Effective Mathematics Instruction: A Focus on Language and Multiple Representations

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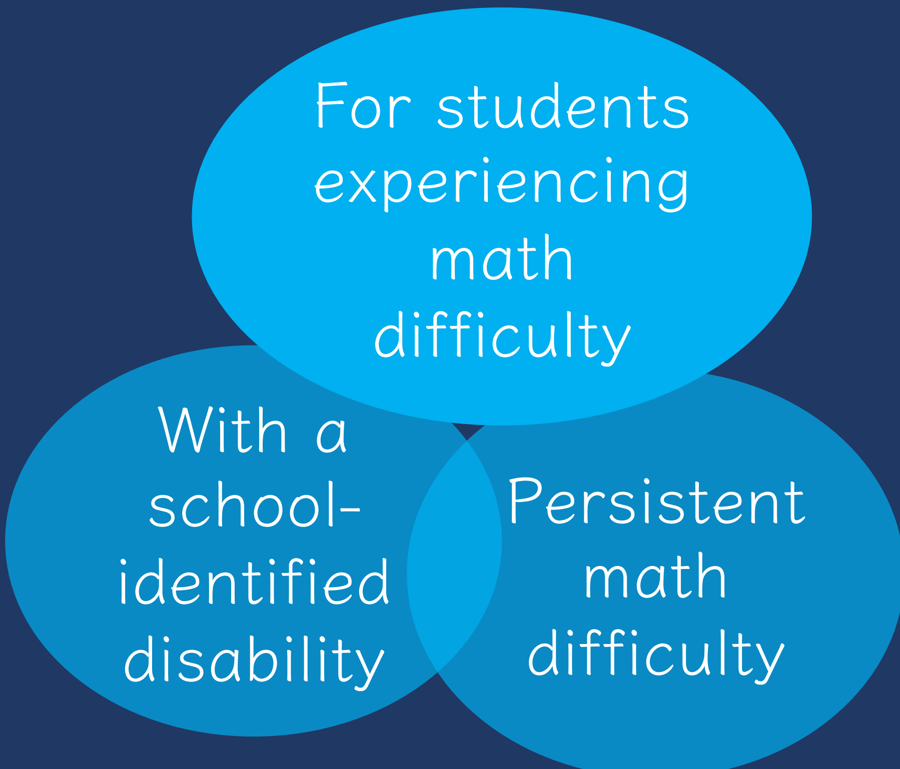
Introduce yourself.

Describe your role as an educator.

Describe the mathematics you support.

Share your Twitter handle!

MATH INTERVENTION



- Tier 2
- Tier 3
- Secondary
- Targeted
- Intensive
- Special Education

1 Systematic Instruction: Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

STRONG EVIDENCE

Show More

2 Mathematical Language: Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.

STRONG EVIDENCE

Show More

3 Representations: Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

STRONG EVIDENCE

Show More

4 Number Lines: Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

STRONG EVIDENCE

Show More

5 Word Problems: Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.

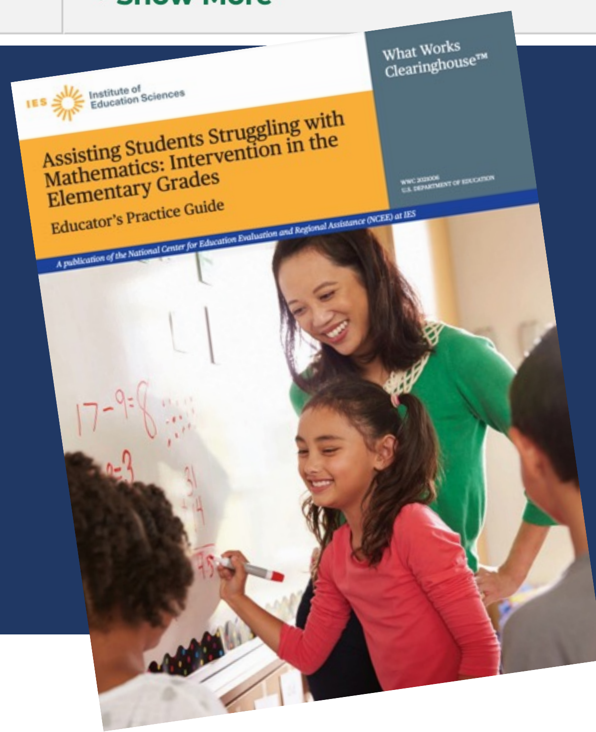
STRONG EVIDENCE

Show More

6 Timed Activities: Regularly include timed activities as one way to build fluency in mathematics.

STRONG EVIDENCE

Show More



https://ies.ed.gov/ncee/wwc/PracticeGuides

Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

Instructional Platform

INSTRUCTIONAL DELIVERY

Explicit instruction

INSTRUCTIONAL STRATEGIES

Explicit Instruction

MODELING	PRACTICE
Step-by-step explanation	Guided practice
Planned examples	Independent practice

SUPPORTS

Ask high-level and low-level questions

Eliciting frequent responses

Providing affirmative and corrective feedback

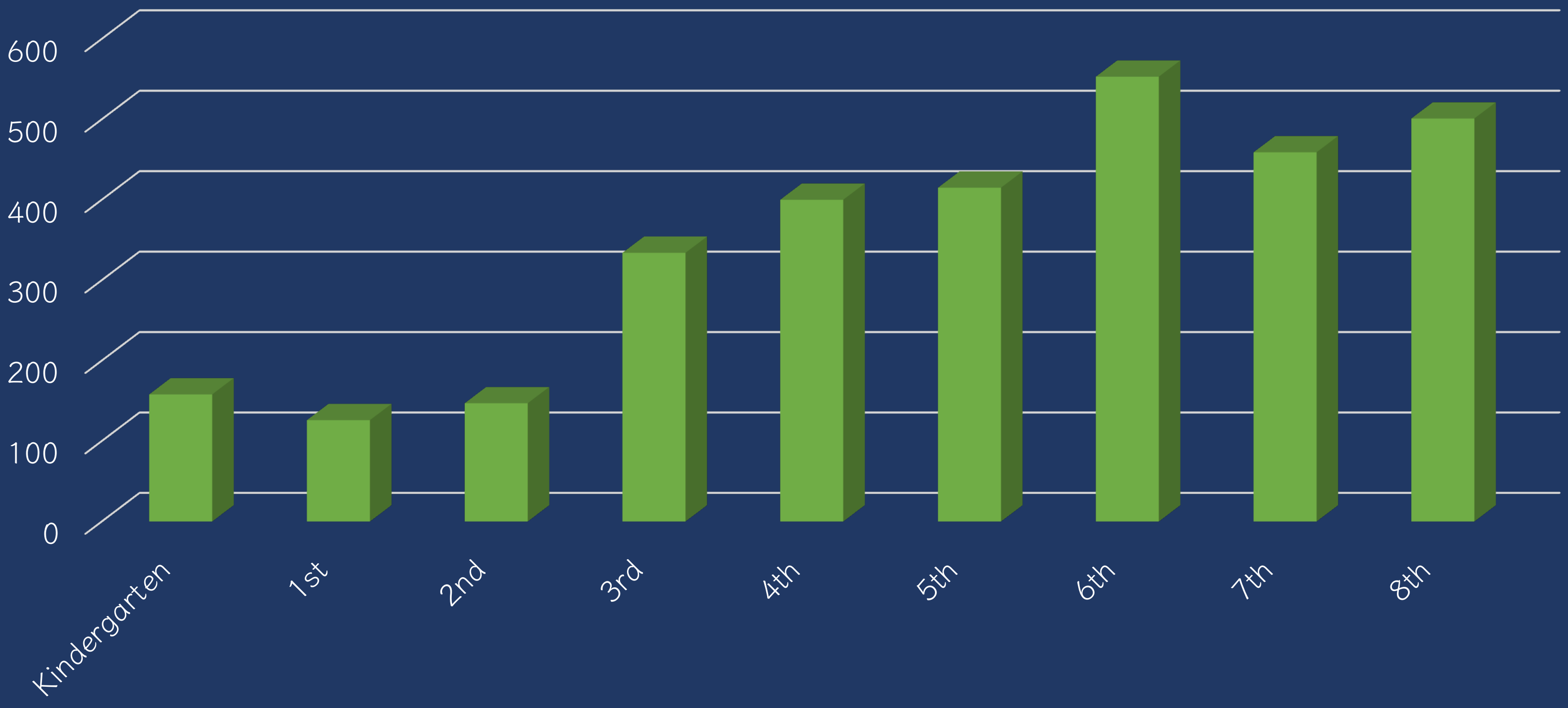
Instructional Platform

INSTRUCTIONAL DELIVERY

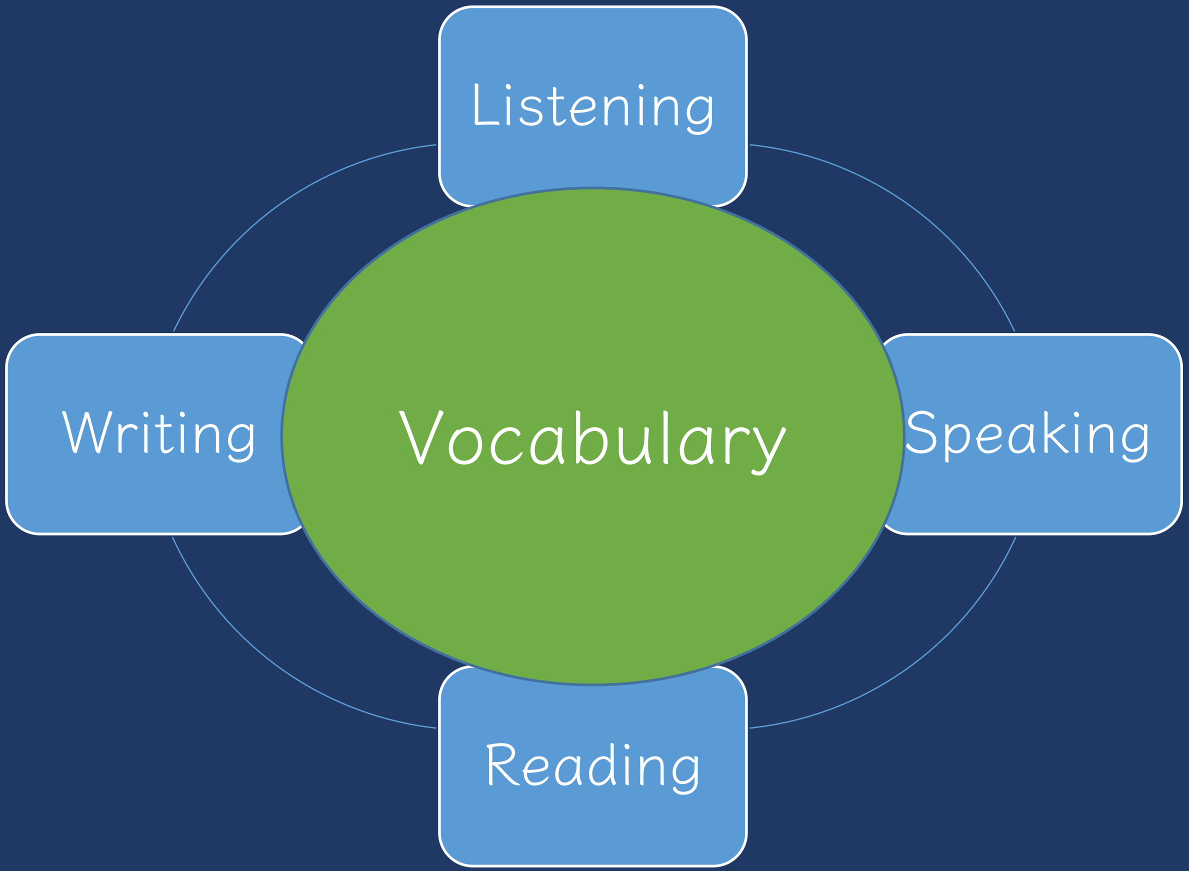
Explicit instruction

Precise language

INSTRUCTIONAL STRATEGIES



Powell, Bos, & Lin (2019)


Listening
Writing
Vocabulary
Speaking
Reading

1. Some math terms are shared with English but have different meanings

base

right

degree

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

difference

even

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings

2. Some math words are shared with English with similar meanings
   (but a more precise math meaning)

3. Some math terms are only used in math

trapezoid

numerator

parallelogram

Rubenstein & Thompson (2002)

1. Some math terms are shared with English but have different meanings
2. Some math words are shared with English with similar meanings (but a more precise math meaning)
3. Some math terms are only used in math
4. Some math terms have more than one meaning

round

square

second

base

Rubenstein & Thompson (2002)

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   (but a more precise math meaning)

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4. Some math terms have more than one meaning

5. Some math terms are similar to other content-area terms with different meanings

divide vs.
Continental
Divide

variable vs.
variably
cloudy

Rubenstein & Thompson (2002)

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6. Some math terms are homographs

eight vs. ate

sum vs. some

rows vs. rose

base vs. bass

Rubenstein & Thompson (2002)

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3. Some math terms are only used in math
4. Some math terms have more than one meaning
5. Some math terms are similar to other content-area terms with different meanings
6. Some math terms are homographs
7. Some math terms are related but have distinct meanings

factor vs. multiple

hundreds vs. hundredths

numerators vs. denominator

Rubenstein & Thompson (2002)

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7. Some math terms are related but have distinct meanings

8. An English math term may translate into another language with different meanings

mesa vs. tabla

Rubenstein & Thompson (2002)

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6. Some math terms are homographs
7. Some math terms are related but have distinct meanings
8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities

four vs. forty

Rubenstein & Thompson (2002)

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3. Some math terms are only used in math
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8. An English math term may translate into another language with different meanings
9. English spelling and usage may have irregularities
10. Some math concepts are verbalized in more than one way

skip count vs. multiples

one-fourth vs. one quarter

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10. Some math concepts are verbalized in more than one way

11. Informal terms may be used for formal math terms

rhombus vs. diamond

vertex vs. corner

1. Some math terms are shared with English but have different meanings

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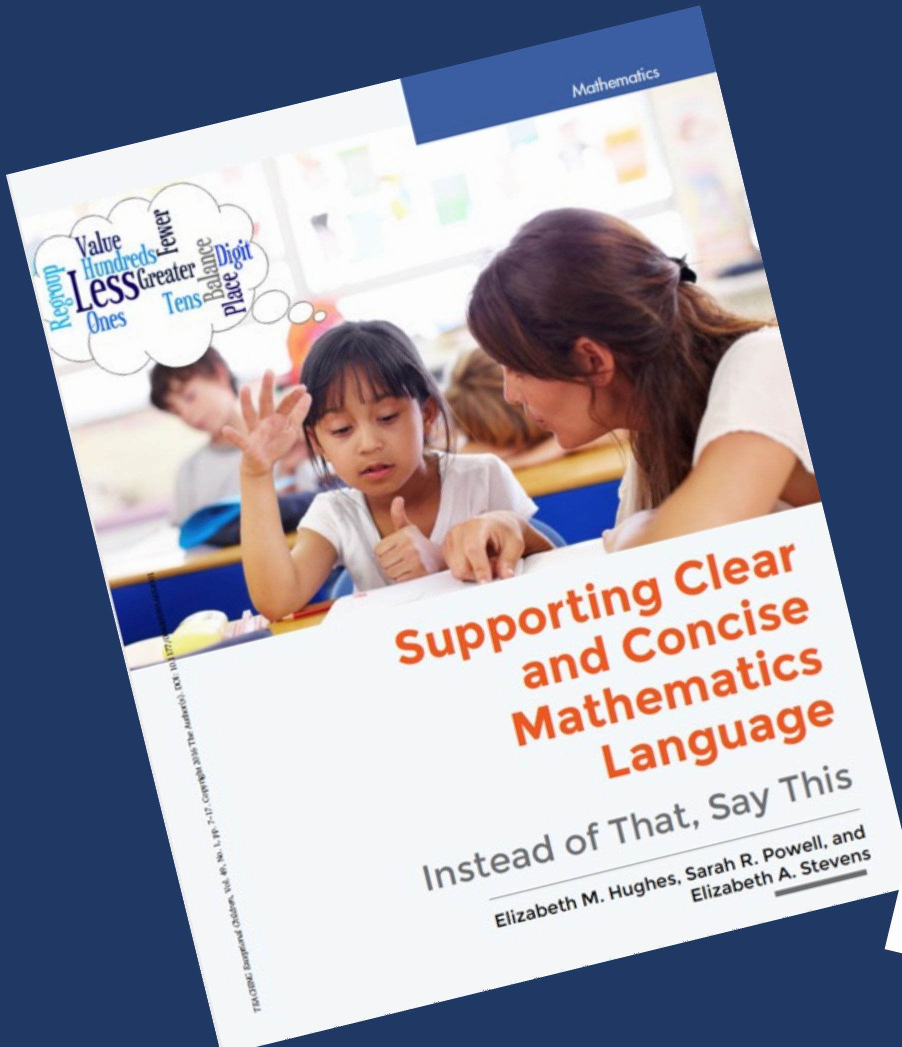
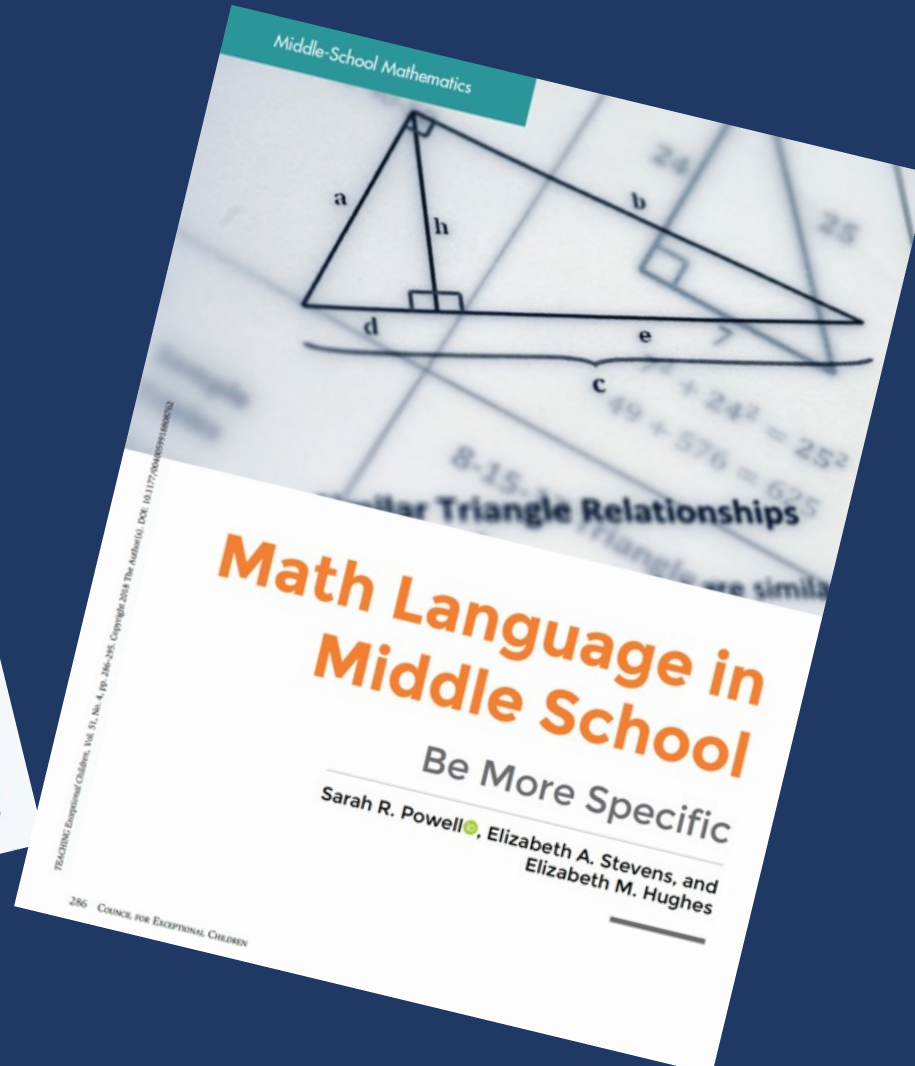
10. Some math concepts are verbalized in more than one way

11. Informal terms may be used for formal math terms

Use formal math language

Use terms precisely

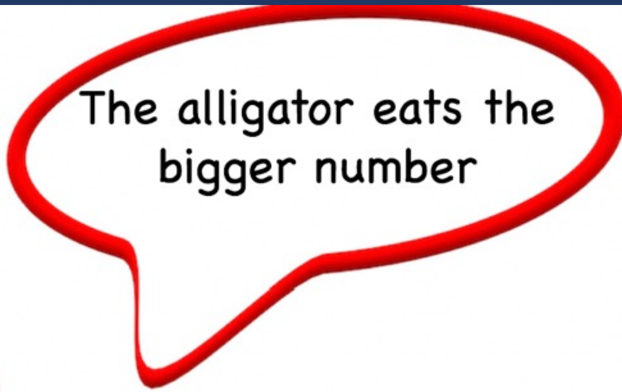

Mathematics
Value
Hundreds
Fewer
Less
Greater
Digit
Regroup
Ones
Tens
Balance
Place
Supporting Clear and Concise Mathematics Language
Instead of That, Say This
Elizabeth M. Hughes, Sarah R. Powell, and Elizabeth A. Stevens

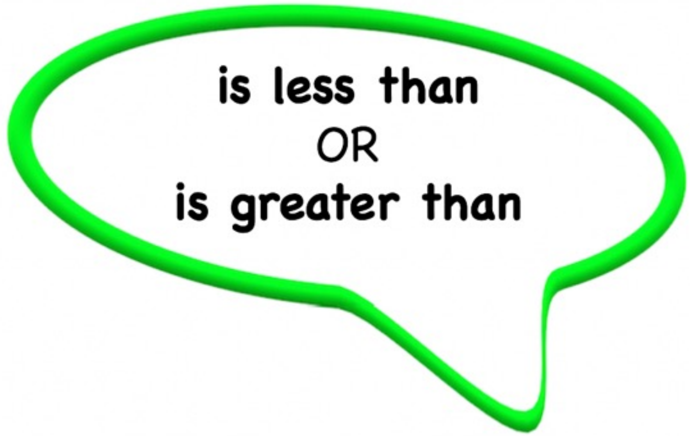

Middle-School Mathematics
a
b
h
d
e
c
Math Language in Middle School
Be More Specific
Sarah R. Powell, Elizabeth A. Stevens, and Elizabeth M. Hughes
286 Council for Exceptional Children

135

Why this is important...

