Calculus II Sample Final Summer 2018

1. Integrate the following

(i) $\displaystyle\int \sin^2 x\, \cos^3 x\, dx$ (ii) $\displaystyle\int x \ln x\, dx$ (iii) $\displaystyle\int \frac{dx}{x^2+4}$

(iv) $\displaystyle\int x \sin 2x\, dx$ (v) $\displaystyle\int \frac{dx}{x^2+3x+2}$ (vi) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\, dx$

(vii) $\displaystyle\int_0^\infty x e^{-x^2}\, dx$ (viii) $\displaystyle\int \frac{dx}{(x^2+1)^{3/2}}$ (ix) $\displaystyle\int \frac{x\, dx}{(x-1)(x-2)^2}$

(x) $\displaystyle\int \frac{dx}{x\,(x^2+1)}$ (xi) $\displaystyle\int x\, e^{-3x}\, dx$ (xii) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

(xiii) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-4}}$

2. Do the following converge

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ (iii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

(iv) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$ (v) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ (vi) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n \ln n}$

(vii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$ (viii) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\sin^2 n}{n}$ (ix) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2}+\frac{1}{n}\right)^n$

3. Calculate the 4^{th} degree Taylor polynomial with remainder for the following. Expand about the point c that is given.

$$(i)\quad f(x) = \sin x, \text{ about } x=\pi \qquad (ii)\quad f(x) = \frac{1}{x+2} \text{ about } x=1$$

4. Determine both the radius and interval of convergence of the following.

$$(i)\quad \sum_{n=1}^{\infty} \frac{(3x)^n}{(n+1)!} \qquad (ii)\quad \sum_{n=1}^{\infty} \frac{(x-1)^n}{n4^n}$$

5. Polar Areas- Find the following areas

(i) inside both $r=\sin\theta, r=\cos\theta$ (ii) outside $r=\dfrac{1}{2}+\cos\theta$ and inside $r=2\cos\theta$

6. Planes and Lines

(i) Find the equation of the plane that contains the lines

$$x=-1+t,\ y=1+t,\ z=2t \text{ and } x=2-s,\ y=s,\ z=2.$$

(ii) That contains the points $P(1,1,3)$, $Q(-2,4,-3)$ and $R(3,-4,4)$

(iii) Find the equation of the line that is perpendicular to the plane in (ii) through the point $(1,2,3)$.

(iv) Find the equation of the line that passes through the points $(1,2,-3)$ and $(1,-2,4)$.

7. Find the projection of $\vec{u}$ onto $\vec{v}$ and the orthogonal complement. Sketch all vectors.

$$(i) \quad \vec{u} = < -1,3 >, \;\; \vec{v} = < 2,2 >$$
$$(ii) \quad \vec{u} = < 5,5 >, \quad \vec{v} = < 1,2 >$$