Effects of Phase Noise on Performance of OFDM Systems Using an ICI Cancellation Scheme

Jinwen Shentu, Kusha Panta, and Jean Armstrong

***Abstract*—This paper investigates the effects of phase noise on the performance of Orthogonal Frequency Division Multiplexing (OFDM) systems using an Intercarrier Interference (ICI) cancellation scheme. In this case, the Common Phase Error (CPE) and ICI caused by phase noise depend on the overall spectrum of each weighted group of subcarriers rather than on the spectrum of each individual subcarrier. This means that the system performance can be improved by filtering the phase noise to fit a particular spectrum. It is shown that the ICI cancellation scheme can significantly improve the Bit Error Rate (BER) performance in the presence of phase noise.**

***Index Terms*—Intercarrier interference (ICI), orthogonal frequency division multiplexing (OFDM), phase noise.**

I. Introduction

PHASE noise causes significant degradation of the performance of Orthogonal Frequency Division Multiplexing (OFDM) systems. The effects of phase noise on OFDM systems have been intensively investigated in the literature [1]–[3]. However, these results cannot be directly used in the case where Intercarrier Interference (ICI) cancellation schemes are used [4], [5]. In this paper, the ICI cancellation technique considered is Polynomial Cancellation Coding (PCC) [6]. ICI cancellation schemes were originally developed to reduce the ICI due to frequency offset rather than the ICI due to phase noise [7]. It will be shown that the ICI cancellation scheme also significantly reduces the ICI caused by phase noise and as a result effectively reduces the impact of phase noise and improves the Bit Error Rate (BER) performance of the system.

In OFDM using PCC (PCC-OFDM), each transmitted data is mapped onto a group of weighted subcarriers [8], [9]. Fig. 1 shows a simplified block diagram of a PCC-OFDM communication system. The high speed Quadrature Amplitude Modulation (QAM) data stream is fed into a serial-to-parallel converter and converted into n lower speed parallel substreams. The ith data block to be transmitted is represented by $\mathbf{d_i} = [d_{0,\,i},\,\ldots,\,d_{n-1,\,i}]'$, where $d_{k,\,i}$, is the kth data value in $\mathbf{d_i}$. n is the number of data values in the block and $[\bullet]'$ represents a column vector. The data block is mapped onto a frequency domain symbol $\mathbf{a_i} = [a_{0,\,i} \cdots a_{N-1,\,i}]'$ with N subcarriers, where $a_{l,\,i}$ is the lth subcarrier in $\mathbf{a_i}$. The Inverse Discrete Fourier Transform (IDFT) of $\mathbf{a_i}$ is given by $\mathbf{b_i} = [b_{0,\,i} \cdots b_{N-1,\,i}]'$, where $b_{k,\,i}$ is the kth sample in $\mathbf{b_i}$. After parallel to serial conversion and filtering, $\mathbf{b_i}$ is up converted to a radio frequency f_c.

Manuscript received November 1, 2002; revised February 3, 2003.

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Digital Object Identifier 10.1109/TBC.2003.810074

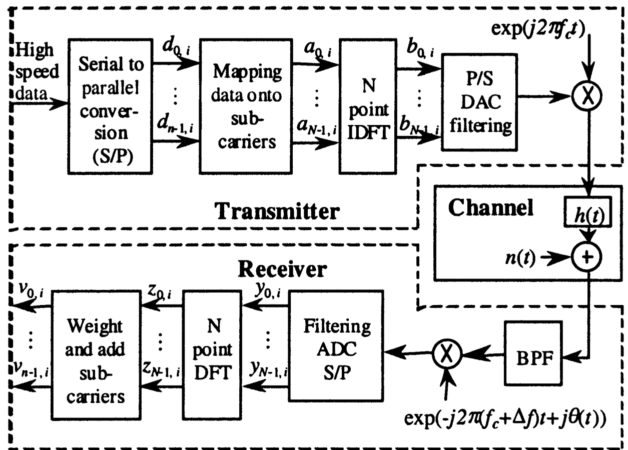

Fig. 1. Simplified block diagram of a PCC-OFDM communication system.

For conventional OFDM, $n = N$ and $a_{k,\,i} = d_{k,\,i}$, there is a simple one-to-one mapping of data values onto the subcarriers. In this paper, the case where each data value to be transmitted is mapped onto pairs of adjacent subcarriers is considered, so that $n = N/2$.