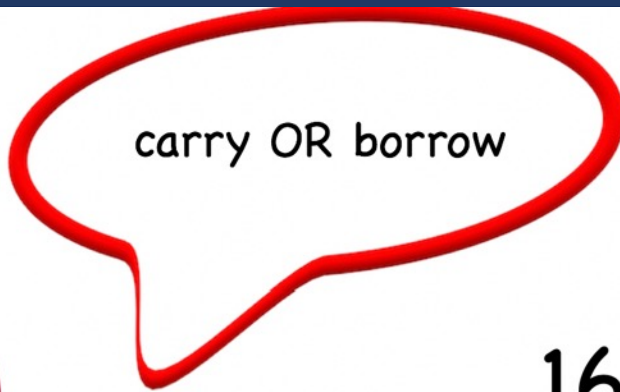
- A number refers to the entire amount.
- The 3 in the tens place value is not a number, but rather a digit in the number 135.
- Reinforces conceptual understanding of place value.
- Emphasizes that 3 is part of the number 135 with a value of 30.





Why this is important…

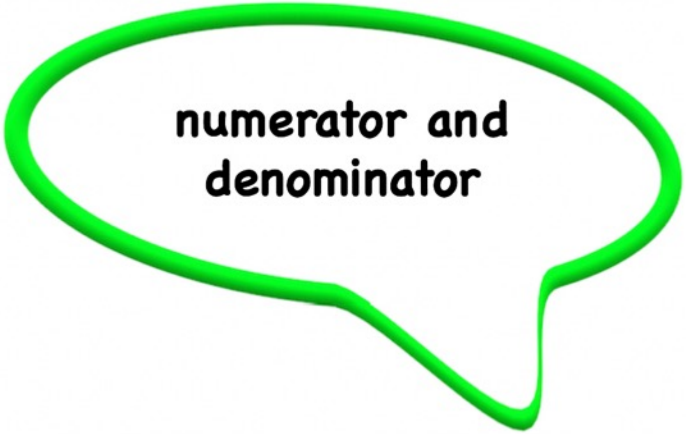
- Students must learn how to read and write the inequality symbols.
- Students must learn to read equations correctly from left to right because < and > are two distinct symbols.



$$\begin{array}{r} 167 \\ +\quad 294 \\ \hline \end{array}$$

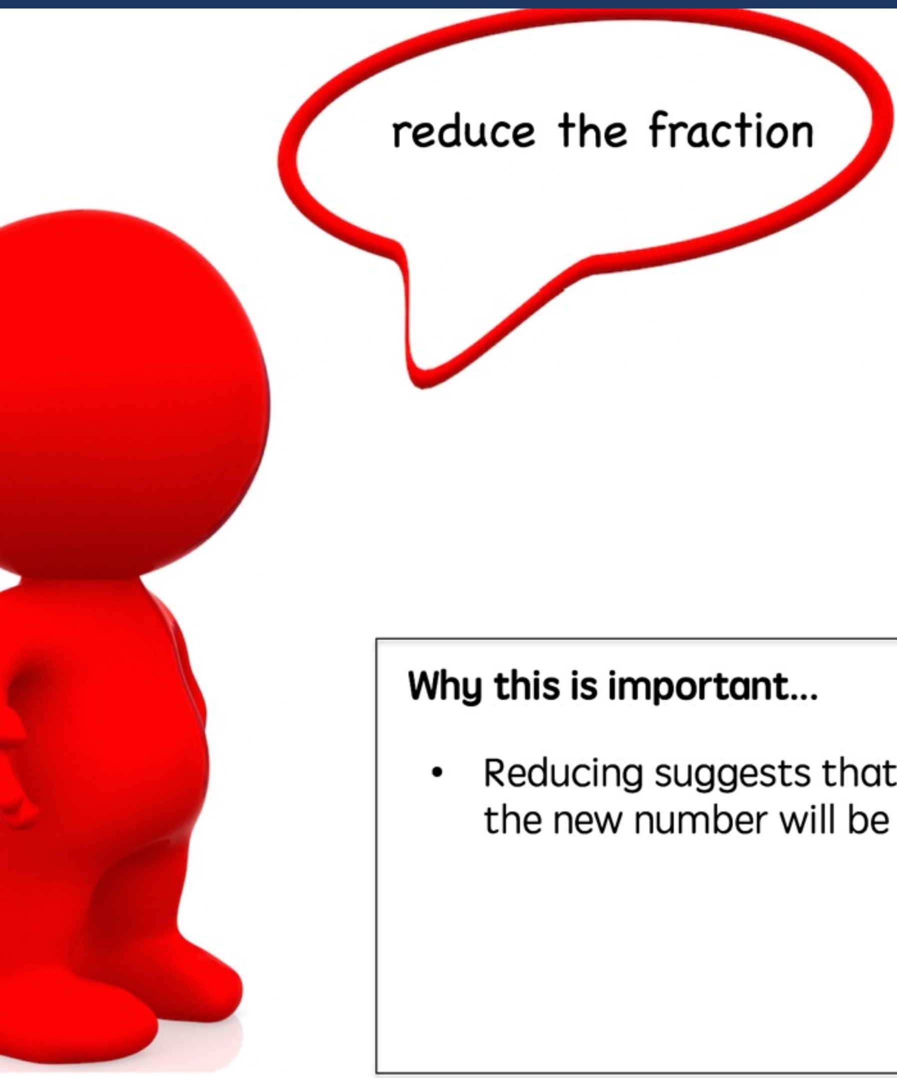
Why this is important…

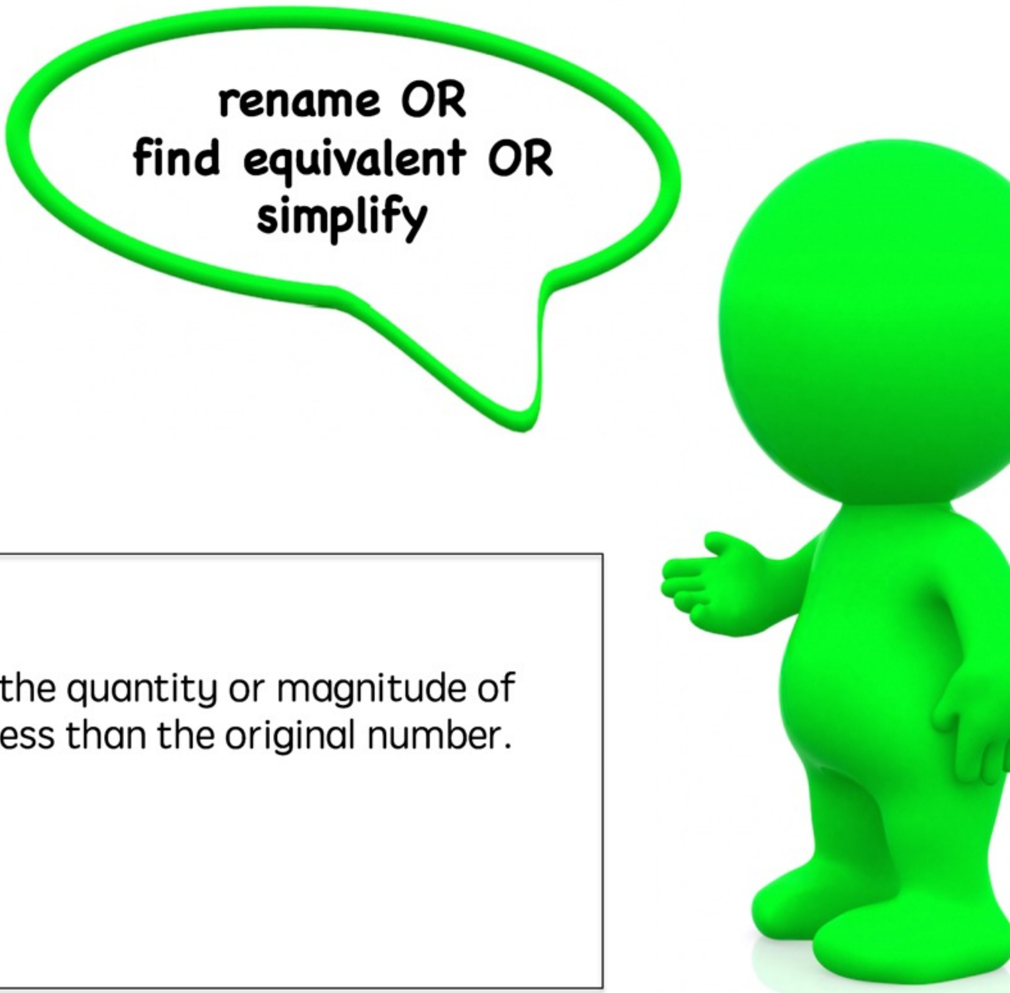
- "Carry" or "borrow" is procedural.
- The other terms reinforce the conceptual understanding or regrouping ones into tens, tens into hundreds, and so on (i.e., the total amount does not change) *or* ungrouping hundreds into tens, tens into ones, and so on.



Why this is important...

- Identifying that there are two separate (whole) numbers suggests that whole number properties can be applied to fractions.
- Emphasizing that a fraction is ONE number with ONE magnitude on a number line that is communicated with a numerator and denominator is important.





Why this is important…

- Reducing suggests that the quantity or magnitude of the new number will be less than the original number.

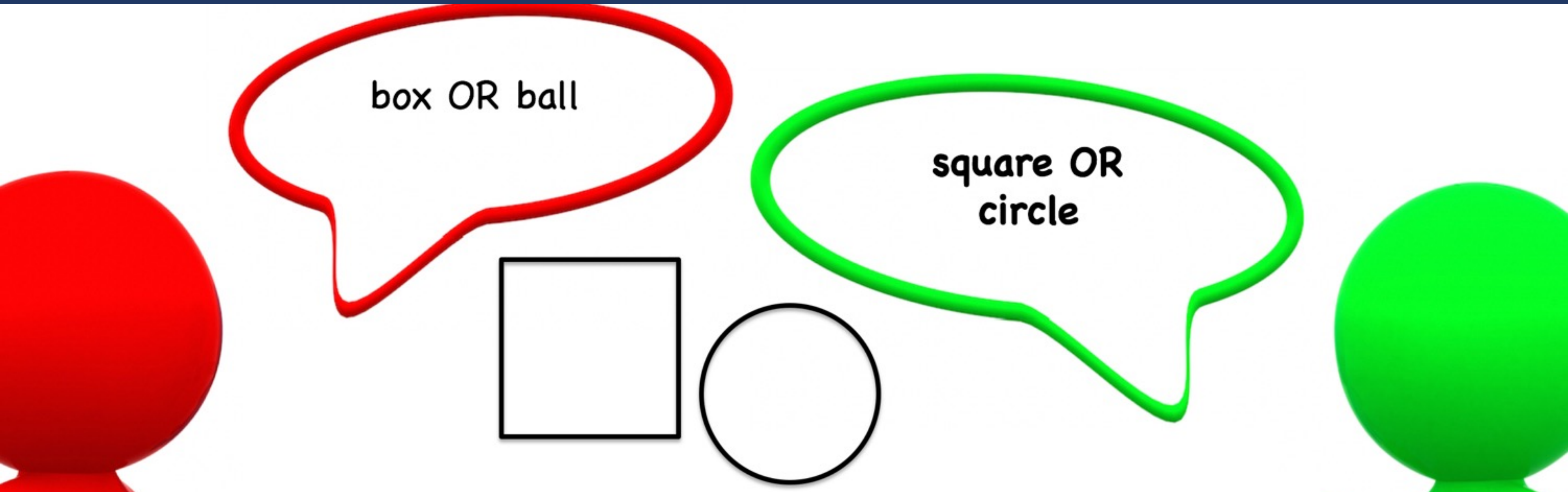
Four point seven
Four point oh seven

Four and seven tenths
Four and seven hundredths

4.7
4.07

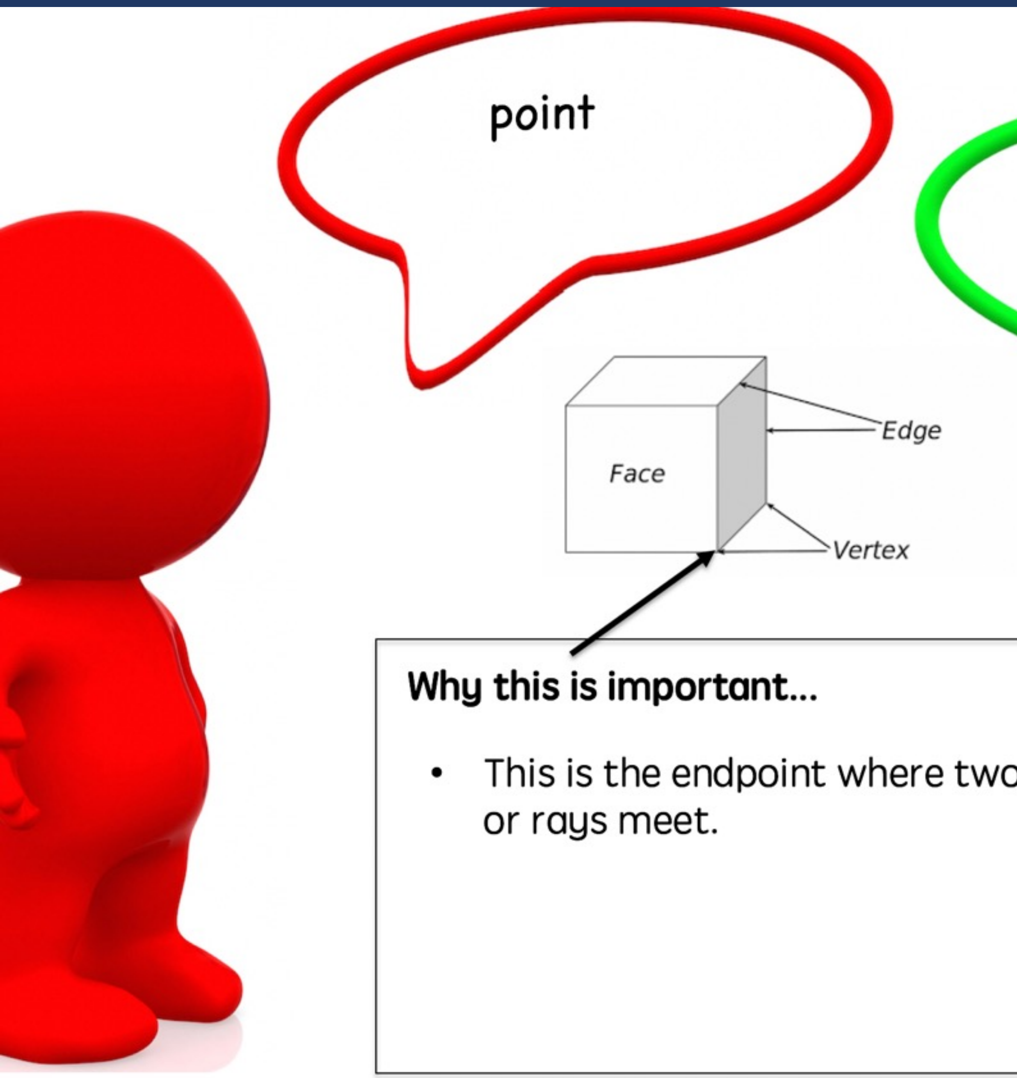
Why this is important…

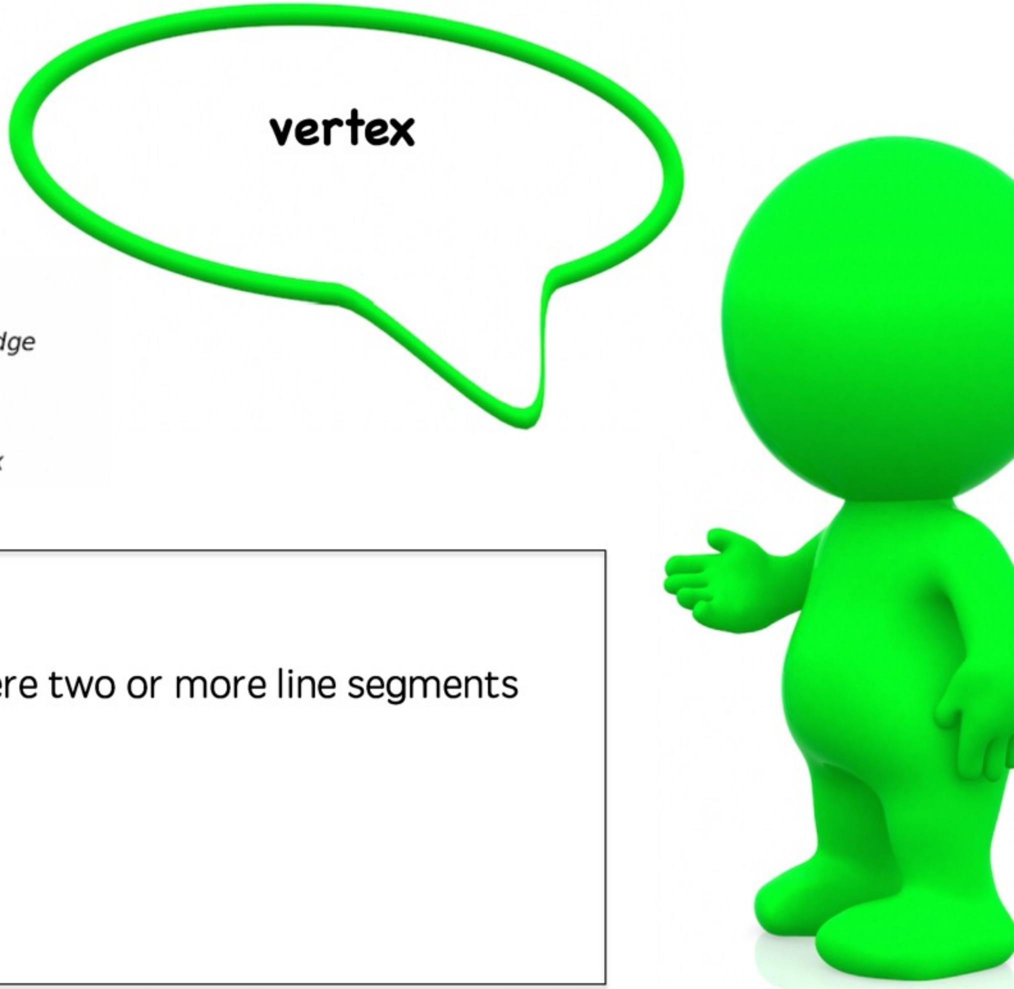
- Accurately shares the magnitude of the decimal.
- Emphasizes place value.



