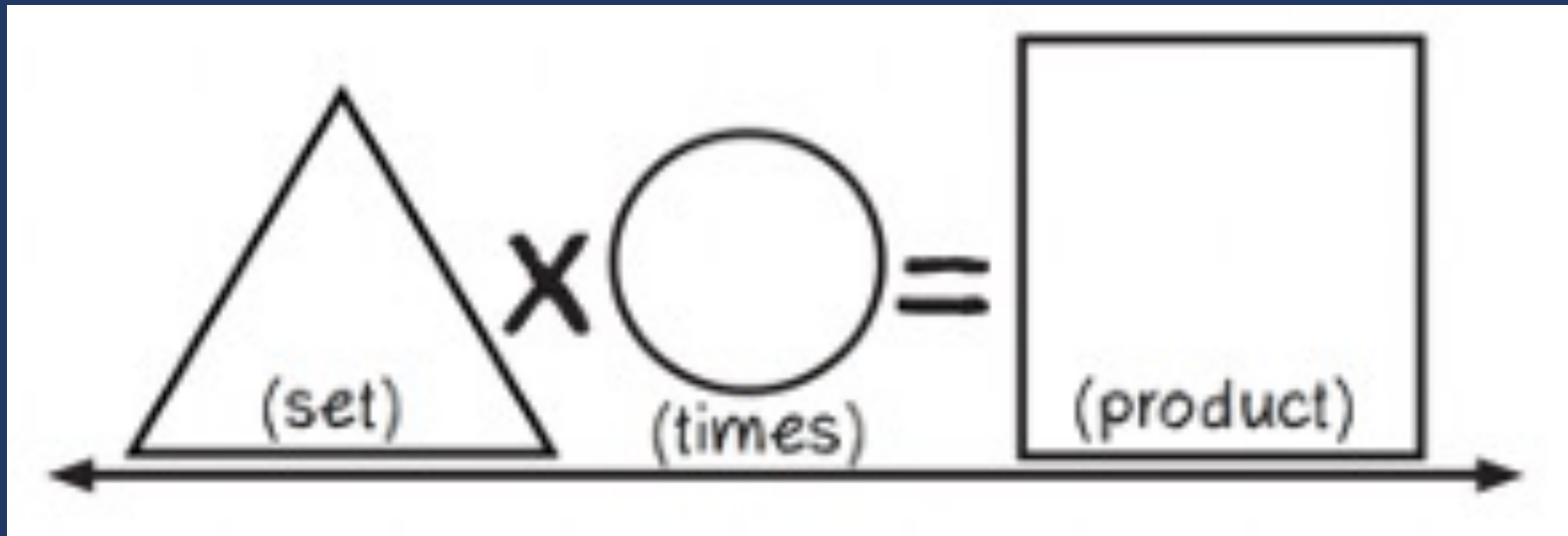
Comparison

“Is a set compared a number of times?”



Comparison

$$S \times T = P$$



Comparison

Multiplicative Word Problems

D.

Enrique has 2 times as many pencils as Ava. Ava has 6 pencils. How many pencils does Enrique have?

E.

Susan has 7 times as many books as Mo. Mo has 18 books. How many books does Susan have?

NOTES ABOUT COMPARISON PROBLEMS:

F.

Sally typed 56 words in 2 minutes. At this rate, how many words could Sally type in 7 minutes?

G.

An airplane's altitude changed -378 feet over 7 minutes. What was the mean change of altitude in feet per minute?



Comparison



Write a Comparison problem.



Ratios/Proportions

Description of **relationships** among quantities

Melissa baked cookies and brownies. The ratio of cookies to brownies was **3:5**. If she baked **25** brownies, how many cookies did she bake?

Emma typed **56** words in **2** minutes. At this rate, how many words could Emma type in **7** minutes?

Xin et al. (2005)



Equal Groups

“Are there groups with an equal number in each group?”

