Least Squares Wavelet Analysis

Ebrahim Ghaderpour and **Spiros D. Pagiatakis**

Department of Earth and Space Science and Engineering

Lassonde School of Engineering, York University, Toronto

Montreal

May 4th 2015

Outline

- Time series and Signals

- Least Squares Spectral Analysis (LSSA)

- Least Squares Wavelet Analysis (LSWA)

- Conclusions

Time series and Signals

- **Time series?**

- **Sequence of measurements over a time interval**

Time series and Signals

- **Time series?**

- **Sequence of measurements over a time interval**

- **Signals?**

- **Constituents of interest in a time series**

Time series and Signals

- **Unequally spaced**

Time series and Signals

- **Unequally spaced**

- **Systematic noise: trends, datum shifts**

Time series and Signals

- **Unequally spaced**

- **Systematic noise: trends, datum shifts**

- **Variable frequency**

Time series and Signals

- **Unequally spaced**

- **Systematic noise: trends, datum shifts**

- **Variable frequency**

- **Variable amplitude**

Time series and Signals

- **Unequally spaced**

- **Systematic noise: trends, datum shifts**

- **Variable frequency**

- **Variable amplitude**

- **Non-stationary**

An example of an unequally spaced and non-stationary time series
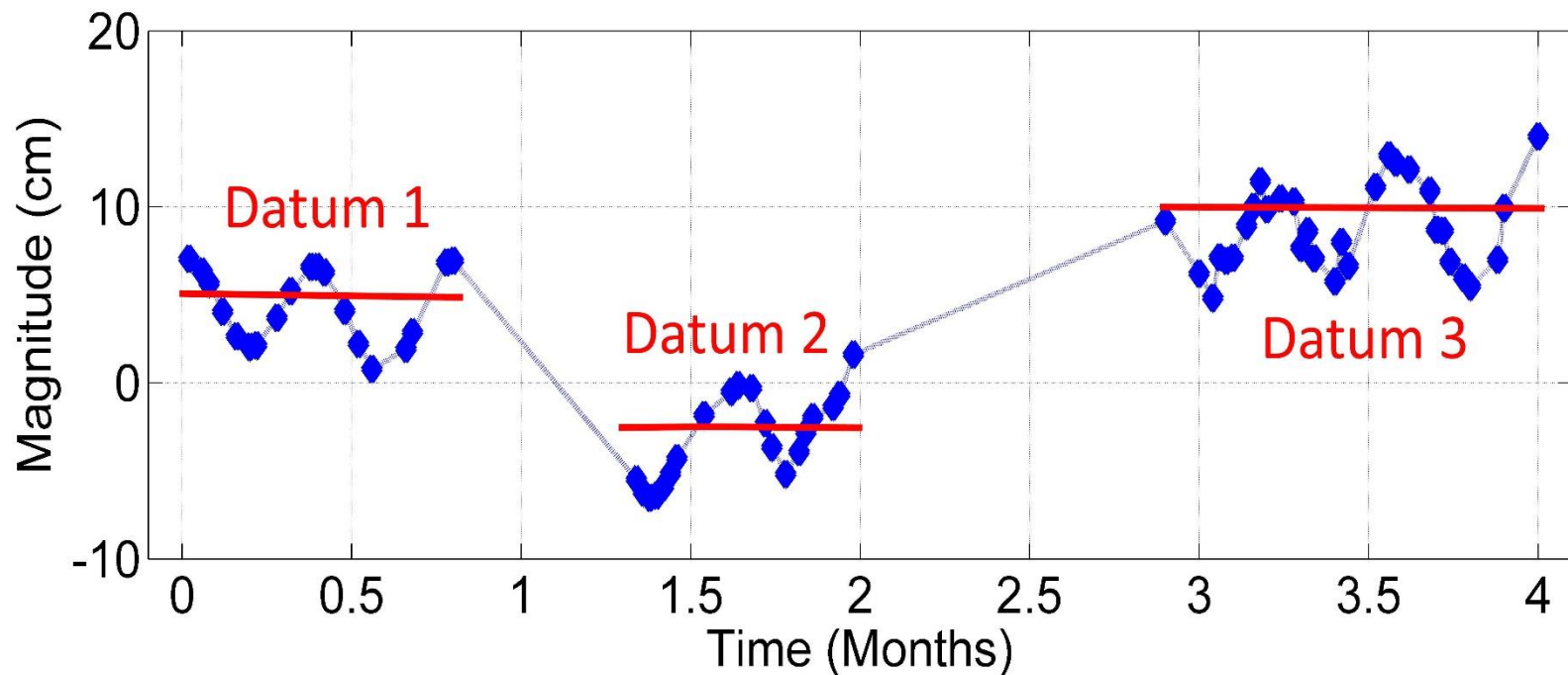
How to analyze these types of time series?