Why this is important…

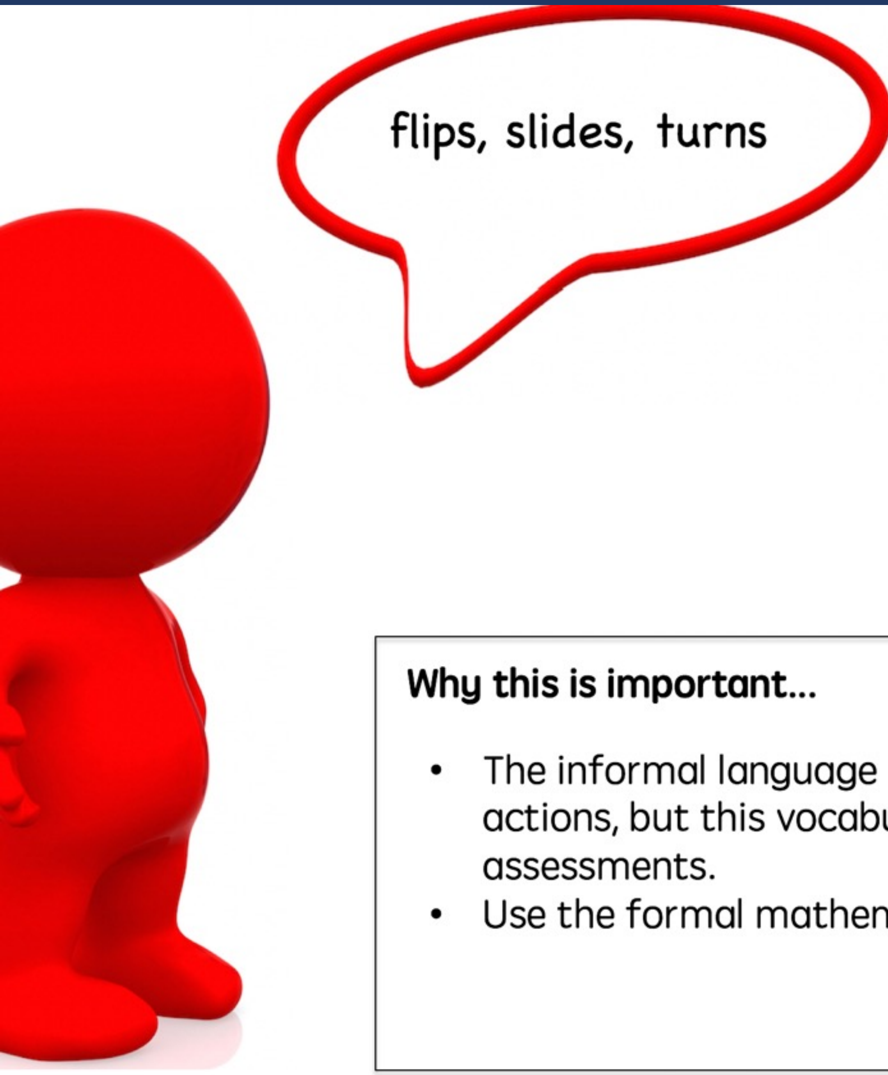
- Use the formal language of shapes to confirm informal language.

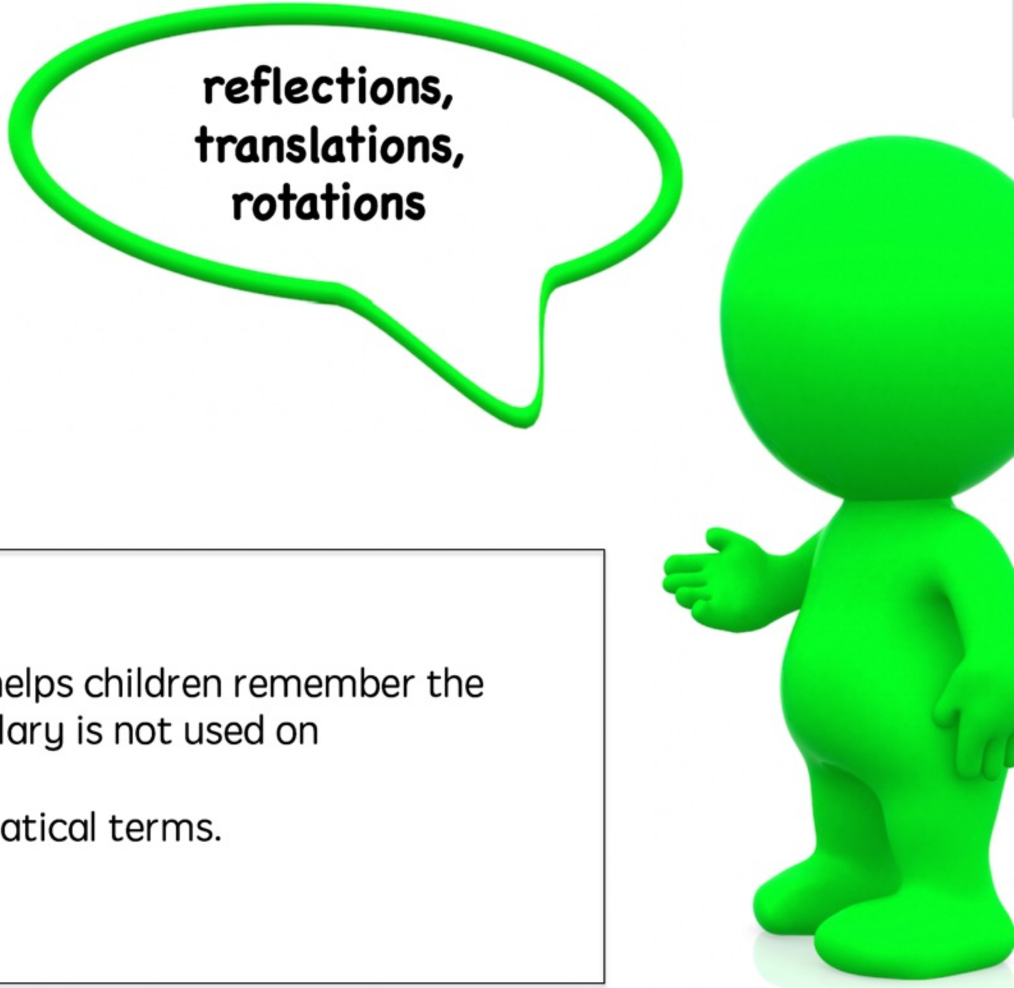




Why this is important…

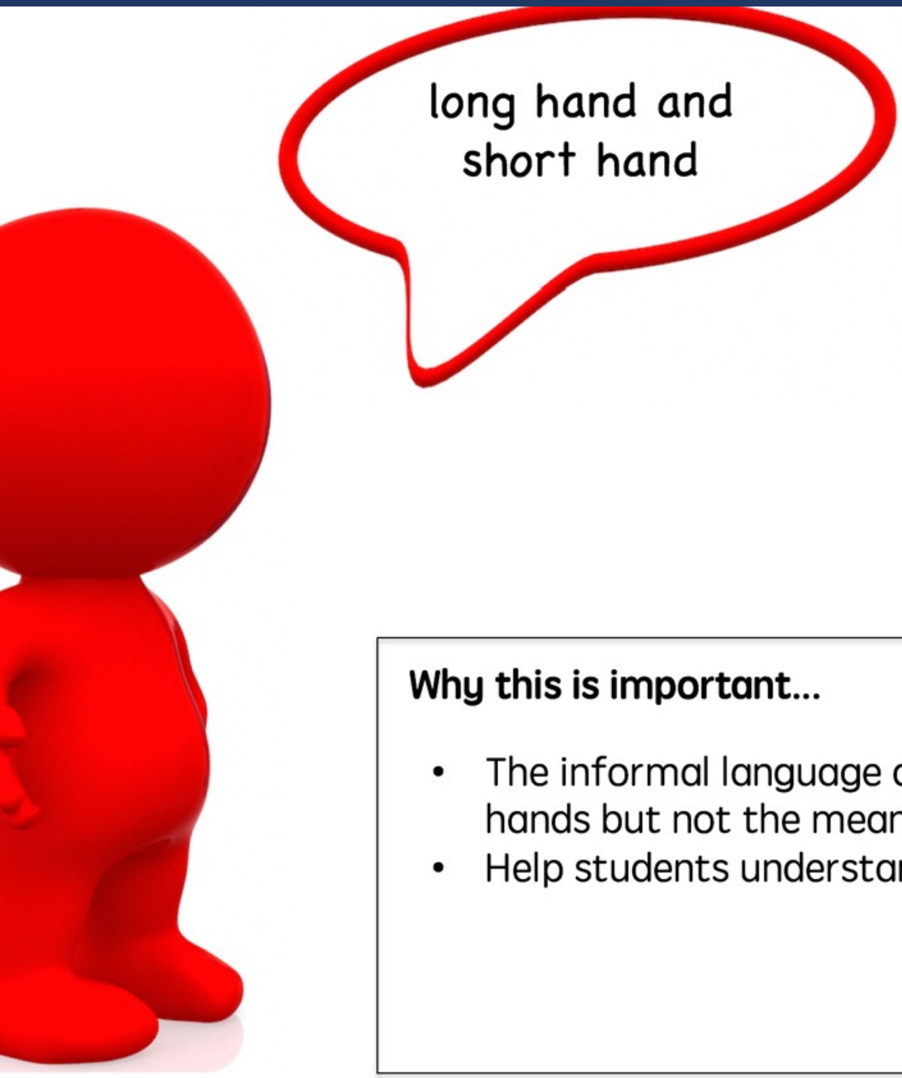
- This is the endpoint where two or more line segments or rays meet.

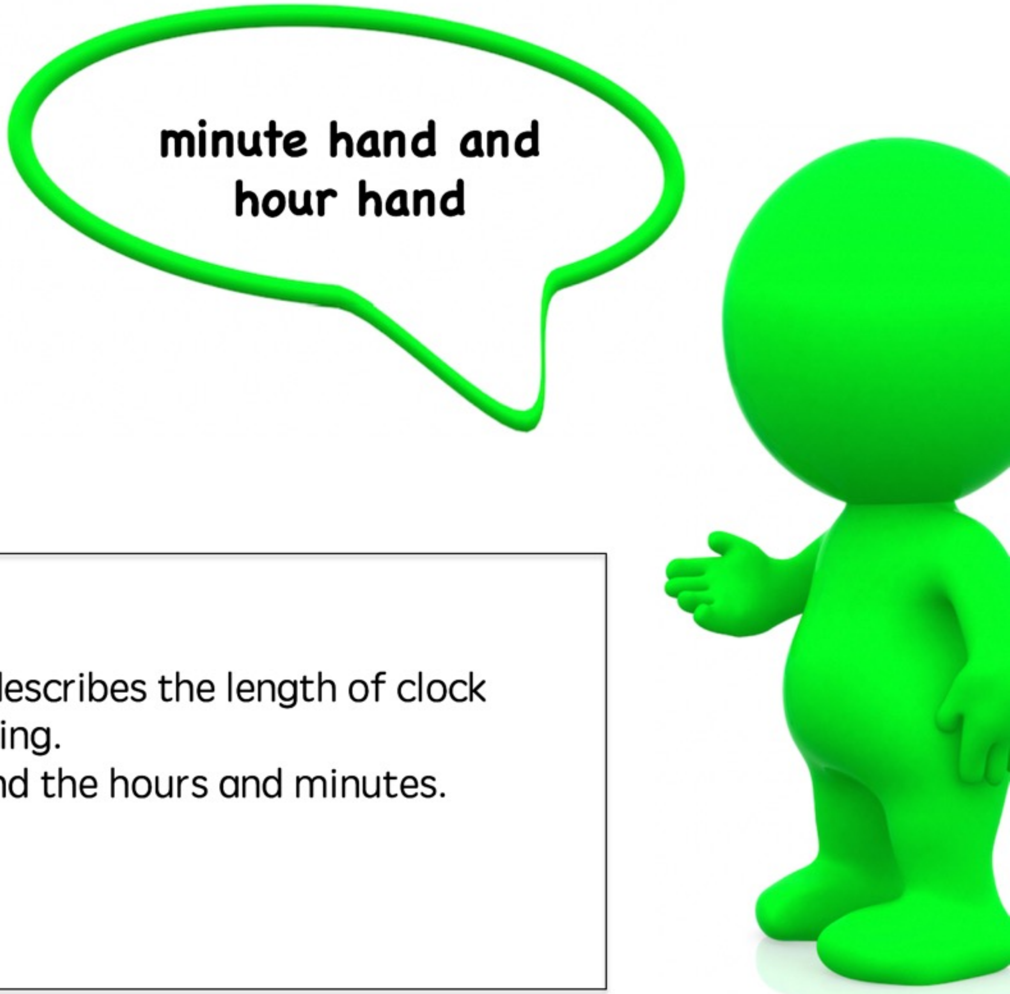




Why this is important…

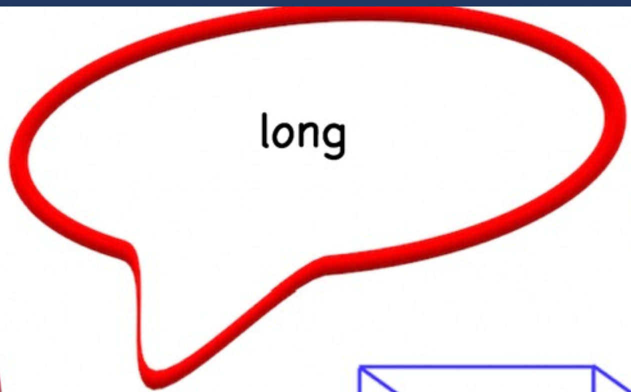
- The informal language helps children remember the actions, but this vocabulary is not used on assessments.
- Use the formal mathematical terms.

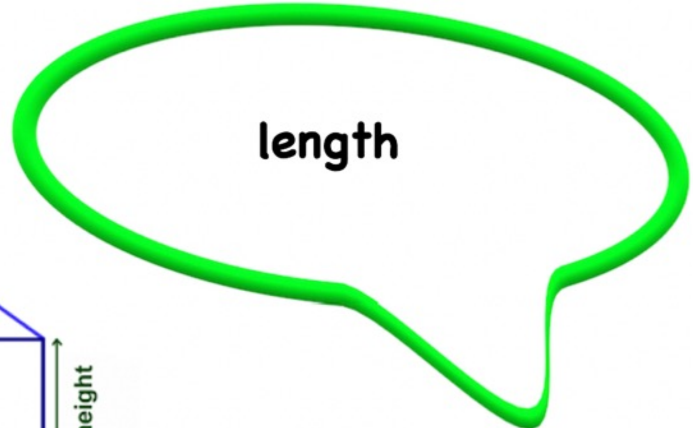


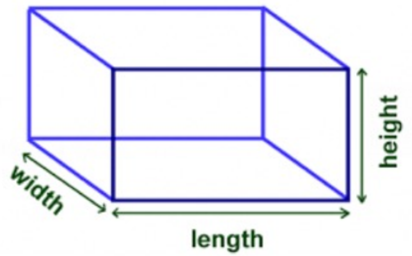


Why this is important...

- The informal language describes the length of clock hands but not the meaning.
- Help students understand the hours and minutes.







Why this is important…

- Using "length" regularly helps when students are introduced to three-dimensional figures.

What are examples of,

"Instead of ____, Say ____?"

Use formal math language

Use terms precisely

Factor

$1 \times \underline{8} = 8$

$\underset{\text{factor}}{2} \times \underset{\text{factor}}{4} = 8$

Multiple

$8 \times 1 = 8$

$8 \times 2 = 16$

multiples of 8

E

Rubenstein & Thompson (2002)

Improper fraction

$$\frac{8}{5}$$

Proportion

$$\frac{2}{5} = \frac{8}{20}$$

Mixed number

$$1\frac{3}{5}$$

Ratio

4:3

Proper fraction

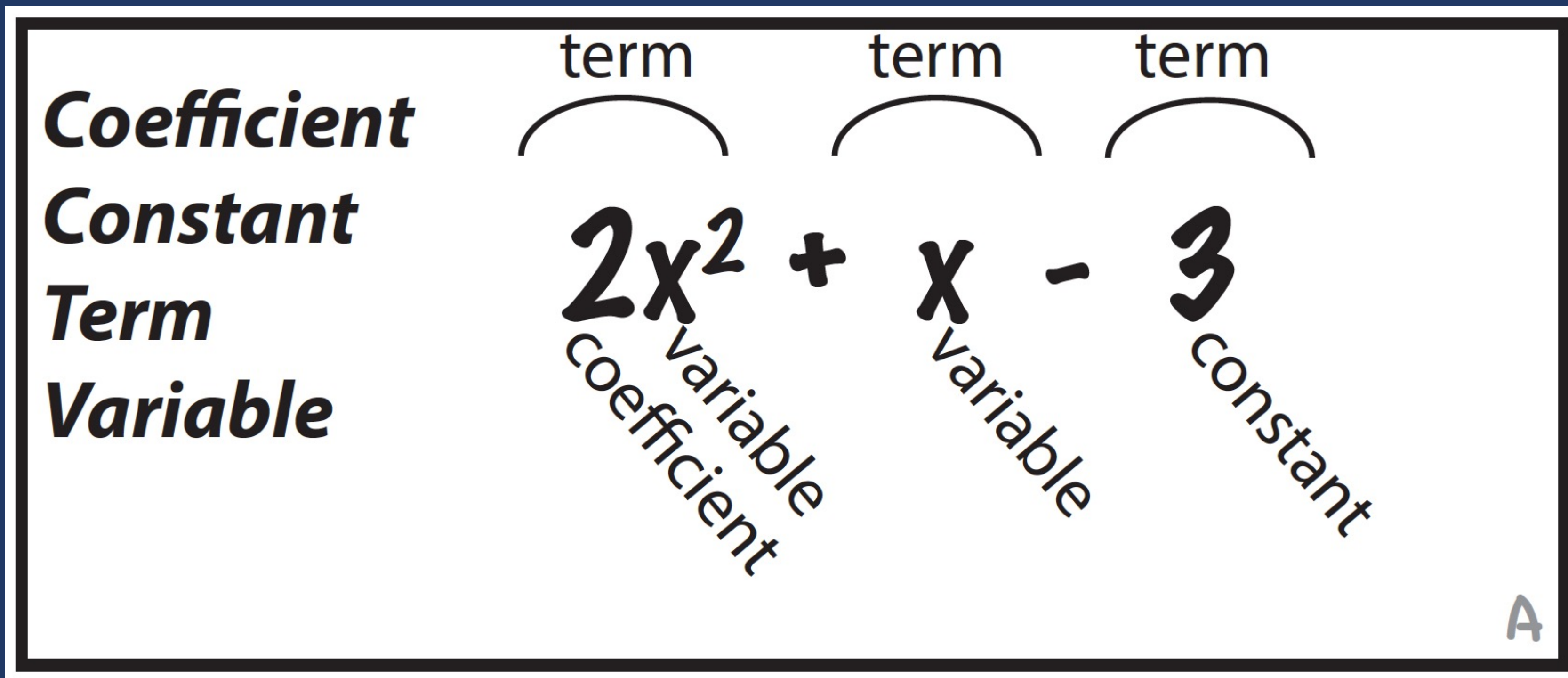
$$\frac{2}{9}$$

Unit fraction

$$\frac{1}{6}$$

D

Rubenstein & Thompson (2002)



Rubenstein & Thompson (2002)

Equation	$9x - 4 = 7x$
Expression	$9x - 4$
Formula	$a^2 + b^2 = c^2$
Function	$f(x)$
Inequality	$9x - 4 > 6x$

C

Rubenstein & Thompson (2002)

