UNIT 2 Stretching and Shrinking

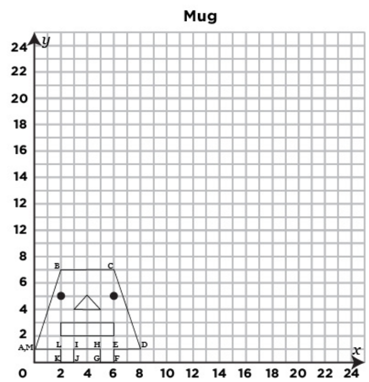
SIMILARITY

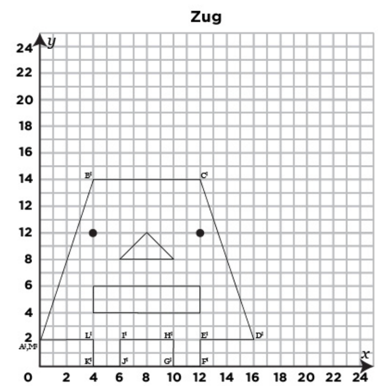
Two figures are considered similar if they have corresponding angles of equal measure and the ratios of each pair of corresponding sides are equivalent.

SAME SHAPE • SAME ANGLES • DIFFERENT SIDE LENGTHS

NOTE CATCHER

Mug and **Zug** are **similar**. They have the same shape and angle measurements, although one is larger than the other.





How can we tell the two figures are definitely similar, other than just looking at them?

Scale factor!

If two figures are similar, there is a special relationship between them called a scale factor. The scale factor is the number multiplied by the side lengths of the one figure to stretch or shrink it to create the similar image. Look at the top of Mug's head, at the line segment $\overline{BC}$. That length is 4 units. Now look at the corresponding top of Zug's head, line segment $\overline{B'C'}$. That length is 8 units. 8 is 2 times larger than 4, so we can say the scale factor from Mug to Zug is 2. Let's check another pair of corresponding side lengths to see if this is consistent. Let's look at the bottom of Mug's one foot, line segment $\overline{KJ}$. That length is 1 unit. Now look at the corresponding side length in Zug's drawing at his foot, line segment $\overline{K'J'}$. That length is 2. 2 is 2 times larger than 1. So once again, we have shown the scale factor is 2. You can do this with every pair of side lengths and this relationship will always be 2, because Zug is 2 times as large as Mug. Try it out!

A scale factor of...

1.5 or $1\frac{1}{2}$	Enlarges the figure by 150%
2	Enlarges the figure by 200%
3	Enlarges the figure by 300%
.25 or $\frac{1}{4}$	Reduces the figure to 25% of the original size
.5 or $\frac{1}{2}$	Reduces the figure to 50% of the original size

+!

SCALE FACTORS are always positive!

How can we make similar figures?

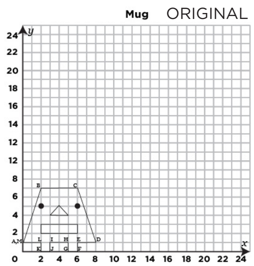
We can take a figure and scale it by applying a rule to it, making it larger or smaller. In the case above, we took our original figure, Mug, and scaled it by a scale factor of 2 to get Zug.

NOTE CATCHER

How can we tell the scale factor when given two figures?

The scale factor show ups in different ways. First make sure you know which figure is the **original** and which figure is the **image**, the resulting figure after you've scaled the original. Let's look at Bug to Zug, an enlargement.

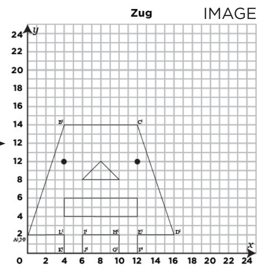
ORIGINAL



ENLARGEMENT

going from smaller to larger

IMAGE



Mug the original's rule is (x,y).
Zug the image's rule is (2x,2y).

	Mug Wum	Zug
Rule	(x, y)	$(2x, 2y)$
Point		
A	(0, 1)	(0, 2)
B	(2, 1)	(4, 2)
C	(2, 0)	(4, 0)
D	(3, 0)	(6, 0)
E	(3, 1)	(6, 2)

A **Compare side lengths on your graphs**, as we did previously. What number is multiplied by the side length in the original figure to get the corresponding side's length in the image? In our example above that number is 2, so 2 is the scale factor. But, be careful! This needs to be true for every pair of side lengths you compare! Every side length in the original needs to be multiplied by 2 to get the corresponding side length in the image. If that number being multiplied isn't consistent then the two images are NOT similar!