Comparison

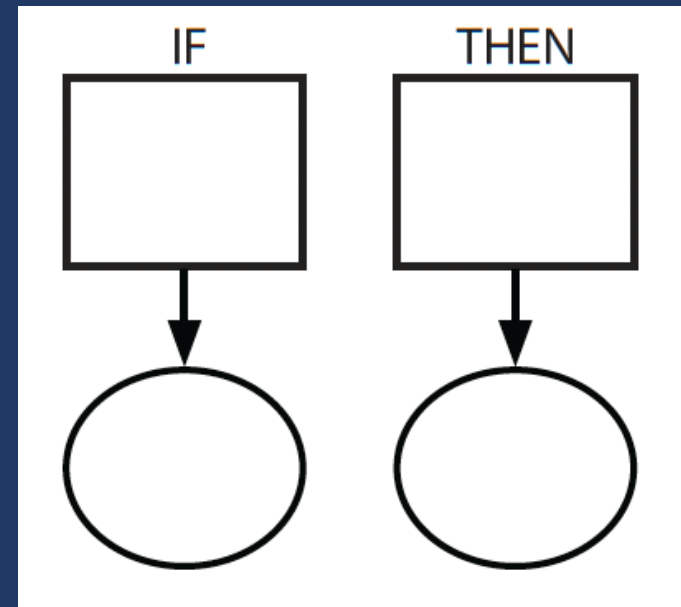
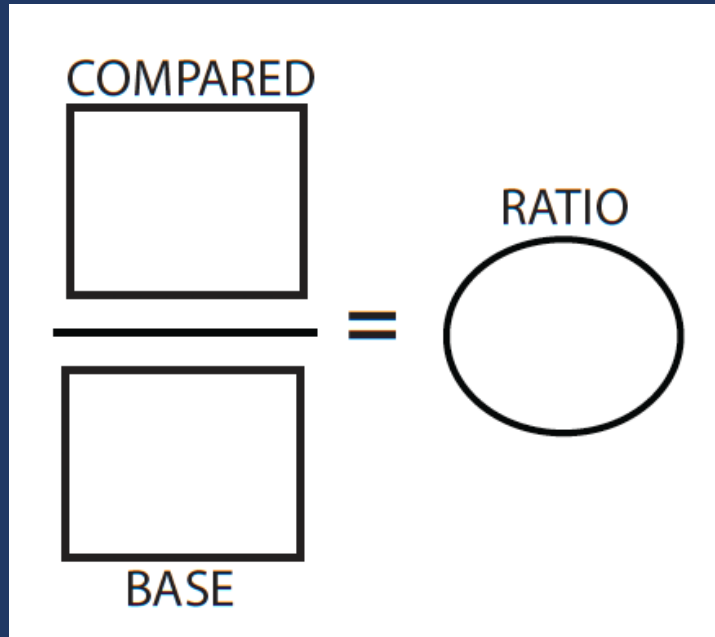
“Is a set compared a number of times?”

Ratios/Proportions

“Are there relationships among quantities – if this, then this?”



Ratios/Proportions



Ratios/Proportions

Multiplicative Word Problems

H.

The number of blueberry muffins that a baker makes each day is 40% of the total number of muffins she makes. On Monday, the baker makes 36 blueberry muffins. What is the total number of muffins that the baker makes on Monday?

I.

Sara buys a sweater at a department store. The sweater costs \$30. The store is having a 25% off sale on everything in the store. Enter the amount of money, in dollars, Sara saves from the sale. Do not consider the sales tax.

J.

Margarita baked cookies and brownies. The ratio of cookies to brownies was 3:5. If she baked 25 brownies, how many cookies did she bake?

NOTES:



Ratios/Proportions



Write a Ratios or Proportions problem.





Schema Check!



Equal Groups

Mr. Kowolski ordered 35 boxes of granola bars. Each box contained 24 granola bars.

What is the total number of granola bars Mr. Kowolski ordered?

Ratios/Proportions

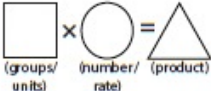
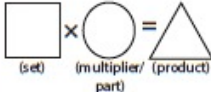
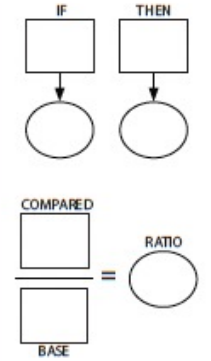
A company makes 625 cell phone cases each day. How many cell phone cases does the company make in 31 days?



Comparison

Danielle's full-grown dog weighs 10 times as much as her puppy. The puppy weighs 9 pounds.

Enter the number of pounds the full-grown dog weighs.

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		Product unknown: Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	Groups unknown: Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	Number unknown: Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	With rate: Maria bought 5 cartons of eggs. Each carton cost \$2.95. How much did Maria spend on eggs?
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Total

Difference

Change

Equal Groups

Comparison

Ratios/Proportions



Teach an attack strategy

Teach about schemas



Word-Problem Solving

Maya has 120 caramel apples to sell. Each caramel apple is covered with one topping.