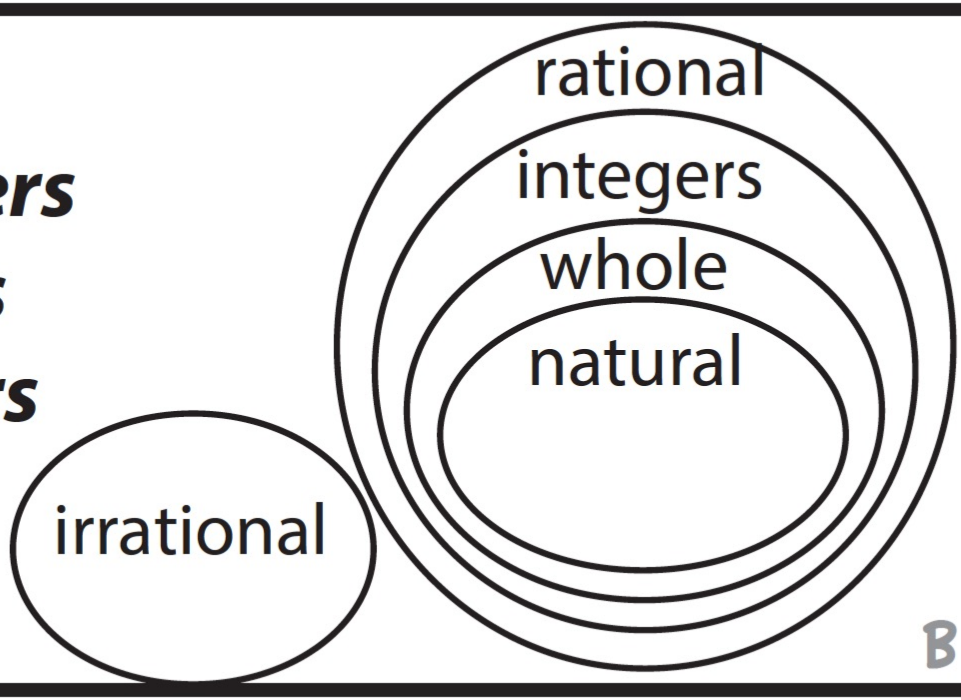
Integers

Irrational numbers

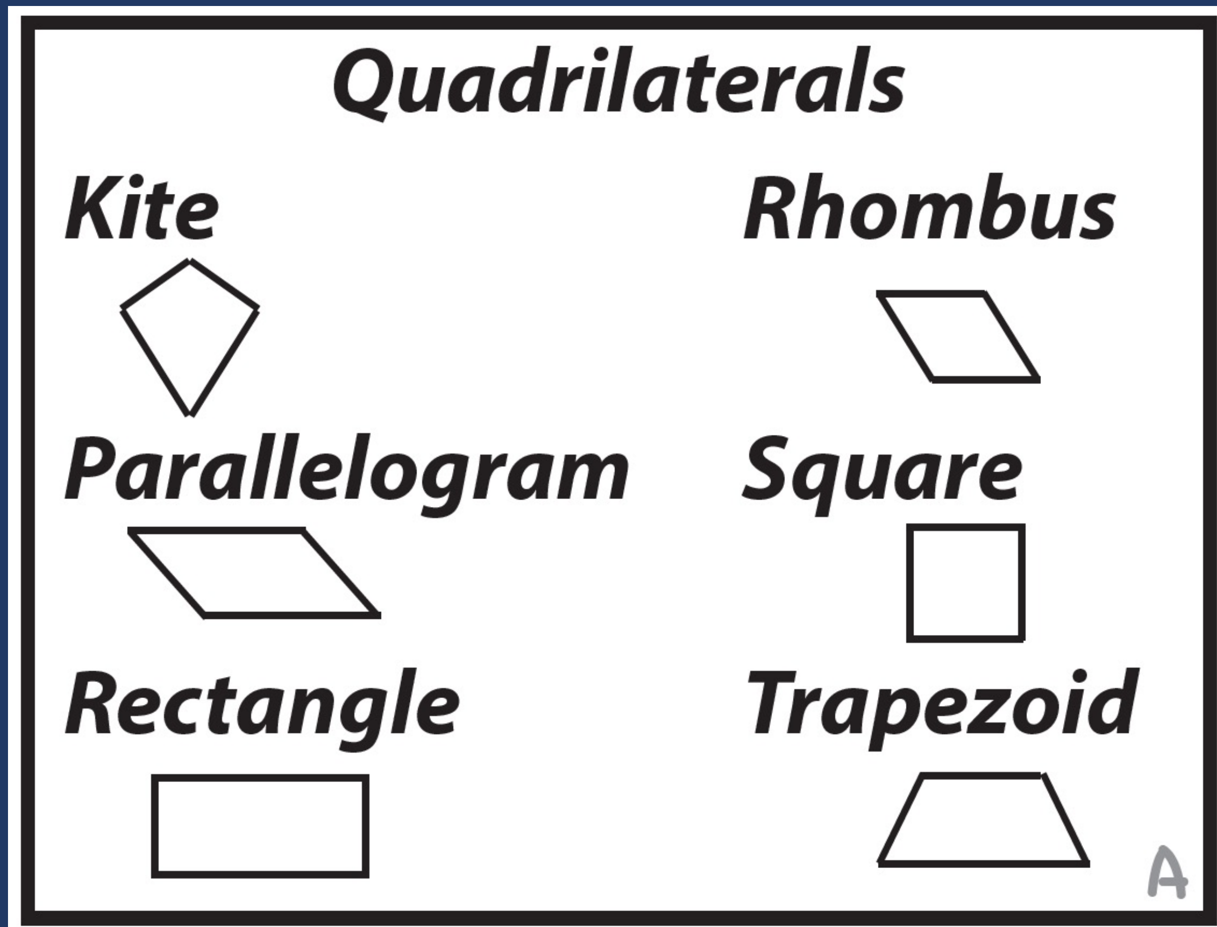
Natural numbers

Rational numbers

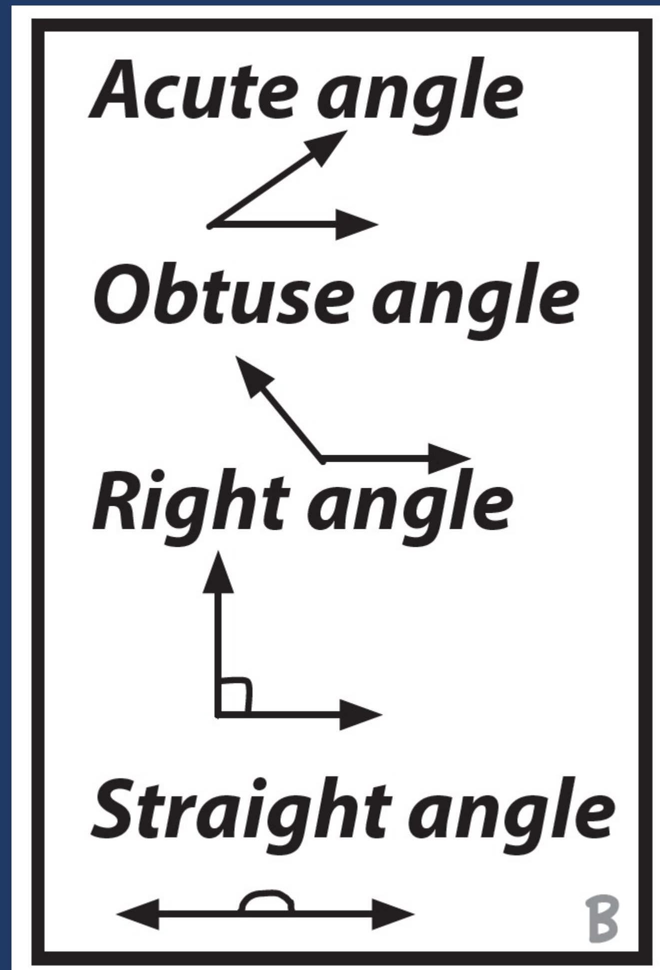
Whole numbers



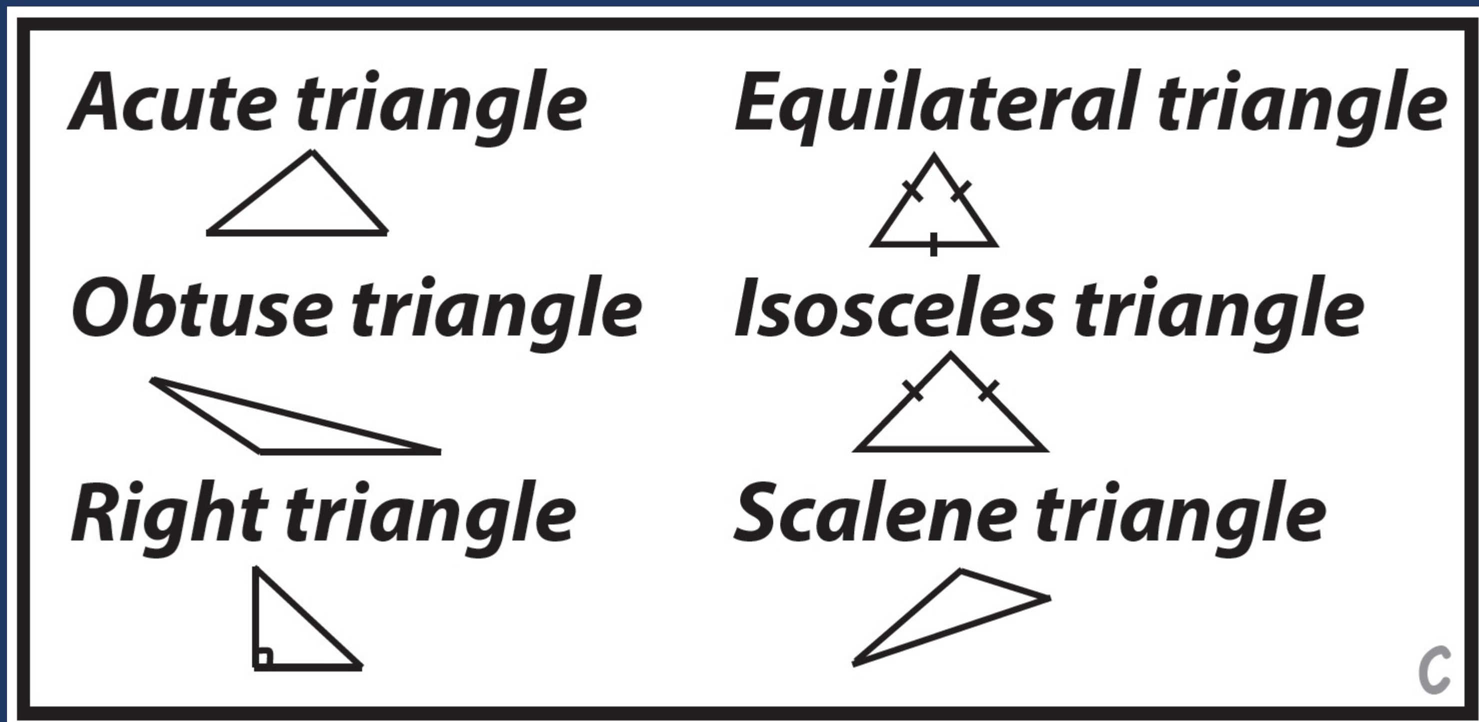
Rubenstein & Thompson (2002)



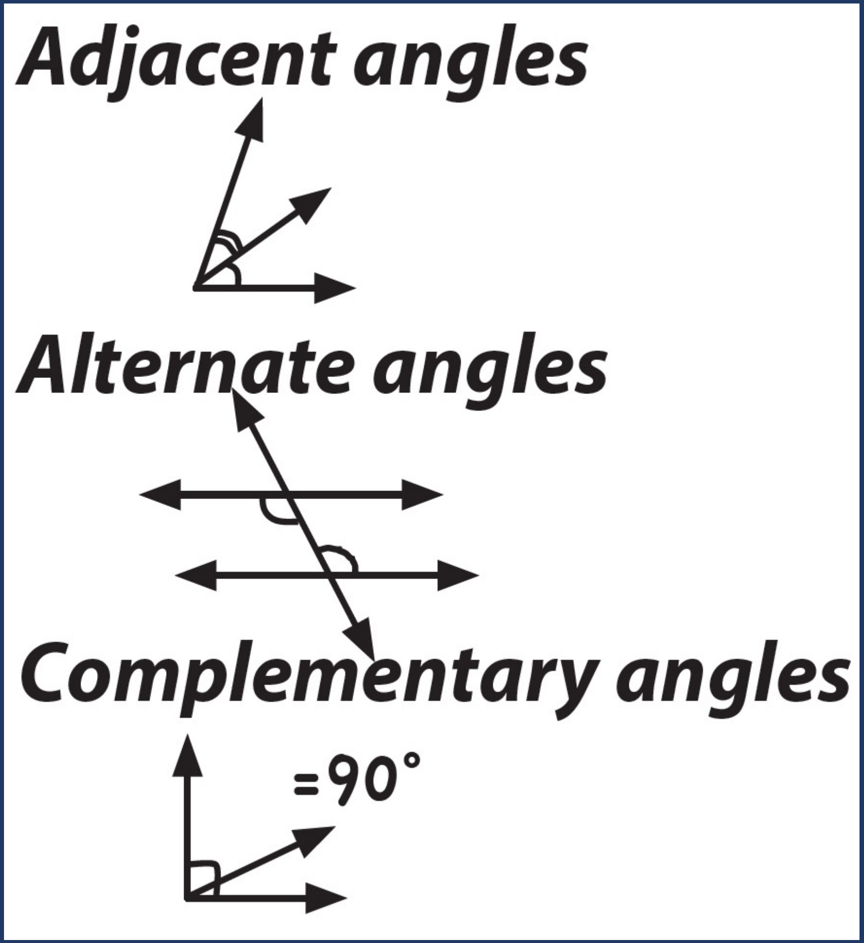
Rubenstein & Thompson (2002)

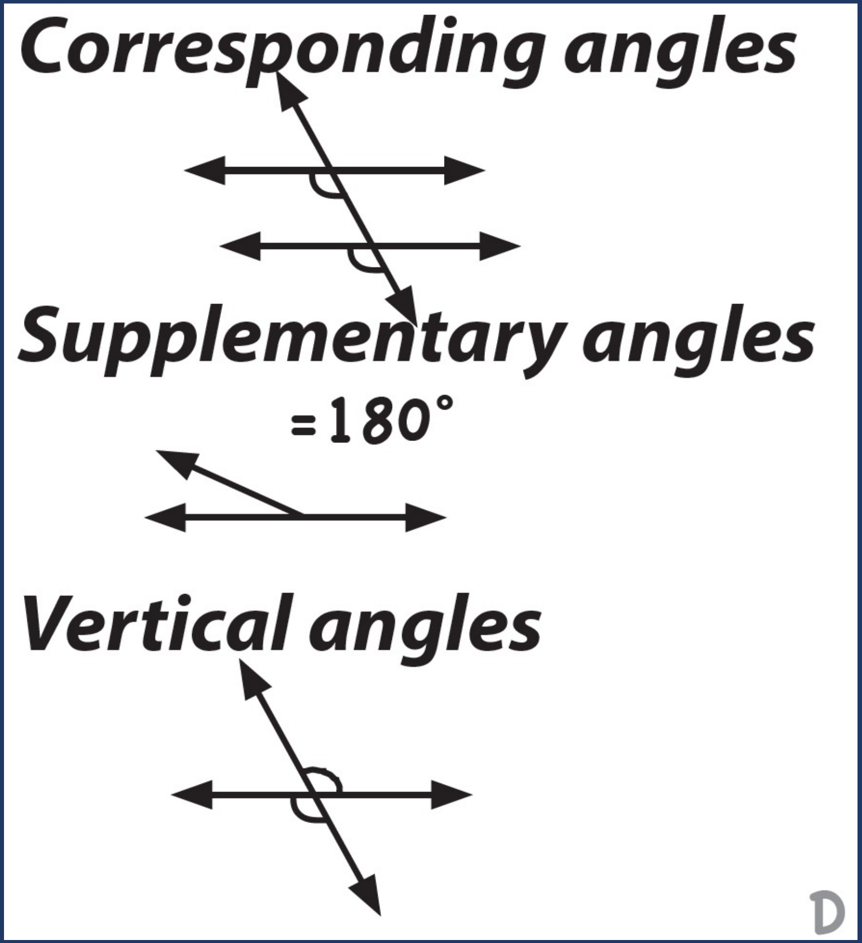


Rubenstein & Thompson (2002)

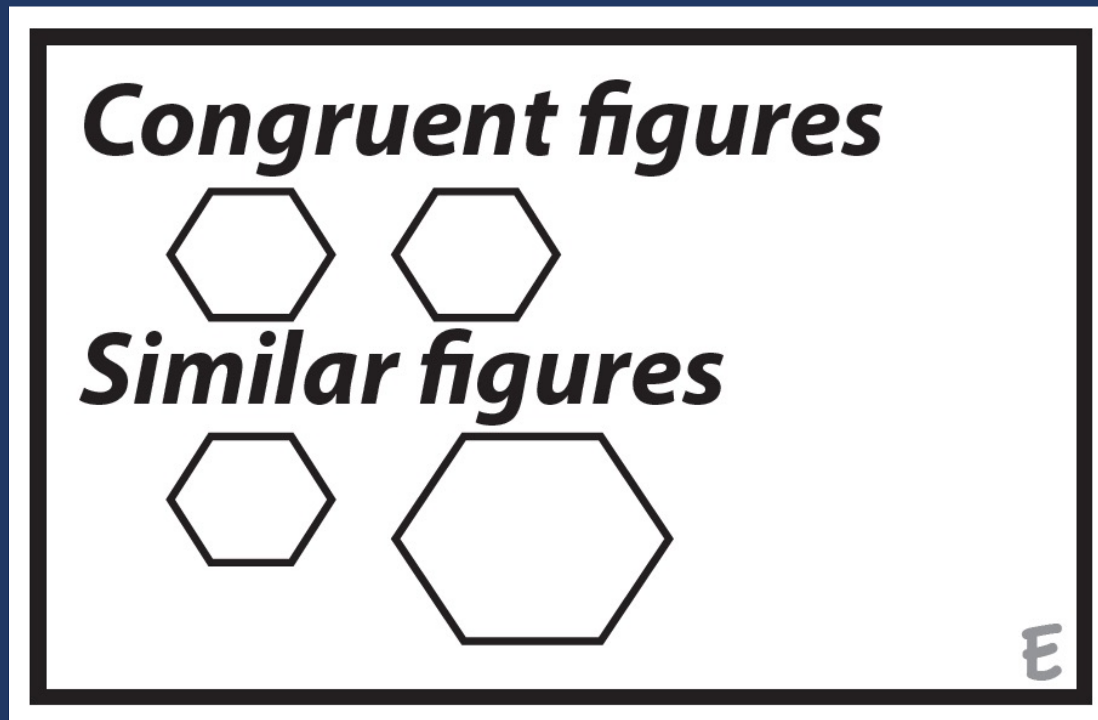


Rubenstein & Thompson (2002)

