Math Language and Fluency

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Say hello.

Describe your role as an educator and the mathematics you support.

Schedule for This Year

September 19	**Mathematics Language and Fluency**
October 17	High-Quality Tier 1
December 5	Leveraging Word Problems – Part 1
January 26	Leveraging Word Problems – Part 2
February 16	High-Quality Mathematics Assessment
March 16	High-Quality Supports in Mathematics – Putting It Together

Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

Mathematical Language

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

INSTRUCTIONAL STRATEGIES

Mathematics Language and Fluency
September 19, 2022

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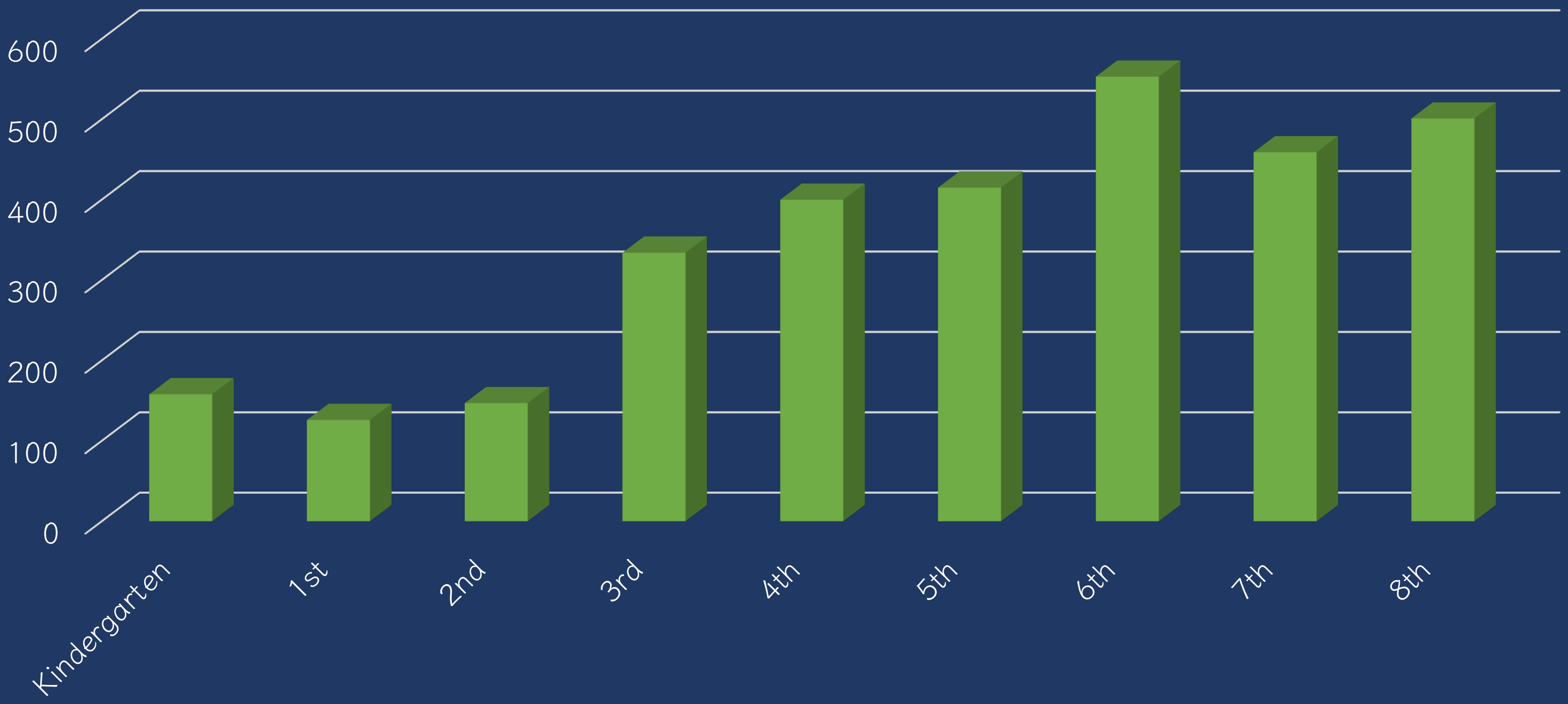
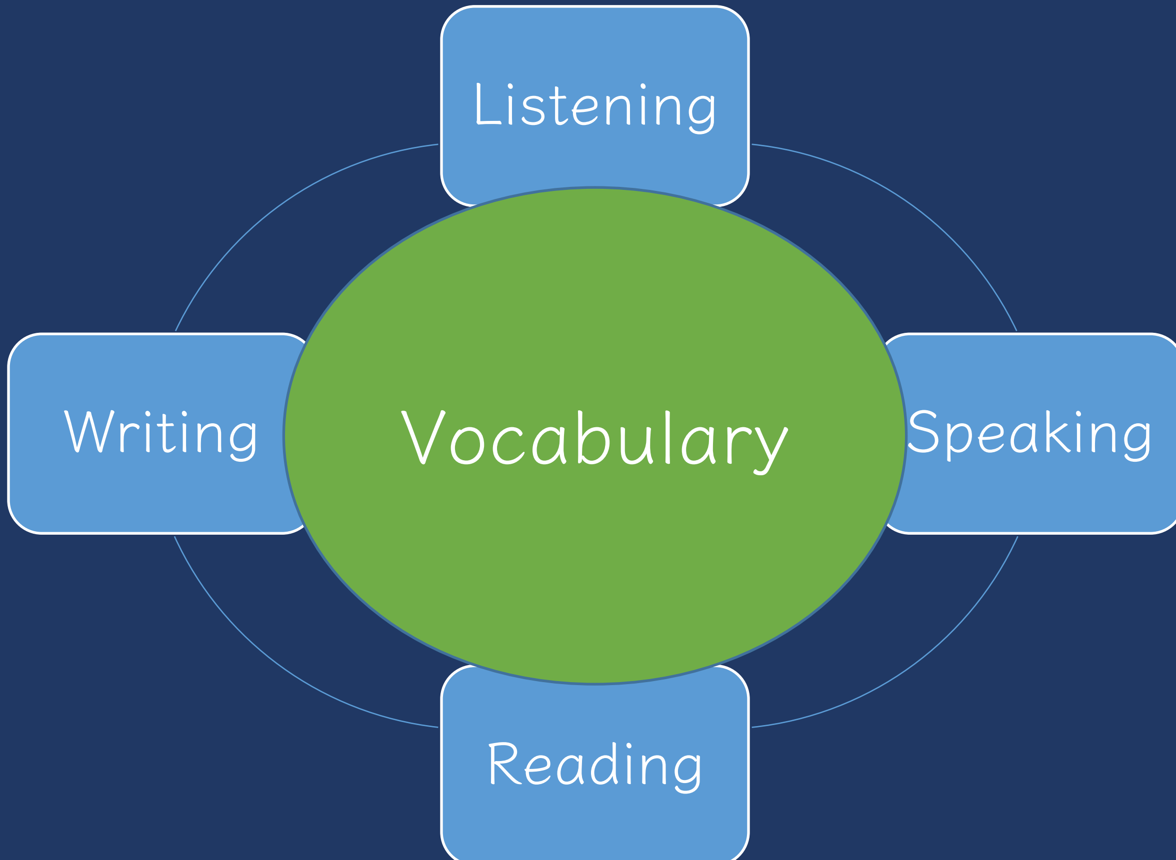
Mathematical Language

Instead of that...	Say this...



Powell, Bos, & Lin (2019)


Listening
Writing
Vocabulary
Speaking
Reading

1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

divide vs.
Continental
Divide

variable vs.
variably
cloudy

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings (but a more precise math meaning)

3. Some math terms are only used in math

4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

6. Some math terms are homographs

7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
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5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way

skip count vs. multiples

one-fourth vs. one quarter

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

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8. An English math term may translate into another language with different meanings

9. English spelling and usage may have irregularities

10. Some math concepts are verbalized in more than one way

11. Informal terms may be used for formal math terms

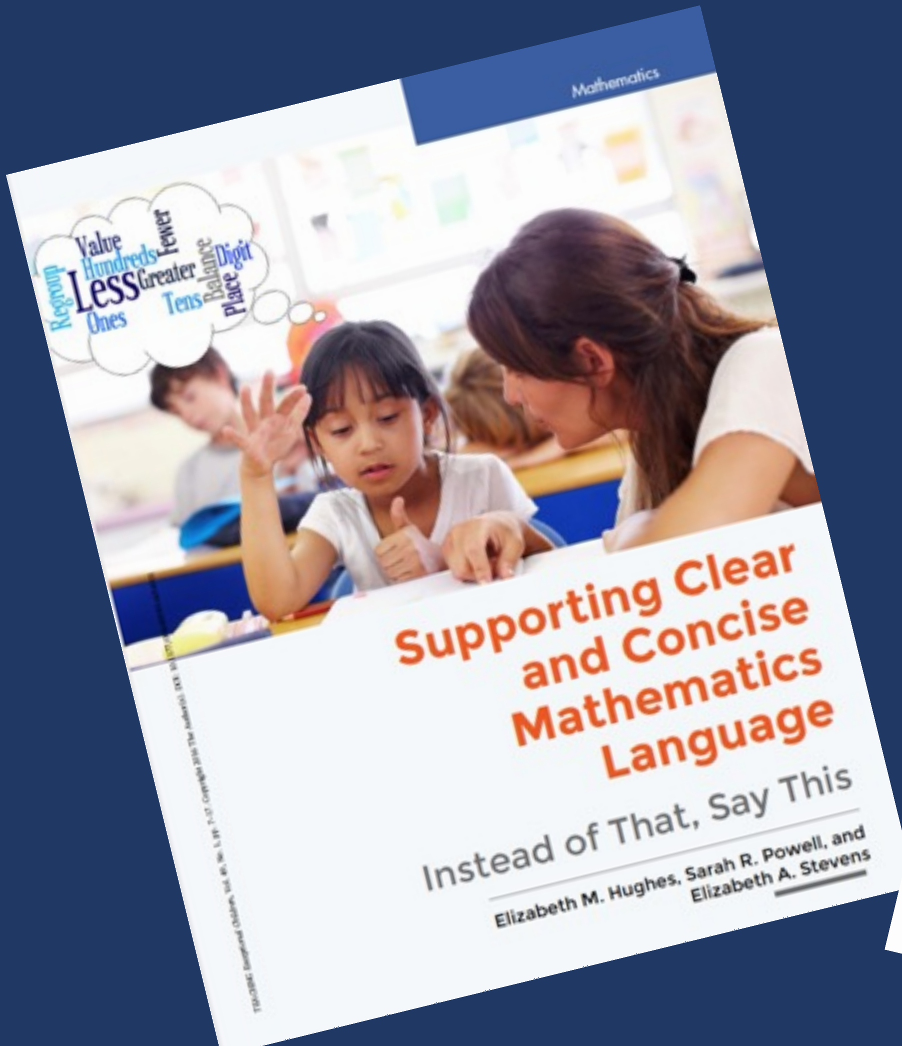
rhombus vs. diamond

vertex vs. corner

What are the ways you see your students experience difficulty with the vocabulary of math?

