Edexcel GCE

Core Mathematics C4

Silver Level S3

(Mark Scheme)

Mr.S.V.Swarnaraja (Marking Examiner, Team Leader & Author)
www.swanash.com, Mobile: +94777304755 , email: swa@swanash.com

Question Number	Scheme		Marks
1.	$\frac{5x+3}{(2x+1)(x+1)^2} \equiv \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$	At least one of "A" or "C" are correct. Breaks up their partial fraction correctly into three terms **and** both "A" $= 2$ and "C" $= 2$.	B1 B1 **cso**
	$A = 2, \quad C = 2$		
	$5x + 3 \equiv A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$ $x = -1 \Rightarrow -2 = -C \Rightarrow C = 2$ $x = -\frac{1}{2} \Rightarrow -\frac{5}{2} + 3 = \frac{1}{4}A \Rightarrow \frac{1}{2} = \frac{1}{4}A \Rightarrow A = 2$	Writes down ***a correct identity*** and attempts to find the value of either one "A" or "B" or "C".	M1
	Either $x^2: \ 0 = A + 2B, \quad$ constant: $3 = A + B + C$ $x: \ 5 = 2A + 3B + 2C$		
	leading to $B = -1$	Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.	A1 **cso**
	So, $\frac{5x+3}{(2x+1)(x+1)^2} \equiv \frac{2}{(2x+1)} - \frac{1}{(x+1)} + \frac{2}{(x+1)^2}$		**[4]**

2. (a)	$\int \frac{1}{x^3}\ln x\,\mathrm{d}x, \quad \left\{\begin{array}{lll} u=\ln x & \Rightarrow & \frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x}=x^{-3} & \Rightarrow & v=\frac{x^{-2}}{-2}=\frac{-1}{2x^2}\end{array}\right\}$		
		In the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu\frac{1}{x^2}.\frac{1}{x}$	M1
	$=\underline{\frac{-1}{2x^2}\ln x}-\underline{\underline{\int \frac{-1}{2x^2}.\frac{1}{x}\,\mathrm{d}x}}$	$\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified.	A1
		$\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ simplified or un-simplified.	A1
	$\left\{=\ \frac{-1}{2x^2}\ln x+\frac{1}{2}\int \frac{1}{x^3}\ \mathrm{d}x\right\}$		
	$=-\frac{1}{2x^2}\ln x+\frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$	$\pm\int \mu\frac{1}{x^2}.\frac{1}{x} \ \rightarrow \pm\beta x^{-2}.$	dM1
		Correct answer, with/without $+c$	A1
			[5]
(b)	$\left\{\left[-\frac{1}{2x^2}\ln x-\frac{1}{4x^2}\right]_1^2\right\}=\left(-\frac{1}{2(2)^2}\ln 2-\frac{1}{4(2)^2}\right)-\left(-\frac{1}{2(1)^2}\ln 1-\frac{1}{4(1)^2}\right)$	Applies limits of 2 and 1 to their part (a) answer and subtracts the correct way round.	M1
	$=\frac{3}{16}-\frac{1}{8}\ln 2$ or $\frac{3}{16}-\ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3-2\ln 2)$, etc, or awrt 0.1	or equivalent.	A1
			[2]
			7

Question Number	Scheme		Marks
3 (a)	$27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$	Forming this identity	M1
	$x=-\frac{2}{3}, \quad 12-\frac{64}{3}+16=\left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3}=\left(\frac{5}{3}\right)B \Rightarrow B=4$ $x=1, \quad 27+32+16=25C \Rightarrow 75=25C \Rightarrow C=3$	Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations. Both $B=4$ and $C=3$ (**Note the A1 is dependent on both method marks in this part.)**	M1 A1
	Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A$ $\Rightarrow A=0$ $x=0, \quad 16=2A+B+4C$ $\Rightarrow 16=2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$	Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A=0$.	B1 (4)
(b)	$f(x)=\frac{4}{(3x+2)^2}+\frac{3}{(1-x)}$		
	$=4(3x+2)^{-2}+3(1-x)^{-1}$ $=1\left(1+\frac{3}{2}x\right)^{-2}+3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$=1\left\{\underline{1+(-2)(\frac{3x}{2});+\frac{(-2)(-3)}{2!}(\frac{3x}{2})^2+\ldots}\right\}$ $+3\left\{\underline{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\ldots}\right\}$	Either $1\pm(-2)(\frac{3x}{2})$ or $1\pm(-1)(-x)$ from either first or second expansions respectively Ignoring 1 and 3, any one correct $\{\underline{\ldots\ldots}\}$ expansion. Both $\{\underline{\ldots\ldots}\}$ correct.	dM1; A1 A1
	$=\left\{1-3x+\frac{27}{4}x^2+\ldots\right\}+3\left\{1+x+x^2+\ldots\right\}$ $=4+0x;+\frac{39}{4}x^2$	$4+(0x)$; $\frac{39}{4}x^2$	A1; A1 (6)