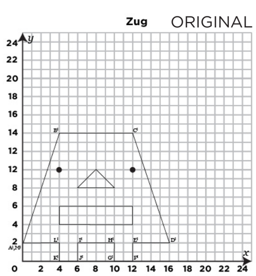
B **Compare their coordinate point rules in their table.** The rule of the original figure is always considered **(x,y)**. Look at the coefficients before the variables in the rule for Zug **(2x,2y)**. They are both the same. That tells us that Zug is similar to Mug. 2 is the consistent coefficient, so 2 is the scale factor. Be careful! If these two coefficients are not the same then the two images are NOT similar!

C **Compare points on the corresponding graphs.** Let's look at a pair of corresponding points on the two graphs. On Mug our original's graph, point L is at (2,1). The corresponding point on Zug the image's graph is L^{I}, or "L prime". *(Notice we can't call it L as well, since that would be confusing.)* That point, L^{I}, is at (4,2). What number did we multiply the x coordinate in the original point L by to get the x coordinate in the image point L^{I}? 2! What number did we multiply the y coordinate in the original point L by to get the y coordinate in the image point L^{I}? 2! Since both those answers were 2, we can say our scale factor is 2. Be careful! If these two answers aren't the same then the two images are NOT similar!

What is the scale factor if our original figure is the larger one, Zug, and we are reducing it to the size of the smaller one, Mug?

Use one of the three strategies above to confirm our scale factor is $\frac{1}{2}$!

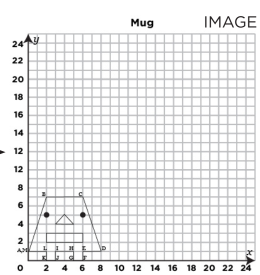
ORIGINAL



REDUCTION

going from larger to smaller

IMAGE



Zug the original's rule is (x,y).
Mug the image's rule is $(\frac{1}{2}x, \frac{1}{2}y)$.

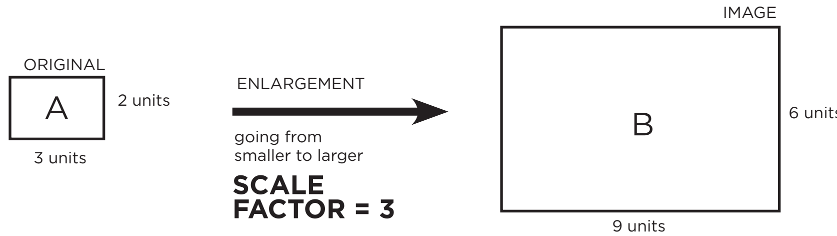
	Zug Wump	Mug
Rule	(x, y)	$(\frac{1}{2}x, \frac{1}{2}y)$
Point		
A	(0, 2)	(0, 1)
B	(4, 2)	(2, 1)
C	(4, 0)	(2, 0)
D	[illegible]	[illegible]

Notice that the scale factor when going from larger to smaller is the RECIPROCAL of the scale factor going from smaller to larger.

$\frac{1}{2}$ & 2 are **reciprocals!**

NOTE CATCHER

How are the scale factor and the perimeters of two similar figures related?



PERIMETER of A: 10 units

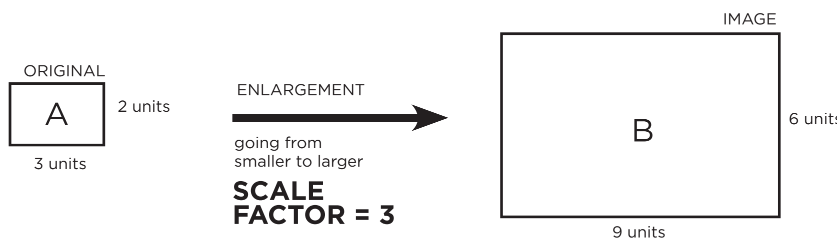
PERIMETER of B: 30 units

SCALE FACTOR

$$\mathbf{10 \times 3 = 30 \text{ units}}$$

The perimeter of the image is the perimeter of the original multiplied by the scale factor!