- $\frac{1}{5}$ of the caramel apples are covered with peanuts.
- $\frac{1}{3}$ are covered with chocolate chips.
- $\frac{3}{10}$ are covered with coconut.
- The rest are covered with sprinkles.

How many caramel apples are covered with sprinkles?

- A** 100
- B** 33
- C** 25
- D** 20

Solve the problem

What skills are necessary to solve this problem?



Revisit this problem.
Discuss the schemas in
the problem.



If needed, number the graph.



U

- Understand by reading
- Underline the label

P

- (Parentheses) needed numbers
- Put the numbers in order

S

- Schema(s)
 - Total Equal Groups
 - Difference Equal Shares
 - Change
- Solve



- Check the number answer
- Check the label answer



Multi-Step Problems

A.

Leslie had 3 pizzas. Each pizza was cut into 8 pieces. Leslie ate 2 pieces. How many pieces were left?

B.

Mr. Kahn gave away 8 blue balloons and 6 red balloons. He gave away 3 times the number of white balloons as red balloons. What was the total number of balloons Mr. Kahn gave away?

C.

An egg farm packages 264 total cartons of eggs each month. The farm has 3 different sizes of cartons.

The small carton hold 8 eggs, and $\frac{1}{6}$ of the total cartons are small.

The medium carton holds 12 eggs, and $\frac{2}{3}$ of the total cartons are medium.

The large carton holds 18 eggs, and the rest of the total cartons are large.

Determine how many each size of carton is needed each month. Then determine how many eggs are needed to fill the 264 cartons.



Teach an attack strategy

Teach about schemas



Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit
instruction

Precise
language

Multiple
representations

INSTRUCTIONAL STRATEGIES

Fluency building

Problem solving
instruction





Pirate Math Equation Quest

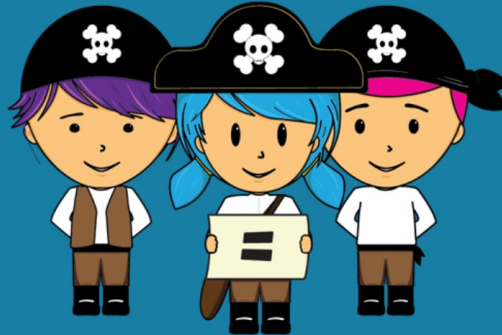
[About](#)[Research](#)[Individual](#)[Small Group](#)[STAAR](#)[Videos](#)

Welcome to Pirate Math Equation Quest!

