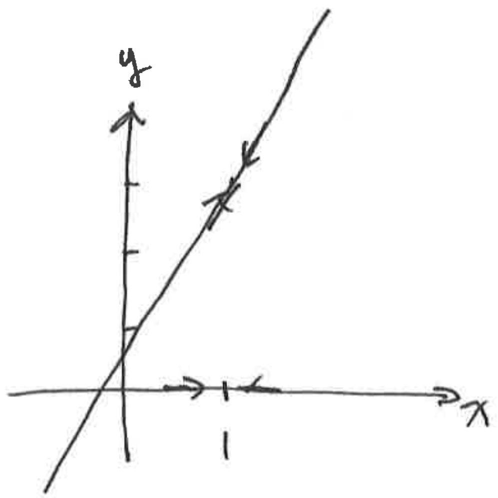
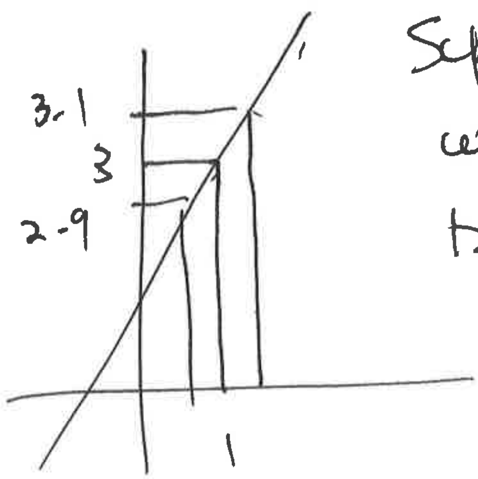
Math 1496 — Calc I

$\lim_{x \to 1} 2x+1 = 3$



As x gets closer to 1
y gets closer to 3

How close? As close as we want



Suppose I wish to be within .1 of 3. How close to $x=1$ must I be?

for $y = 3.1$ so $2x+1 = 3.1$ $\quad 2x = 2.1$

$x = 1.05$

$y = 2.9$ so $2x+1 = 2.9$ $\quad 2x = 1.9$

$x = .95$

so if $.95 \le x \le 1.05$ then $2.9 \le y \le 3.1$

or $-.05 \le x-1 \le .05$ $\quad -.1 \le y-3 \le .1$

a $|x-1| \le .05$ $\quad |y-3| \le .1$

suppose we wish to be with .002 of 3

we repeat

$2x+1 = 3.002$ $\quad 2x = 2.002$ $\quad x = 1.001$

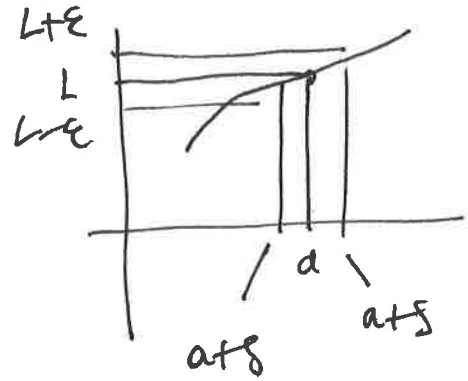
$2x+1 = 2.998$ $\quad 2x = 1.998$ $\quad x = .999$

so $.999 < x < 1.001$ then $2.998 < y < 3.002$

or $|x-1| < .001$ then $|y-3| < .002$

suppose $|y-3| < \varepsilon$ then $|x-1| < \varepsilon/2$

Formal Definition of a Limits



For every $\varepsilon > 0$ there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$$\lim_{x \to 0} f(x) = L$$

Previous Ex

$$\lim_{x \to 1} 2x + 1 = 3$$

For every $\varepsilon > 0$ there exists a $\delta > 0$

such that $|2x+1-3| < \varepsilon$ when $|x-1| < \delta$

Aside

$|2x+1-3| < \varepsilon$

$|2x-2| < \varepsilon$

$2|x-1| < \varepsilon$

$|x-1| < \varepsilon/2$

choose

$\delta = \varepsilon/2$

$|x-1| < \varepsilon/2$

$2|x-1| < \varepsilon$

$|2x-2| < \varepsilon$

$|2x+1-3| < \varepsilon$

$|f - L| < \varepsilon$ ✓

Ex 2

$$\lim_{x \to -1} 3x+4 = 1$$

For every $\varepsilon \geq 0$ there exists $\delta > 0$ such that $|3x+4-1| < \varepsilon$ whenever $|x+1| < \delta$

Aside

$|3x+4-1| < \varepsilon$

$|3x+3| < \varepsilon$

$3|x+1| < \varepsilon$

$|x+1| < \varepsilon/3$

choose $\delta = \varepsilon/3$

If $|x+1| < \varepsilon/3$

$3|x+1| < \varepsilon$

$|3x+3| < \varepsilon$

$|3x+4-1| < \varepsilon$

$|f-L| < \varepsilon$ ✓

Ex 3

$$\lim_{x \to 4} -2x+5 = -3$$

For every $\varepsilon \geq 0$ there exists a $\delta \geq 0$

such that

$|-2x+5+3| < \varepsilon$ whenever $|x-4| < \delta$

Aside

$|-2x+5+3| < \varepsilon$

$|-2x+8| < \varepsilon$

$|-2(x-4)| < \varepsilon$

$|x-4| < \varepsilon/2$

if $|x-4| < \varepsilon/2$

$2|x-4| < \varepsilon$

$|2x-8| < \varepsilon$

$|-2x+5+3| < \varepsilon$

$|f-L| < \varepsilon$ ∎

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