ARML Problem Set 9
Due date: Sunday, March 6, 2016

Complete the following problems individually. These are within the difficulty of individual and team problems. You do not need a calculator. Email your solution to *xiangtaoliu@gmail.com*. You may hand write and scan them. However, LaTeX is preferred. Please ensure your solution is legible. The solution to each problem should begin with you stating the final answer and followed by the justification of how you found your answer. Remember to prove anything you do not think is obvious. Each question is worth 10 points. Problem set solutions will be posted the day after the due date. I will try to have everything graded by the due date of the following problem set.

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1. Given that the equation
$$\left(m^2-1\right)x^2-2(m+2)x+1=0$$
has at least one real root. Determine the possible values of m.

2. The sum of three consecutive prime number is 173. Determine the largest of these 3 numbers.

3. Let $f(x)=x^3-49x^2+623x-2015$ and $g(x)=f(x+5)$. Determine the sum and product of the roots of $g(x)$.

4. Determine the largest integer k such that k^2 divides
$$\underbrace{33\ldots33}_{2n}-\underbrace{66\ldots66}_{n}$$

5. Let $f(x)$ be a polynomial such that when divided by $(x-1),(x-2),(x-3)$, the remainders are $1,2,3$, respectively. Find the remainder of $f(x)$ when it is divided by $(x-1)(x-2)(x-3)$.

6. Let ABC be a triangle with $A=70°$, D is on AC, and the angle bisector of A intersect BD at H such that $AH:HE=3:1$ and $BH:HD=5:3$. Determine the measure of angle C.

7. Consider a sphere with n great circles such that no three of the great circles intersect at a point. Determine the number of regions that these great circles partition the sphere.

8. Determine all positive integer n such that 2^n is divisible by $n!$.

9. In rectangle $WASH$, point E lies on SH such that $\angle AWS=\angle HWE$. Point D lies on WS such that $ED\perp WS$. Given that the area of $WASH$ is 100 and the area of SED is 32. Determine the value of SWE.

10. Let $a_1=a_2=a_3=1$. For $n>3$, let a_n be the number of real numbers x such that
$$x^4-2a_{n-1}x^2+a_{n-2}a_{n-3}=0$$
Determine the value of $\sum_{i=1}^{1000}a_i$.