How are the scale factor and the areas of two similar figures related?



AREA of A: 6 units^2

AREA of B: 54 units^2

SCALE FACTOR^2

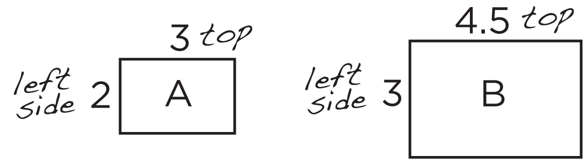
$$\mathbf{6 \times 3^2 = 6 \times 9 = 54 \text{ units}^2}$$

The area of the image is the area of the original multiplied by the square of the scale factor!

NOTE CATCHER

How can we use ratios of side lengths to determine if two figures are similar?

1. Label the sides of your two figures so you can determine corresponding side lengths. For example, use labels like **S** (short), **M** (medium), and **L** (long), or **SHORT SIDE** and **LONG SIDE**, etc.



2. Create a ratio in fractional form of adjacent side lengths (sides that are right next to each other) in the **first** figure.

$$\frac{\text{top}}{\text{left side}} \qquad \frac{3}{2}$$

3. Create a ratio in fractional form of corresponding adjacent side lengths in the **second** figure.

$$\frac{\text{top}}{\text{left side}} \qquad \frac{4.5}{3}$$

4. If those two ratios are equivalent/equal and can be reduced to the same number (HINT: use a calculator!), then the two figures are similar.

$$\frac{3}{2} \stackrel{?}{=} \frac{4.5}{3}$$

$$1.5 \overset{\checkmark}{=} 1.5$$

What are the four tests we can perform to determine if two figures are similar? *(What are the characteristics of two similar figures?)*

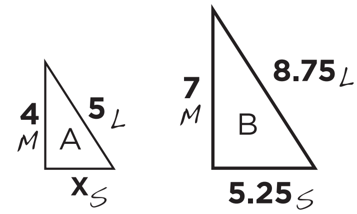
- ☑ The figures have the same **shape**.
- ☑ Corresponding **angle** measures in the figures are equal. (This is true for EVERY pair of corresponding angles.)
- ☑ Corresponding side lengths grow by the same **scale factor** from one figure to the other. (This is true for EVERY pair of corresponding sides.)
- ☑ The ratios of corresponding adjacent sides within each shape are proportional. (The ratios are equal since they each can be reduced to an equivalent fraction.)

NOTE CATCHER

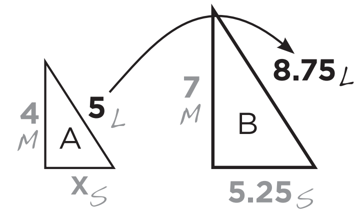
SOLVING FOR MISSING SIDE LENGTHS IN SIMILAR FIGURES

STRATEGY 1: USING SCALE FACTOR

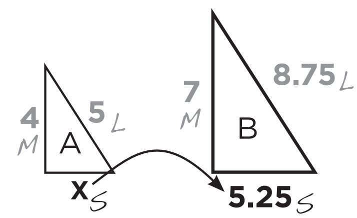
1. Label the sides of your two figures so you can determine corresponding side lengths. For example, use labels like **S** (short), **M** (medium), and **L** (long), or **SHORT SIDE** and **LONG SIDE**, etc.



2. Choose a side length in the first figure and its corresponding side length in the second figure. Determine the **scale factor**.
   5 • SF = 8.75
   So by fact families,
   8.75÷5 = SF
   SF = 1.75



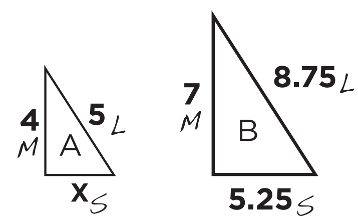
3. Use the scale factor to determine the missing side length.
   x • SF = 5.25
   x • 1.75 = 5.25
   So by fact families,
   5.25÷1.75 = x
   x = 3



SOLVING FOR MISSING SIDE LENGTHS IN SIMILAR FIGURES

STRATEGY 2: USING RATIOS

1. Label the sides of your two figures so you can determine corresponding side lengths. For example, use labels like **S** (short), **M** (medium), and **L** (long), or **SHORT SIDE** and **LONG SIDE**, etc.



