Math 1497 Calc 2

Integration by Parts

Formula $\int u\,dv = uv - \int v\,du$

Ex 1 $\int_0^1 x e^{-x}\,dx$

Note $\int e^{ax}\,dx = \frac{e^{ax}}{a} + C$

$u = x \quad v = -e^{-x}$

$du = 1\,dx \quad dv = e^{-x}\,dx$

$= -xe^{-x}\Big|_0^1 - \int_0^1 -e^{-x}\,dx$

$= -xe^{-x}\Big|_0^1 - e^{-x}\Big|_0^1$

$= (-1e^{-1} + 0) - (e^{-1} - e^{0}) = 1 - 2e^{-1}$

Ex2 $\int_0^{1/2} \sin^{-1}x\,dx$

$u = \sin^{-1}x \qquad v = x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \qquad dv = dx$

$= x\sin^{-1}x \Big|_0^{1/2} - \int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\,dx$

$w = 1-x^2 \qquad x=0 \quad w=1$

$dw = -2x\,dx \qquad x=\frac{1}{2} \quad w=\frac{3}{4}$

$= \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right) - 0 - \int_1^{3/4} \frac{-\frac{1}{2}\,du}{\sqrt{u}}$

$= \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right) + \frac{1}{2}\int_1^{3/4} u^{-1/2}\,du$

$= \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right) + u^{1/2}\Big|_1^{3/4}$

$= \frac{\pi}{12} + \sqrt{\frac{3}{4}} - 1$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

Ex 3 $\int \sin^2 x \, dx$

$\int \sin x \sin x \, dx$ parts

$u = \sin x \qquad v = -\cos x$

$du = \cos x \, dx \quad dv = \sin x \, dx$

$= -\sin x \cos x - \int -\cos^2 x \, dx$

$= -\sin x \cos x + \int 1 - \sin^2 x \, dx$

$2 \int \sin^2 x \, dx = -\sin x \cos x + x$

$\int \sin^2 x \, dx = -\frac{1}{2} \sin x \cos x + \frac{1}{2} x + c$

Note! $\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x \quad \rightarrow \quad \sin^2 x = \frac{1 - \cos 2x}{2}$

$\int \sin^2 x \, dx = \int \frac{1}{2} - \frac{\cos 2x}{2} \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$ Easier!