Use formal math language

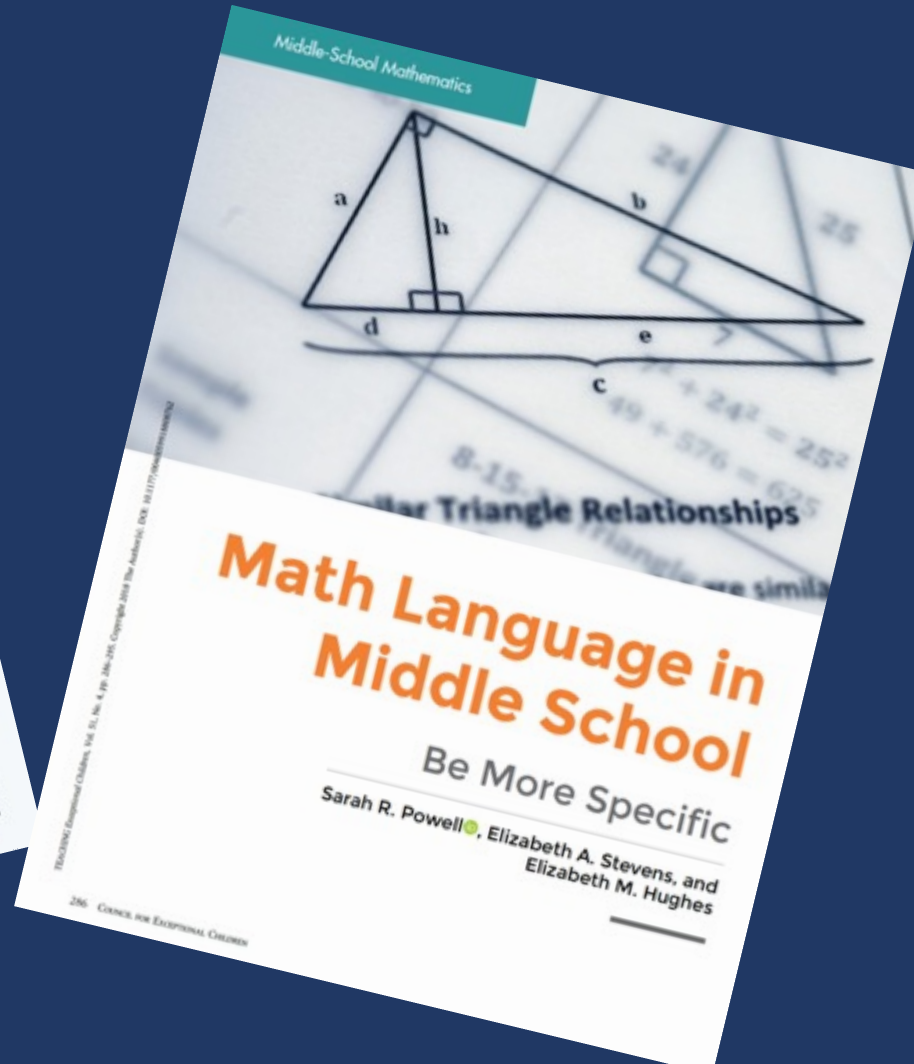
Use terms precisely


Mathematics

Supporting Clear and Concise Mathematics Language

Instead of That, Say This

Elizabeth M. Hughes, Sarah R. Powell, and Elizabeth A. Stevens

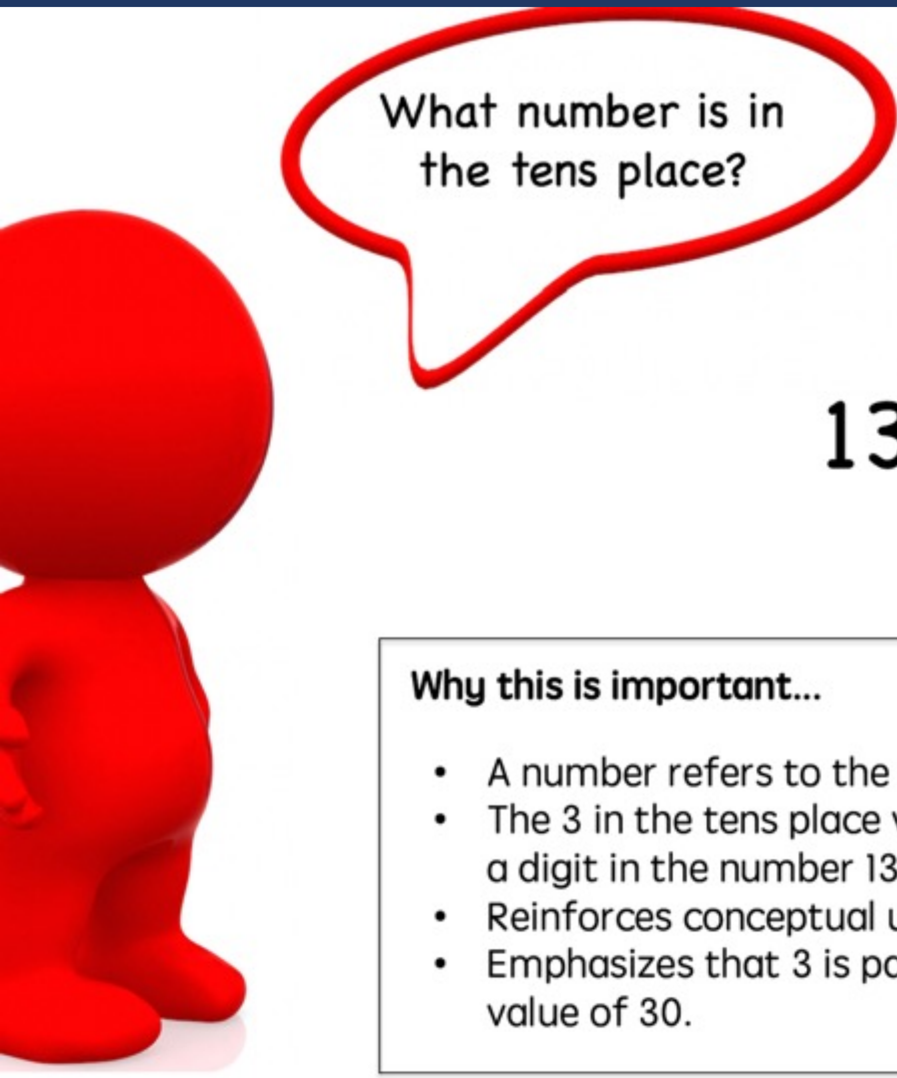

Middle-School Mathematics

Math Language in Middle School

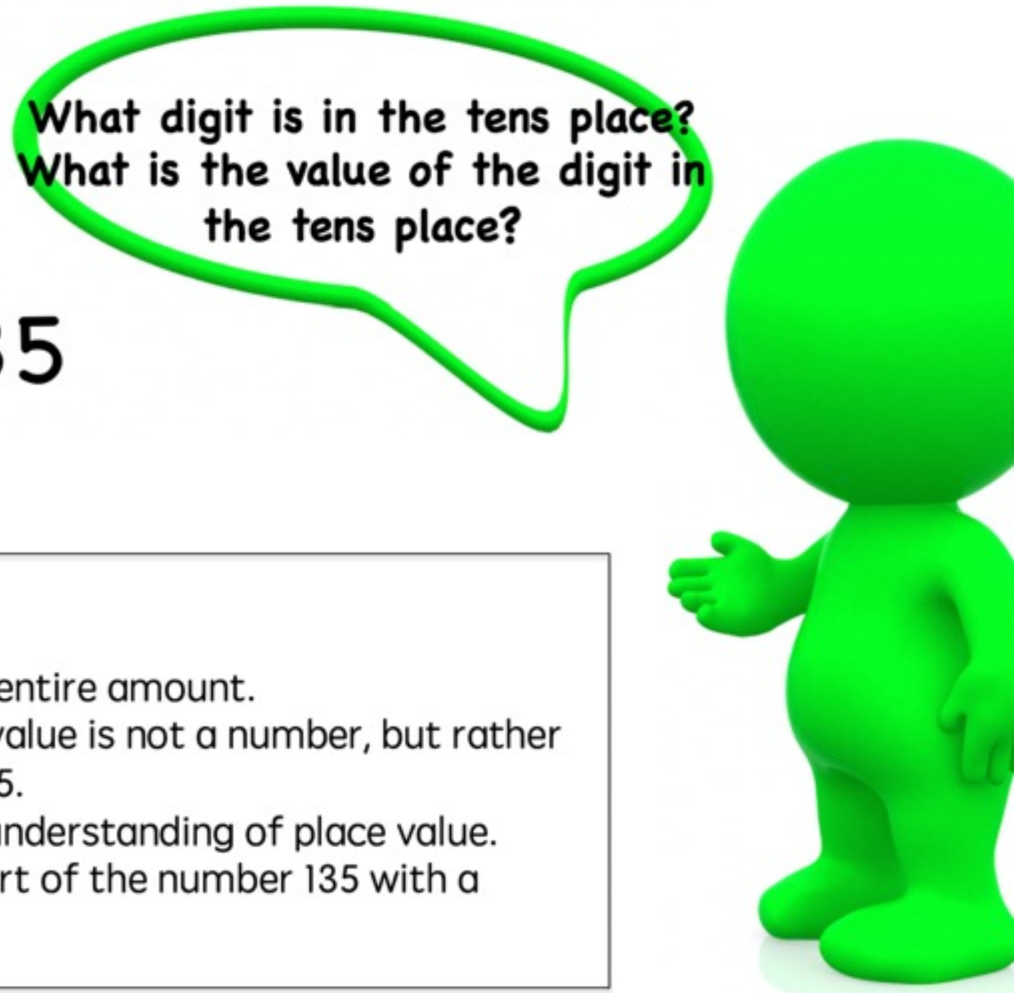
Be More Specific

Sarah R. Powell, Elizabeth A. Stevens, and Elizabeth M. Hughes

286 Council for Exceptional Children

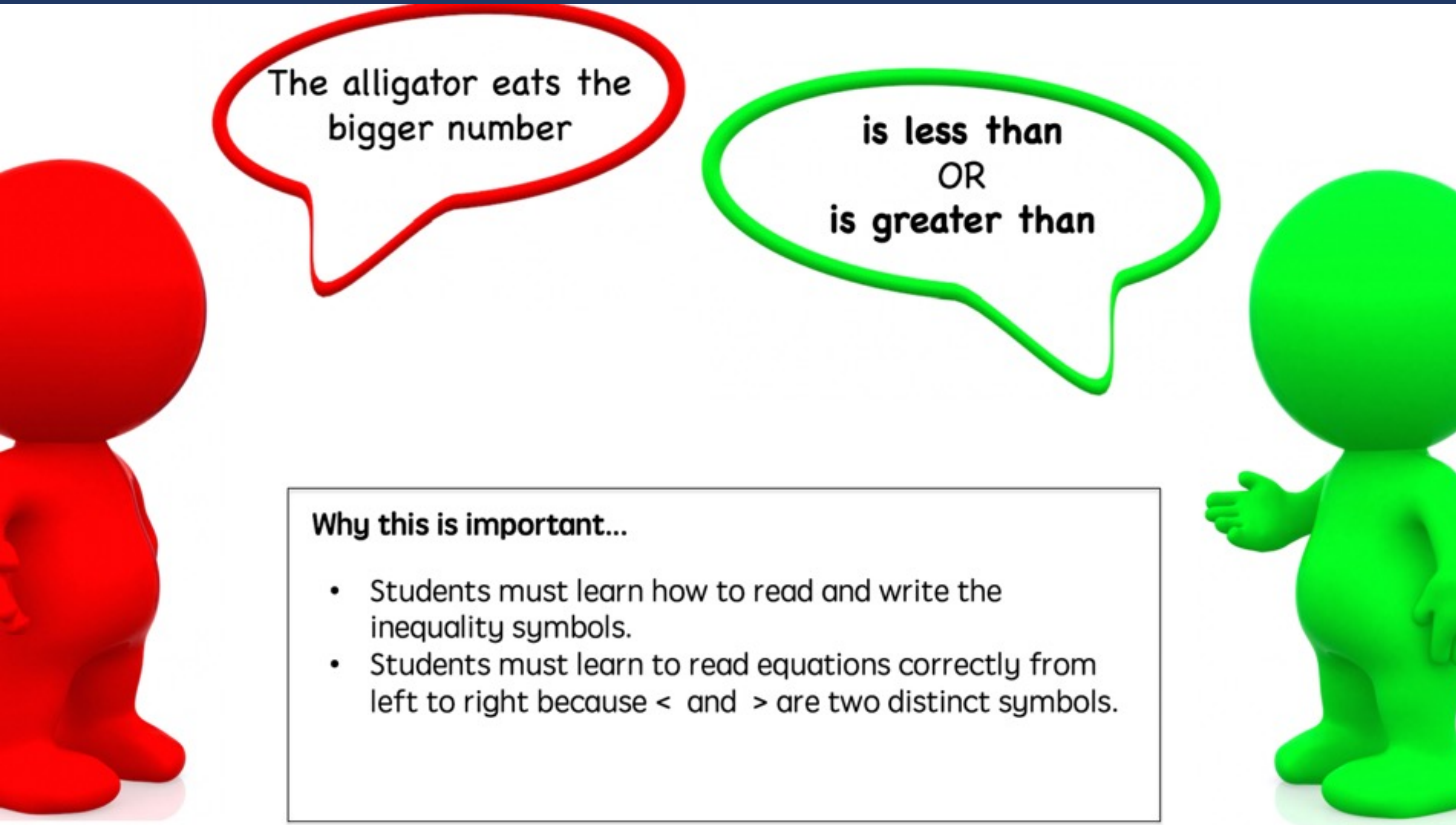


135



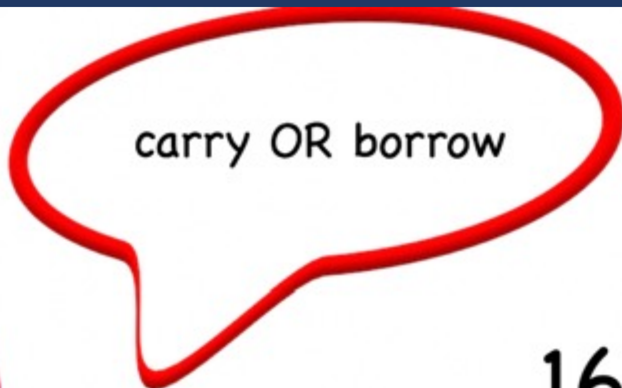
Why this is important...

- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.



Why this is important…

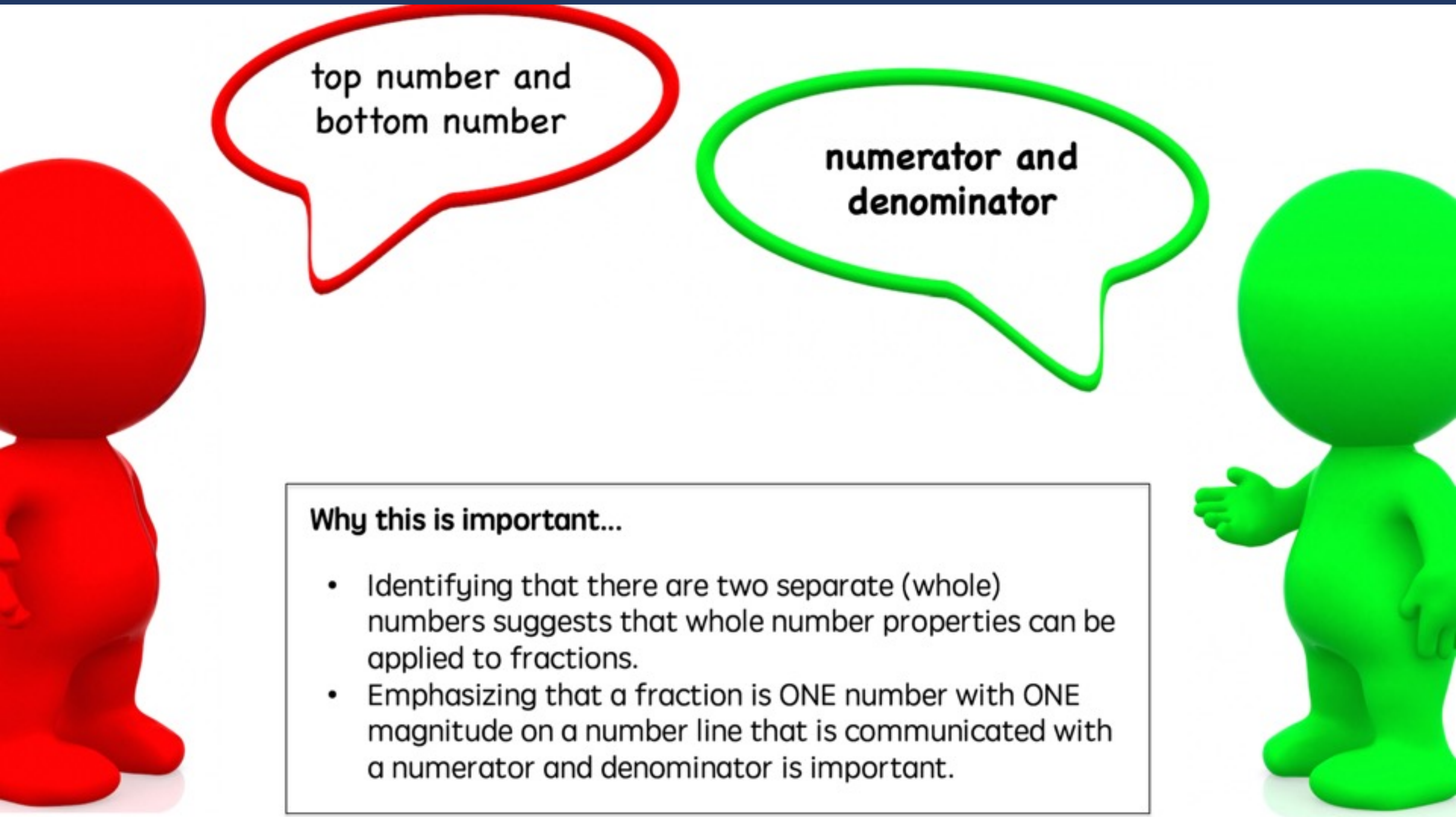
- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.