2. Create a ratio in fractional form of adjacent side lengths (sides that are right next to each other) in the **first** figure. $\frac{S}{L}$ $\frac{x}{5}$

3. Create a ratio in fractional form of corresponding adjacent side lengths in the **second** figure. $\frac{S}{L}$ $\frac{5.25}{8.75}$

4. Since the figures are similar, we can set those two ratios equal to each other in a proportion and solve for the missing side, using **cross multiplication**. **x = 3**
   (See next page)

$$\frac{x}{5} = \frac{5.25}{8.75}$$

Remember... make sure you include the variable in one of your two ratios. Disregard side lengths that only appear in one of the two figures.

NOTE CATCHER

How do we use cross multiplication to solve for the variable in a proportion?

$$\frac{x}{5} = \frac{5.25}{8.75}$$

1. Cross multiply!

numerator of left fraction X denominator of right fraction = numerator of right fraction X denominator of left fraction

$$\frac{x}{5} = \frac{5.25}{8.75}$$

$$8.75x = (5.25)(5)$$

2. Simplify!

$$8.75x = 26.75$$

3. Divide both sides by the coefficient in front of the variable or use **fact families**!

$$\frac{8.75x}{8.75} = \frac{26.25}{8.75}$$

$$\frac{\cancel{8.75}x}{\cancel{8.75}} = \frac{26.25}{8.75}$$

1

$$\mathbf{x = 3}$$

$$8.75x = 26.25$$

$$x = 26.25 \div 8.75$$

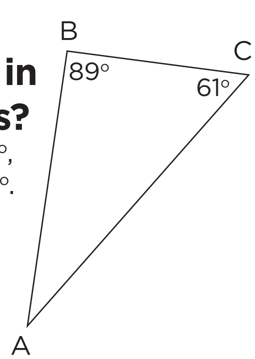
$$\mathbf{x = 3}$$

Remember... any number divided by itself equals 1!

$$\frac{5}{5} = 1$$

How can you determine the missing angle in a triangle when given the two other angles?

Since the sum of the angles of each and every triangle is 180°, add up the two angles given and subtract that sum from 180°.



89 + 61 = 150 **180 – 150 = 30** **So angle A is 30°**

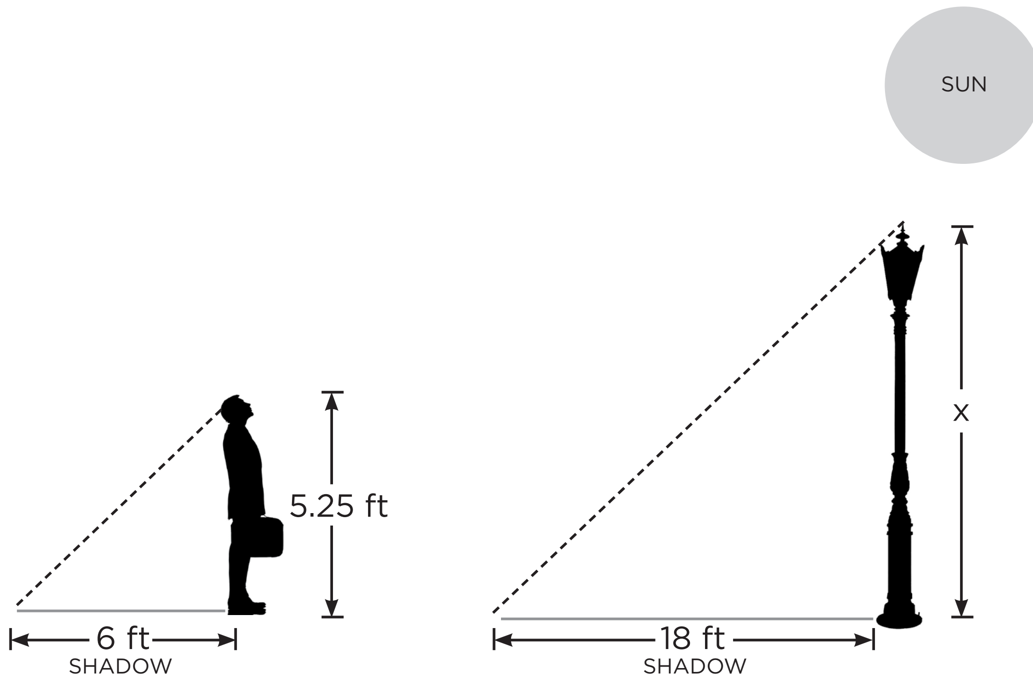
Remember... the sum of the angles in any triangle will always be 180°!