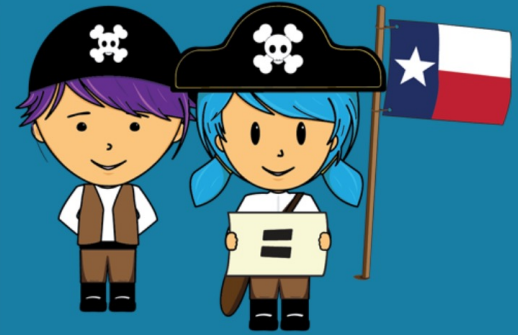
Individual Word-Problem Intervention



Small-Group Word-Problem Intervention

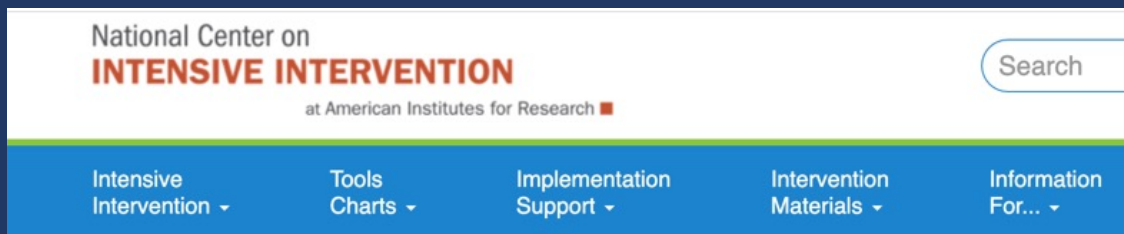


Small-Group Word-Problem Intervention for STAAR



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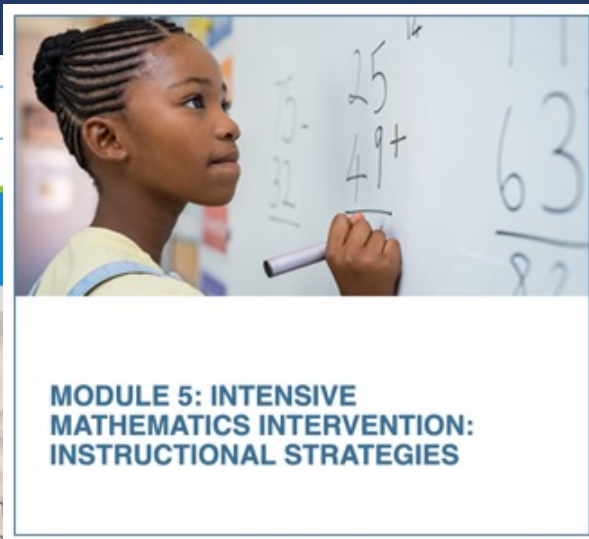


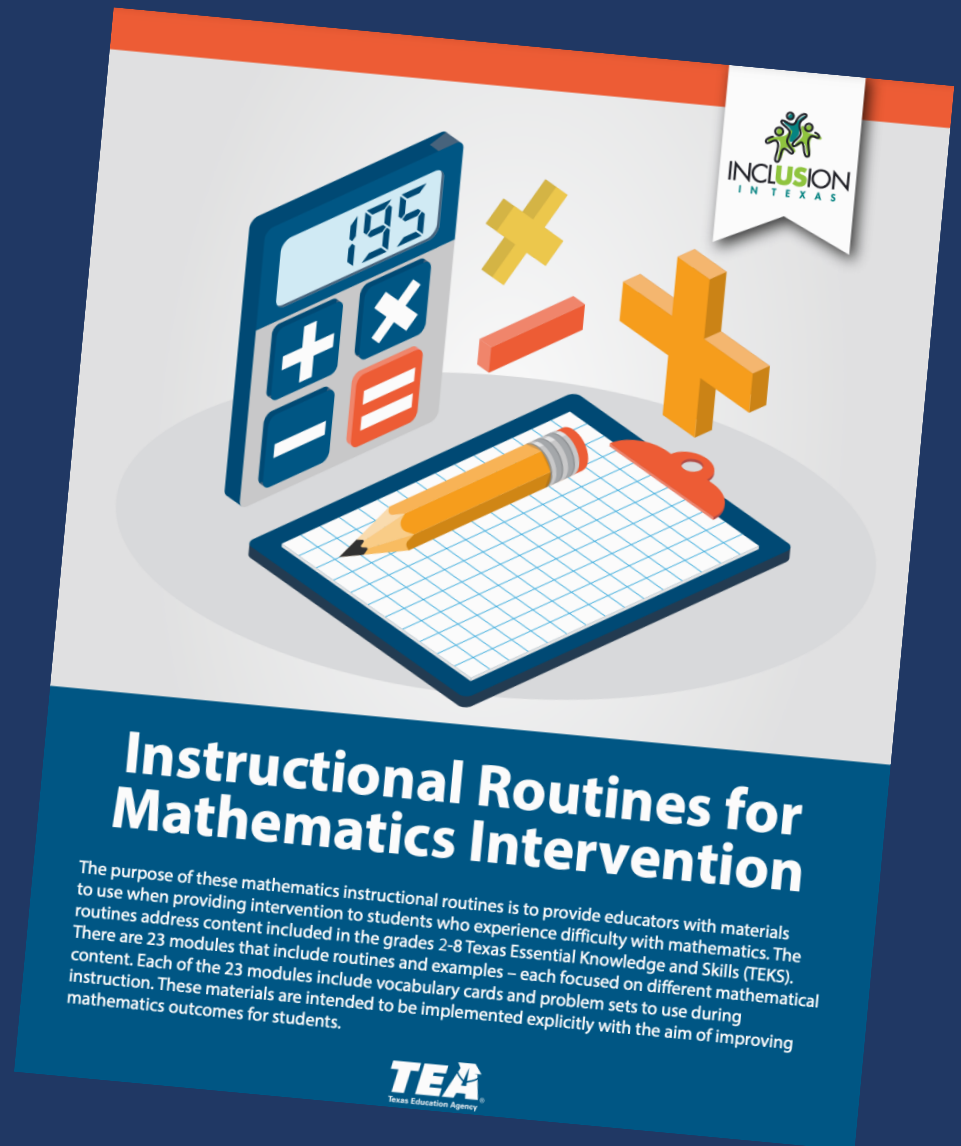
Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