$$\begin{array}{r} 167 \\ +\ \ 294 \\ \hline \end{array}$$

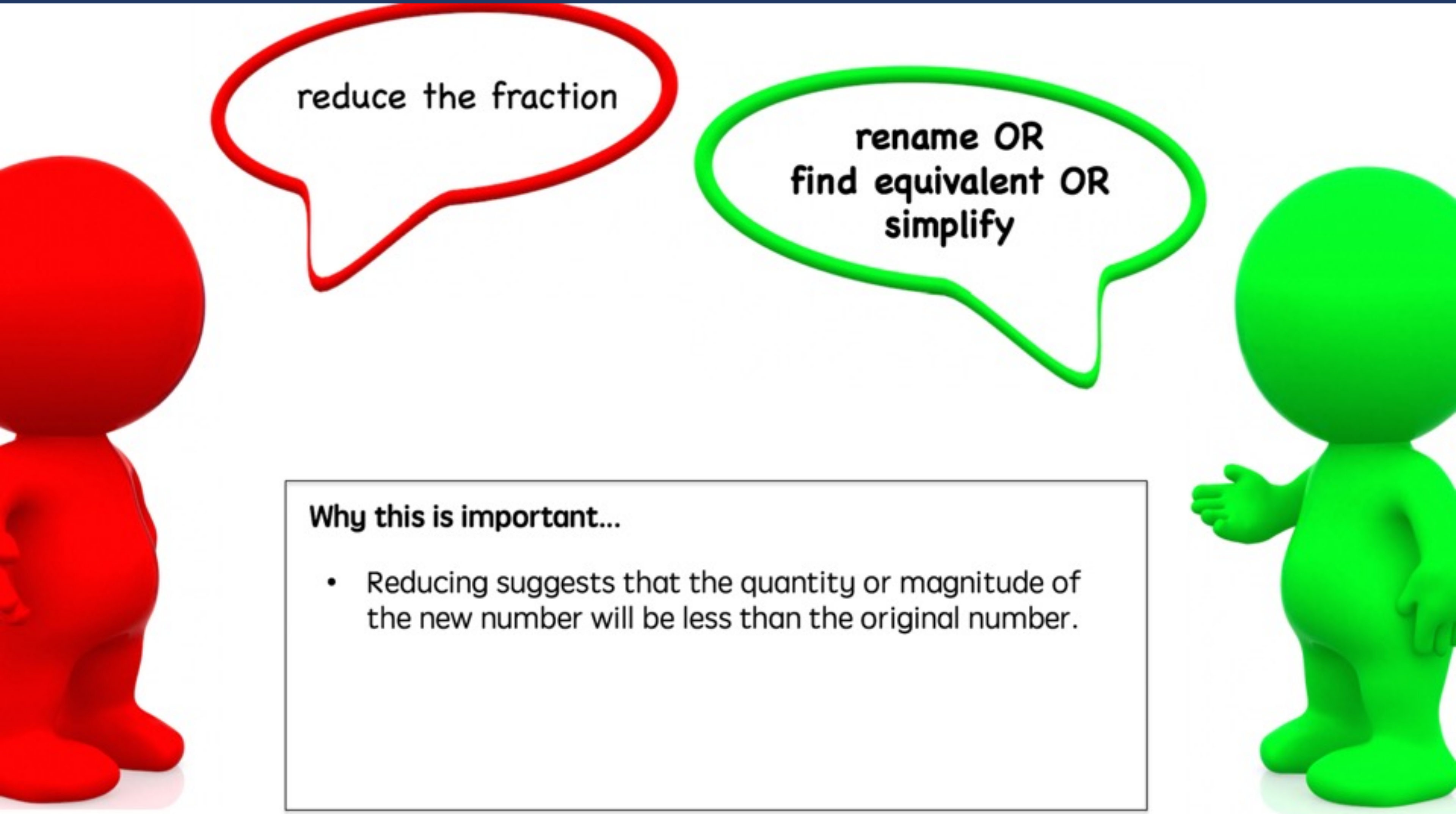
Why this is important…

- “Carry” or “borrow” is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



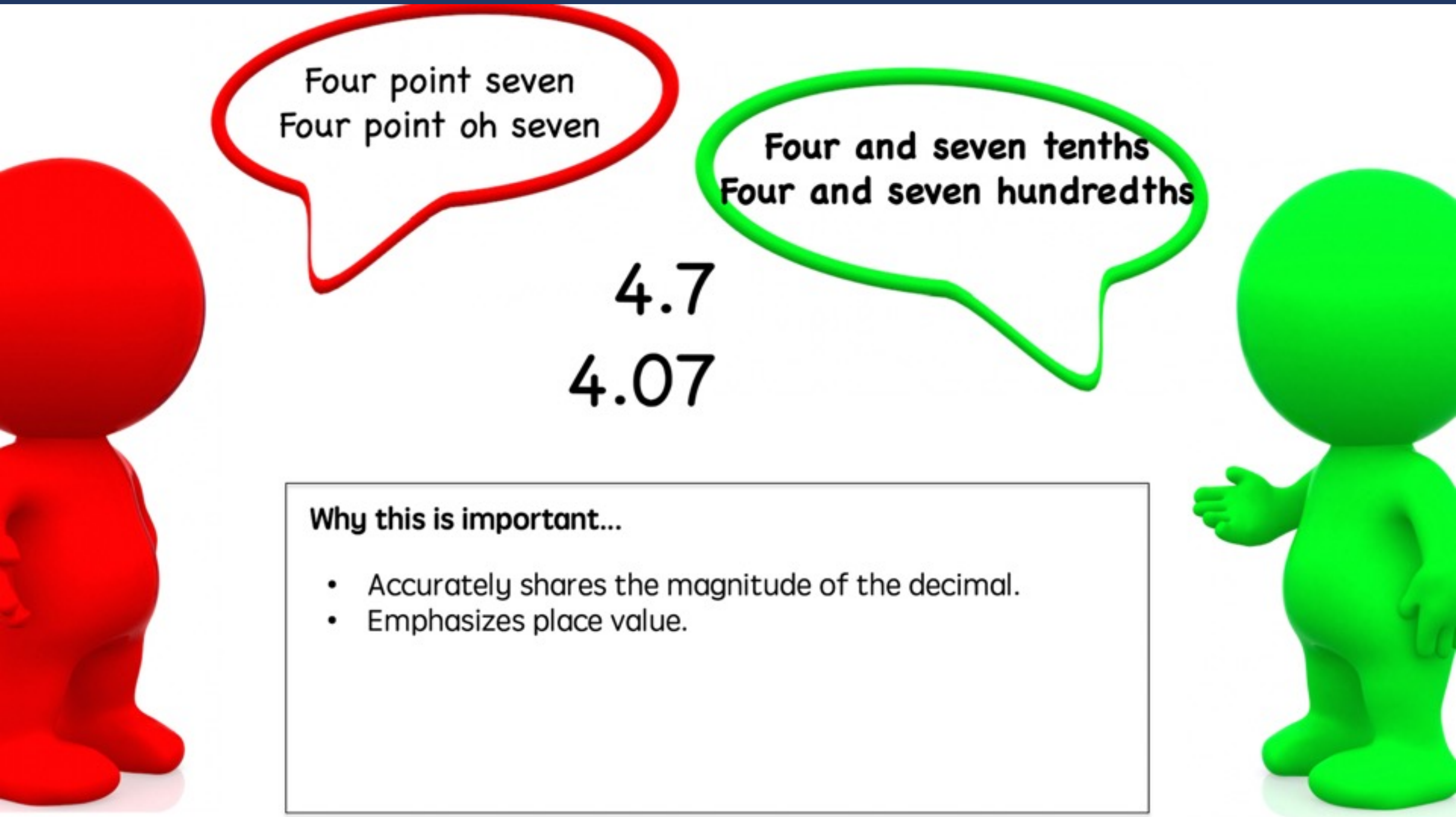
Why this is important...

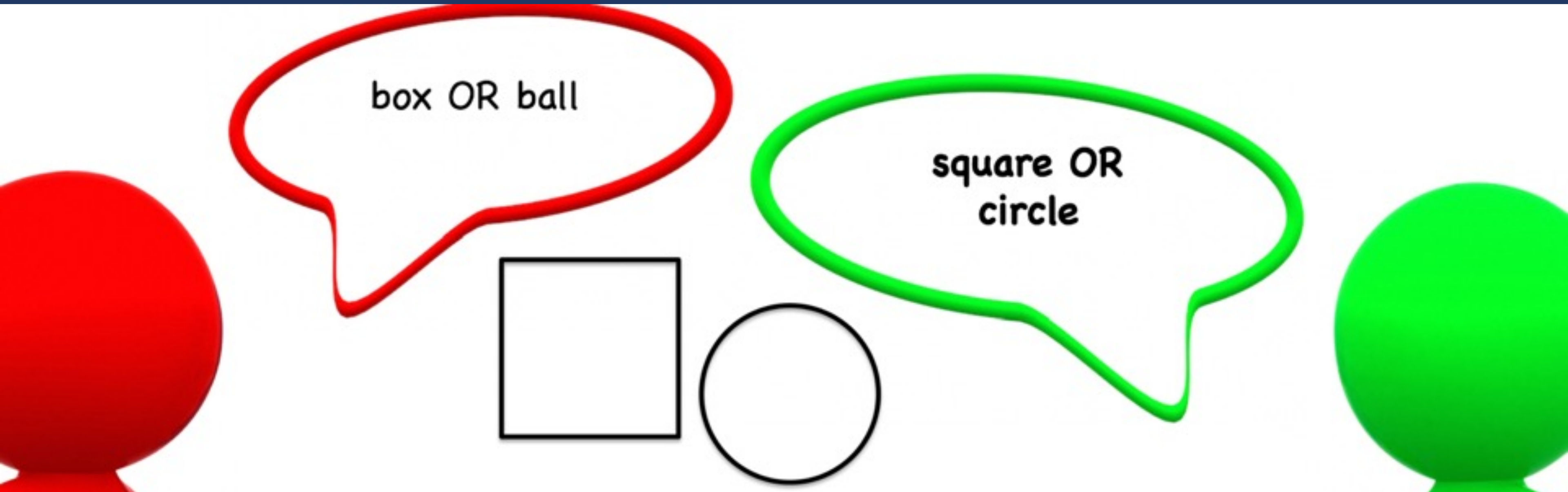
- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.



Why this is important...

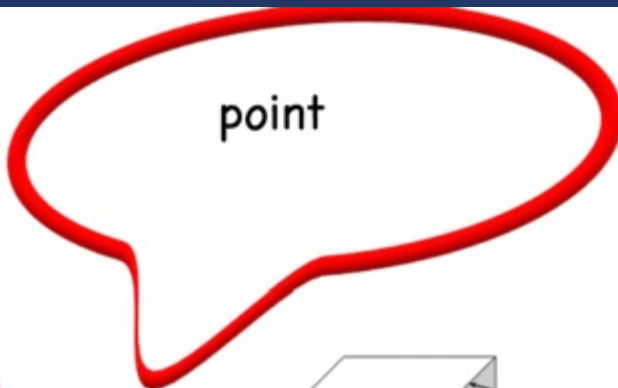
- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

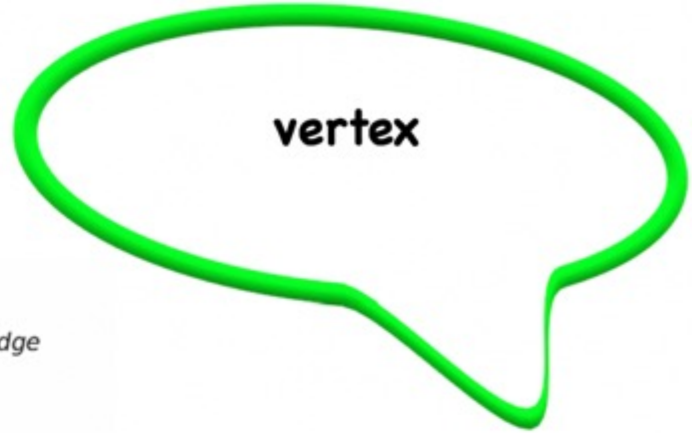

Four point seven
Four point oh seven
Four and seven tenths
Four and seven hundredths
4.7
4.07
Why this is important...
• Accurately shares the magnitude of the decimal.
• Emphasizes place value.

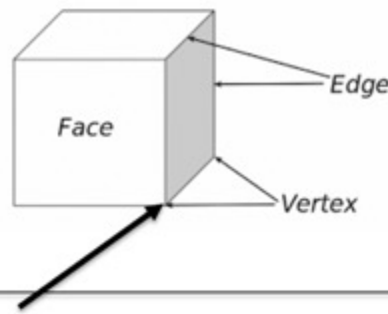


Why this is important...

- Use the formal language of shapes to confirm informal language.







Why this is important...

- This is the endpoint where two or more line segments or rays meet.

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Mathematical Language

Instead of that…	Say this…

Identify examples of "Instead of ____, say ____."

Use formal math language

Use terms precisely

Use Terms With Precision

Strategies for Teaching Mathematics Language

MATH

Factor

$1 \times 8 = 8$

$2 \times 4 = 8$

factor factor

Multiple

$8 \times 1 = 8$

$8 \times 2 = 16$

multiples of 8

E

Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

4:3

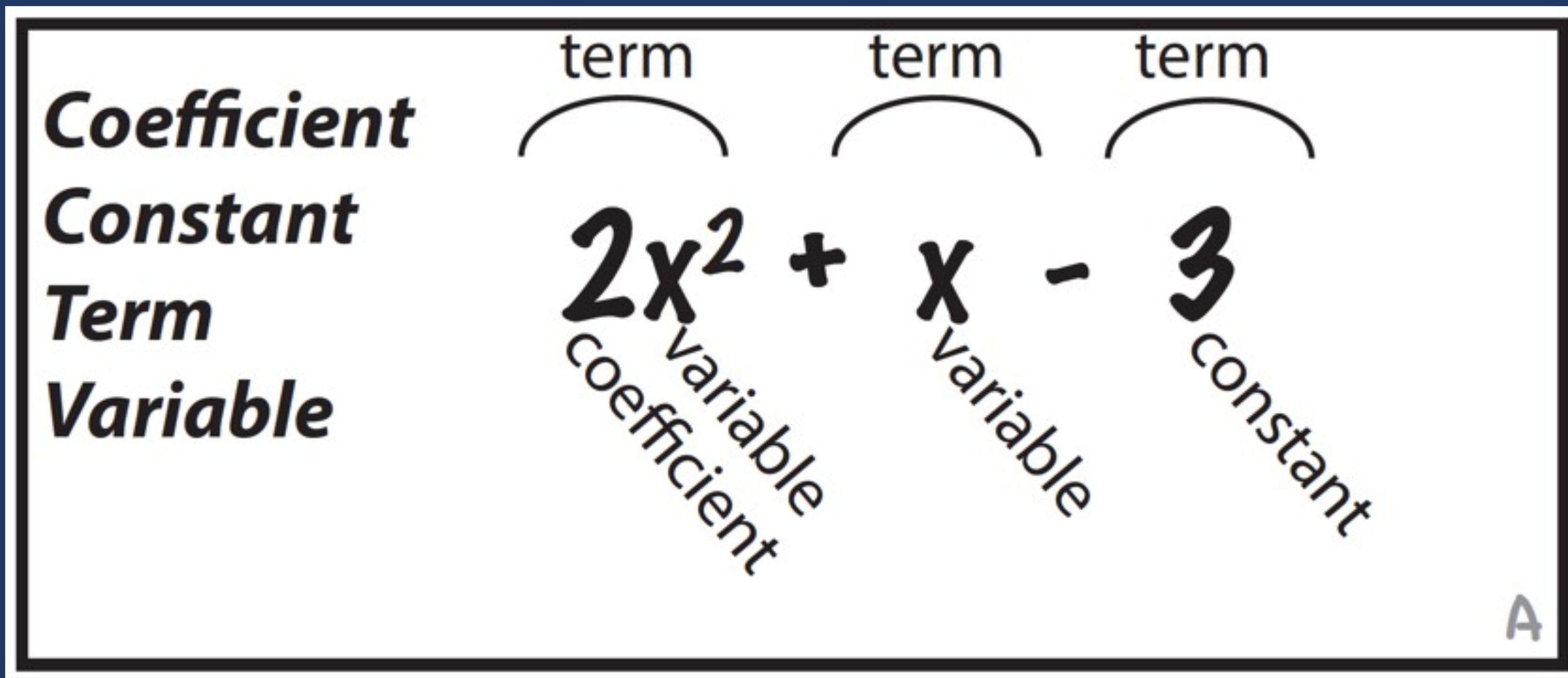
Proper fraction

$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

D


Coefficient
Constant
Term
Variable
term
term
term
$2x^2 + x - 3$
coefficient
variable
variable
constant
A