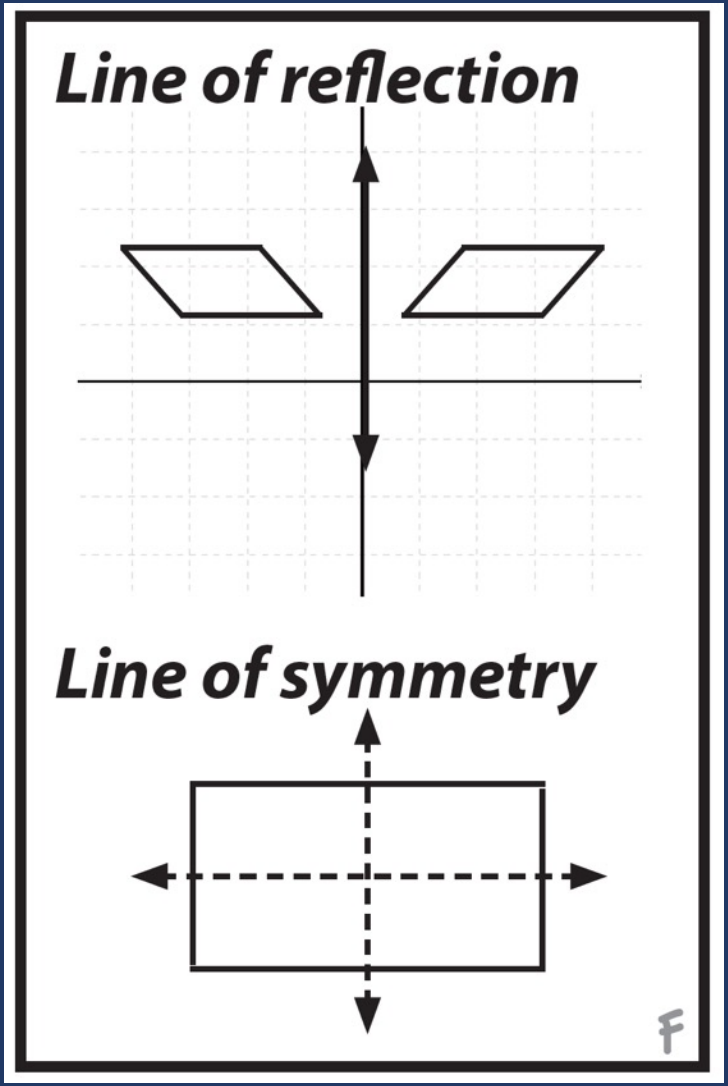




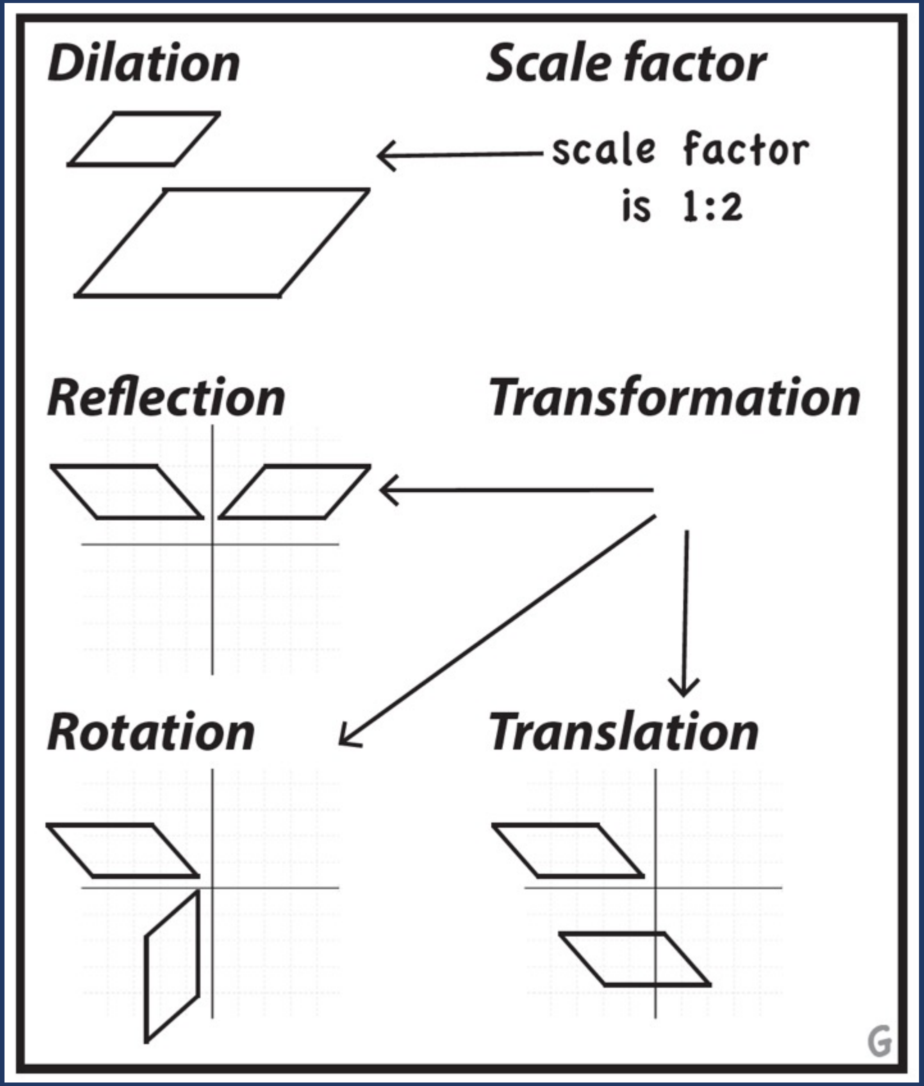
Rubenstein & Thompson (2002)



Rubenstein & Thompson (2002)



Rubenstein & Thompson (2002)



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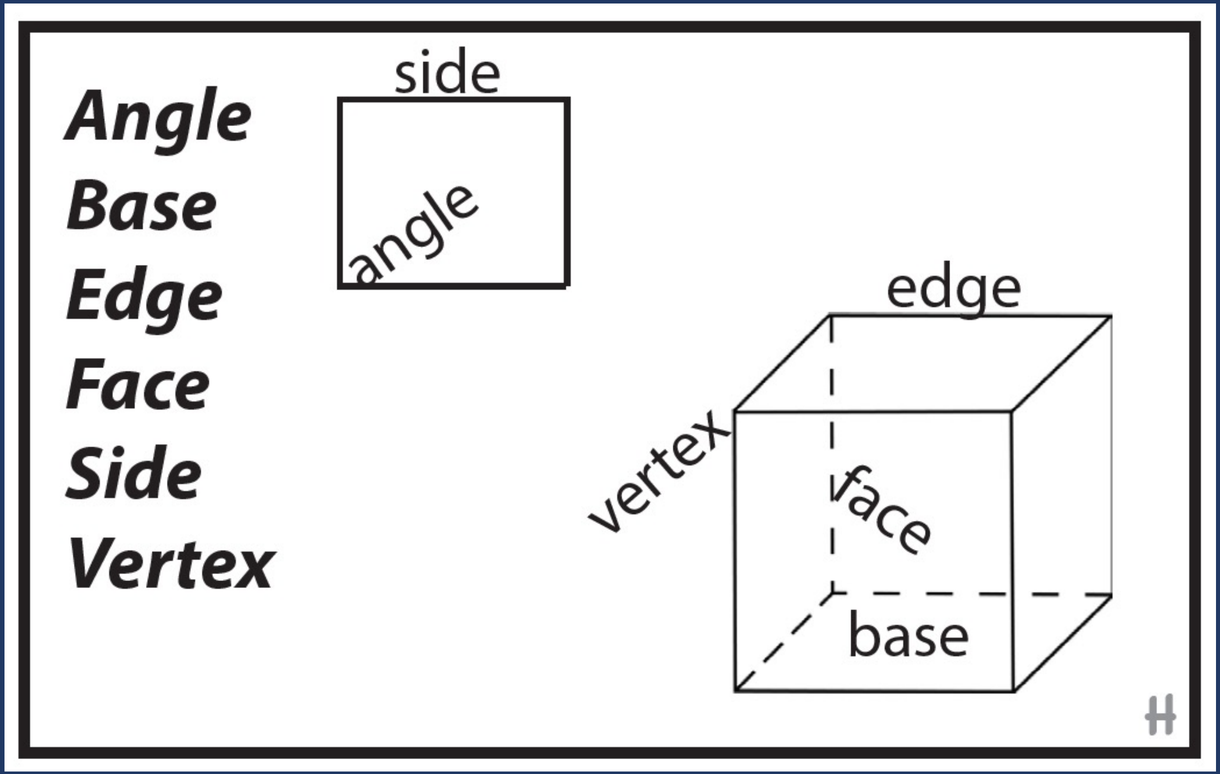

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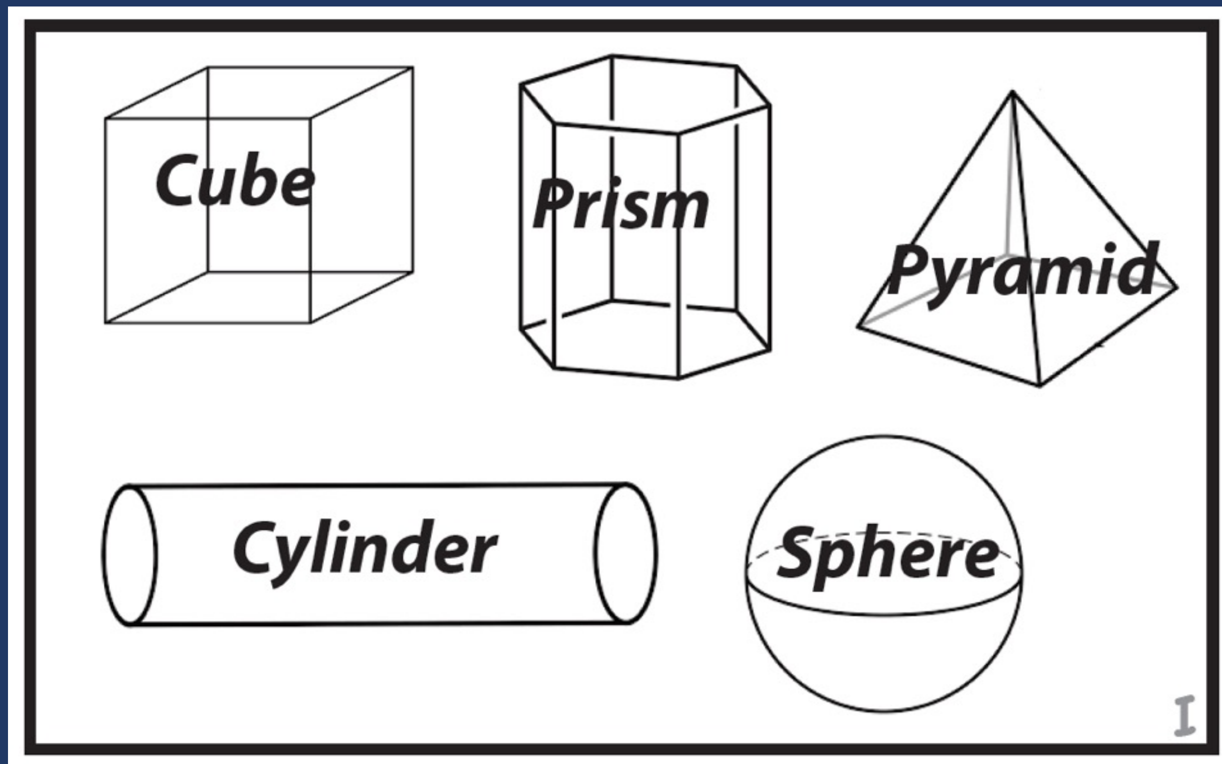
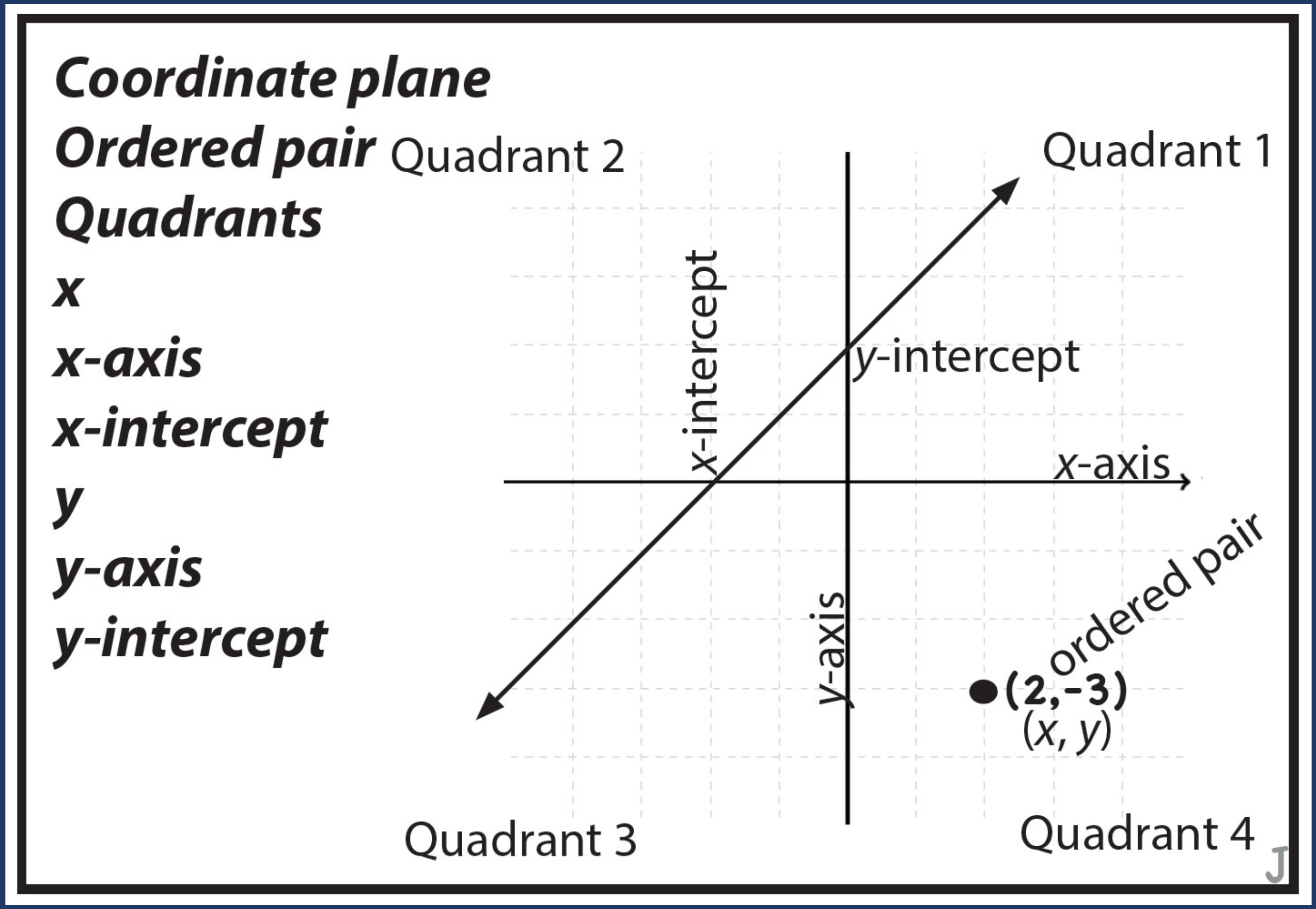


Rubenstein & Thompson (2002)



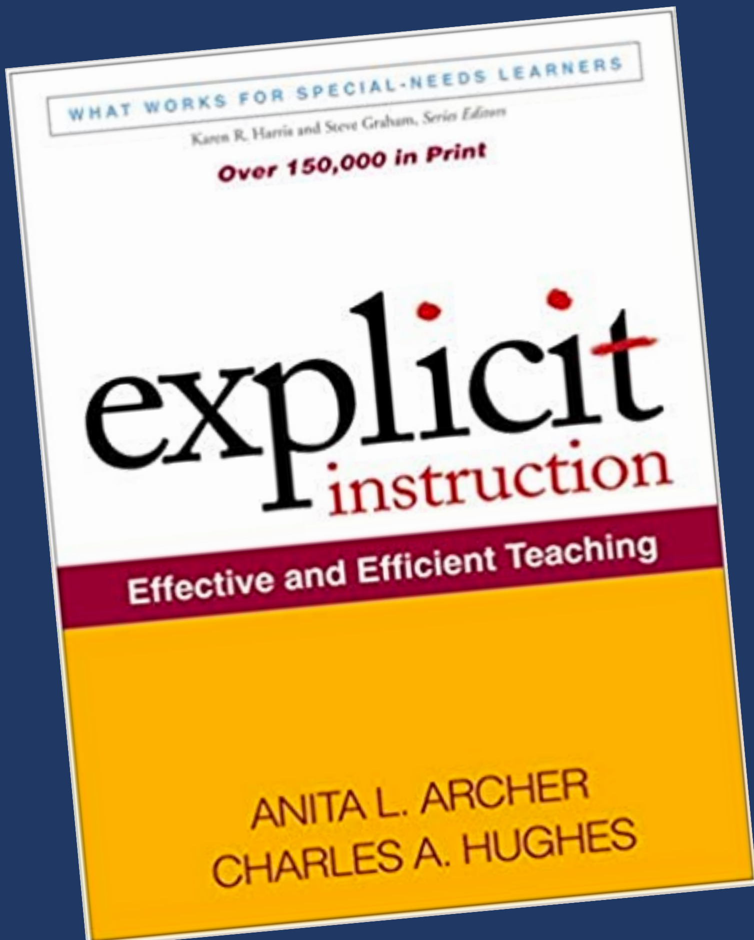
Rubenstein & Thompson (2002)

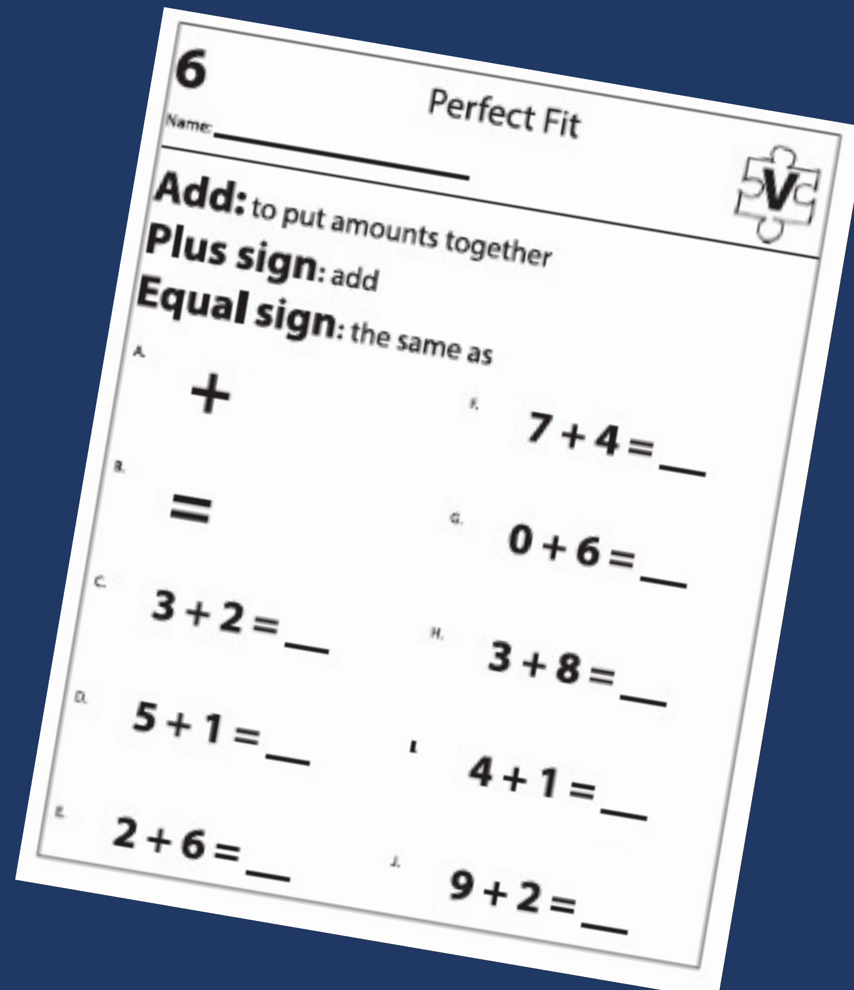
Which terms do your students not use precisely?