In the channel, the transmitted signal is filtered by the channel response $h(t)$. Additive noise $n(t)$ is injected. At the receiver, the received signal is band-pass filtered and the phase noise $\theta(t)$ is introduced. The band-pass signal is then converted into baseband signal $\mathbf{y_i} = [y_{0,\,i} \cdots y_{N-1,\,i}]'$, where $y_{k,\,i}$ is the kth subcarrier in $\mathbf{y_i}$. The Discrete Fourier Transform (DFT) of $\mathbf{y_i}$ is given by $\mathbf{z_i} = [z_{0,\,i} \cdots z_{N-1,\,i}]'$, where $z_{l,\,i}$ represents the lth demodulated subcarrier in $\mathbf{z_i}$. The demodulated subcarriers are then weighted and added to generate the ith data block of estimates $\mathbf{v_i} = [v_{0,\,i} \cdots v_{n-1,\,i}]'$, where $v_{k,\,i}$ represents the estimate of $d_{k,\,i}$. For a pair of subcarriers $z_{2M,\,i}$, $z_{2M+1,\,i}$, an estimate is calculated using $v_{M,\,i} = (z_{2M,\,i} - z_{2M+1,\,i})/2$.

Phase noise is normally modeled as an ideal phase modulation in the oscillator's signal. Phase noise in an OFDM system results from the instabilities in both oscillators of the transmitter and receiver. Without loss of generality, in this study the local oscillator in the receiver will be considered as the phase noise source.

This paper is organized as follows. In Section II, the expression for the output of the weighting and adding block is derived in terms of the wanted signal, the Common Phase Error (CPE) and the ICI. In Section III, the generation of the phase noise for simulations is described. In Section IV, numerical simulation results are presented and the performance of PCC-OFDM in combating phase noise is evaluated. Conclusions are drawn in Section V.

II. System Analysis

As described above, the IDFT of $\mathbf{a_i}$ is given by $\mathbf{b_i}$. The kth sample of $\mathbf{b_i}$ can be represented by

$$b_{k,i} = \frac{1}{N} \sum_{l=0}^{N-1} a_{l,i} \exp\left(\frac{j2\pi kl}{N}\right), \qquad k = 0, 1, \ldots, N-1 \tag{1}$$

where $a_{l,i}$ is the signal on the lth subcarrier in the ith OFDM symbol. Assuming the channel noise $n(t)$ is Additive White Gaussian Noise (AWGN) with zero mean and finite variance, the channel response is unity and the signal is sampled at the Nyquist rate, then the kth sample in received baseband signal $\mathbf{y_i}$ is given by

$$y_{k,i} = \exp(j\theta(k))\, b_{k,i} + n_{k,i} \tag{2}$$

where $\theta(k)$ is the discrete phase noise. $n_{k,i}$ is the kth sample of $n(t)$ with a phase rotation caused by the phase noise. Thus the mth demodulated subcarrier in $\mathbf{z_i}$ is given by

$$z_{m,i} = \sum_{k=0}^{N-1} y_{k,i} \exp\left(\frac{-j2\pi km}{N}\right) + w_{m,i} \tag{3}$$

where $w_{m,i}$ is the DFT of $n_{k,i}$ with zero mean and finite variance. Substituting (1) and (2) to (3) gives

$$z_{m,i} = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} a_{l,i} \exp\left(\frac{j2\pi k(l-m)}{N}\right) \cdot \exp(j\theta(k)) + w_{m,i}. \tag{4}$$

Using $a_{2M,i} = -a_{2M+1,i} = d_{M,i}$ for PCC-OFDM in (4), the $2M$th demodulated subcarrier can be expressed as

$$z_{2M,i} = \frac{1}{N} \sum_{L=0}^{N/2-1} d_{L,i} \sum_{k=0}^{N-1} \left\{ \exp\left(\frac{j4\pi k(L-M)}{N}\right) - \exp\left(\frac{j2\pi k(2(L-M)+1)}{N}\right) \right\} \cdot \exp(j\theta(k)) + w_{2M,i}. \tag{5}$$

Let

$$\psi_p = \frac{1}{N} \sum_{k=0}^{N-1} \exp\left(\frac{-j2\pi kp}{N}\right) \exp(j\theta(k)) \tag{6}$$

where $p = -(N-1), \ldots, 0, \ldots, (N-1)$, (5) can be written as

$$z_{2M,i} = d_{M,i}(\psi_0 - \psi_{-1}) + \sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i} \cdot \left(\psi_{2(M-L)} - \psi_{2(M-L)-1}\right) + w_{2M,i}. \tag{7}$$

The quantity ψ_p is the DFT of $\exp(j\theta(k))$, evaluated at frequency p/N. The properties of each term in (7) will be analyzed later after we have derived the expression of the relevant output of the weighting and adding block for the subcarrier pair. If $\theta(k)$ is a constant, then $\psi_p = 0$, for $p \neq 0$, there is no ICI. Furthermore, for $\theta(k) = 0$, we obtain $\psi_0 = 1$, the demodulated subcarrier is equal to the original data transmitted plus AWGN.