Equation	$9x - 4 = 7x$
Expression	$9x - 4$
Formula	$a^2 + b^2 = c^2$
Function	$f(x)$
Inequality	$9x - 4 > 6x$

C

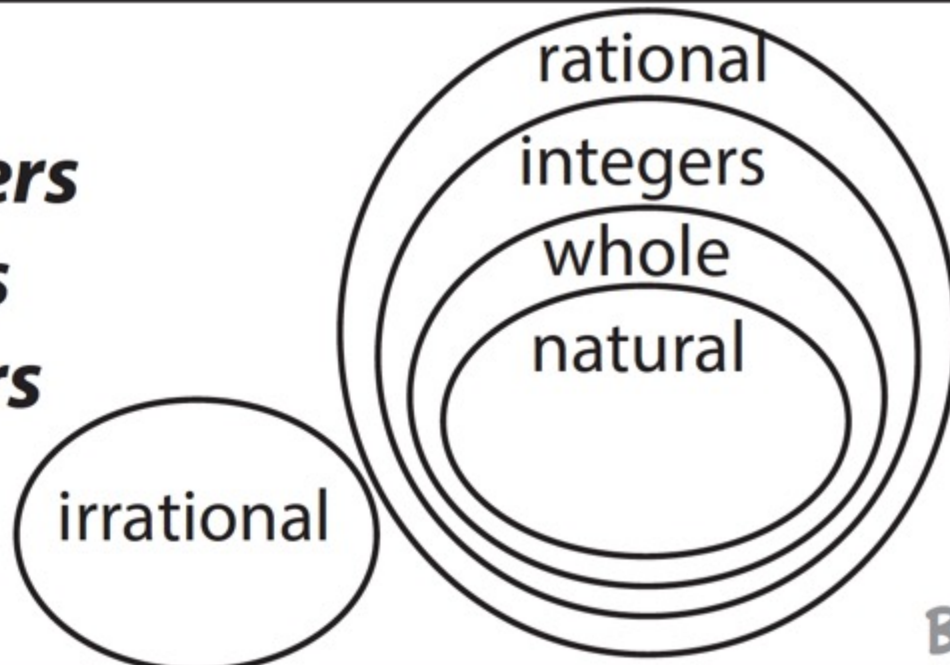
Integers

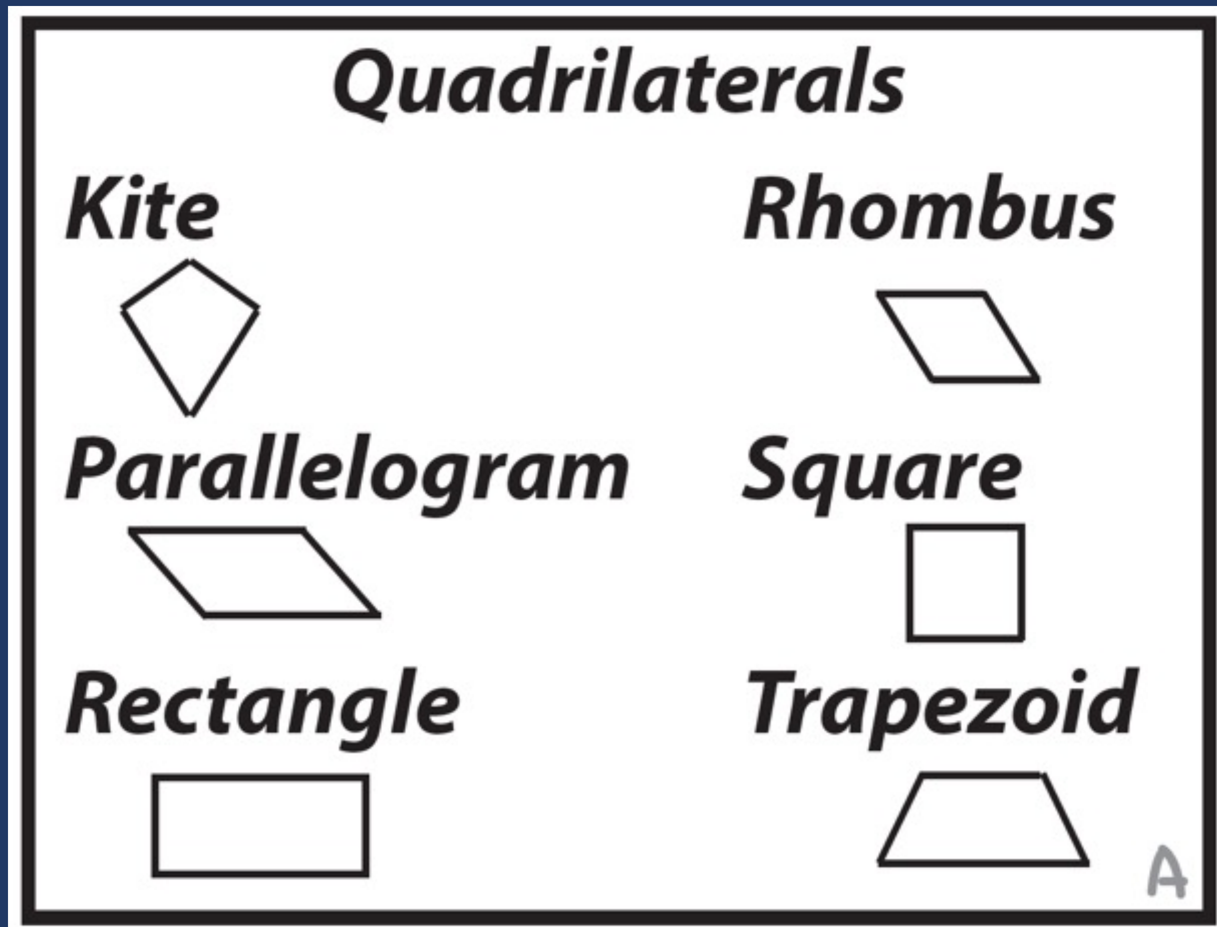
Irrational numbers

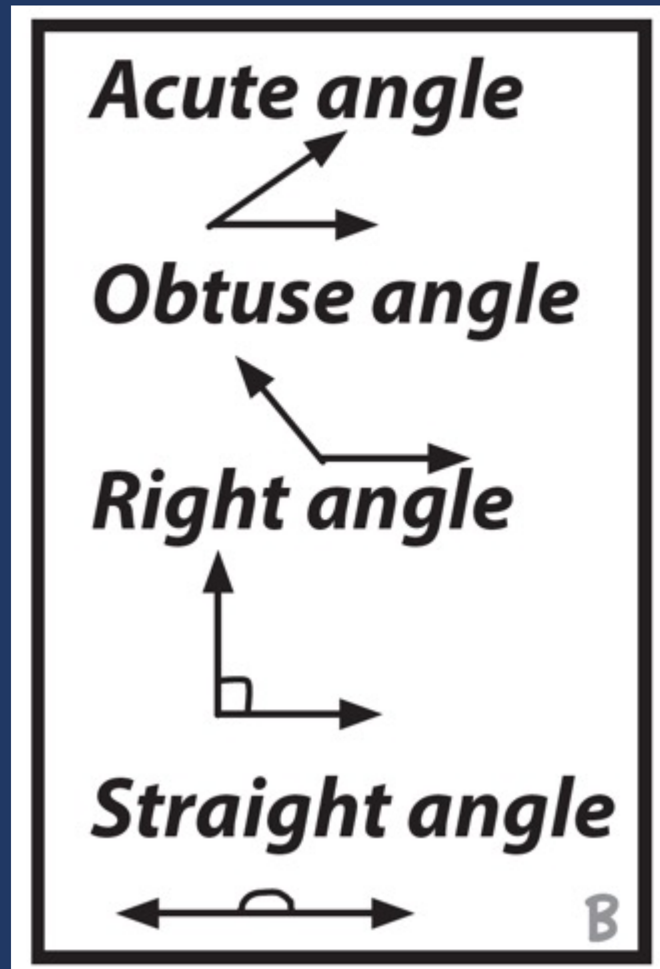
Natural numbers

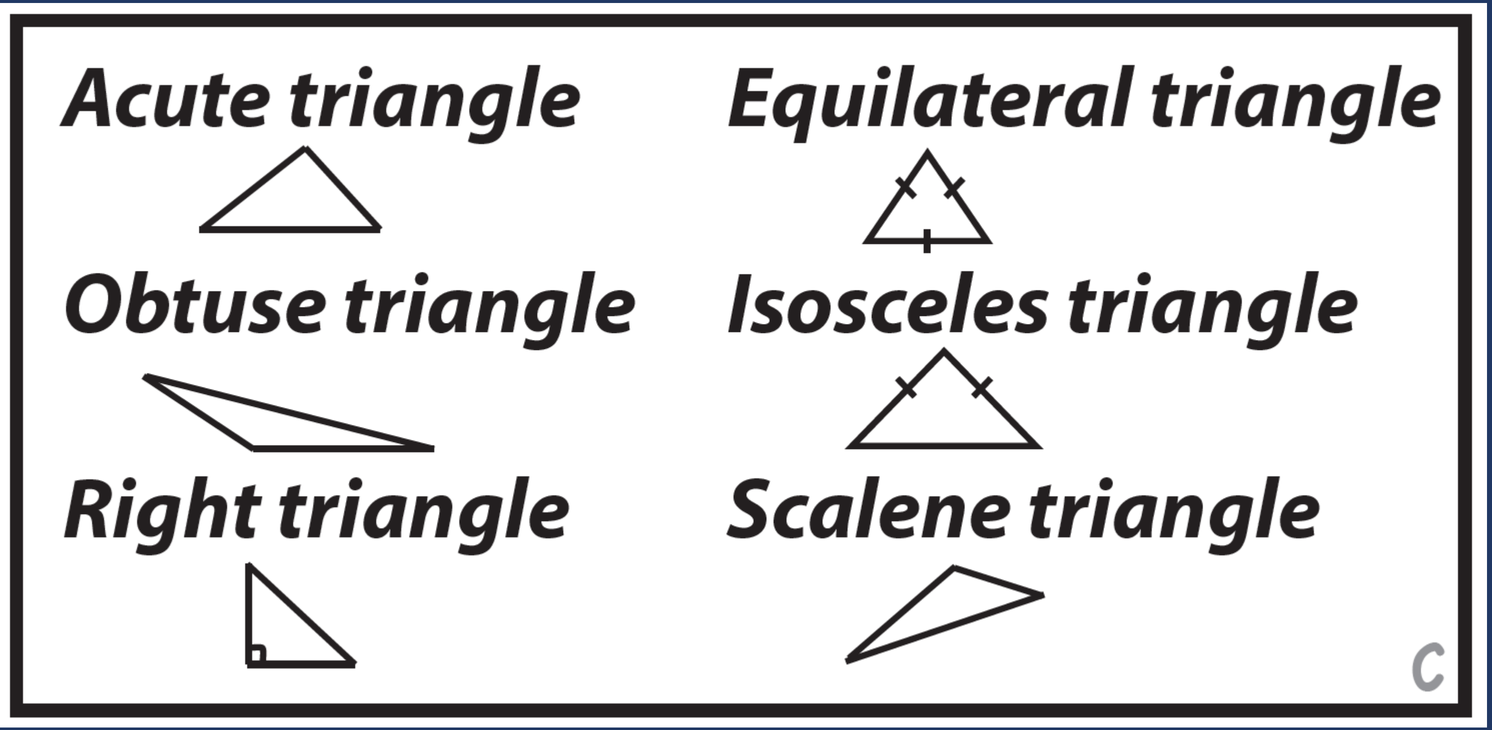
Rational numbers

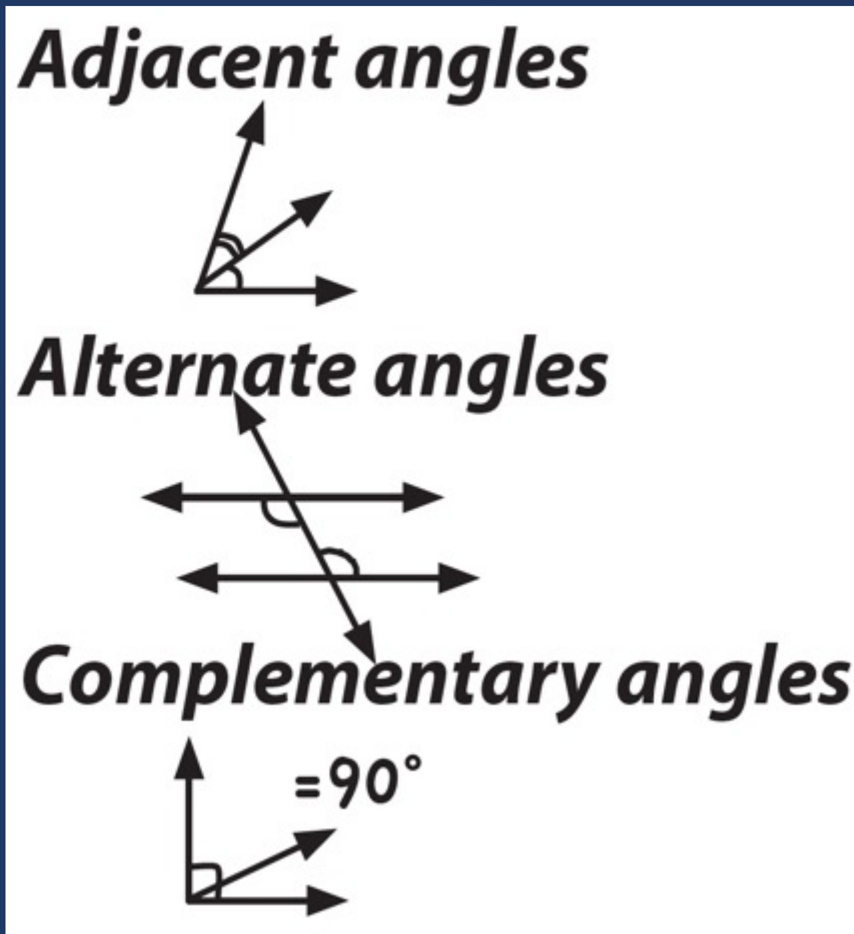
Whole numbers

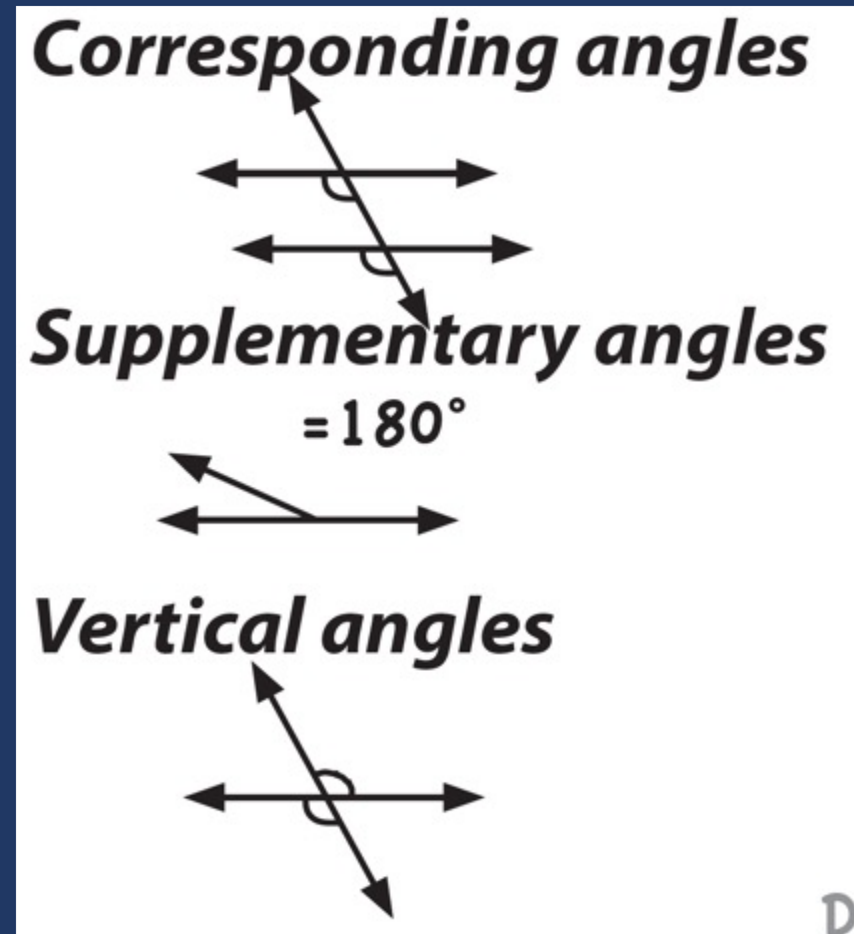



Quadrilaterals
Kite
Rhombus
Parallelogram
Square
Rectangle
Trapezoid


Acute angle
Obtuse angle
Right angle
Straight angle
