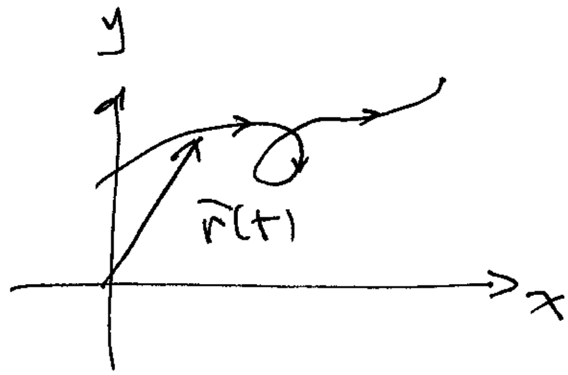
Math 2471 — Calc 3

Vector Functions

position vector $\quad \bar{r}(t) = \langle f(t), g(t) \rangle$



Tangent Vector

$$\bar{T} = \frac{\bar{r}'}{\|\bar{r}'\|}$$

Normal Vector

$$\bar{N} = \frac{\bar{T}'}{\|\bar{T}'\|}$$

Binormal Vector

$$\bar{B} = \bar{T} \times \bar{N}$$

An application we saw in Calc I was that if $s = S(t)$ is position, then velocity is

$$\vec{v} = \frac{ds}{dt}$$

and acceleration

$$a = \frac{dv}{dt}$$

so we can define the velocity vector $\bar{v}$ and acceleration vector $\bar{a}$ as

$$\bar{v} = \frac{d\bar{r}}{dt}, \qquad \bar{a} = \frac{d\bar{v}}{dt}$$

Ex $\bar{r} = \langle 2t, -16t^2 + t + 1 \rangle$

parabola

if $x = 2t$, $y = -16t^2 + t + 1$

$$= -16\left(\frac{x}{2}\right)^2 + \frac{x}{2} + 1 = -4x^2 + x + 1$$

$\bar{v} = \frac{d\bar{r}}{dt} = \langle 2, -32t+1 \rangle$ note $\|\bar{v}\| =$ speed.

$\bar{a} = \frac{d\bar{v}}{dt} = \langle 0, -32 \rangle \leftarrow$ so acceleration vector is straight down

another thing we did in calc I was integrate so we can do so with vector functions

Ex Suppose $\bar{a} = \langle 0, t, t \rangle$

with $\bar{v}(1) = \langle 0, 5, 0 \rangle$ $\bar{r}(1) = \langle 0, 9, 0 \rangle$

so we integrate

$$\bar{v} = \int \bar{a}(t)\,dt = \left\langle \int 0\,dt, \int t\,dt, \int t\,dt \right\rangle$$

$$= \left\langle c_1, \frac{t^2}{2} + c_2, \frac{t^2}{2} + c_3 \right\rangle$$

note: 3 constants of integration.

so $\bar{v}(1) = \langle c_1, c_2 + \frac{1}{2}, c_3 + \frac{1}{2} \rangle = \langle 0, 5, 0 \rangle$

$\Rightarrow \quad c_1 = 0 \quad c_2 = \frac{9}{2} \quad c_3 = -1/2$

so

$\bar{v}(t) = \langle 0, \frac{1}{2}t^2 + \frac{9}{2}, \frac{1}{2}t^2 - \frac{1}{2} \rangle$

$\bar{r}(t) = \int \bar{v}\,dt$

$= \langle c_4, \frac{t^3}{6} + \frac{9}{2}t + c_5, \frac{t^3}{6} - \frac{t}{2} + c_6 \rangle$

$\bar{r}(1) = \langle c_4, \frac{1}{6} + \frac{9}{2} + c_5, \frac{1}{6} - \frac{1}{2} + c_6 \rangle = \langle 0, 0, 0 \rangle$

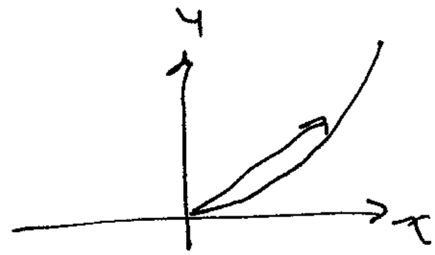
$\Rightarrow \quad c_4 = 0, \quad c_5 = -\frac{14}{3} \quad c_6 = \frac{1}{3}$

so

$\bar{r}(t) = \langle 0, \frac{t^3}{6} + \frac{9}{2}t - \frac{14}{3}, \frac{t^3}{6} - \frac{t}{2} + \frac{1}{3} \rangle$

Consider

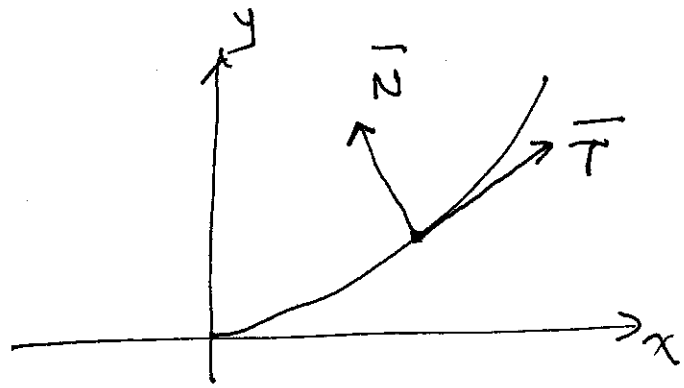
$$\bar{r}(t) = \langle t, \tfrac{1}{2}t^2 \rangle$$



$$x = t,\ y = \tfrac{1}{2}t^2 \Rightarrow y = \tfrac{1}{2}x^2$$

$$\bar{T} = \frac{\bar{r}'}{\|\bar{r}'\|} = \frac{\langle 1, t \rangle}{\sqrt{1+t^2}}$$

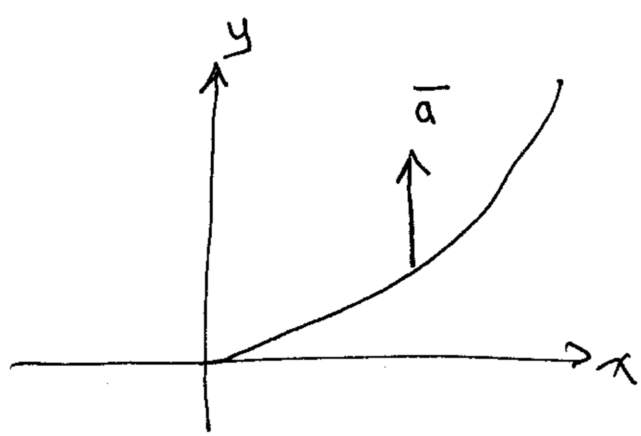
$$\bar{N} = \frac{\bar{T}'}{\|\bar{T}'\|} = \frac{\langle -t, 1 \rangle}{\sqrt{1+t^2}}$$



Also

$$\bar{v} = \bar{r}' = \langle 1, t \rangle$$

$$\bar{a} = \bar{v}' = \langle 0, 1 \rangle$$



There must be a connection between

$\bar{a}$, $\bar{T}$ & $\bar{N}$

HW pg 826-827

\# 7, 9, 11, 31, 33, 35

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