Question Number	Scheme	Marks
3 (c)	Actual $= \mathrm{f}(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2) or seeing awrt 4.3 and believing it is candidate's actual f(0.2). M1
	Or Actual $= \mathrm{f}(0.2) = \dfrac{4}{(3(0.2)+2)^2} + \dfrac{3}{(1-0.2)}$ $= \dfrac{4}{6.76} + 3.75 = 4.341715976... = \dfrac{2935}{676}$	Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$ with their A, B and C.
	Estimate $= \mathrm{f}(0.2) = 4 + \frac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b) M1$\surd$
	%age error $= \dfrac{\lvert 4.39 - 4.341715976... \rvert}{4.341715976...} \times 100$	$\left\lvert \dfrac{\text{their estimate - actual}}{\text{actual}} \right\rvert \times 100$ M1
	$= 1.112095408... = 1.1\%\,(2\text{sf})$	1.1% A1 **cao** (4)
		[14]

Question Number	Scheme		Marks
4.	(a) $\dfrac{dx}{dt} = 2\sin t\cos t,\ \dfrac{dy}{dt} = 2\sec^2 t$		B1 B1
	$\dfrac{dy}{dx} = \dfrac{\sec^2 t}{\sin t\cos t}\left(= \dfrac{1}{\sin t\cos^3 t}\right)$	or equivalent	M1 A1 **(4)**
	(b) At $t = \dfrac{\pi}{3},\ x = \dfrac{3}{4},\ y = 2\sqrt{3}$		B1
	$\dfrac{dy}{dx} = \dfrac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \dfrac{16}{\sqrt{3}}$		M1 A1
	$y - 2\sqrt{3} = \dfrac{16}{\sqrt{3}}\left(x - \dfrac{3}{4}\right)$		M1
	$y = 0 \ \Rightarrow\ x = \dfrac{3}{8}$		M1 A1 **(6)**
			[10]

Question Number	Scheme		Marks
5.	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$		B1
	$\ldots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$		M1 A1
At $x = 2$,	$\ln y = 2(2)\ln 2$		M1
leading to	$y = 16$	Accept $y = \mathrm{e}^{4\ln 2}$	A1
At $(2, 16)$	$\frac{1}{16}\frac{\mathrm{d}y}{\mathrm{d}x} = 2\ln 2 + 2$		M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 16(2 + 2\ln 2)$		A1 **(7)**
			[7]

Question Number	Scheme	Marks
6.	(a) **i**: $6-\lambda=-5+2\mu$ **j:** $-3+2\lambda=15-3\mu$ Any two equations leading to $\lambda=3,\ \mu=4$ $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+3\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}-5\\15\\3\end{pmatrix}+4\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ **k:** LHS $=-2+3(3)=7$, RHS $=3+4(1)=7$ (As LHS = RHS, lines intersect)	M1 M1 A1 M1 A1 B1 **(6)**
	(b) $\begin{pmatrix}-1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\1\end{pmatrix}=-2-6+3=\sqrt{14}\sqrt{14}\cos\theta \quad (\theta\approx 110.92^\circ)$ Acute angle is 69.1° awrt 69.1	M1 A1 A1 **(3)**
	(c) $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+1\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}5\\-1\\1\end{pmatrix}$ $(\Rightarrow B$ lies on $l_1)$	B1 **(1)**
	(d) Let d be shortest distance from B to l_2 $\overrightarrow{AB}=\begin{pmatrix}5\\-1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\7\end{pmatrix}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$ $\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{\left(2^2+(-4)^2+(-6)^2\right)}=\sqrt{56}$ awrt 7.5 $\frac{d}{\sqrt{56}}=\sin\theta$ $d=\sqrt{56}\sin 69.1^\circ\approx 6.99$ awrt 6.99	M1 A1 M1 A1 **(4)** **[14]**