B


Acute triangle
Equilateral triangle
Obtuse triangle
Isosceles triangle
Right triangle
Scalene triangle
C

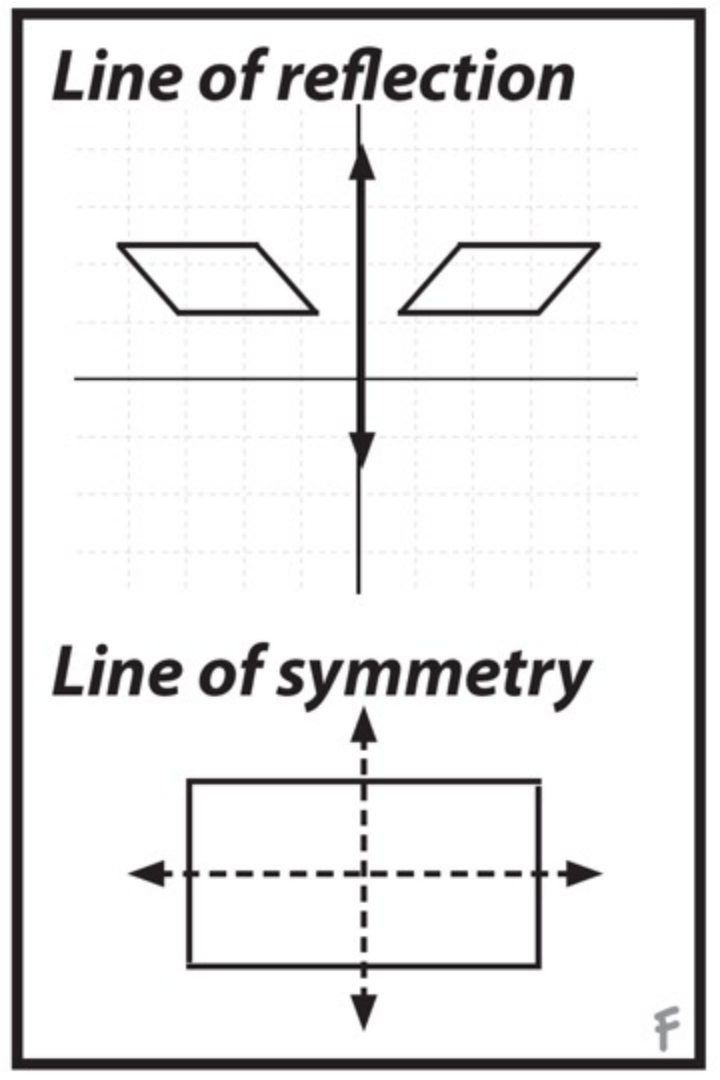

Adjacent angles
Alternate angles
Complementary angles
=90°

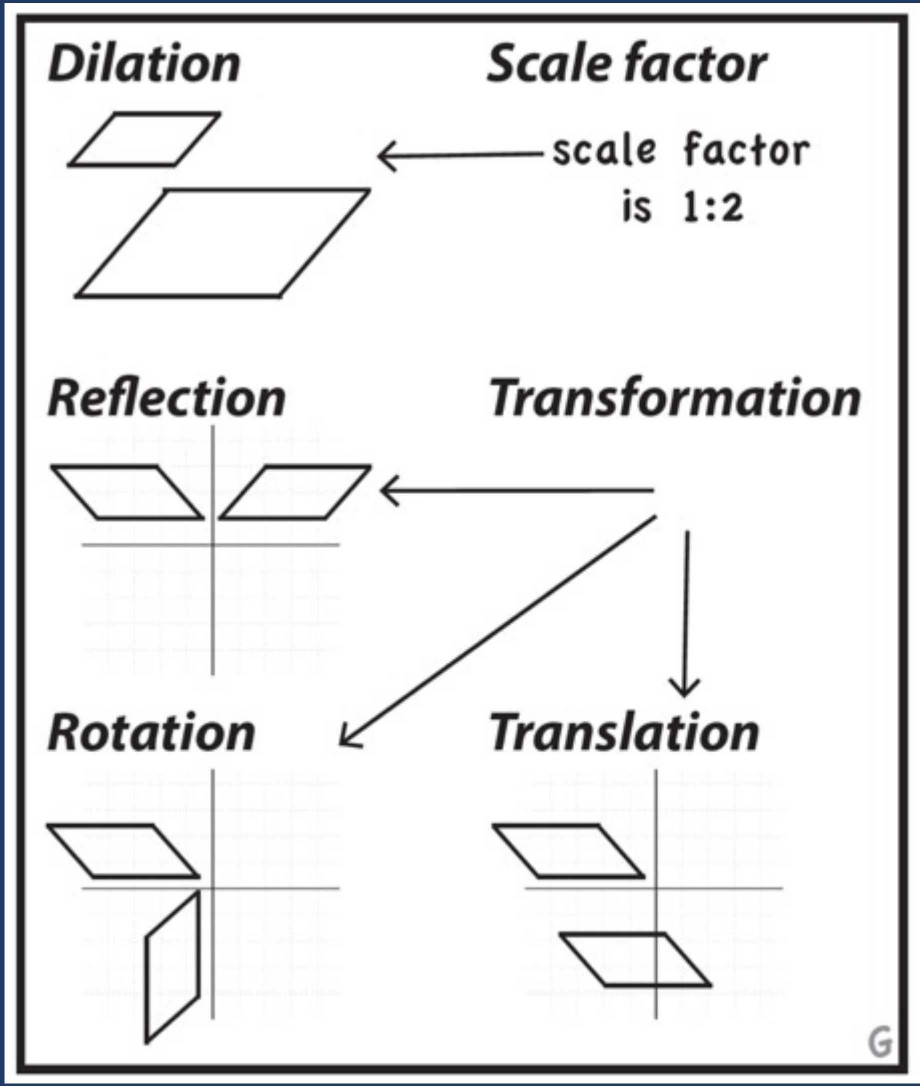

Corresponding angles
Supplementary angles
=180°
Vertical angles

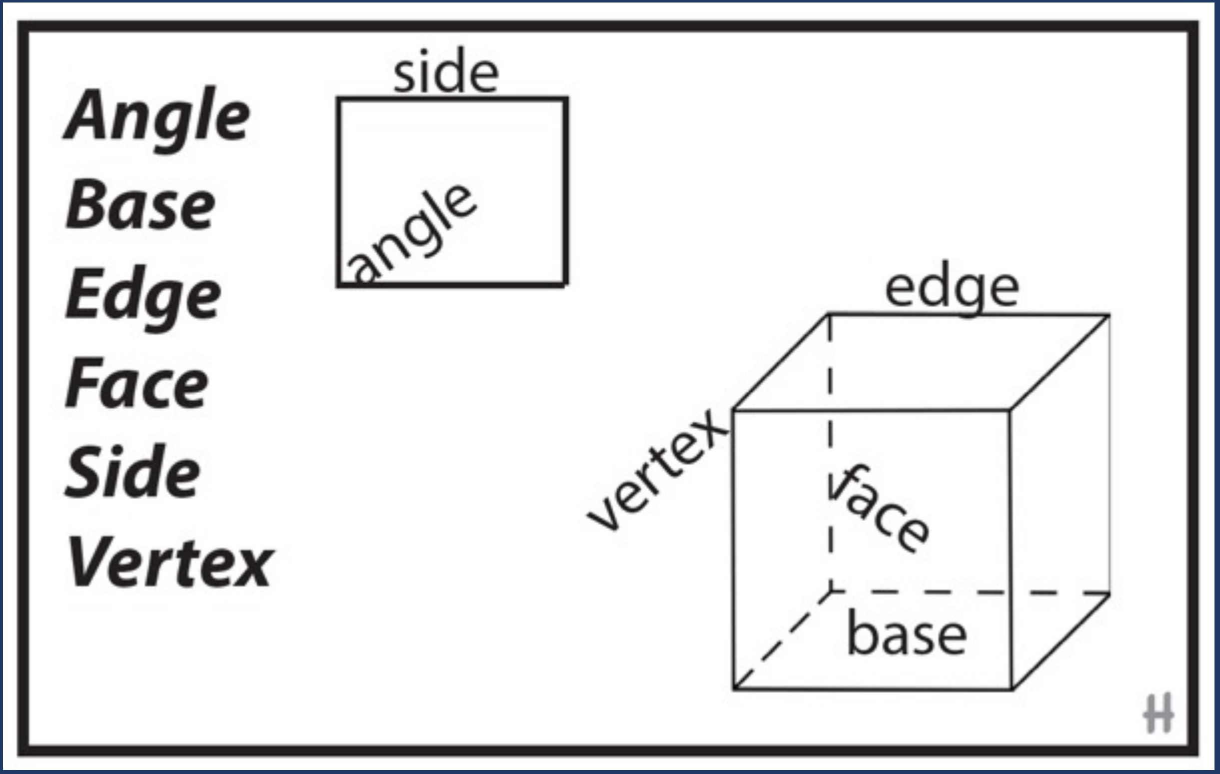
Congruent figures

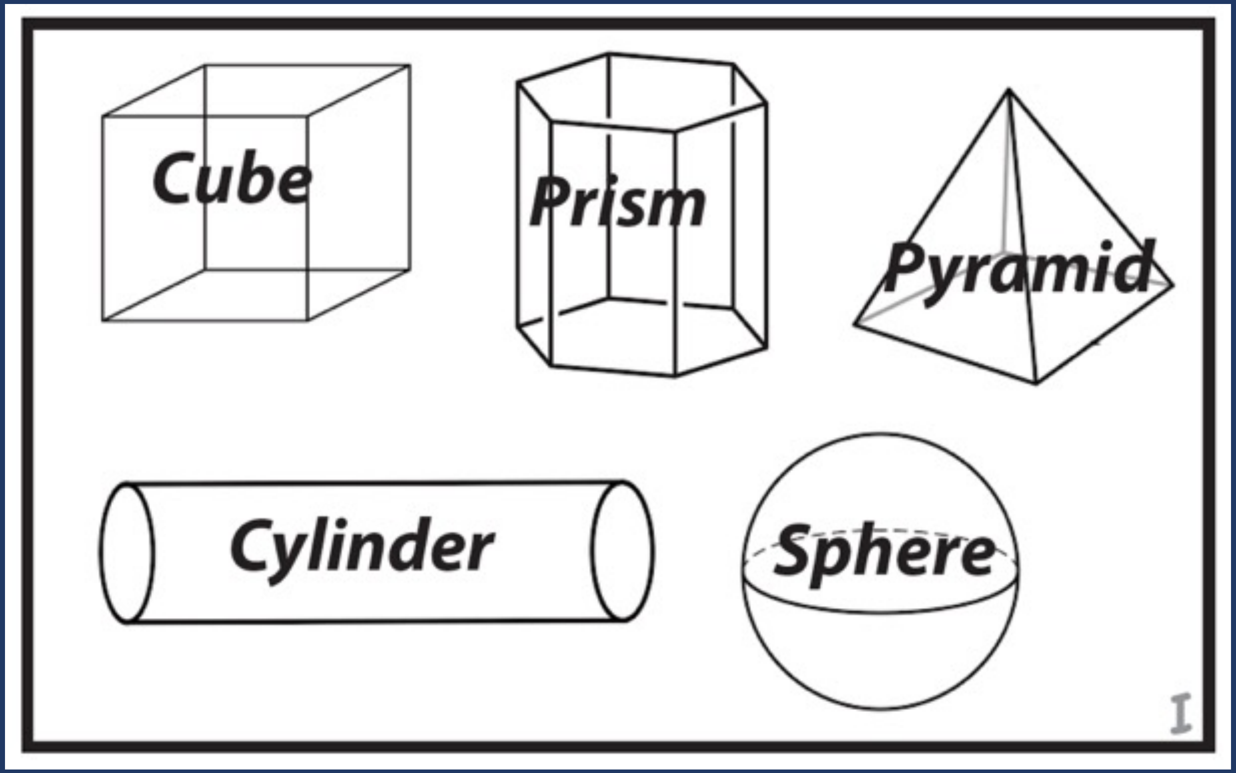
Similar figures

E


Line of reflection
Line of symmetry


Dilation
Scale factor
scale factor is 1:2
Reflection
Transformation
Rotation
Translation