Intensive instruction was recently identified as a [high-leverage practice in special education](#), and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

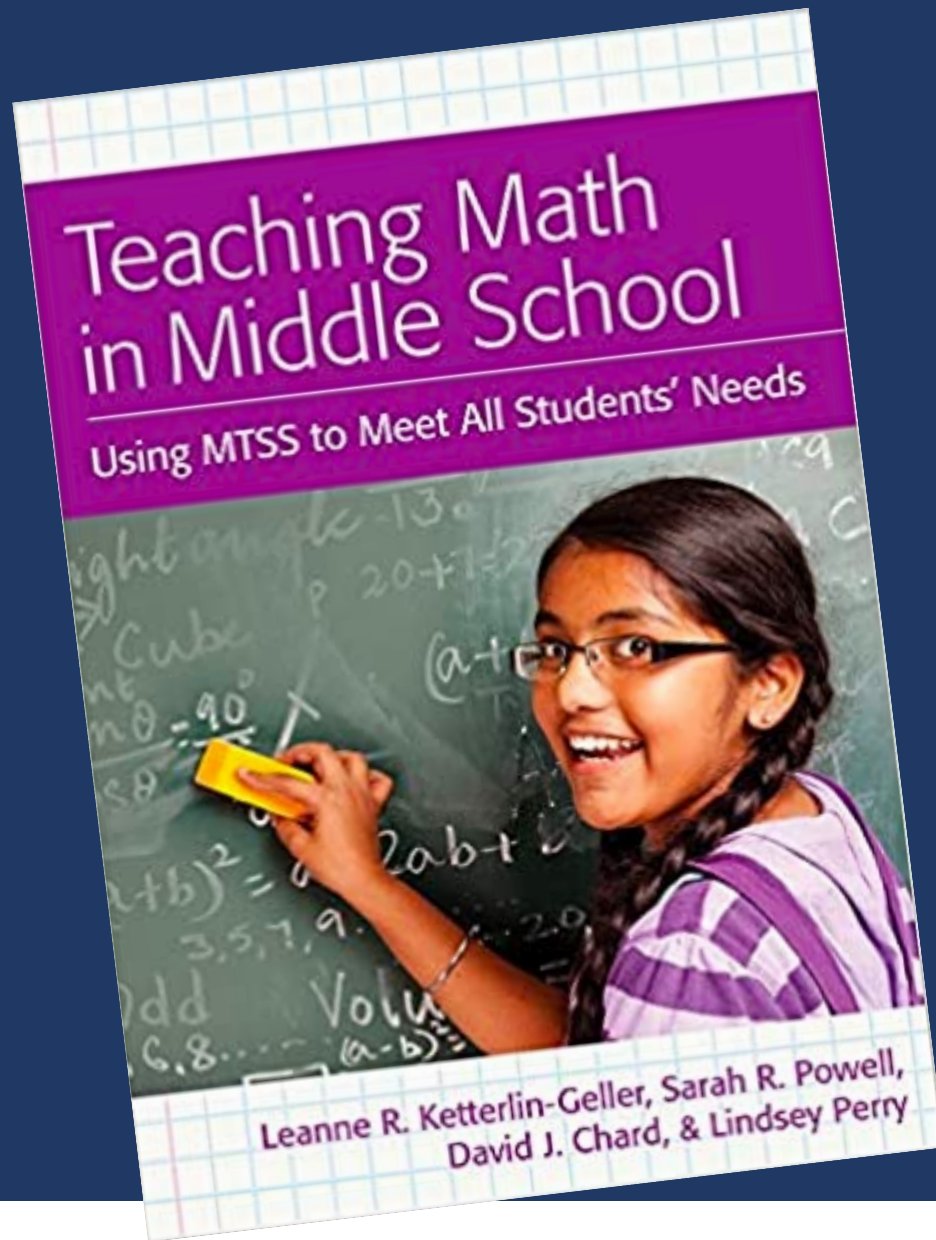
NCII, through a collaboration with the University of Connecticut and the [National Center on Leadership in Intensive Intervention](#) and with support from the [CEEDAR Center](#), developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the [Features of Explicit Instruction Course](#) and so we suggest that users complete both courses.





https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859







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