NOTE CATCHER

How do we use shadows to find heights of objects?

Sometimes when objects are too tall to measure, we can use their shadows to help us determine their heights.

Because the sun is so far from the earth, the sun's rays are parallel by the time they reach us. This effect creates equivalent angles and therefore similar triangles built from the height of nearby objects and their shadows. Look at the situation below:



How tall is the lamp post? **Create a proportion!**

$$\frac{\text{OBJECT HEIGHT}}{\text{OBJECT SHADOW}} \qquad \frac{5.25}{6} = \frac{x}{18}$$

Solve for x, the height of the lamp post.

x = 15.75 ft

NOTE CATCHER

Remember...

(x,y)

horizontal first

then vertical

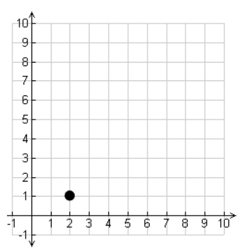
So, to plot (2,1)...

(2,1)

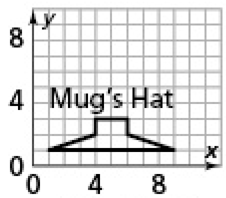
1. Count two spaces to the right

2. Count one space up

3. Place a dot at that position



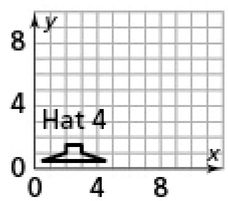
How can we look at the coordinate rules of two figures to determine if they are similar?



RULE:
(x,y)

The coordinate rule of your original figure will always be **(x,y)**.

NOTE: The lefthand corner of Mug's hat is at **(1,1)**.



RULE:
(.5x,.5y)

scale factor

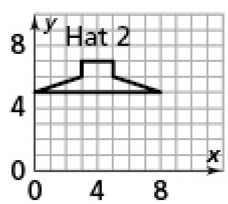
Similar images will always have the same **coefficient** in front of both the **x** and **y**. That number represents the **scale factor** from the original figure to the image.

> Remember, if the scale factor is a whole number greater than 1, the figure is an **enlargement**. If it's a fraction, it will be a **reduction**. You will never have a negative scale factor!

Is **Hat 4** similar to Mug's original hat?

SIMILAR ✓

NOTE: The lefthand corner of Hat 4 is at **(.5,.5)** (1,1) → (.5x,.5y) → (.5,.5)



RULE:
(x–1,y+4)

As long as the coefficients in front of our image's **x** and **y** are the same, we have similar images. In this case, the coefficients in front of **x** and **y** are both **1**, so this image is similar to the original.

When numbers are **added to** or **subtracted from** our **x** and **y**, we are merely shifting the image around the coordinate plane. In this case, our image will be **shifted 1 space to the left** and **4 spaces up** on the coordinate plane.

Is **Hat 2** similar to Mug's original hat?

SIMILAR ✓

NOTE: The lefthand corner of Hat 2 is at **(0,5)** (1,1) → (x-1,y+4) → (0,5)

(x+1,y+2)

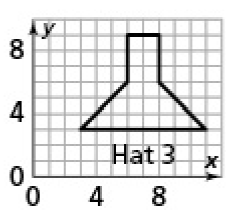
move to the right one space

move up two spaces

(x–1,y–2)

move to the left one space

move down two spaces



RULE:
(x+2,3y)

The coefficient in front of **x** is 1 and the coefficient in front of **y** is 3. Therefore, this image is not similar to the original. This image does move two spaces to the right and is stretched vertically by 300%.

REMEMBER...always look at the coefficients to determine similarity.

Is **Hat 3** similar to Mug's original hat?

NOT SIMILAR ✖

NOTE: The lefthand corner of Hat 3 is at **(3,3)** (1,1) → (x+2,3y) → (3,3)