Angle
Base
Edge
Face
Side
Vertex
side
angle
edge
vertex
face
base


Cube
Prism
Pyramid
Cylinder
Sphere

Coordinate plane

Ordered pair

Quadrants

x

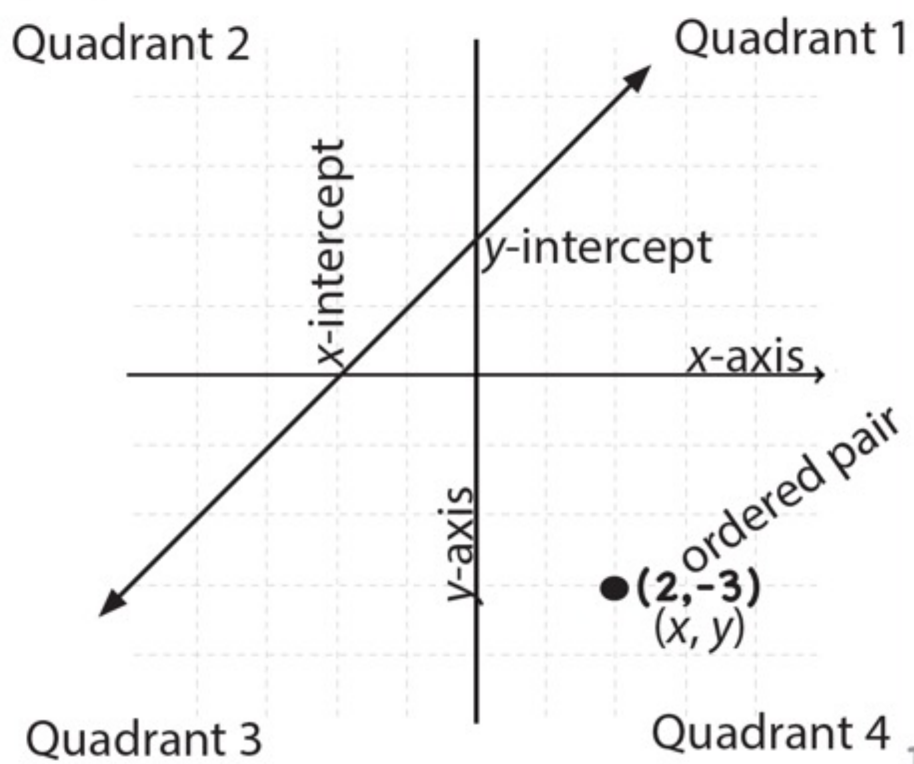
x-axis

x-intercept

y

y-axis

y-intercept



Use Terms With Precision

Strategies for Teaching Mathematics Language

Discuss terms you want your students to use with precision.

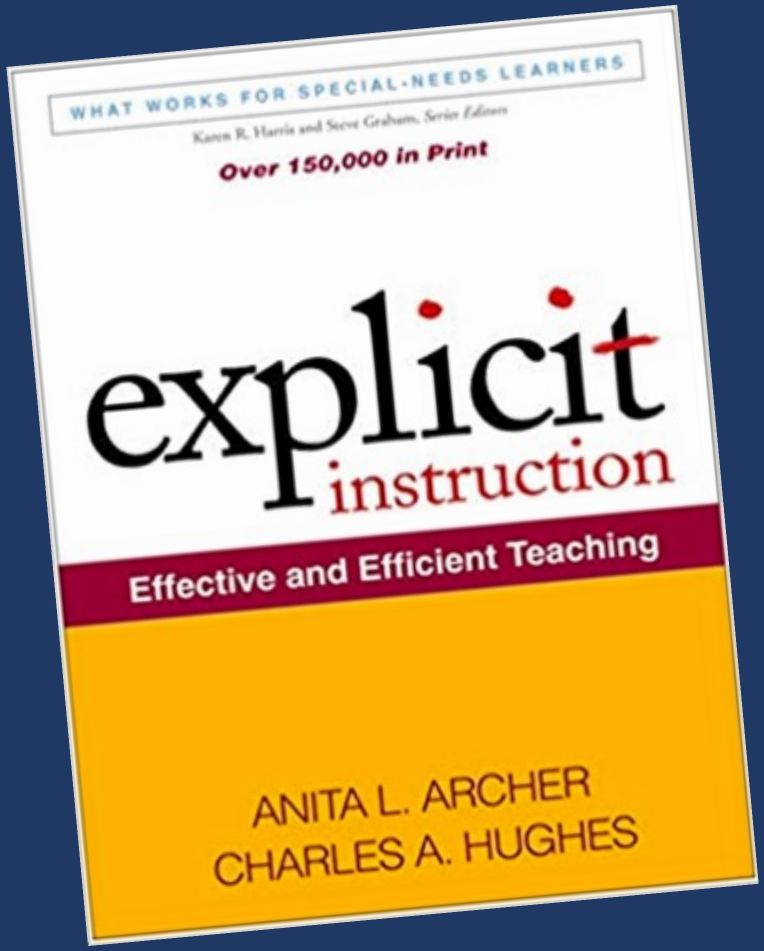
Use formal math language

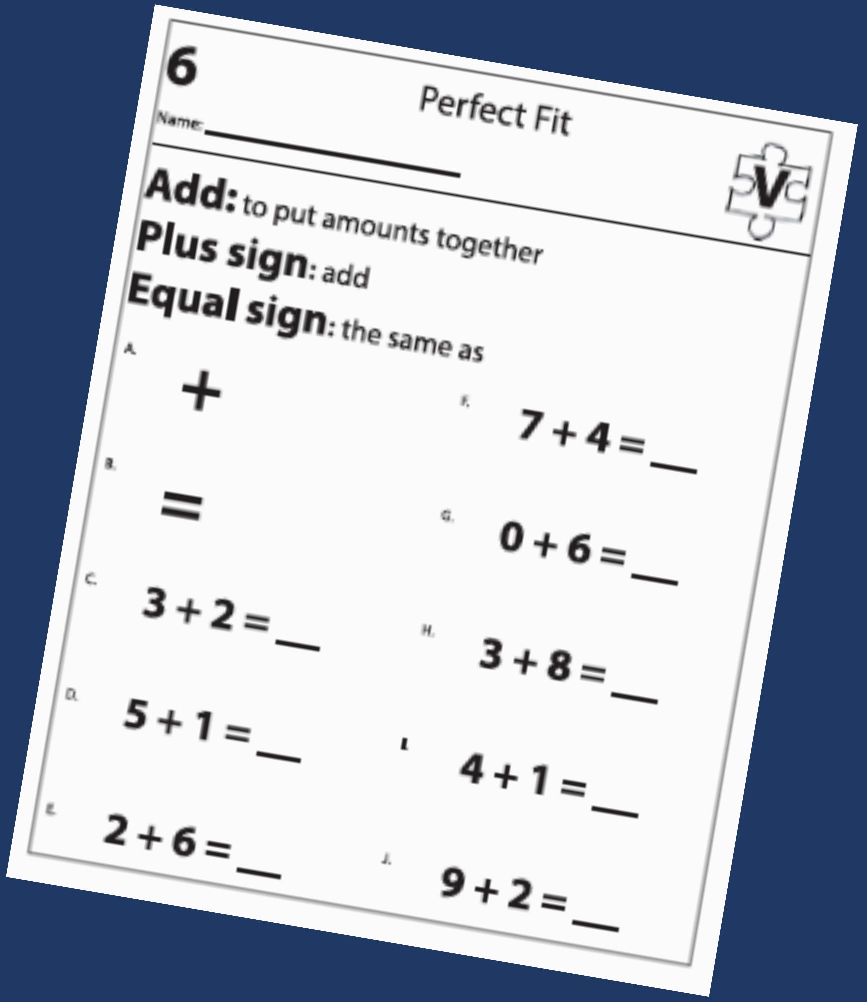
Use terms precisely

Use Terms With Precision

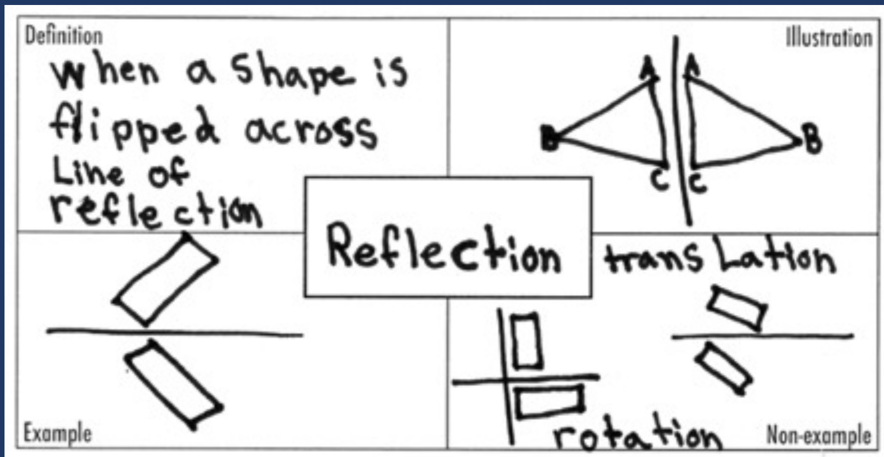
Strategies for Teaching Mathematics Language

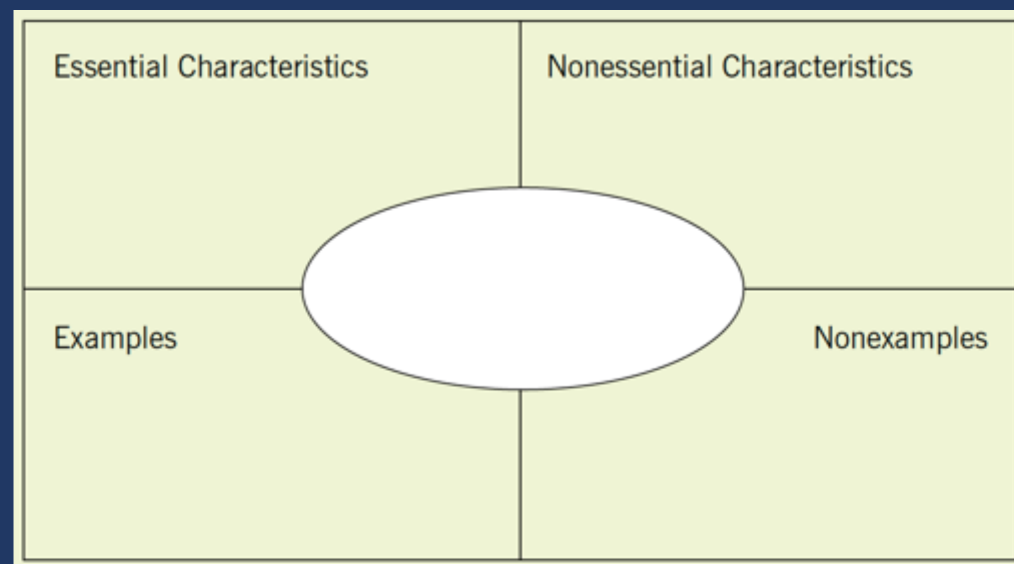
1. Use explicit instruction





2. Use graphic organizers





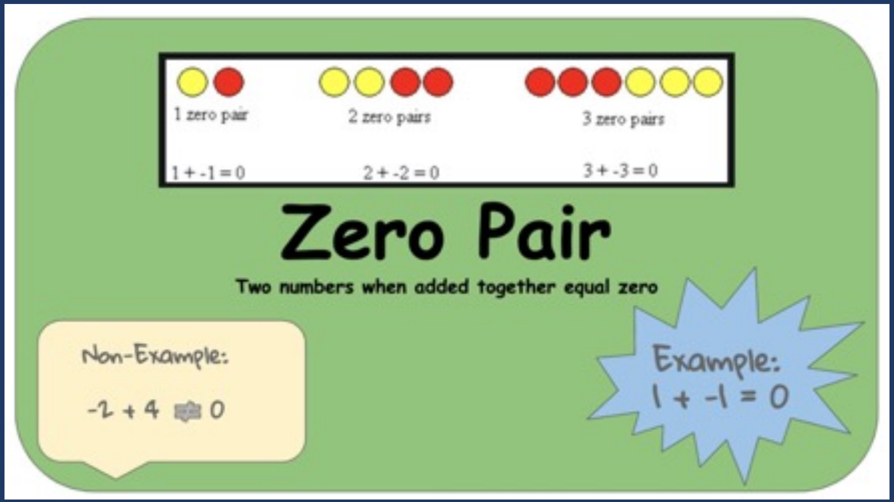
Dunston & Tyminski (2013)

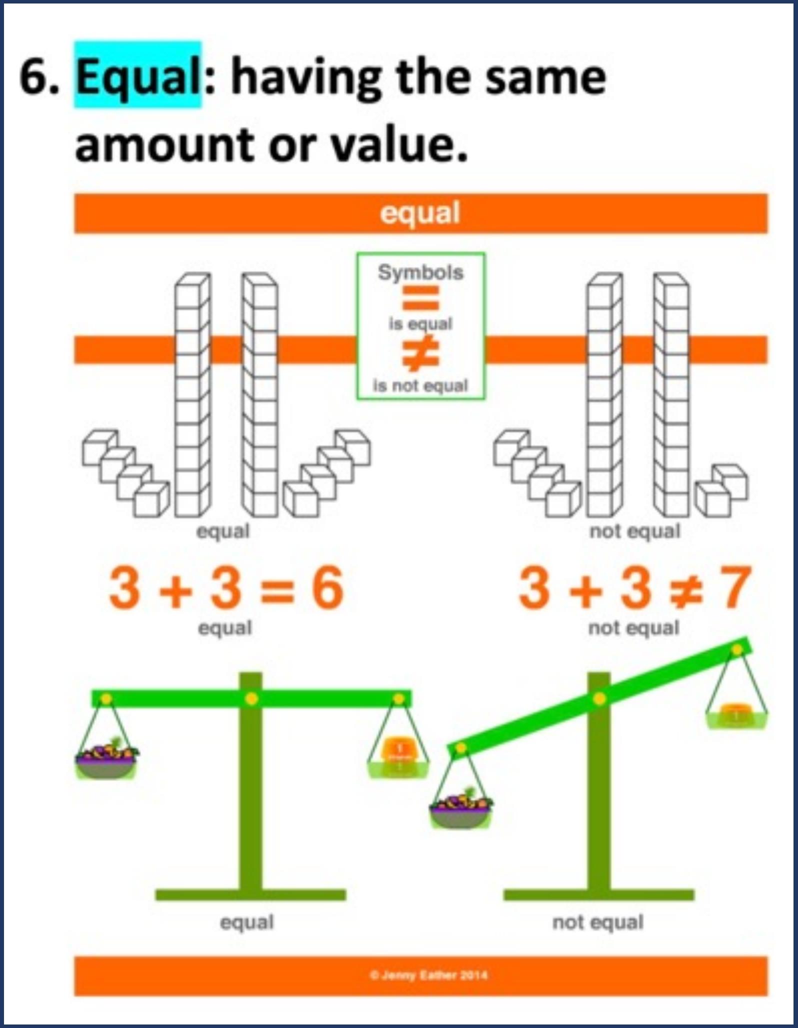
2. Use graphic organizers

Word	Lightbulb Word
Definition	Picture

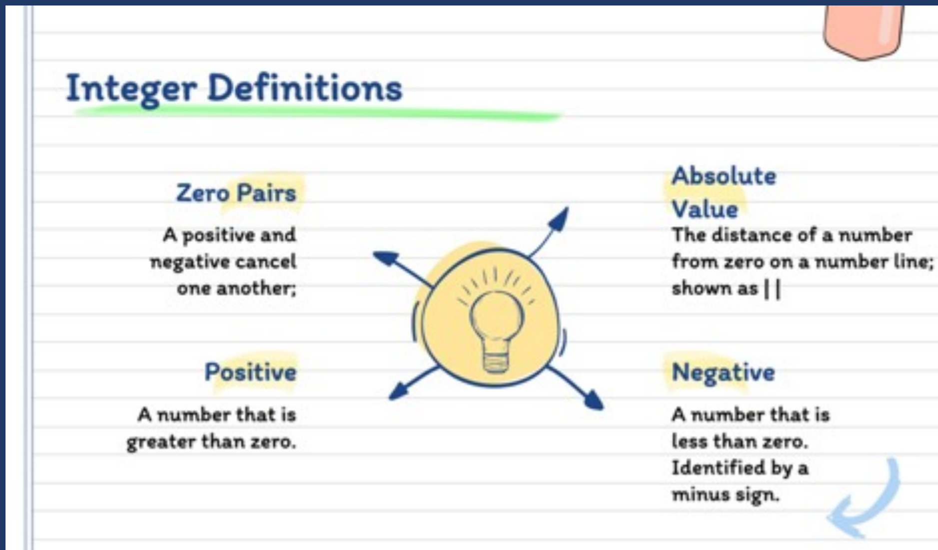
Dunston & Tyminski (2013)