For small phase noise $\theta(k)$, using the approximation $\exp(j\theta(k)) \approx 1 + j\theta(k)$, (6) can be written as

$$\psi_p \approx \frac{1}{N} \sum_{k=0}^{N-1} \exp\left(\frac{-j2\pi kp}{N}\right)(1 + j\theta(k)). \tag{8}$$

For $p \neq 0$, (8) becomes

$$\psi_p \approx \frac{j}{N} \sum_{k=0}^{N-1} \exp\left(\frac{-j2\pi kp}{N}\right)\theta(k). \tag{9}$$

ψ_p is the DFT of the discrete phase noise or the frequency spectrum of the discrete phase noise.

For $p = 0$,

$$\psi_0 \approx 1 + \frac{j}{N} \sum_{k=0}^{N-1} \theta(k). \tag{10}$$

Similarly, we can get the $(2M+1)$th demodulated subcarrier

$$z_{2M+1,i} = d_{M,i}(\psi_1 - \psi_0) + \sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i} \cdot \left(\psi_{2(M-L)+1} - \psi_{2(M-L)}\right) + w_{2M+1,i}. \tag{11}$$

The Mth data value in the ith data block after PCC weighting and adding is given by

$$v_{M,i} = (z_{2M,i} - z_{2M+1,i})/2. \tag{12}$$

Substituting (7) and (11) to (12) gives

$$\begin{aligned} v_{M,i} = {} & d_{M,i} + \tfrac{1}{2} d_{M,i}[-\psi_{-1} + 2(\psi_0 - 1) - \psi_1] \\ & + \tfrac{1}{2} \sum_{\substack{L=0 \\ L\neq M}}^{N/2-1} d_{L,i}\left(-\psi_{2(M-L)+1} + 2\psi_{2(M-L)} - \psi_{2(M-L)-1}\right) + (w_{2M,i} - w_{2M+1,i})/2 \end{aligned} \tag{13}$$

where the first term at the right hand is the wanted signal, the second term is the CPE, the third term is the ICI and the last is the weighted and added Gaussian noise. In OFDM, the CPE and ICI depend on the individual frequency spectrum of the phase noise [2]. In contrast, the CPE and ICI in PCC-OFDM depend on the combinations of phase noise spectra rather than individual spectra. This makes it possible to reduce the effects of phase noise by using a Phase Locked Loop (PLL) that can fit the overall phase noise frequency spectrum to a particular pattern. For the special case where the individual frequency spectrum ψ_p has a linear relationship with frequency, the ICI caused by the phase noise can be completely cancelled. Substituting ψ_{-1}, ψ_0 and ψ_1 from (9) and (10) to the second term in (13), the CPE in the Mth data value of the ith data block of estimates can be obtained by

$$\begin{aligned} \xi_{M,i} &= \tfrac{1}{2} d_{M,i}[-\psi_{-1} + 2(\psi_0 - 1) - \psi_1] \\ &= \frac{j2d_{M,i}}{N} \sum_{k=0}^{N-1} \sin^2\left(\frac{\pi k}{N}\right)\theta(k) = j\, d_{M,i}\Theta_0 \end{aligned} \tag{14}$$

where Θ_0 is a constant, $\Theta_0 = (2/N)\sum_{k=0}^{N-1} \sin^2(\pi k/N)\,\theta(k)$. This result indicates that as in OFDM all PCC-OFDM subcarriers experience a common phase rotation. This rotation can be detected and therefore compensated using techniques provided in the literature [10]. One simple way to do this is to insert pilot tones in a symbol and estimate the rotation angle. Once the common phase rotation is corrected, the only remaining effect of the phase noise is the ICI.