Use formal math language

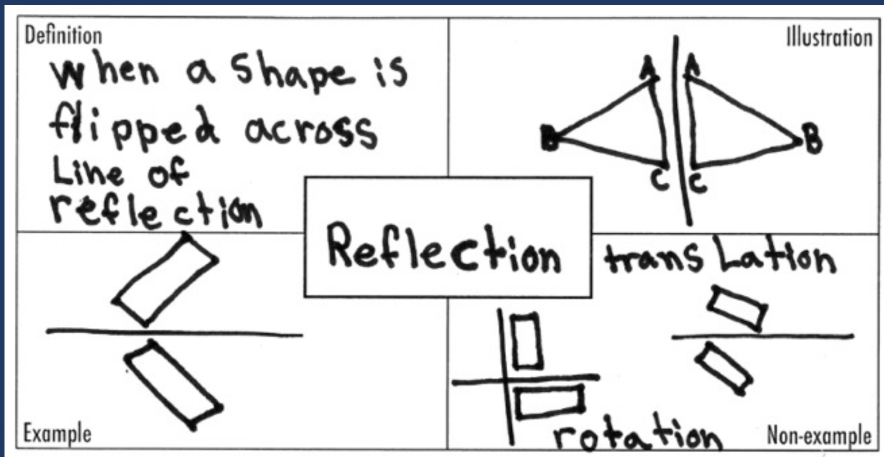
Use terms precisely

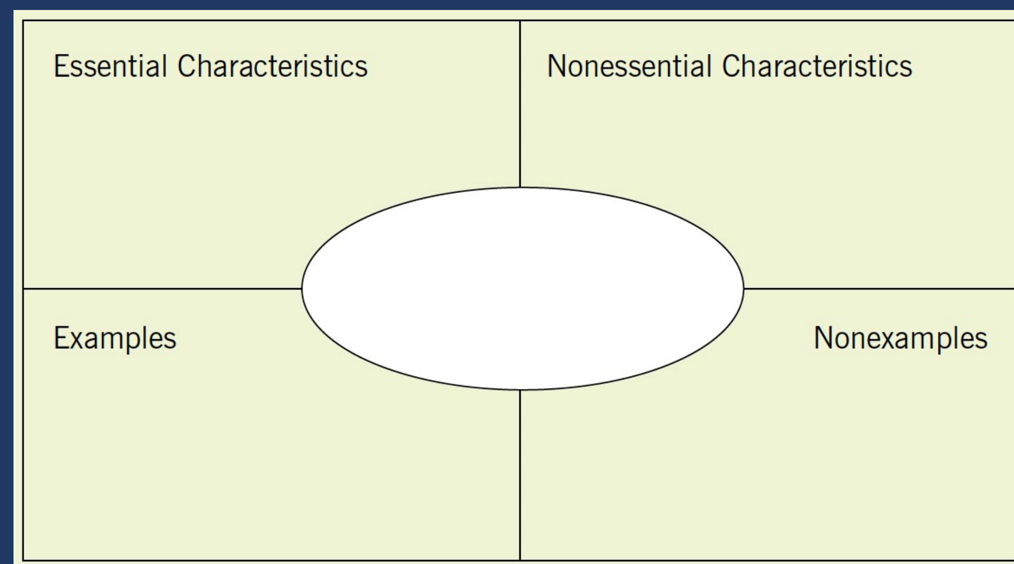
1. Use explicit instruction





2. Use graphic organizers





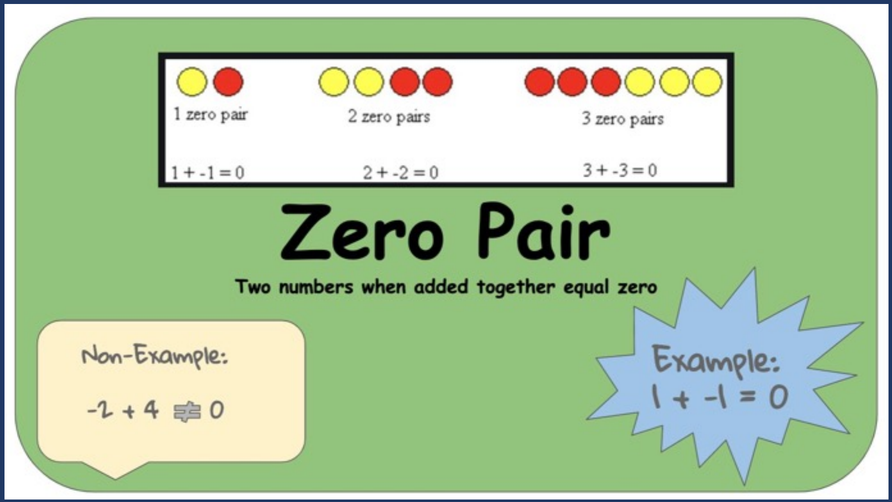
Dunston & Tyminski (2013)

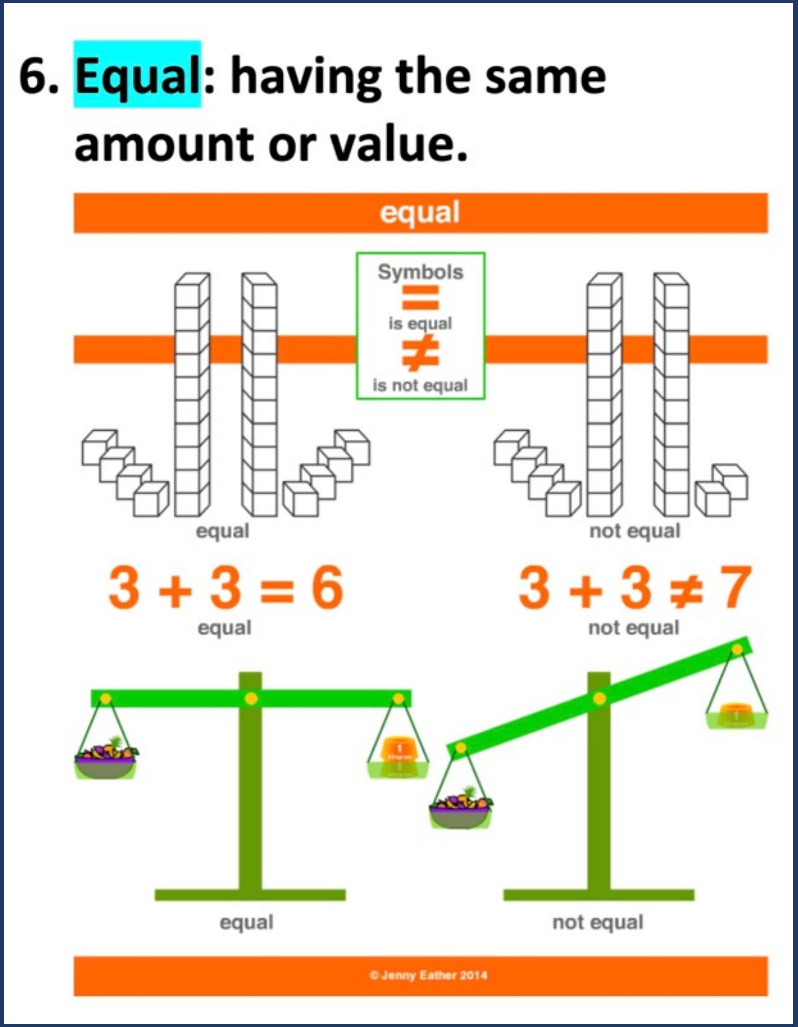
2. Use graphic organizers

Word	Lightbulb Word
Definition	Picture

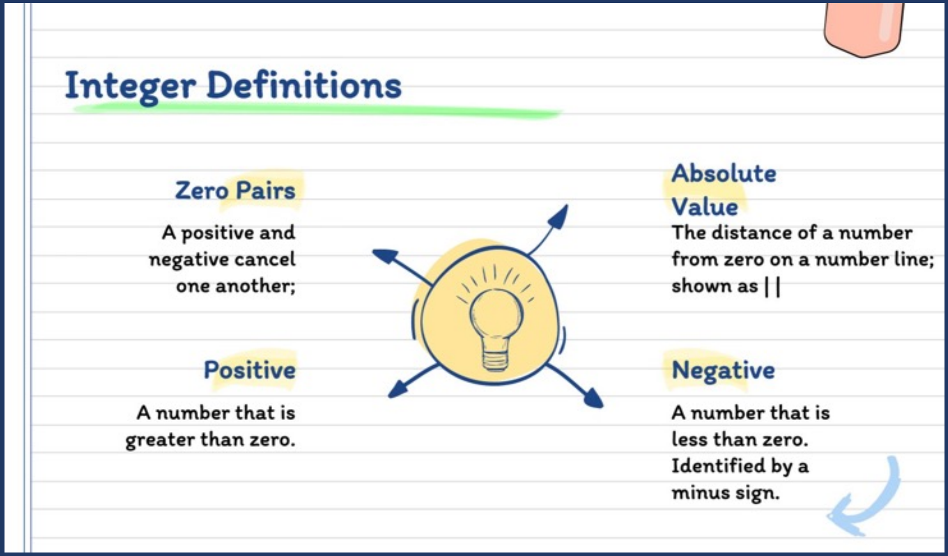
Dunston & Tyminski (2013)

3. Have students create vocabulary cards





4. Have students create glossaries



Numerator: how many parts of the whole

- Ex. $\frac{4}{10}$

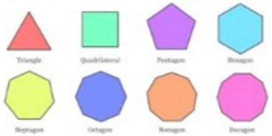
Odd number: a number not divided evenly by 2

- Ex. 1, 3, 5, 7, 9....

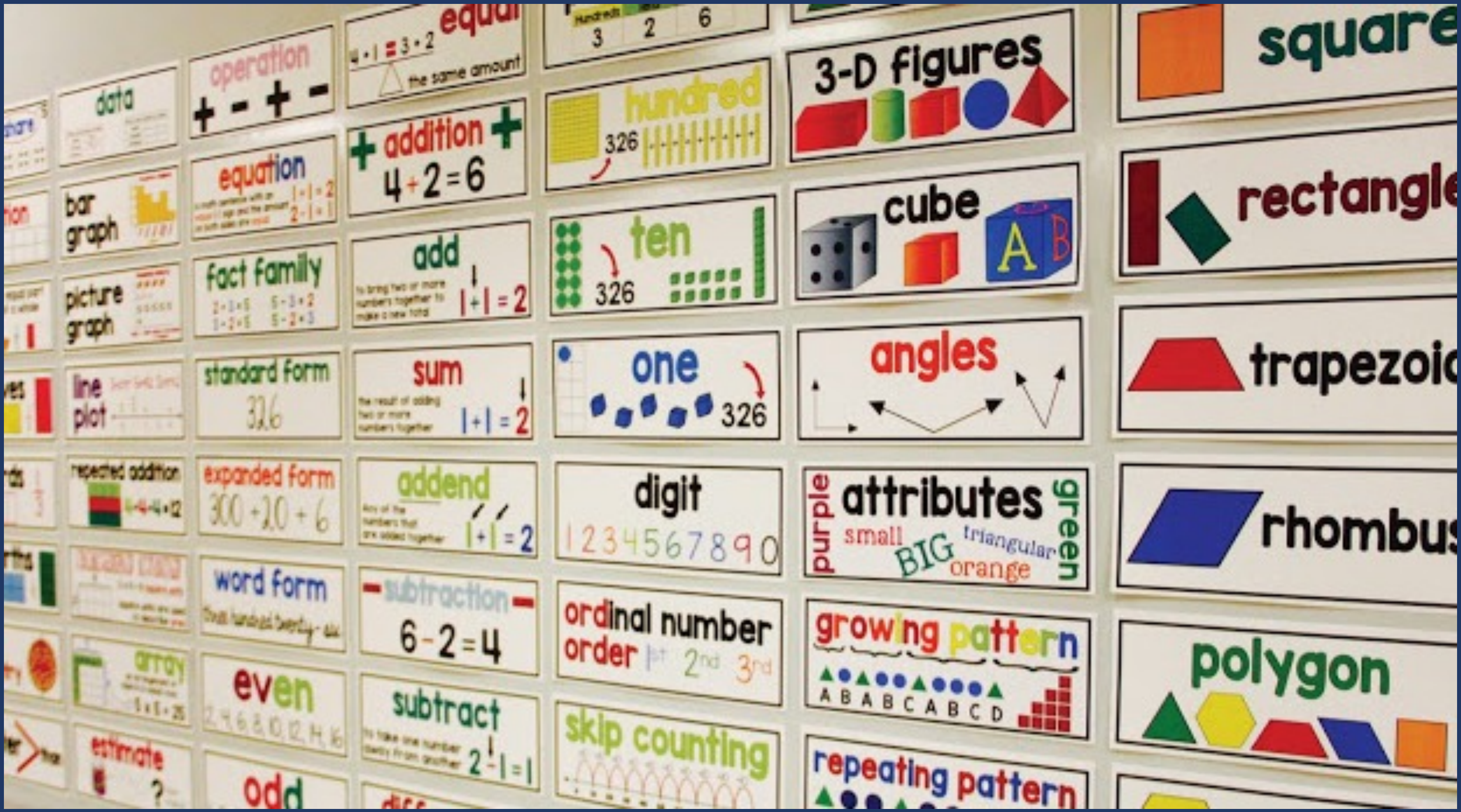
Percent: a specific number in comparison to 100

- 74%

Polygon: any enclosed shape that is made up of 3 or more straight lines

- Ex. 

5. Create a word wall



6. Preview vocabulary

Dear Feisty Fifth Graders,

Today we have multiple opportunities to do exciting projects! For example, we are going to be doing a science experiment to see how the tilt of a ramp relates to how far a matchbox car will roll. There are several factors we will be looking at in this experiment. I look forward to hearing multiple ideas on how to set up this experiment.

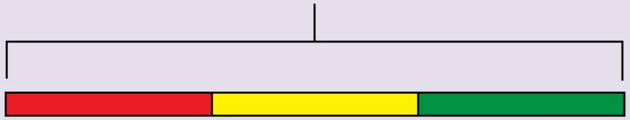
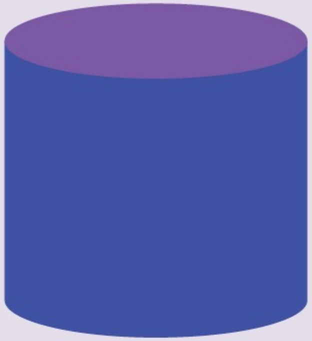
One other thing that factors into our day is that we have an assembly before lunch. We will get to hear music from the high school play. I think we will hear multiple songs.

Sincerely, Ms. Livers

Here is a problem to start your day... in my letter I have used two words that are important math words for today's lesson. Can you find them and tell what they mean in this letter and what they mean when talking about numbers? (Answer this in your math notebook)

Bay-Williams & Livers (2009)

7. Cluster vocabulary

	Length	Weight
Meaning	How long something is	How heavy something is
Visual	1 Yard  1 Foot 1 Foot 1 Foot	2000 pounds = 1 ton 

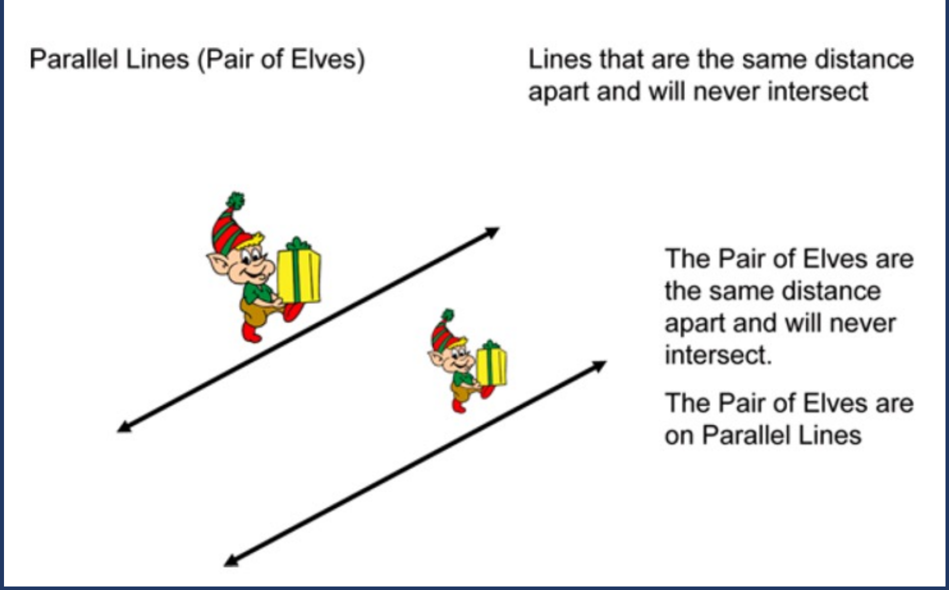
Livers & Bay-Williams (2014)

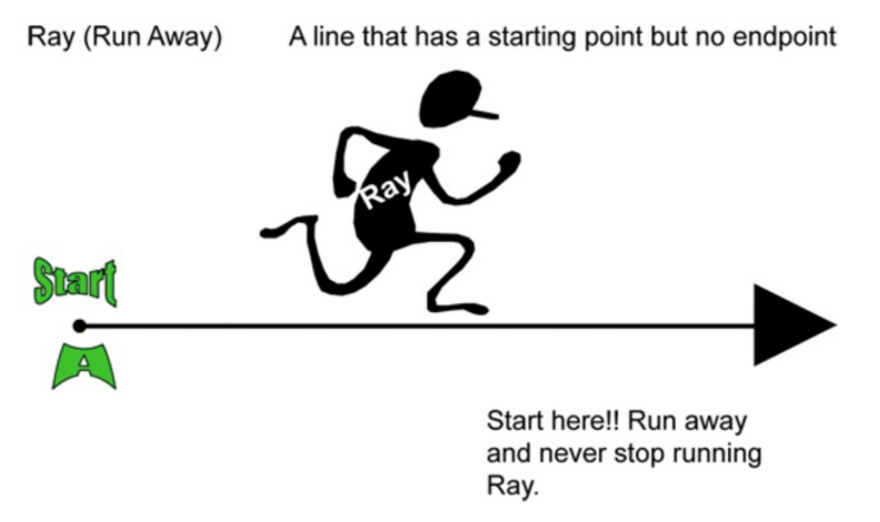
7. Cluster vocabulary

Rating	Word	Definition	Synonym(s)	Example	Sample Problem
2	expression	a mathematical phrase combining operations, numbers and/or variables.	phrase algebraic expression	6 $6n$ $6+n$ no equal sign	Lucia earns \$8 per hour for babysitting and gets a \$5 tip. Write an expression to represent the amount she would earn if she worked for x hours.
2	variable	a quantity that can change or take many values. (refers to the letter or symbol representing the quantity)	unknown	x D y T	The variable x represents the number of hours Charlie works in a week. Write an expression to represent his earnings if he earns \$9 per hour.
1	product	the result when two or more numbers are multiplied	total answer	$3 \times 2 = 6$ product	The product of 6 and a number is 24. What is the number?
3	quotient	the result of a division (refers to the number of times the divisor divides the dividend)	answer	$18 \div 2 = 9$ quotient $\begin{array}{r} 9 \\ 2\overline{)18} \end{array}$	Estimate the quotient when 365 is divided by 12.

Marin (2018)

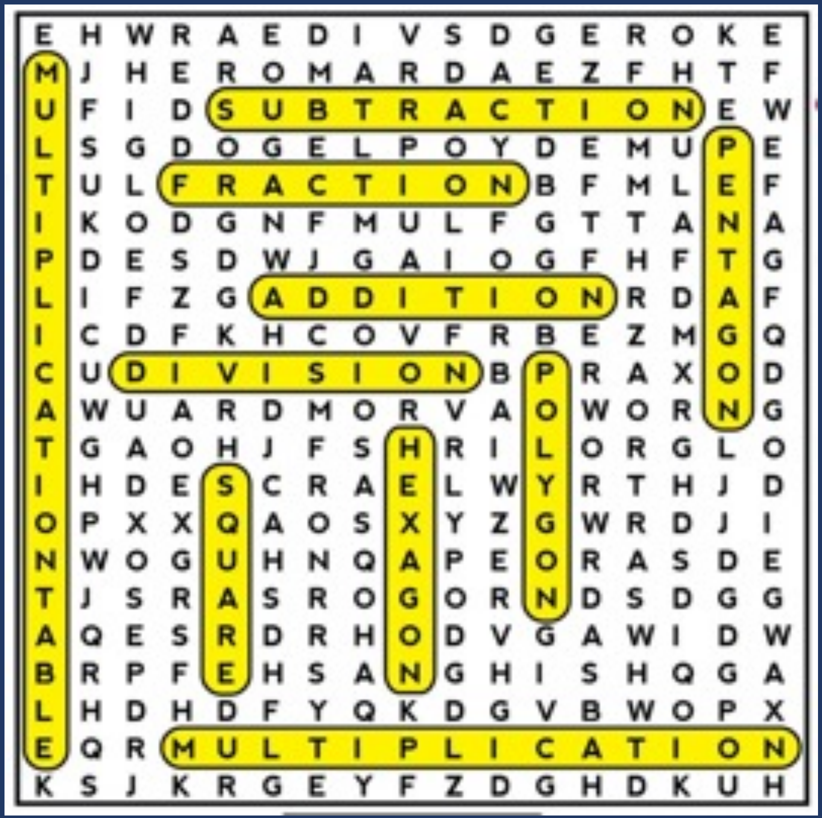
8. Use mnemonics



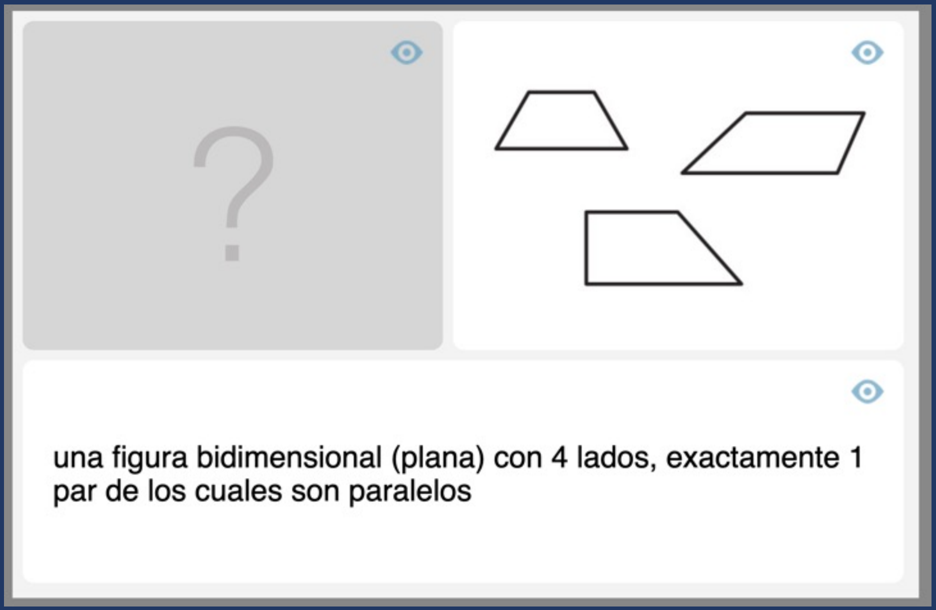


Riccomini et al. (2015)

9. Do word games



10. Use technology



Math Learning Center

Math Lingo

What are other ways to support learning mathematics vocabulary?