3. Have students create vocabulary cards





4. Have students create glossaries



Numerator: how many parts of the whole

- Ex. $\frac{4}{10}$

Odd number: a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9….

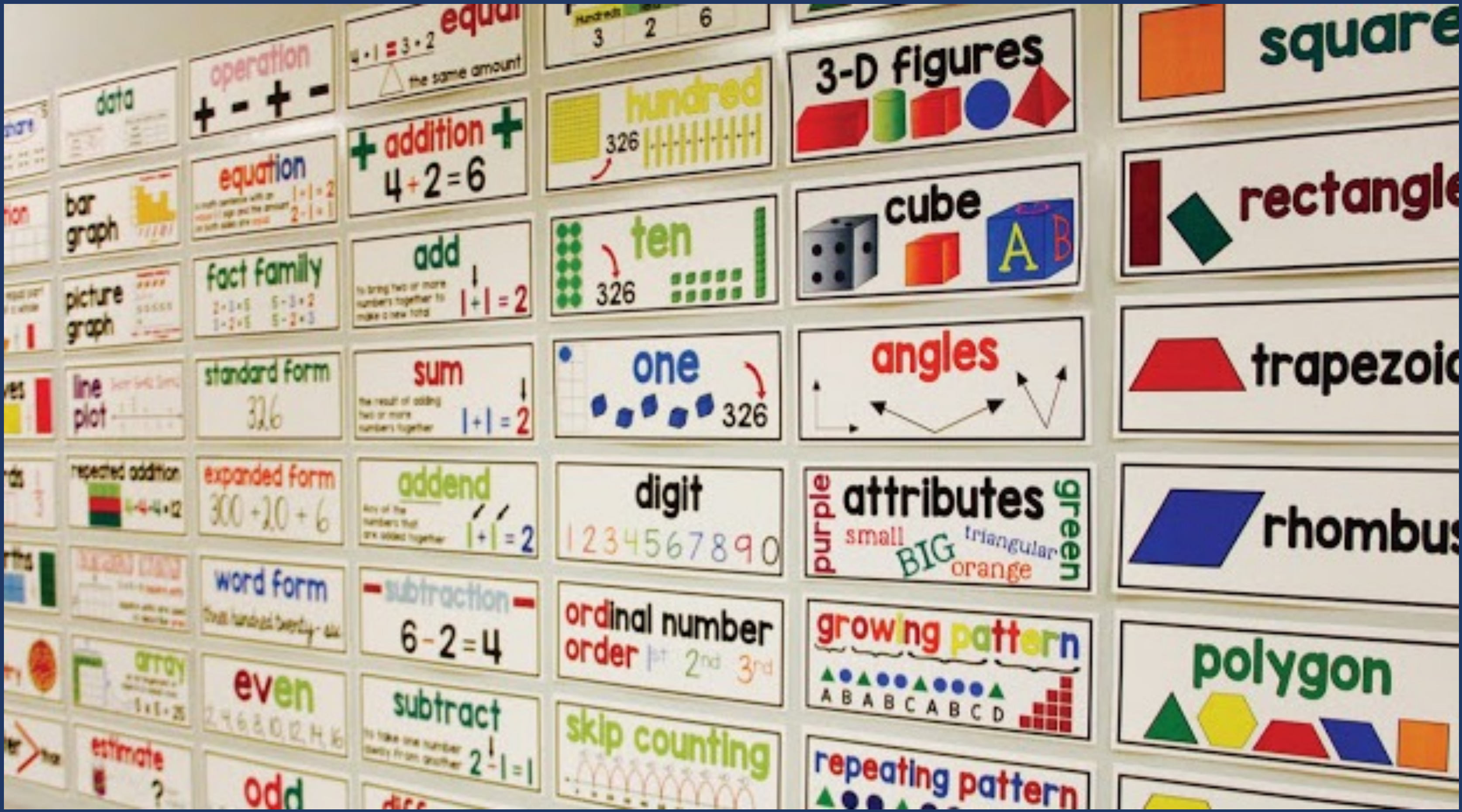
Percent: a specific number in comparison to 100

- 74%

Polygon: any enclosed shape that is made up of 3 or more straight lines

- Ex. 

5. Create a word wall



6. Preview vocabulary

Dear Feisty Fifth Graders,

Today we have multiple opportunities to do exciting projects! For example, we are going to be doing a science experiment to see how the tilt of a ramp relates to how far a matchbox car will roll. There are several factors we will be looking at in this experiment. I look forward to hearing multiple ideas on how to set up this experiment.

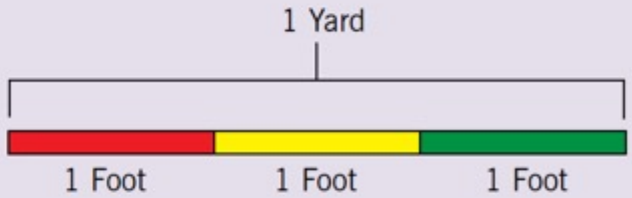
One other thing that factors into our day is that we have an assembly before lunch. We will get to hear music from the high school play. I think we will hear multiple songs.

Sincerely, Ms. Livers

Here is a problem to start your day... in my letter I have used two words that are important math words for today's lesson. Can you find them and tell what they mean in this letter and what they mean when talking about numbers? (Answer this in your math notebook)

Bay-Williams & Livers (2009)

7. Cluster vocabulary

	Length	Weight
Meaning	How long something is	How heavy something is
Visual		2000 pounds = 1 ton 

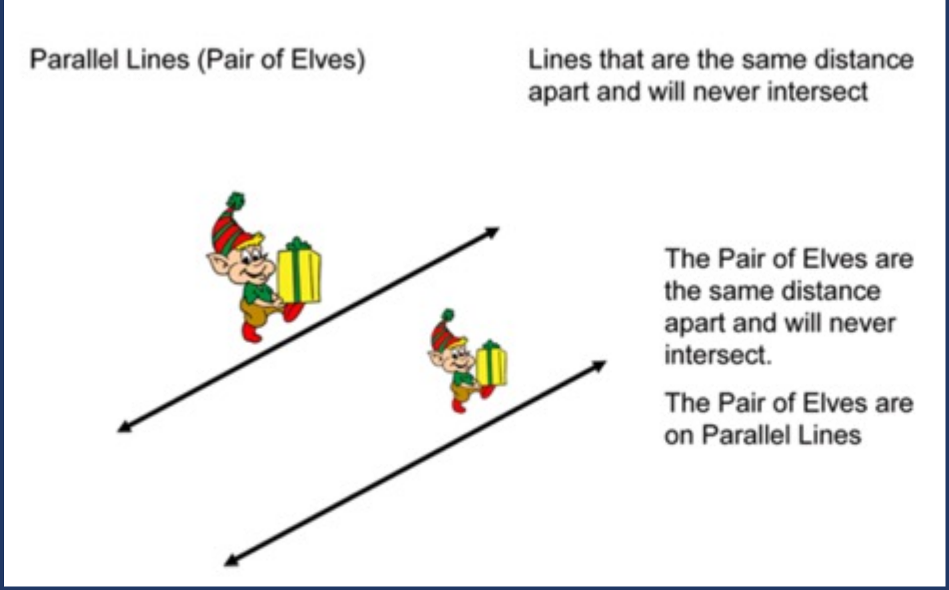
Livers & Bay-Williams (2014)

7. Cluster vocabulary

Rating	Word	Definition	Synonym(s)	Example	Sample Problem
2	expression	a mathematical phrase combining operations, numbers and/or variables.	phrase algebraic expression	6 $6n$ $6+n$ no equal sign	Lucia earns \$8 per hour for babysitting and gets a \$5 tip. Write an expression to represent the amount she would earn if she worked for x hours.
2	variable	a quantity that can change or take many values. (refers to the letter or symbol representing the quantity)	unknown	x D y T	The variable x represents the number of hours Charlie works in a week. Write an expression to represent his earnings if he earns \$9 per hour.
1	product	the result when two or more numbers are multiplied	total answer	$3 \times 2 = 6$ product	The product of 6 and a number is 24. What is the number?
3	quotient	the result of a division (refers to the number of times the divisor divides the dividend)	answer	$18 \div 2 = 9$ $2\overline{)18}$ with 9 above quotient	Estimate the quotient when 365 is divided by 12.

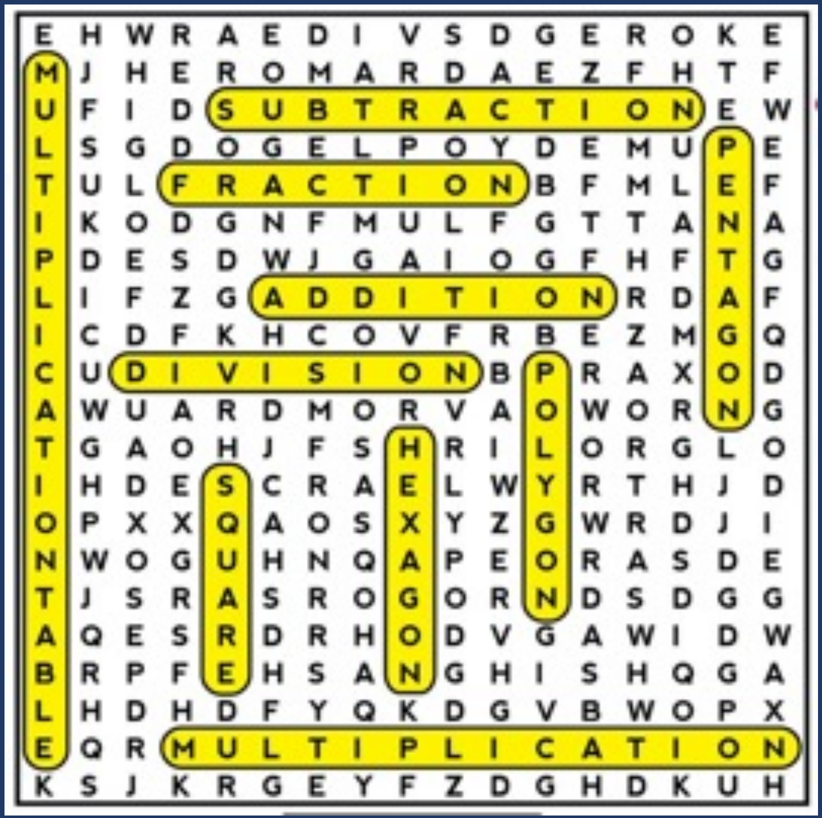
Marin (2018)

8. Use mnemonics

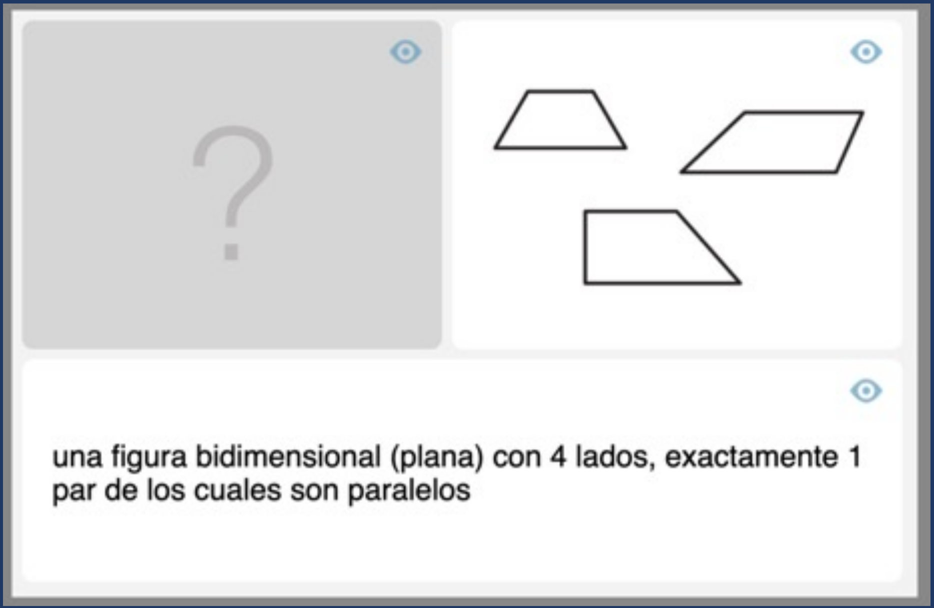


Riccomini et al. (2015)