Question Number	Scheme	Marks
Q7	(a) $y=0 \Rightarrow t\left(9-t^2\right)=t(3-t)(3+t)=0$	
	$t=0, 3, -3$ Any one correct value	B1
	At $t=0$, $x=5(0)^2-4=-4$ Method for finding one value of x	M1
	At $t=3$, $x=5(3)^2-4=41$	
	$\left(\text{At } t=-3,\ x=5(-3)^2-4=41\right)$	
	At A, $x=-4$; at B, $x=41$ Both	A1 (3)
	(b) $\dfrac{\mathrm{d}x}{\mathrm{d}t}=10t$ Seen or implied	B1
	$\int y\,\mathrm{d}x=\int y\dfrac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t=\int t\left(9-t^2\right)10t\,\mathrm{d}t$	M1 A1
	$=\int\left(90t^2-10t^4\right)\mathrm{d}t$	
	$=\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\ (+C)\qquad \left(=30t^3-2t^5\ (+C)\right)$	A1
	$\left[\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\right]_0^3=30\times3^3-2\times3^5\quad(=324)$	M1
	$A=2\int y\,\mathrm{d}x=648\quad\left(\text{units}^2\right)$	A1 (6)
		[9]

Question Number	Scheme	Marks
Q8	(a) $\dfrac{\mathrm{d}x}{\mathrm{d}u} = -2\sin u$	B1
	$\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x = \int \frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}} \times -2\sin u\,\mathrm{d}u$	M1
	$\displaystyle= \int \frac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\,\mathrm{d}u$ Use of $1-\cos^2 u = \sin^2 u$	M1
	$\displaystyle= -\frac{1}{4}\int \frac{1}{\cos^2 u}\,\mathrm{d}u$ $\pm k\displaystyle\int \frac{1}{\cos^2 u}\,\mathrm{d}u$	M1
	$\displaystyle= -\frac{1}{4}\tan u\ (+C)$ $\pm k\tan u$	M1
	$x=\sqrt{2} \Rightarrow \sqrt{2} = 2\cos u \Rightarrow u = \dfrac{\pi}{4}$	
	$x=1 \Rightarrow 1 = 2\cos u \Rightarrow u = \dfrac{\pi}{3}$	M1
	$\displaystyle\left[-\frac{1}{4}\tan u\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}} = -\frac{1}{4}\left(\tan\frac{\pi}{4} - \tan\frac{\pi}{3}\right)$	
	$\displaystyle= -\frac{1}{4}\left(1-\sqrt{3}\right)\ \left(= \frac{\sqrt{3}-1}{4}\right)$	A1 (7)
	(b) $\displaystyle V = \pi\int_1^{\sqrt{2}} \left(\frac{4}{x\left(4-x^2\right)^{\frac{1}{4}}}\right)^2 \mathrm{d}x$	M1
	$\displaystyle= 16\pi\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x$ $16\pi\times$ integral in (a)	M1
	$\displaystyle= 16\pi\left(\frac{\sqrt{3}-1}{4}\right)$ $16\pi\times$ their answer to part (a)	A1ft (3)
		[10]

Statistics for C4 Practice Paper Silver Level S3

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	4		82	3.29	3.87	3.51	3.24	2.89	2.23	1.97	0.81
2	7	7	71	4.98	6.86	5.97	4.94	3.90	2.81	2.09	0.94
3	14		72	10.04		12.02	9.57	8.16	6.42	5.42	2.69
4	10		65	6.50	9.62	8.38	6.73	4.96	3.47	2.03	0.93
5	7		64	4.47	6.79	5.76	4.56	3.36	2.24	1.42	0.70
6	14		62	8.70	12.92	10.84	8.84	6.78	4.88	3.35	1.75
7	9		60	5.36		6.92	4.68	3.52	2.62	1.58	1.25
8	10		41	4.07		5.71	2.63	1.67	0.85	0.41	0.20
	75		**63**	**47.41**		**59.11**	**45.19**	**35.24**	**25.52**	**18.27**	**9.27**