Instructional Platform

INSTRUCTIONAL DELIVERY

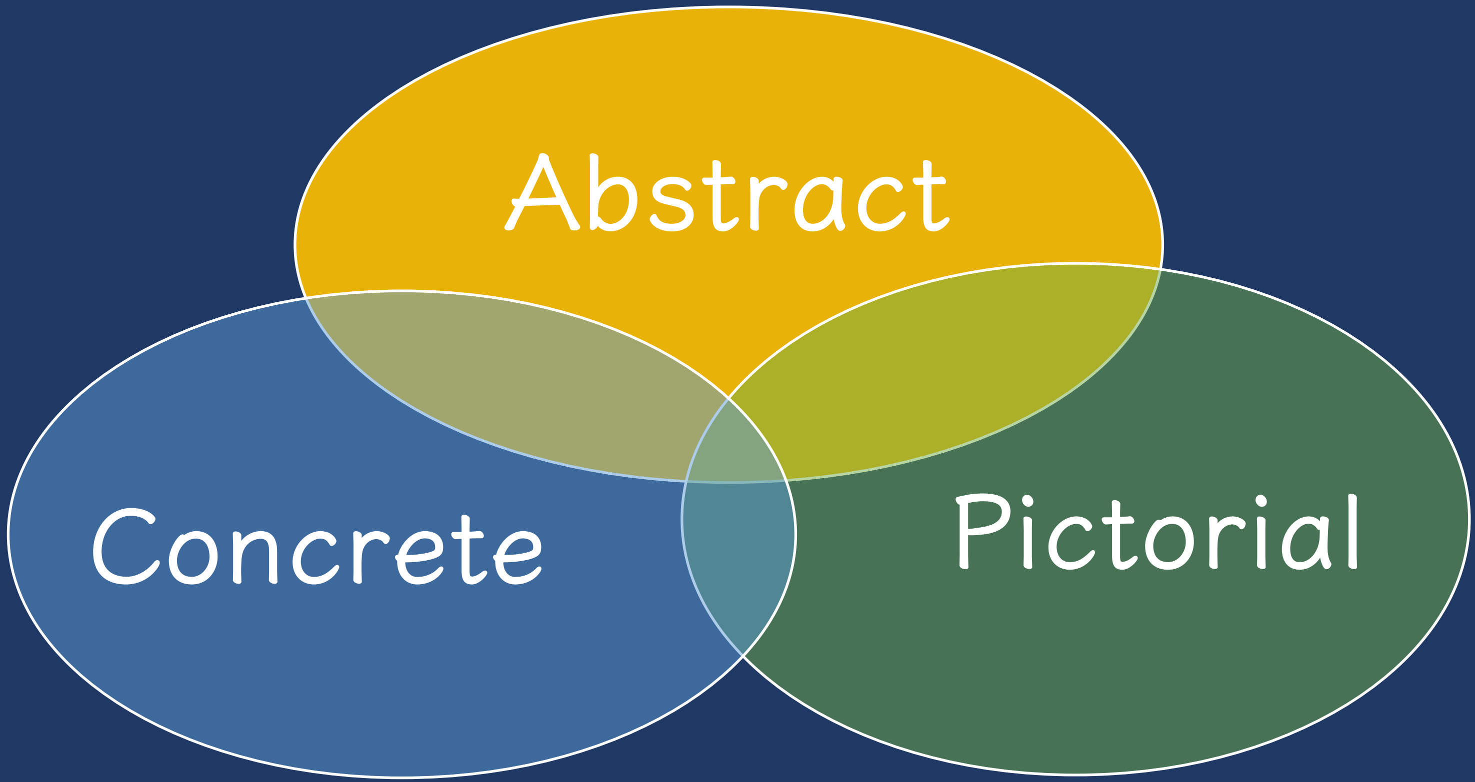
Explicit instruction

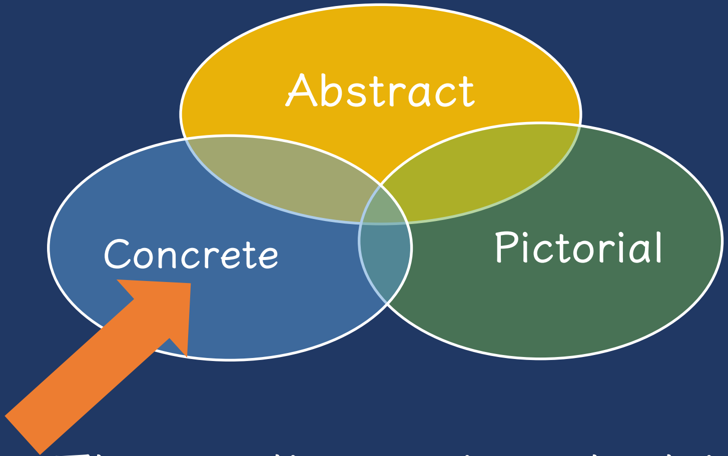
Precise language

Multiple representations

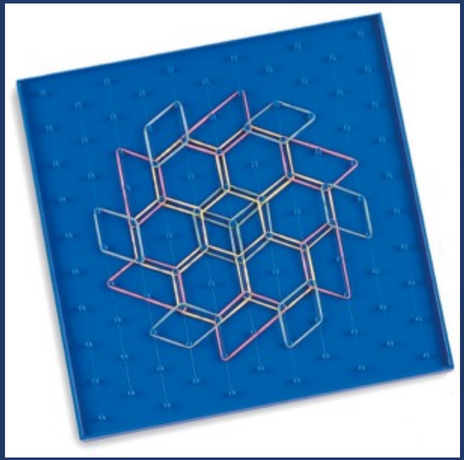
INSTRUCTIONAL STRATEGIES

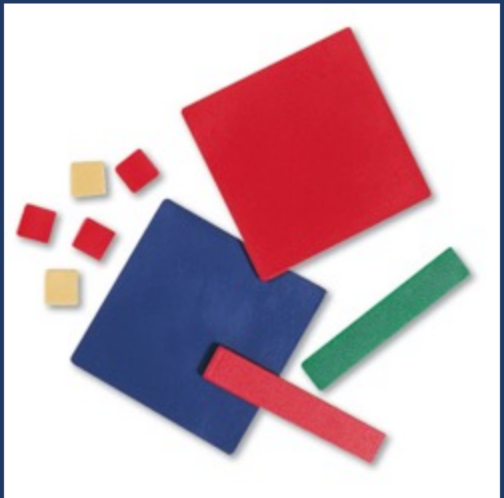
Multiple Representations

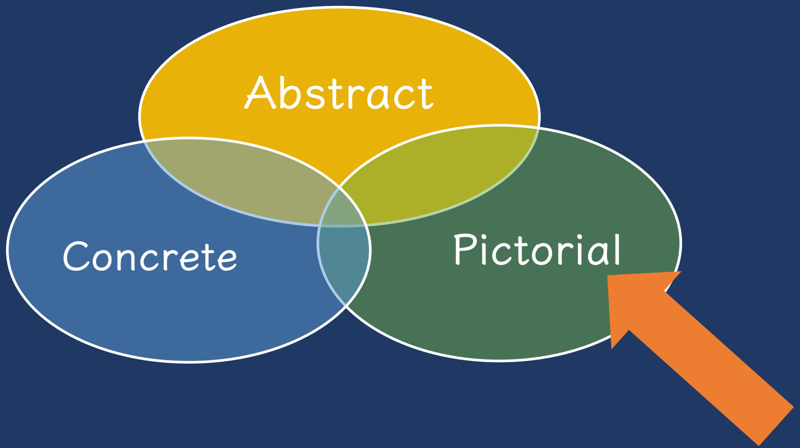


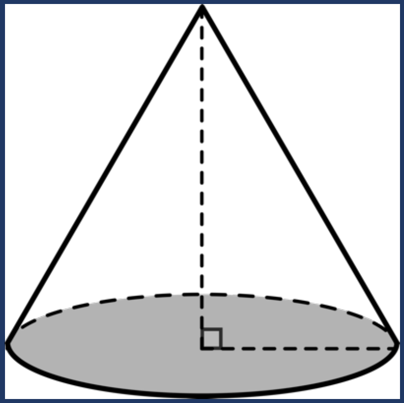


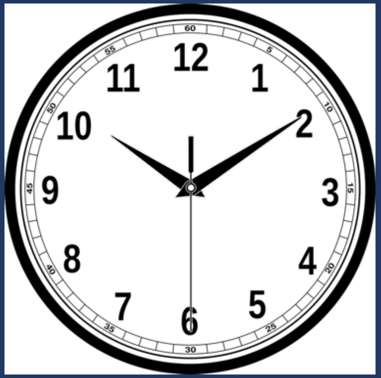
Three-dimensional objects



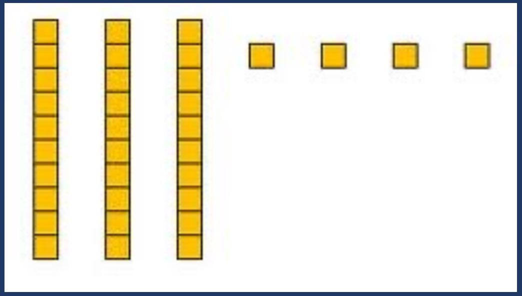


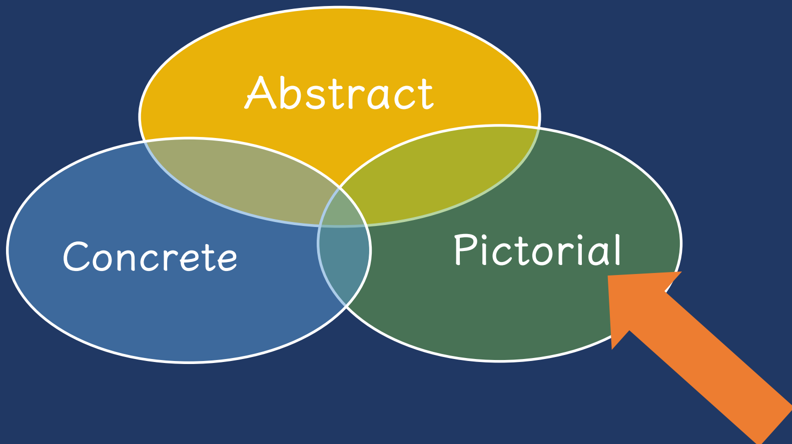




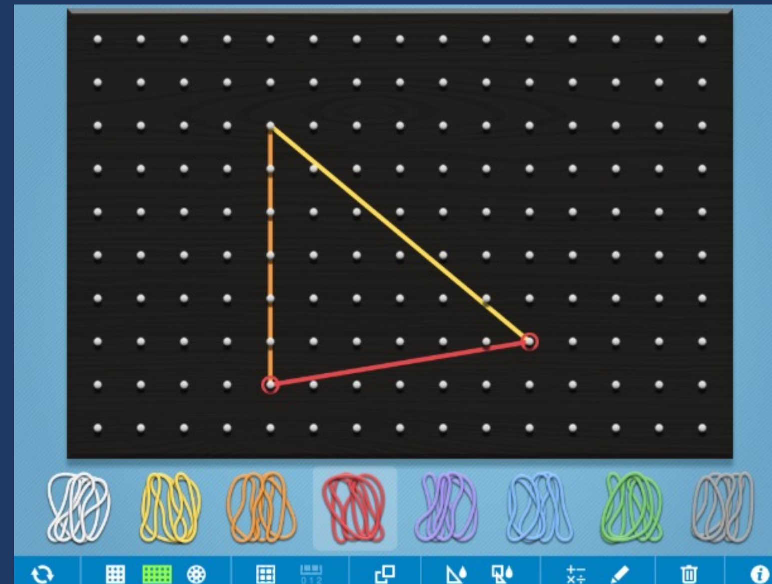


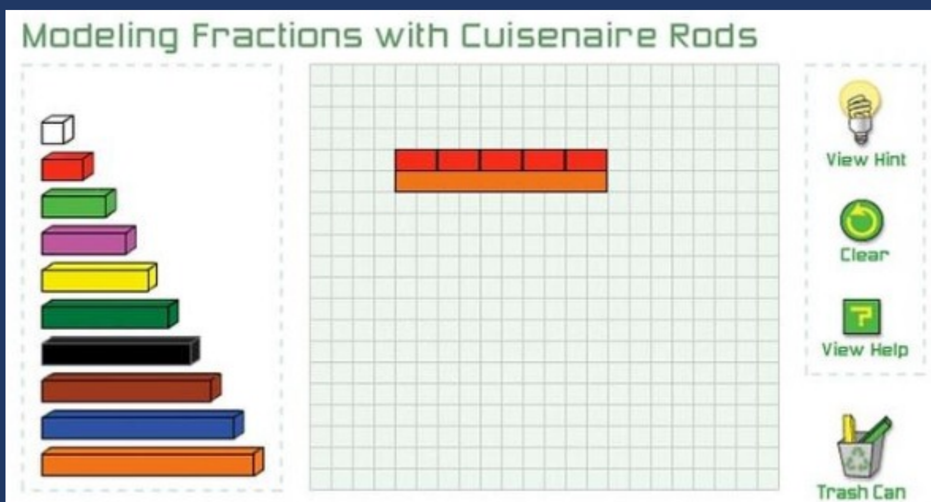
Two-dimensional images

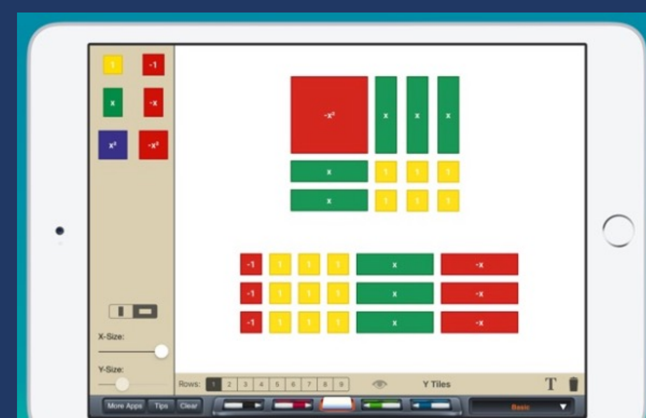


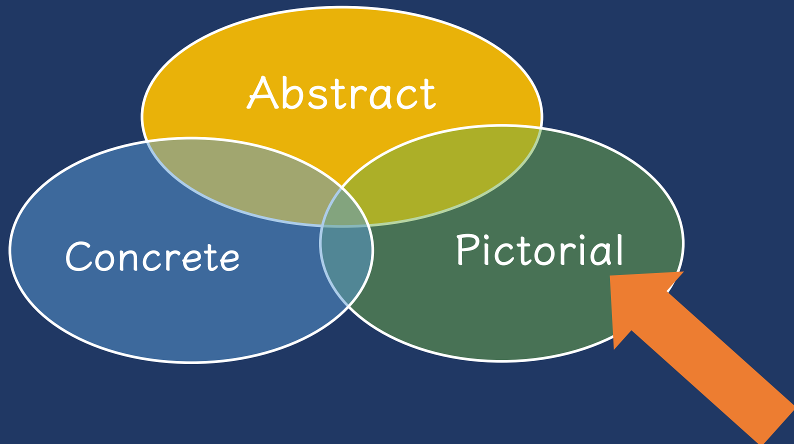


Two-dimensional images

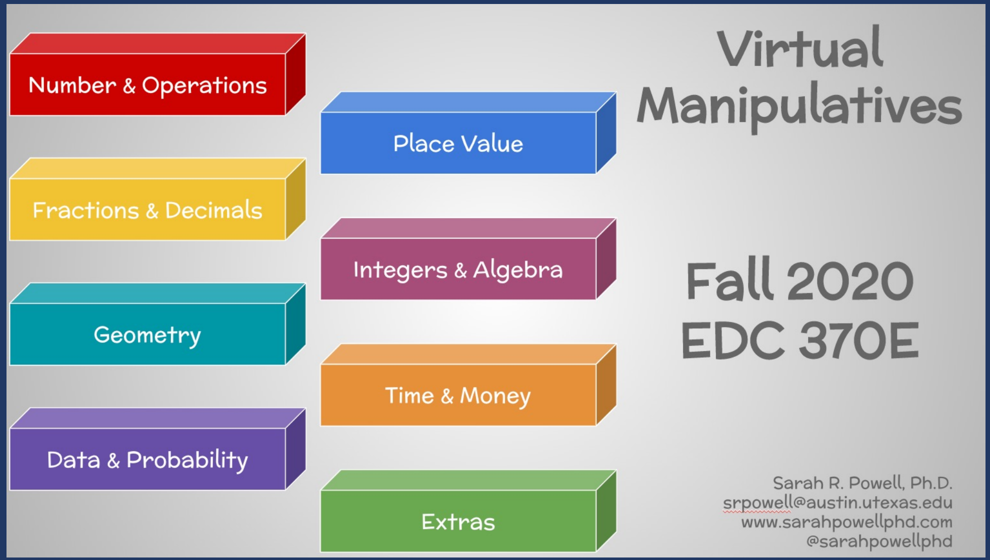


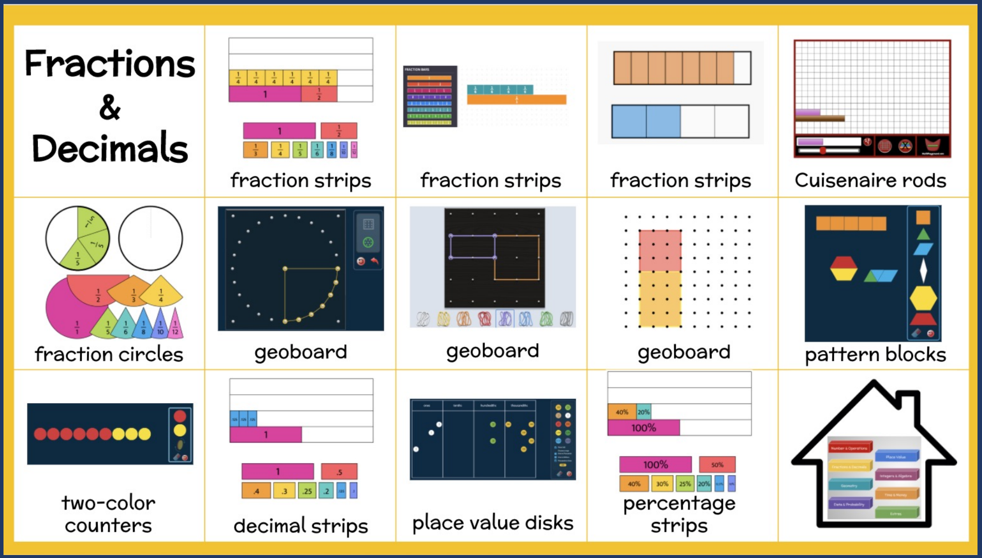


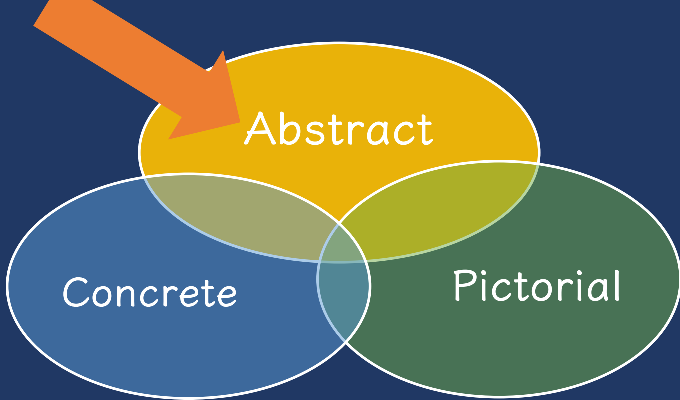




Two-dimensional images







Numerals and symbols and words

$2 + 8 = 10$

34 = 3 tens and 4 ones

$x - 6 = 8$

$$\begin{array}{r} 4{,}179 \\ +\quad 569 \\ \hline \end{array}$$

If you are left handed:

What's one of your favorite hands-on manipulatives?

If you are right handed:

What's one of your favorite virtual manipulatives?