Similarly, the ICI in the Mth data value in the ith data block of estimates in (13) is given by

$$\begin{aligned}\zeta_{M,i} &= \frac{1}{2}\sum_{\substack{L=0\\L\neq M}}^{N/2-1} d_{L,i}\left(-\psi_{2(M-L)+1}+2\psi_{2(M-L)}-\psi_{2(M-L)-1}\right)\\ &= \frac{2j}{N}\sum_{\substack{L=0\\L\neq M}}^{N/2-1} d_{L,i}\sum_{k=0}^{N-1}\sin^2\left(\frac{\pi k}{N}\right)\\ &\quad\cdot\exp\left(\frac{j4\pi k(L-M)}{N}\right)\theta(k). \qquad (15)\end{aligned}$$

Let

$$\Theta_{L-M} = \frac{2}{N}\sum_{k=0}^{N-1}\sin^2\left(\frac{\pi k}{N}\right)\exp\left(\frac{j4\pi k(L-M)}{N}\right)\theta(k). \qquad (16)$$

Equation (15) can be written as

$$\zeta_{M,i} = j\sum_{\substack{L=0\\L\neq M}}^{N/2-1} d_{L,i}\Theta_{L-M}. \qquad (17)$$

This term is the contribution of the phase noise to all other subcarriers in a symbol. This result shows that the phase noise introduces ICI and causes loss of orthogonality. Equations (14) and (16) show that the effects of the phase noise depend on the shape of the frequency spectrum and bandwidth of the phase noise.

III. Oscillator Phase Noise

Phase noise can be characterized in the frequency domain by the power spectral density (PSD), $S_\theta(f)$, where f stands for frequency. An approximation for the PSD of a free-running oscillator can be found in [10]. In the PSD, roll-off factors of $1/f$, $1/f^2$ and $1/f^3$ can be observed. However, a PLL is always used in practice, so that the PSD depends also on the characteristics of the PLL. In our simulation studies, a PLL is used so that the high order roll-off factors in the PSD will be removed. Phase noise $\theta(t)$ is a zero-mean and wide-sense stationary process with a narrow-band PSD and a finite variance σ_θ^2. A phase noise sequence can be obtained by passing a white Gaussian noise sequence through a low-pass filter [2]. A simplified method to get this is to define an equivalent low-pass process with the PSD given by [11]

$$S_\theta(f) = 10^{-c} + \begin{cases} 10^{-a} & |f| \le f_l \\ 10^{-(f-f_l)(b/(f_h-f_l))-a} & f > f_l \\ 10^{(f+f_l)(b/(f_h-f_l))-a} & f < -f_l \end{cases} \qquad (18)$$

where f_l is the PLL bandwidth and f_h is the phase noise bandwidth with $f_h > f_l$. Parameter c determines a white noise floor and a determines the noise level in the frequency ranges from the center frequency to $\pm f_l$. Parameter b determines the noise roll-off rate from the noise floor at f_l to the noise level at f_h. Fig. 2 shows a visualized version of the phase noise PSD given by (18). The variance of the phase noise is given by

$$\sigma_\theta^2 = \int_{-\infty}^{\infty} S_\theta(f)\,df. \qquad (19)$$

A typical set of parameters based on a real 5.2 GHz frequency synthesizer is $a = 8$, $b = 2$, $c = 12$, $f_l = 10$ kHz, $f_h = 100$ kHz [11]. In later simulations, the parameters a, b, c and $(f_h - f_l)$ will be set to these values while f_h varies to meet the requirements of phase noise power.

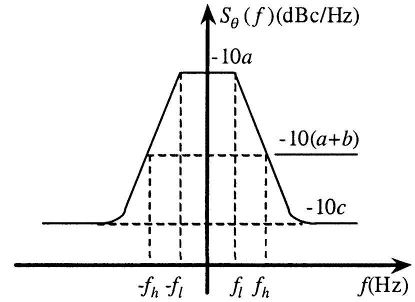

Fig. 2. A PSD of low pass filtered phase noise.

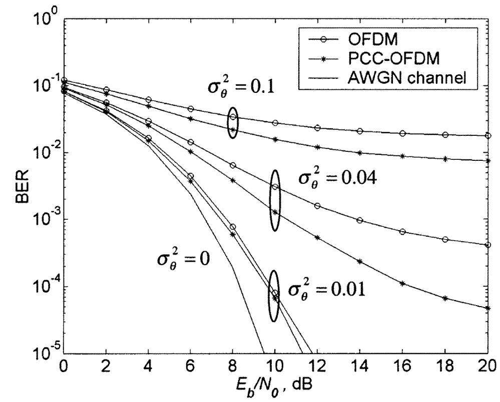

Fig. 3. BER performance as a function of E_b/N_0 for PCC-OFDM and OFDM for phase noise $\sigma_\theta^2 = 0$, 0.01, 0.04 and 0.1 rad^2, $N = 128$.