9. Do word games

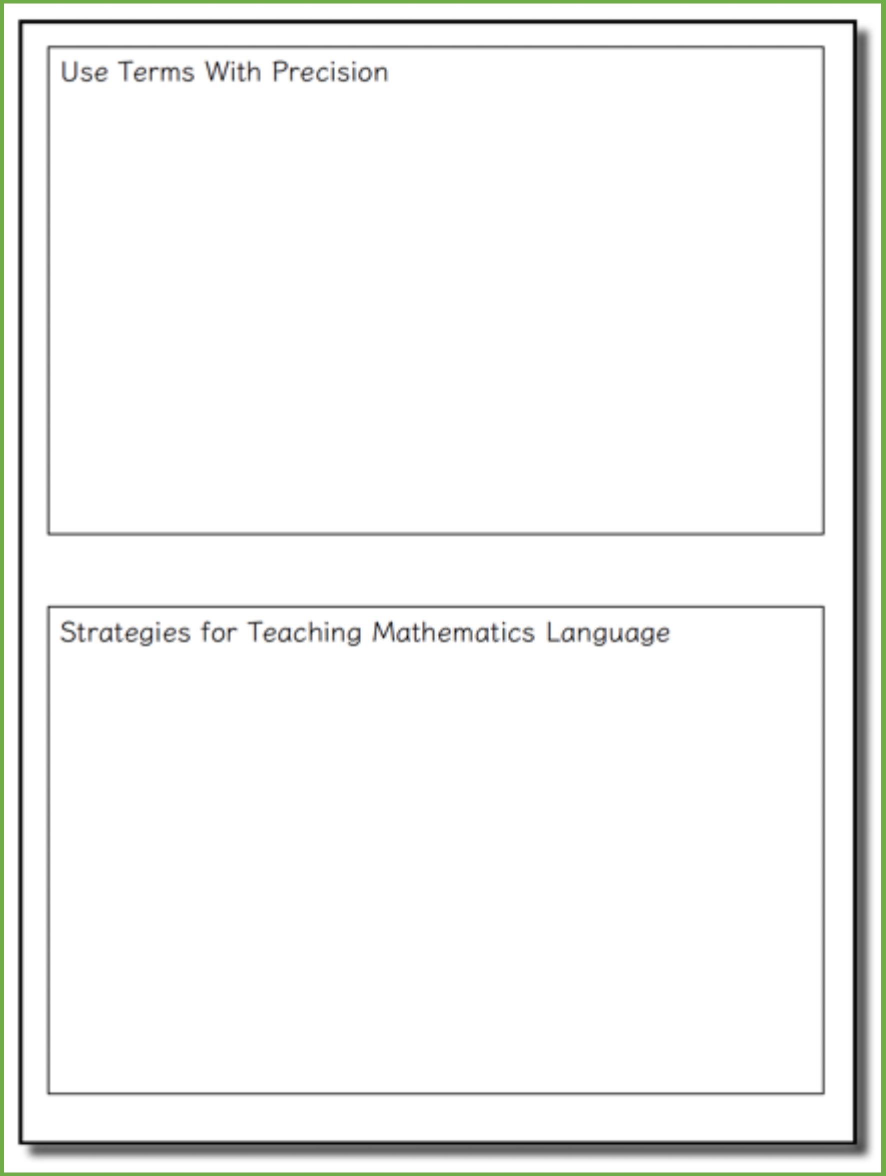


10. Use technology



Math Learning Center

Math Lingo


Use Terms With Precision

Strategies for Teaching Mathematics Language

Discuss your strategy for focusing on mathematical language in your teaching.

Fluency

Fluency

What is your mathematical language goal for the next 4 weeks?

What is your fluency goal for the next 4 weeks?

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

Precise language

Multiple representations

INSTRUCTIONAL STRATEGIES

Fluency building

Building Fluency

Fluency is doing mathematics *easily* and *accurately*.

Fluency in mathematics makes mathematics *easier*.

Fluency provides less stress on working memory.

Fluency helps students build confidence with mathematics.

With fluency, it is important to *emphasize* both conceptual learning *and* procedural learning.

Addition	Subtraction
Multiplication	Division

Counting

Comparing numbers

Counting coins

Telling time

Identifying equivalent fractions

Knowing multiples

Identifying shapes

Knowing formulas

Addition

100 addition facts

Single-digit *addends* sum to *a* single- or double-digit number

$$\begin{array}{r} 5 \\ +\ 4 \\ \hline 9 \end{array}$$

(addend)
(addend)
(sum)

Subtraction

100 subtraction facts

Subtrahend and difference are single-digit numbers and minuend is single- or double-digit number

$$\begin{array}{r} 16 \\ -\ \ 8 \\ \hline 8 \end{array}$$

(minuend)
(subtrahend)
(difference)

Multiplication

100 multiplication facts

Multiplication of single-digit factors results in a single- or double-digit product

$$\begin{array}{r} 2 \\ \times\ 3 \\ \hline 6 \end{array}$$

(factor)
(factor)
(product)

Division

90 division facts

Divisor and quotient are single-digit numbers and dividend is single- or double-digit number

8	÷	4	=	2
(dividend)		(divisor)		(quotient)

Addition	Subtraction
Multiplication	Division

Build fluency with math facts.

- Addition: single-digit addends
- Subtraction: single-digit subtrahend
- Multiplication: single-digit factors
- Division: single-digit divisor

$$\begin{array}{r} 5 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \div\ 8 \\ \hline \end{array}$$

Cover, Copy, Compare

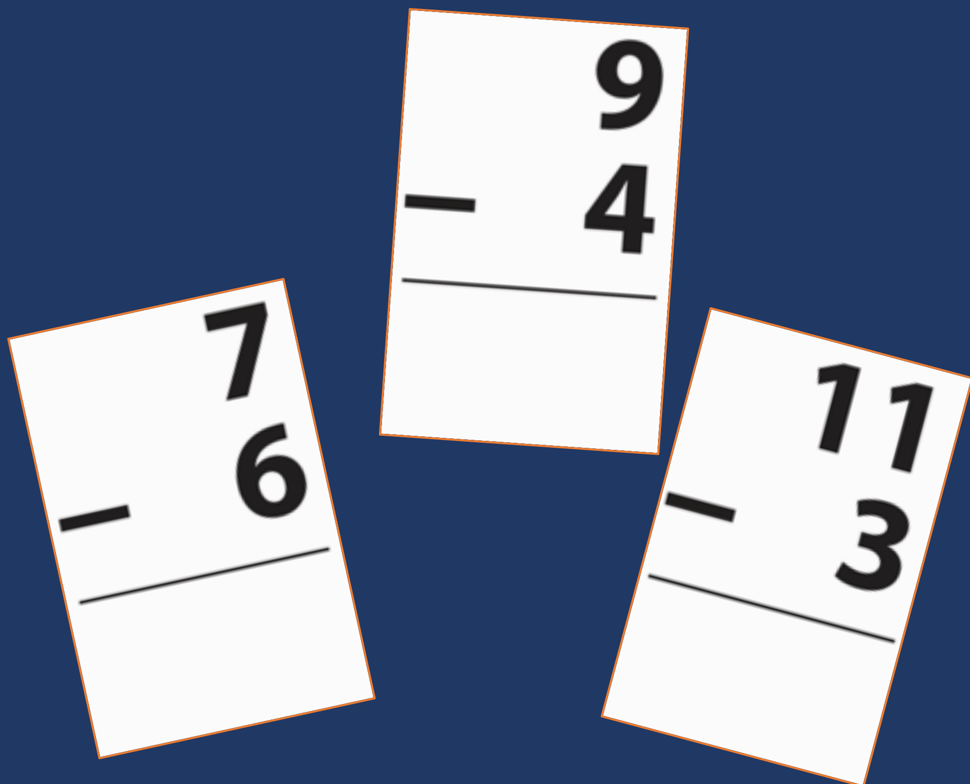
9 × 6 = 54		8 × 6 = 48	
7 × 8 = 56		6 × 5 = 3	
9 × 9 = 81		×	
6 × 7 = 42			
8 × 8 = 64			

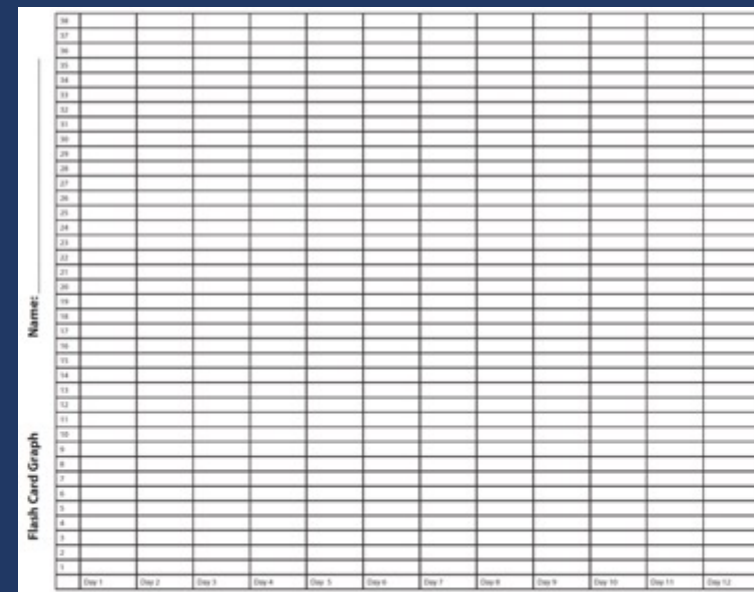
File Folder

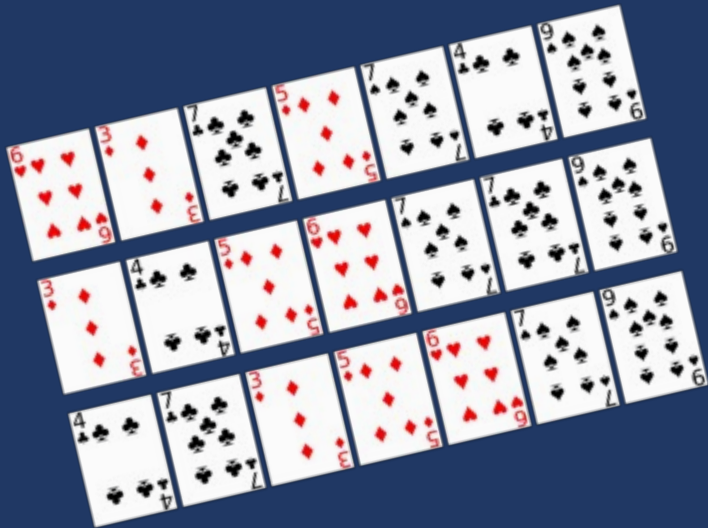
6 + 3 =	
1 + 7 =	8
6 + 4 =	1
7 + 3 =	1
2 + 7 =	9
5 + 6 =	11
4 + 7 =	11
7 + 8 =	15
6 + 7 =	13
7 + 9 =	16
7 + 6 =	13
8 + 7 =	15
7 + 0 =	7
9 + 6 =	15
6 + 0 =	6
6 + 8 =	14

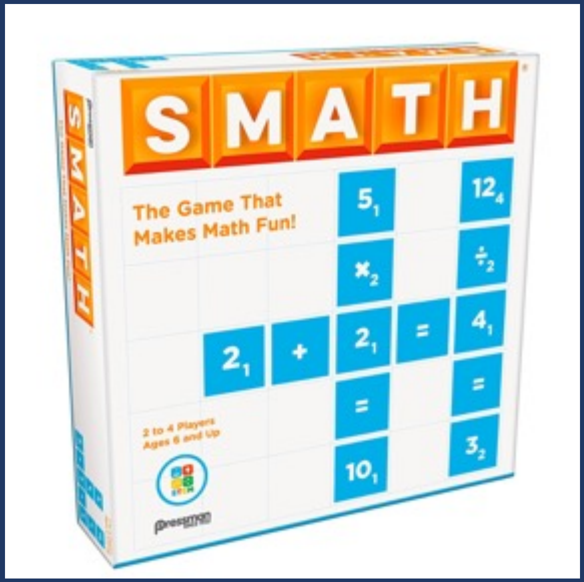
Taped Problems

6 × 5	8 × 6	7 × 9	6 × 8
9 × 8	8 × 5	7 × 8	6 × 6
7 × 7	6 × 9	5 × 9	8 × 4
9 × 4	6 × 9	9 × 5	8 × 7
6 × 7	8 × 8	4 × 8	5 × 7

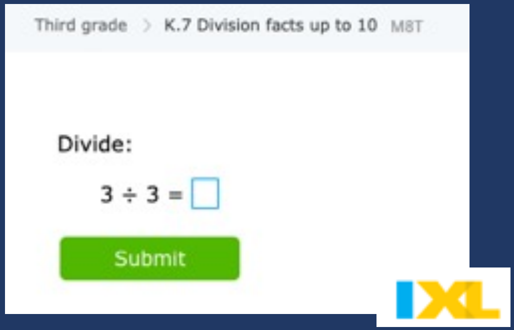

9
− 4
7
− 6
11
− 3

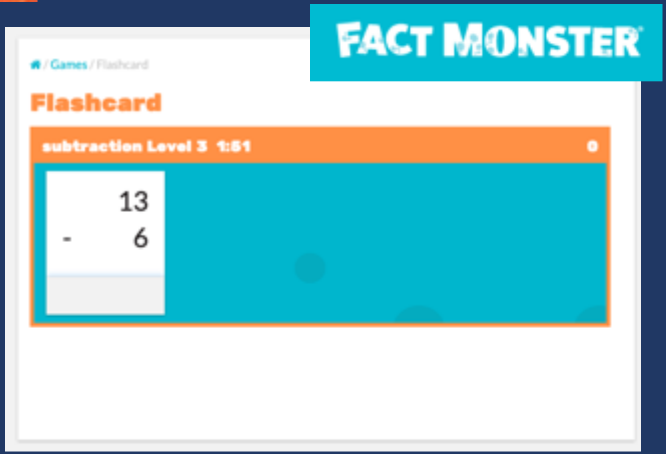

Flash Card Graph
Name:




SMATH
The Game That
Makes Math Fun!


möbi





DAILY and BRIEF

Addition	Subtraction
Multiplication	Division

Build fluency with whole-number computation

$$\begin{array}{r} 15 \\ +\ \ 28 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ \ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 1009 \\ -\ \ 724 \\ \hline \end{array}$$

$$\begin{array}{r} 7250 \\ \div\ \ 15 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with rational-number computation

$$\begin{array}{r} 1.4 \\ +\ \ 3.9 \\ \hline \end{array}$$

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{9}{4} - \frac{3}{8}$$

$$\begin{array}{r} 7.892 \\ \div\ \ 0.14 \\ \hline \end{array}$$

Addition	Subtraction
Multiplication	Division

Build fluency with integer computation

$$\begin{array}{r} 1.4 \\ \underline{+\ \ -3.9} \end{array}$$

$$\begin{array}{r} 6 \\ \underline{\times\ \ -12} \end{array}$$

$$-14 - (-7) =$$

$$-135 \div 2 =$$

Addition	Subtraction
Multiplication	Division

What fluency practice do your students need?

Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

What is your mathematical language goal for the next 4 weeks?

What is your fluency goal for the next 4 weeks?

Schedule for This Year

September 19	**Mathematics Language and Fluency**
October 17	High-Quality Tier 1
December 5	Leveraging Word Problems – Part 1
January 26	Leveraging Word Problems – Part 2
February 16	High-Quality Mathematics Assessment
March 16	High-Quality Supports in Mathematics – Putting It Together

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