Addition: Total (Part-Part-Whole, Combine)

Count one set, count another set, put sets together, count sum

$$2 + 3 = 5$$

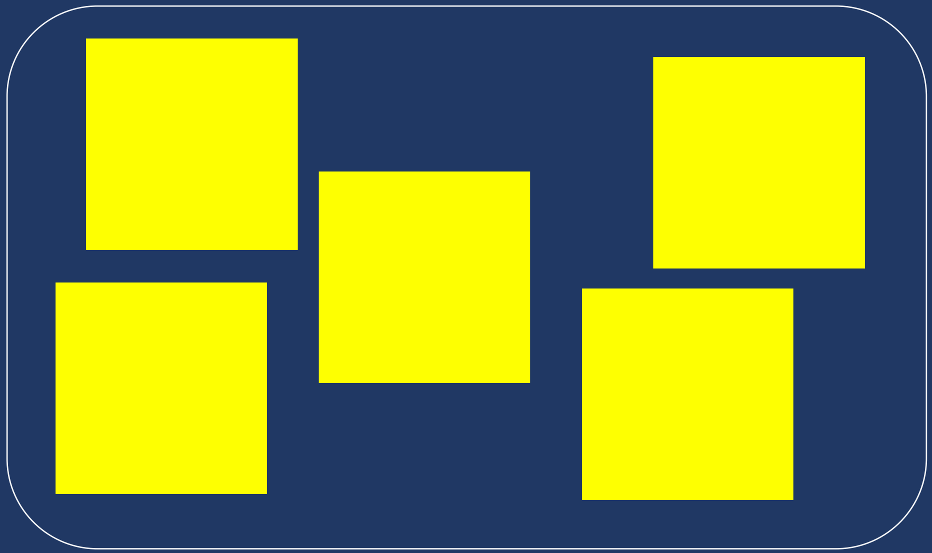
Addition: Join (Change Increase)

Start with a set, add the other set, count sum

$$2 + 3 = 5$$

Subtraction: Separate (Change Decrease)

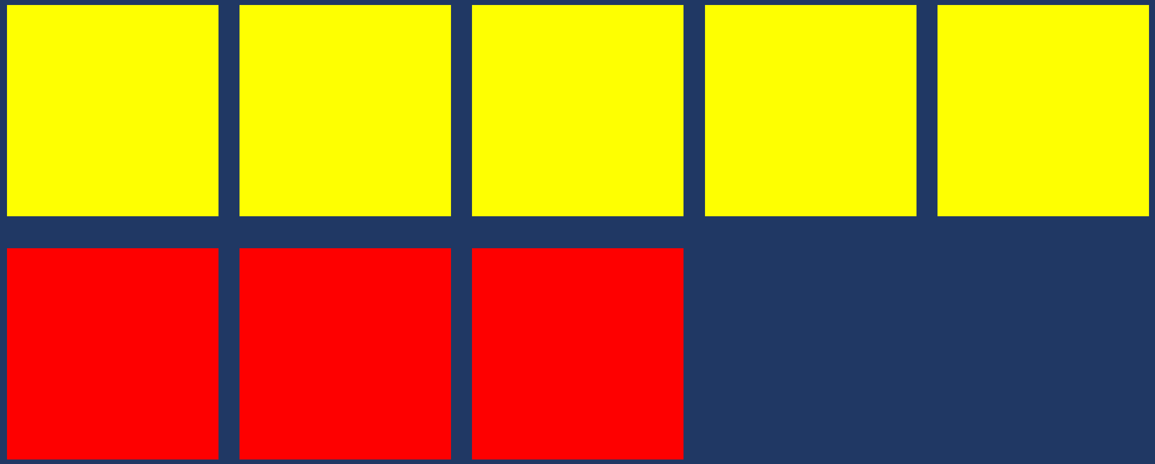
Start with a set, take away from that set, count difference



$$5 - 3 = 2$$

Subtraction: Difference (Compare)

Compare two sets, count difference



$$5 - 3 = 2$$

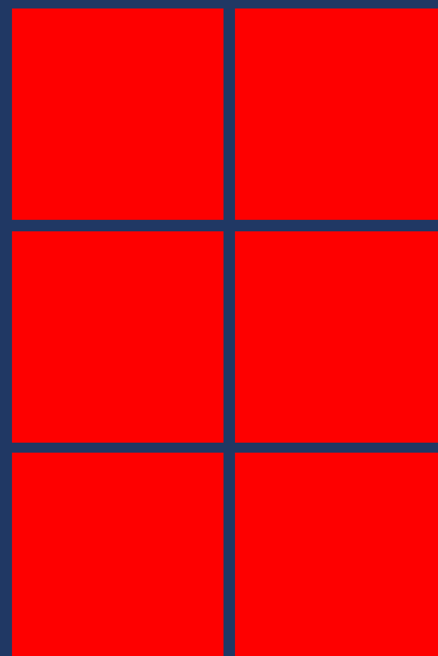
Multiplication: Equal Groups

Show the groups, show the amount for *each* group, count product

$$3 \times 2 = 6$$

Multiplication: Equal Groups

Show the groups, show the amount for *each* group, count product



$$3 \times 2 = 6$$

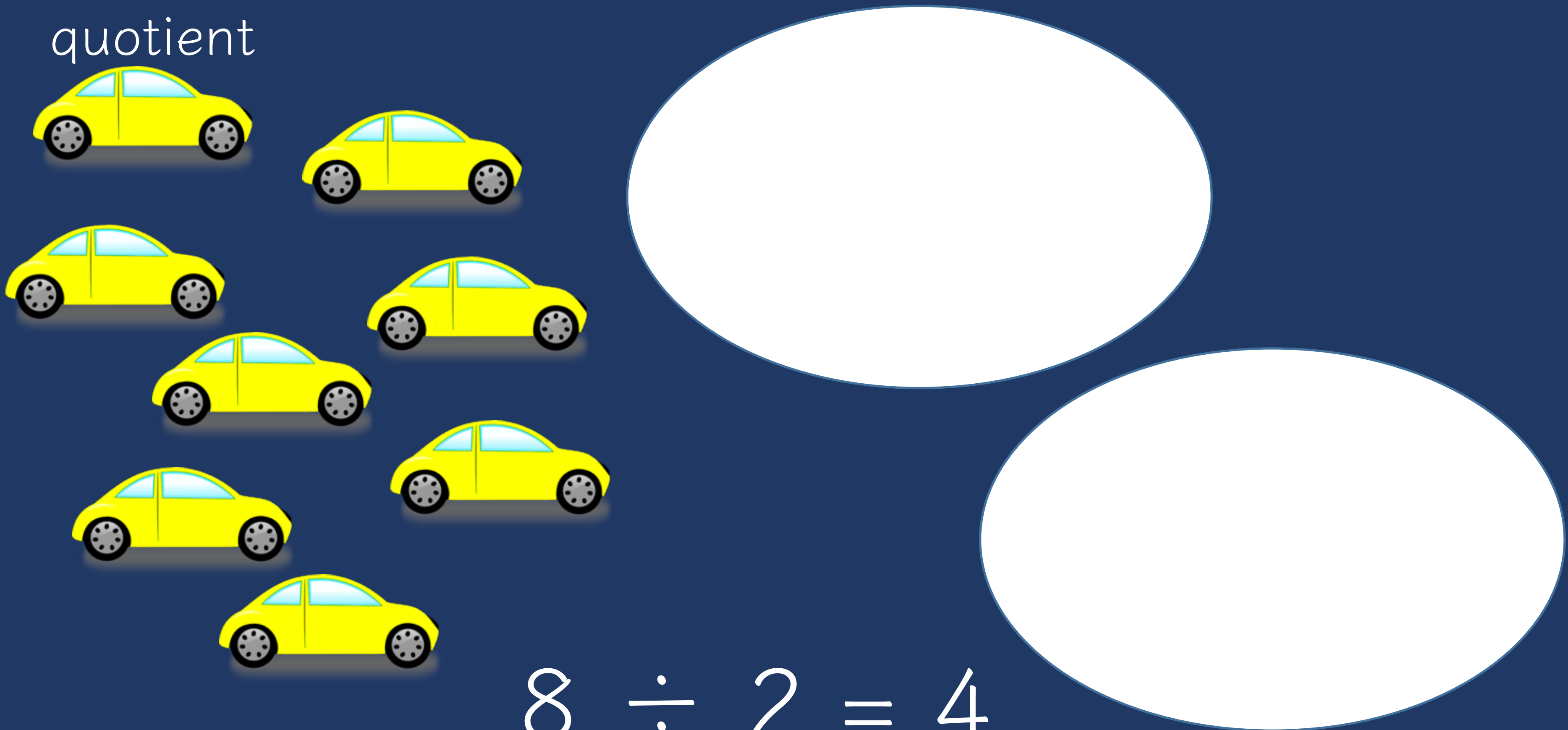
Multiplication: Comparison

Show a set, then multiply the set

$$3 \times 2 = 6$$

Division: Equal Groups (Partitive Division)

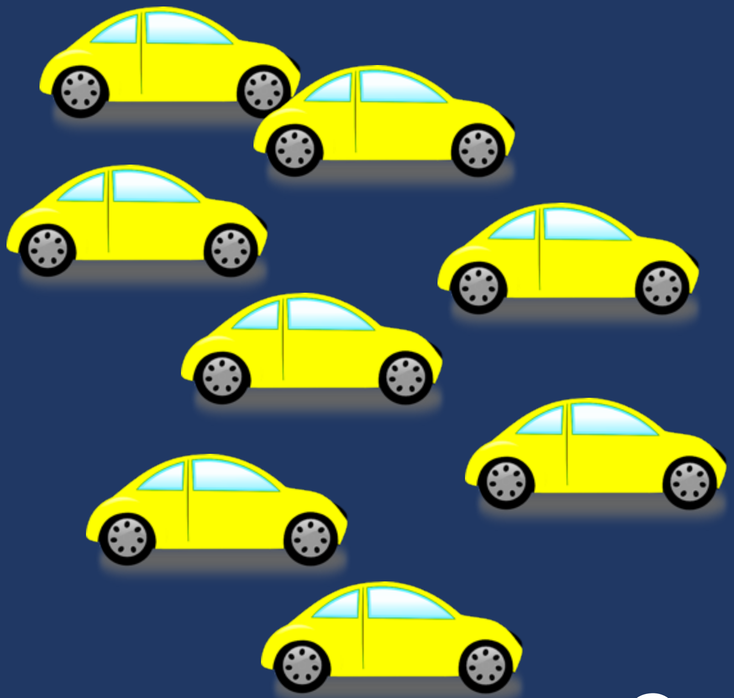
Show the dividend, divide *equally* among divisor, count quotient



$$8 \div 2 = 4$$

Division: Equal Groups (Quotative Division)

Show the dividend, make groups of the divisor, count groups



$$8 \div 2 = 4$$

Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building

Building Fluency

Addition	Subtraction
Multiplication	Division

- Fluency is doing mathematics *easily* and *accurately*.
- Fluency in mathematics makes mathematics *easier*.
- Fluency provides less stress on working memory.
- Fluency helps students build confidence with mathematics.

Instructional Platform

INSTRUCTIONAL DELIVERY

- Explicit instruction
- Precise language
- Multiple representations

INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

Have an attack strategy

UPS✓

UNDERSTAND
Read and explain.

PLAN
How will you solve the problem?

SOLVE
Set up and do the math!

✓CHECK
Does your answer make sense?

Created by: Sarah Powell (srpowell@austin.utexas.edu)

Teach word-problem schemas

Total

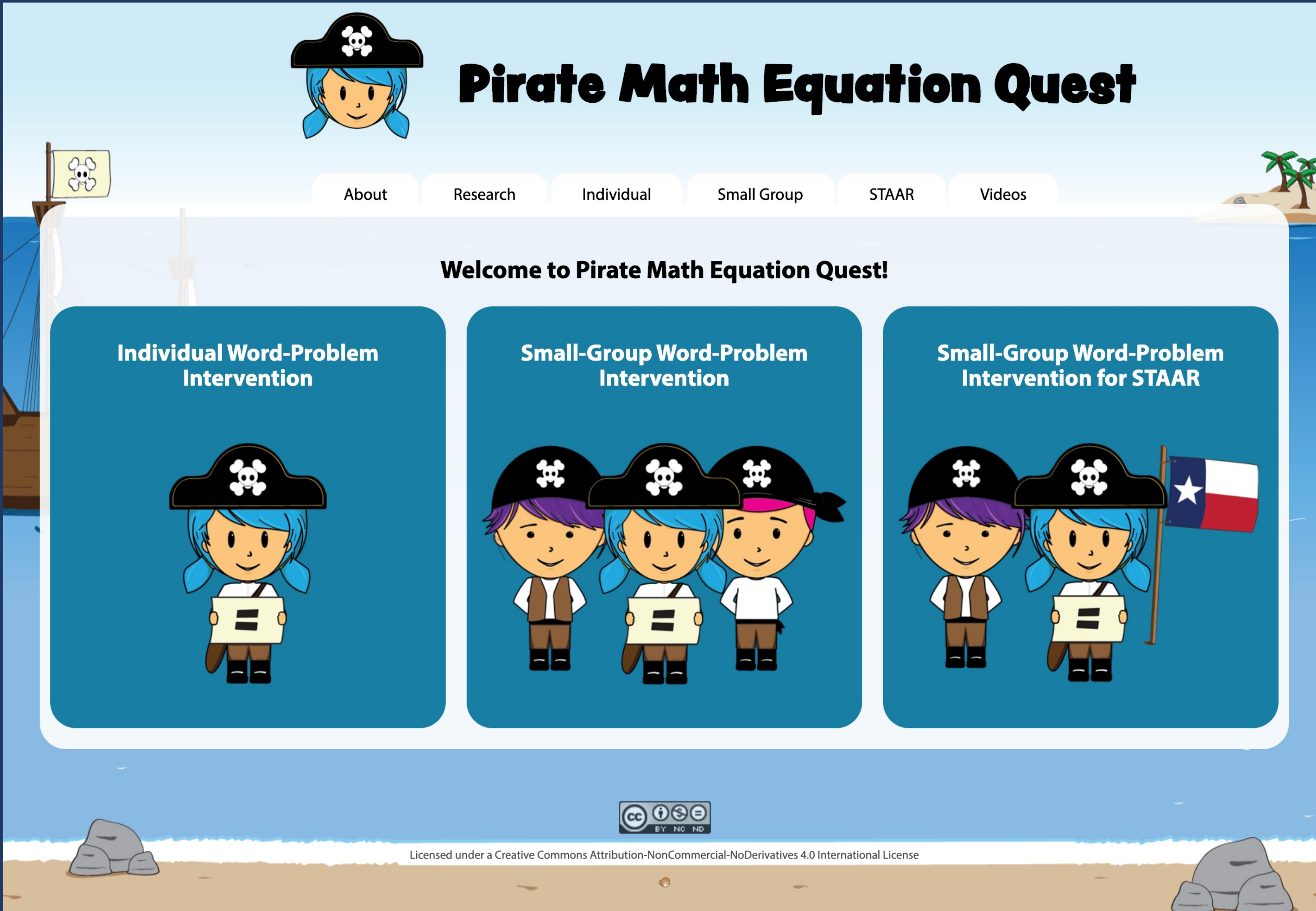
Difference

Change

Equal Groups

Comparison

Ratios/Proportions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Instructional Platform

INSTRUCTIONAL DELIVERY

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- Precise language
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INSTRUCTIONAL STRATEGIES

- Fluency building
- Problem solving instruction

https://intensiveintervention.org/intensive-intervention-math-course

National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

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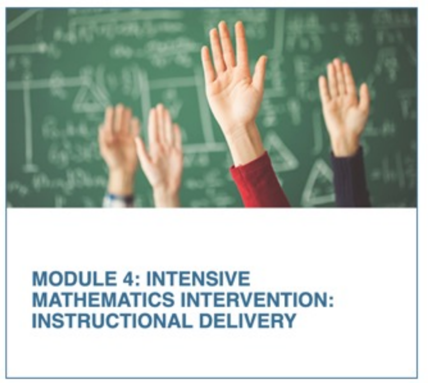
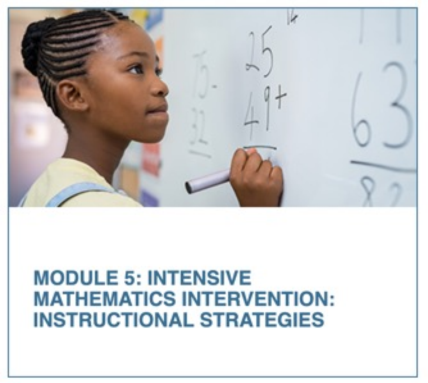
Intensive Intervention in Mathematics Course Content

NCII, through a collaboration with the University of Connecticut, developed a set of course content focused on developing educators' skills in designing and delivering intensive mathematics instruction. This content is designed to support faculty and professional development providers with instructing pre-service and in-service educators who are developing and/or refining their implementation of intensive mathematics intervention.

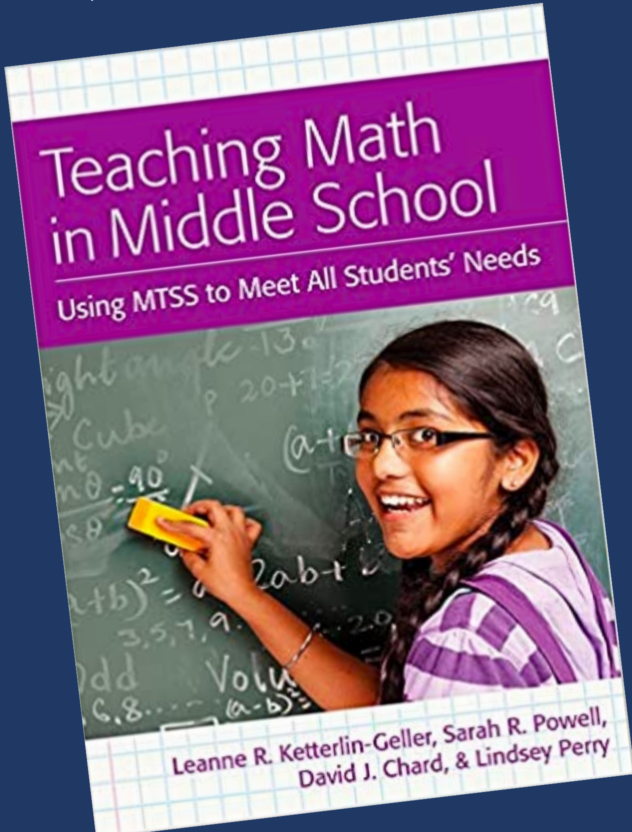
Intensive instruction was recently identified as a high-leverage practice in special education , and DBI is a research based approach to delivering intensive instruction across content areas (NCII, 2013). This course provides learners with an opportunity to extend their understanding of intensive instruction through in-depth exposure to DBI in mathematics, complete with exemplars from actual classroom teachers.

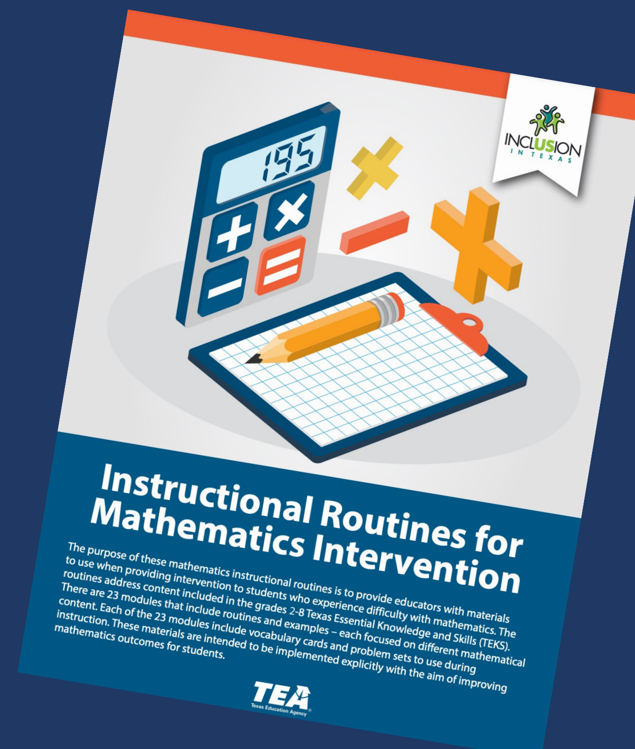
NCII, through a collaboration with the University of Connecticut and the National Center on Leadership in Intensive Intervention and with support from the CEEDAR Center , developed course content focused on enhancing educators' skills in intensive mathematics intervention. The course includes eight modules that can support faculty and professional development providers with instructing pre-service and in-service educators who are learning to implement intensive mathematics intervention through data-based individualization (DBI). The content in this course complements concepts covered in the Features of Explicit Instruction Course and so we suggest that users complete both courses.





https://www.amazon.com/Teaching-Math-Middle-School-Students/dp/1598572741





https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859

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