IV. Simulation Results

To demonstrate the effects of phase noise on the performance of PCC-OFDM systems, computer simulations were performed. In all simulations, the number of symbols simulated was 100 000 and no CPE correction was used. Fig. 3 shows the BER performance as a function of bit energy to noise ratio E_b/N_0 for PCC-OFDM and OFDM, with phase noise of variance $\sigma_\theta^2 = 0$, 0.01, 0.04 and 0.1 rad^2, and $N = 128$. The modulation scheme was 4 QAM and an AWGN channel was assumed. It is evident that PCC-OFDM is much less sensitive to phase noise than OFDM. For $\sigma_\theta^2 = 0.04$ rad^2 and a BER of 10^{-3}, PCC-OFDM requires about 3 dB lower on E_b/N_0 than OFDM. As E_b/N_0 increases, the difference between PCC-OFDM and OFDM increases because phase noise becomes dominant. The BER performance for PCC-OFDM and OFDM in an ICI free AWGN channel is also given by the AWGN curve. We note that PCC-OFDM and OFDM have the same performance in this case. The simulation results were obtained using the same number of subcarriers for PCC-OFDM and OFDM. In this case, the spectrum of each subchannel in PCC-OFDM is larger because of the mapping of data onto pairs of subcarriers. However, this will not contribute to the improvement in system BER performance because for a given variance of phase noise of an OFDM system using a PLL, the system degradation is independent of the number of subcarriers [12]. In other words, the performance improvement is due to the ICI cancellation property of PCC [4].

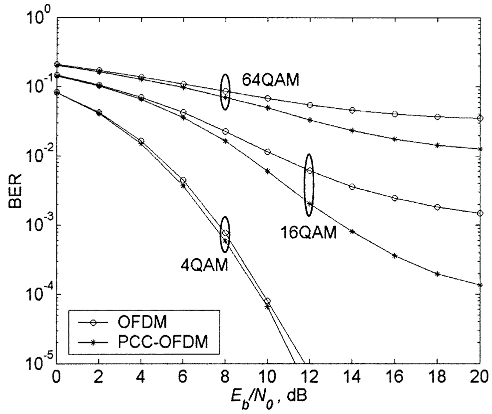

Fig. 4. BER performance as a function of E_b/N_0 for PCC-OFDM and OFDM for 4 QAM, 16 QAM, and 64 QAM where $\sigma_\theta^2 = 0.01$, $N = 128$.

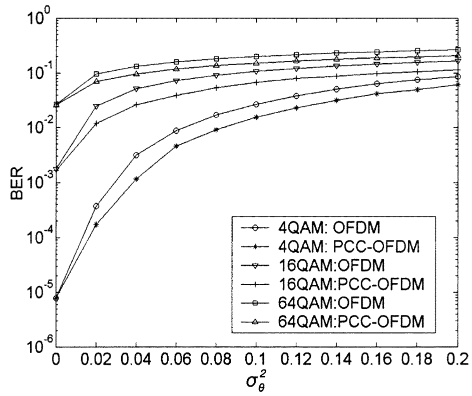

Fig. 5. BER performance as a function of phase noise for PCC-OFDM and OFDM, $E_b/N_0 = 10$ dB and $N = 128$.

Fig. 4 shows the BER performance as a function of phase noise power for PCC-OFDM and OFDM for 4 QAM, 16 QAM and 64 QAM, where $\sigma_\theta^2 = 0.01$ rad^2, and $N = 128$. It is shown that a higher order QAM scheme is more sensitive to phase noise than a lower order QAM scheme. The performance improvement by PCC-OFDM follows the same trend as shown in Fig. 3. Higher order QAM schemes get larger improvement. For 16 QAM and a BER of 10^{-3}, the required E_b/N_0 value for PCC-OFDM is about 6 dB lower than for OFDM. At the given phase noise level, 64 QAM is unable to conduct a reliable data transmission even if E_b/N_0 is high. However, the performance improvement by PCC-OFDM is significant.

Fig. 5 shows the BER performance as a function of phase noise for PCC-OFDM and OFDM where $E_b/N_0 = 10$ dB and $N = 128$. It is shown that the performance improvement by PCC-OFDM becomes more significant as the phase noise power increases. For 4 QAM and a BER of 10^{-4}, PCC-OFDM can tolerate a phase noise level of 0.03 rad^2 while OFDM can only support 0.02 rad^2.

V. Conclusions

The effects of phase noise on the performance of OFDM systems using an ICI cancellation scheme have been analytically evaluated. The expressions for CPE and ICI caused by phase noise have been derived. It is shown that both CPE and ICI depend on the overall spectrum of each weighted group of subcarriers rather than on the spectrum of each individual subcarrier. Since the characteristics of phase noise can be modified by a PLL, the system performance can be improved by carefully choosing a proper PSD of phase noise. It is shown that the ICI cancellation scheme can significantly improve the BER performance in the presence of phase noise.

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