

## Projectile Motion Notes

Projectile- defined as an object or mass traveling through space without the aid of an engine or self initiated power. Exp. Baseball, football, golf ball, tennis, volleyball, projectile vomiting.

A projectile's path is the shape of a parabola first identified by Galileo.  
Its path has motion in the x axis and in the y axis.

Motion in the x axis has no effect on motion in the y axis. They are independent motions.

The kinematic equations apply but are written in respect to the axis of travel.

Moving in the x axis

$$\Delta x = \frac{1}{2} (v_{fx} + v_{ix}) \Delta t$$

$$\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x \Delta t$$

$$v_{fx}^2 = v_{ix}^2 + 2 a_x \Delta x$$

where:

$v_{ix}$  = horizontal velocity initially

$a_x$  = horizontal acceleration

$v_{fy}$  = vertical final velocity

$\Delta x$  = change in horizontal displacement

Moving in the y axis

$$\Delta y = \frac{1}{2} (v_{fy} + v_{iy}) \Delta t$$

$$\Delta y = v_{iy} \Delta t + \frac{1}{2} g (\Delta t)^2$$

$$v_{fy} = v_{iy} + g \Delta t$$

$$v_{fy}^2 = v_{iy}^2 + 2 g \Delta y$$

$v_{fx}$  = horizontal velocity final

$v_{iy}$  = vertical initial velocity

$g$  = acceleration due to gravity  $-9.8 \text{ m/s}^2$

$\Delta y$  = change in vertical displacement

### HORIZONTALLY LAUNCHED PROJECTILES

1. to find time in motion, use  $\Delta y = v_{iy} \Delta t + \frac{1}{2} g \Delta t^2$  where  $v_{iy} = 0$  and solve for time  $\Delta t$
2. to find the distance from base or  $\Delta x$ , use  $\Delta x = v_{ix} \Delta t + \frac{1}{2} a_x \Delta t^2$  where  $a_x = 0$
3. to find how high or tall something is use  $\Delta y = v_{iy} \Delta t + \frac{1}{2} g \Delta t^2$  where  $v_{iy} = 0$  and you have to have time or calculate it. Possibly  $\Delta t = \Delta x / v_{ix}$
4. to find horizontal velocity.  $v_{ix} = \Delta x / \Delta t$

### PROJECTILES LAUNCHED AT AN ANGLE

1. If the velocity is given  $v_0$  and the angle, find the horizontal and vertical velocity components using trig.
2. To solve for time of flight, assume that  $v_{fy} = -v_{iy}$ , use  $v_{fy} = v_{iy} + g \Delta t$  and solve for  $\Delta t = -2 v_{iy} / g$  or if landing at different elevations, Use the quadratic formula  $ax^2 + bx + c = 0$  or  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  where  $x = \Delta t$ ,  $a = \frac{1}{2} g$ , and  $b = v_{iy}$ .
3. To solve for maximum height, assume  $v_{fy} = 0$  at top, use  $v_{fy}^2 = v_{iy}^2 + 2 g \Delta y$ , solve for  $\Delta y$ .
4. To solve for total horizontal distance,  $\Delta x$  or Range, use  $\Delta x = v_{ix} \Delta t$  or only on a level surface use Range (R) =  $v_0^2 \sin 2(\Theta) / g$  where  $g$  is the absolute value, no negative.

## Projectile Practice

## HORIZONTALS

1. A stone is thrown horizontally at 15m/s from the top of a cliff 44 m high.
  - a. How long does the stone take to reach the bottom of the cliff?
  - b. How far from the base of the cliff does the stone strike the ground?
2. A physics book is thrown horizontally at a velocity of 10.0 m/s from the top of a cliff 78.4 m high.
  - a. How long does the stone take to reach the bottom of the cliff?
  - b. How far from the base of the cliff does the stone strike the ground?
3. Wyle E. Coyote is now in the real world. He goes running off a cliff and becomes a human projectile. It takes him 12.5 seconds to hit the bottom of the canyon. He falls to his cartoon death and lands 82 m from the wall of the canyon.
  - a. How far did he fall? (Or how high is the cliff?)
  - b. What was his cartoon velocity as he ran horizontally off a cliff?
4. The longest shot on a golf tournament was made by Mike Austin in 1974. The ball went a distance of 471 m. Suppose the ball was shot horizontally off a cliff at 80.0 m/s. Calculate the height of the cliff.
5. A movie director is shooting a scene that involves dropping a stunt dummy out of an airplane and into a swimming pool. The plane is 10.0 m above the ground, traveling at a velocity of 22.5 m/s in the positive  $x$  direction. The director wants to know where in the plane's path the dummy should be dropped so that it will land in the pool. What is the dummy's horizontal displacement?

## ANGLES

1. A player kicks a football from ground level with a velocity of magnitude 27.0 m/s at an angle of  $30.0^\circ$  above the horizontal.
  - a. What is its "hang-time?"
  - b. How far does the ball travel before it hits the ground?
  - c. What is the maximum height the ball reaches?
2. A kicker now kicks the football with the same velocity, but at  $60^\circ$  from the horizontal.
  - a. What is its "hang-time?"
  - b. How far does the ball travel before it hits the ground?
  - c. What is the maximum height the ball reaches?
3. The narrowest strait on earth is Seil Sound in Scotland, which lies between the mainland and the island of Seil. The strait is only about 6.0 m wide. Suppose an athlete wanting to jump "over the sea" leaps at an angle of  $35^\circ$  with respect to the horizontal. What is the minimum initial speed that would allow the athlete to clear the gap? Neglect air resistance.
4. In 1993, Wayne Brian threw a spear a record distance of 201.24 m. (This is not an official sport record because a special device was used to "elongate" Brian's hand.) Suppose Brian threw the spear at a  $35.0^\circ$  angle with respect to the horizontal. What was the initial speed of the spear?