- **Fast Fourier Transform (FFT)**

- **FFT : stationary and equally spaced!**

How to analyze these types of time series?

- **Fast Fourier Transform (FFT)**

- **FFT : stationary and equally spaced!**

- **Least Squares Spectral Analysis (LSSA)**

- **How does the LSSA work?**

Least Squares Spectral Analysis (LSSA)

- **Fits sinusoids to the entire time series**

Least Squares Spectral Analysis (LSSA)

- **Fits sinusoids to the entire time series**

- **Equally spaced & unequally spaced!**

Least Squares Spectral Analysis (LSSA)

- **Fits sinusoids to the entire time series**

- **Equally spaced & unequally spaced!**

- **Trends, datum shifts, …**

Least Squares Spectral Analysis (LSSA)

- **Fits sinusoids to the entire time series**

- **Equally spaced & unequally spaced!**

- **Trends, datum shifts, …**

- **Correlation among sinusoids**

Example: $f(t) = 1.5 \sin 5t + 2 \sin 10t$
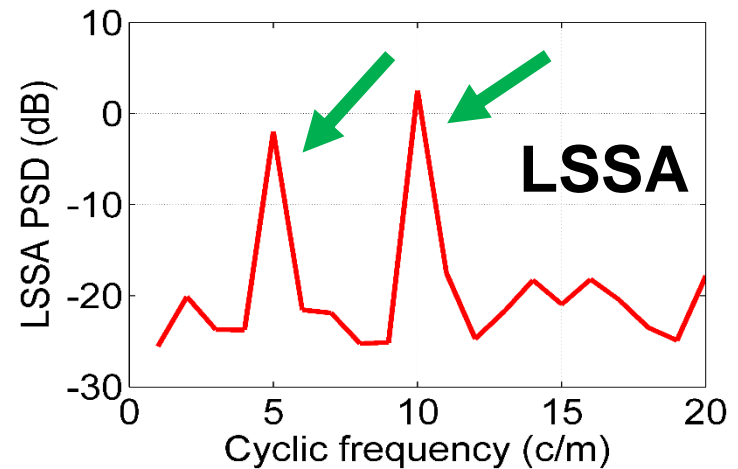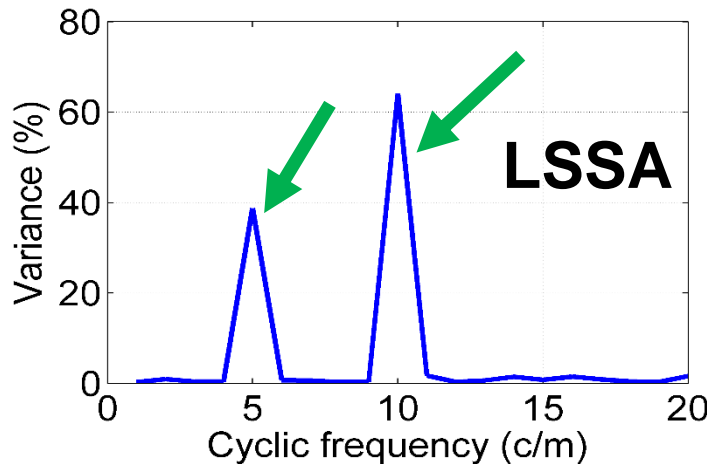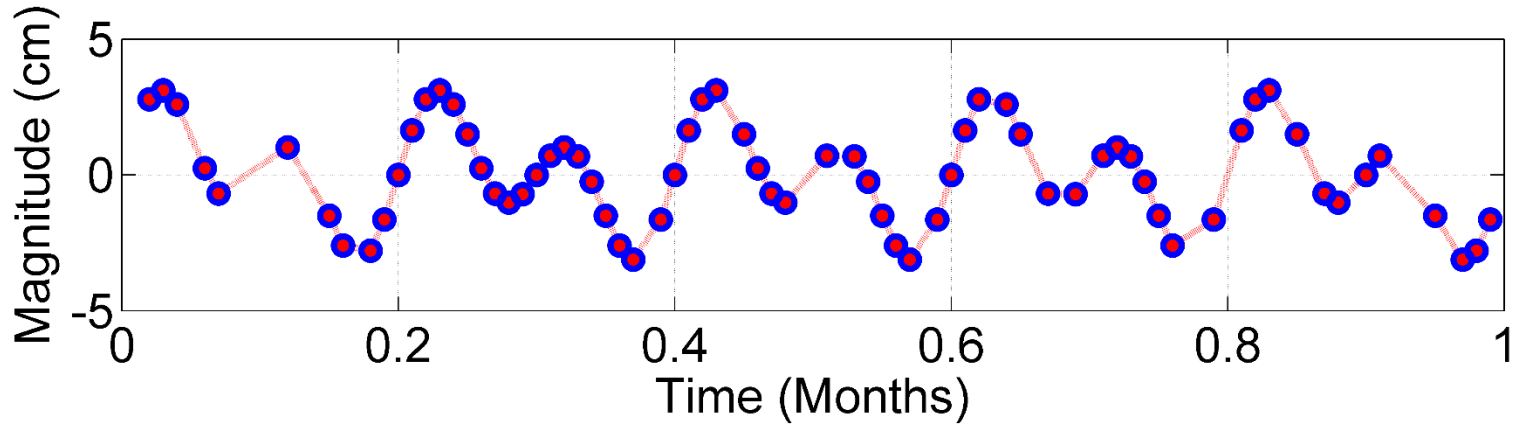
Shortcomings of the LSSA

Time series having constituents of :

