Math 1496 Calc 1

The derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad (1)$$

and derivative at a point

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad (2)$$

with the derivative which gives the slope of the tangent, a pt the will give us the tangent line

Ex Find the eq^n of the tangent to

$y = 2x^2 + 3x - 1$ at $(1, 4)$

Sol^n So we need a slope. We will use the derivative for this. We will actually use both def^n's to see the difference.

Def^n (1)

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) - 1 - (2x^2+3x-1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h - \cancel{1} - \cancel{2x^2} - \cancel{3x} + \cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x+2h+3)}{\cancel{h}}$$

$$= \lim_{h\to 0} 4x+2h+3 = 4x+3$$

so $f'(x) = 4x+3$ then $f'(1) = 7$

Tangent $y - 4 = 7(x-1)$

Def^n (2) $f'(1) = \lim_{x\to 1} \frac{2x^2+3x-1-4}{x-1}$ ← $2x^2+3x-5$

$$= \lim_{x\to 1} \frac{(2x+5)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= \lim_{x\to 1} 2x+5 = 7$$ Same slope

so we would get the same tangent

<u>Continuity</u>

A function is cont^s at $x=a$ if

(1) $\lim_{x\to a} f(x)$ exists

(2) $f(a)$ defined

(3) $\lim_{x\to a} f(x) = f(a)$

We say a function is <u>differentiable</u> at $x=a$ if

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} \text{ exists}$$

and if so we define this to be $f'(a)$

<u>Ex</u> is $f(x) = |x| = \begin{cases} -x & x<0 \\ x & x\geq 0 \end{cases}$

diff^{ble} at $x=0$?

Here we will use def^n (2). Note $f(0)=0$

(1) $\lim_{x\to 0^-} \frac{f(x)-f(0)}{x} = \lim_{x\to 0} \frac{-x}{x} = -1$

(2) $\lim_{x\to 0^+} \frac{f(x)-f(0)}{x} = \lim_{x\to 0} \frac{x}{x} = 1 \neq -1$ so no not diff^{ble}

Ex2 Is $f(x) = \sqrt{x}$ diff$^{\text{ble}}$ at $x = 0$

$$\lim_{x \to 0^+} \frac{\sqrt{x} - 0}{x - 0} = \lim_{x \to 0^+} \frac{1}{\sqrt{x}} \text{ DNE so No!}$$

Ex3 Is $f(x) = \begin{cases} 2x - 1 & x < -1 \\ x^2 + 4x & x \geq -1 \end{cases}$

cont^{s} and diff$^{\text{ble}}$ at

(1) Continuity

(a) $\lim_{x \to -1^-} 2x - 1 = -3$

$\lim_{x \to -1^+} x^2 + 4x = 1 - 4 = -3$ $\lim_{x \to -1} f(x)$ exists

(b) $f(-1) = -3$

(c) $\lim_{x \to -1} f(x) = f(-1)$ so Yes f is cont^{s} at $x = 3$

(2) Differentiability

$$\lim_{x \to -1^-} \frac{2x - 1 + 3}{x + 1} = \lim_{x \to -1} \frac{2\cancel{(x+1)}}{\cancel{x+1}} = 2$$

$$\lim_{x \to -1^+} \frac{x^2 + 4x + 3}{x + 1} = \lim_{x \to -1} \frac{\cancel{(x+1)}(x+3)}{\cancel{(x+1)}} = 2$$

same so yes diff & $f'(-1) = 2$

Some Standard Derivatives

$f(x) = \sin(x)$

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \sin x \frac{\cos h - 1}{h} + \cos x \frac{\sin h}{h}$$

$$= \sin x \lim_{h\to 0} \frac{\cos h - 1}{h} + \cos x \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \sin x\,(0) + \cos x\,(1)$$

so ~~##~~ ~~$\frac{}{}$~~

$$\boxed{\frac{d}{dx} \sin x = \cos x}$$

$f(x) = \cos x \quad f'(x) = -\sin x$

$$f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \cos x \frac{\cos h - 1}{h} - \sin x \frac{\sin h}{h}$$

$$= \cos x \lim_{h \to 0} \frac{\cos h - 1}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= -\sin x \quad \square \qquad \text{so } \boxed{\frac{d}{dx} \cos x = -\sin x}$$

$f(x) = e^x \quad f'(x) = e^x$

$$f'(x) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= e^x \lim_{h \to 0} \frac{e^h - 1}{h} = e^x \cdot 1$$

so $\boxed{\frac{d}{dx} e^x = e^x}$