- **Variable frequency?!**

- **Variable amplitude?!**

Example: $f(t) = \cos(\frac{230}{2.3-t}) + \cos(\frac{500}{2.3-t})$
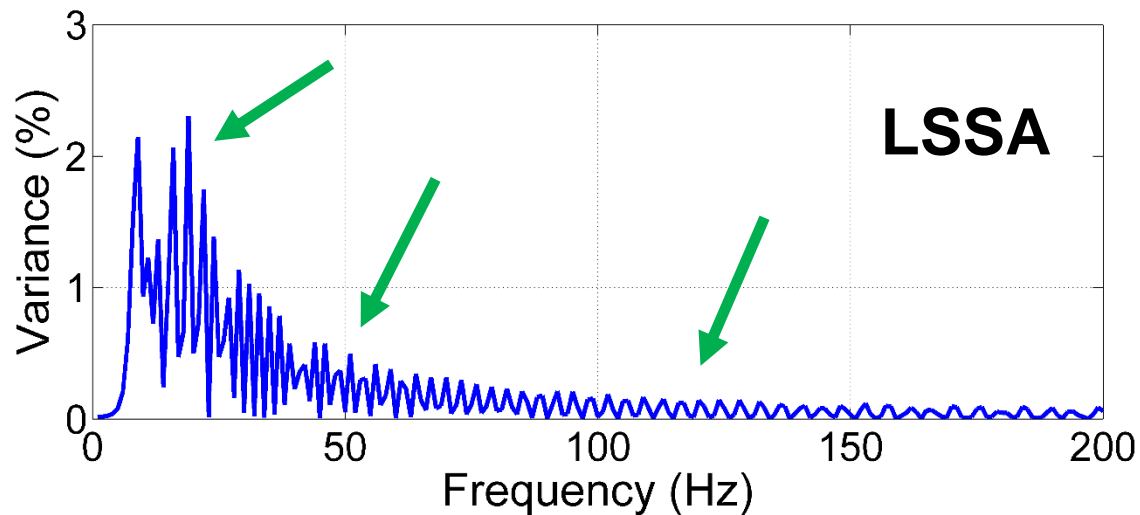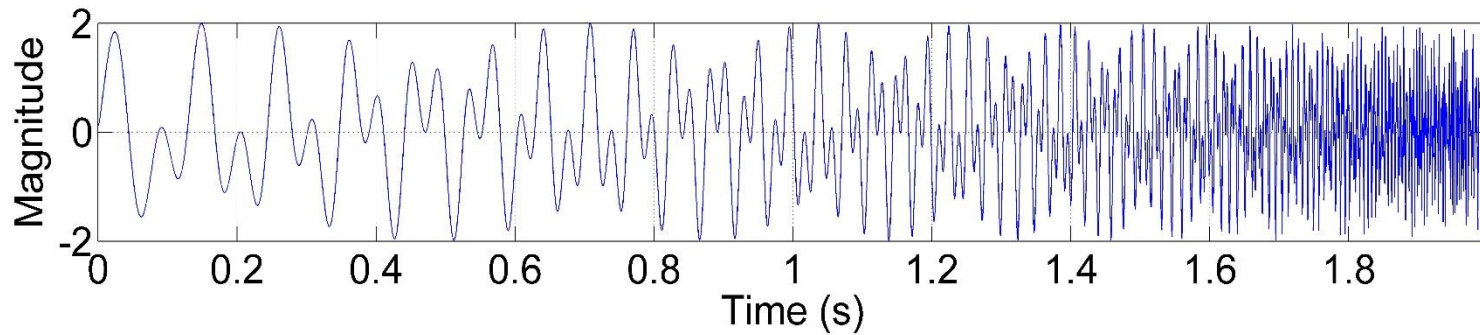
How to analyze these types of time series?

- **Continuous Wavelet Transform (CWT)**

How to analyze these types of time series?

- **Continuous Wavelet Transform (CWT)**
    - **Not defined for unequally spaced**

How to analyze these types of time series?

- **Continuous Wavelet Transform (CWT)**
    - **Not defined for unequally spaced**
    - **No trends, datum shifts, …**

How to analyze these types of time series?

- **Continuous Wavelet Transform (CWT)**
  - **Not defined for unequally spaced**
  - **No trends, datum shifts, …**
  - **Loses power!**

Least Squares Wavelet Analysis (LSWA)

- **Fits sinusoids like LSSA**

Least Squares Wavelet Analysis (LSWA)

- **Fits sinusoids like LSSA**

- **But to segments NOT the entire time series**

Least Squares Wavelet Analysis (LSWA)

- **Fits sinusoids like LSSA**

- **But to segments NOT the entire time series**

- **How to segment?**

Least Squares Wavelet Analysis (LSWA)

- **Segment : window**

Least Squares Wavelet Analysis (LSWA)

- **Segment : window**
  - **Place** (location)

Least Squares Wavelet Analysis (LSWA)

- **Segment : window**
    - **Place** (location)
    - **Length**

Least Squares Wavelet Analysis (LSWA)

- **Segment : window**
    - **Place** (location)
    - **Length**
    - **Size**

Least Squares Wavelet Analysis (LSWA)

- **Window:**
  - **Dilates and shrinks**

Least Squares Wavelet Analysis (LSWA)

- **Window:**
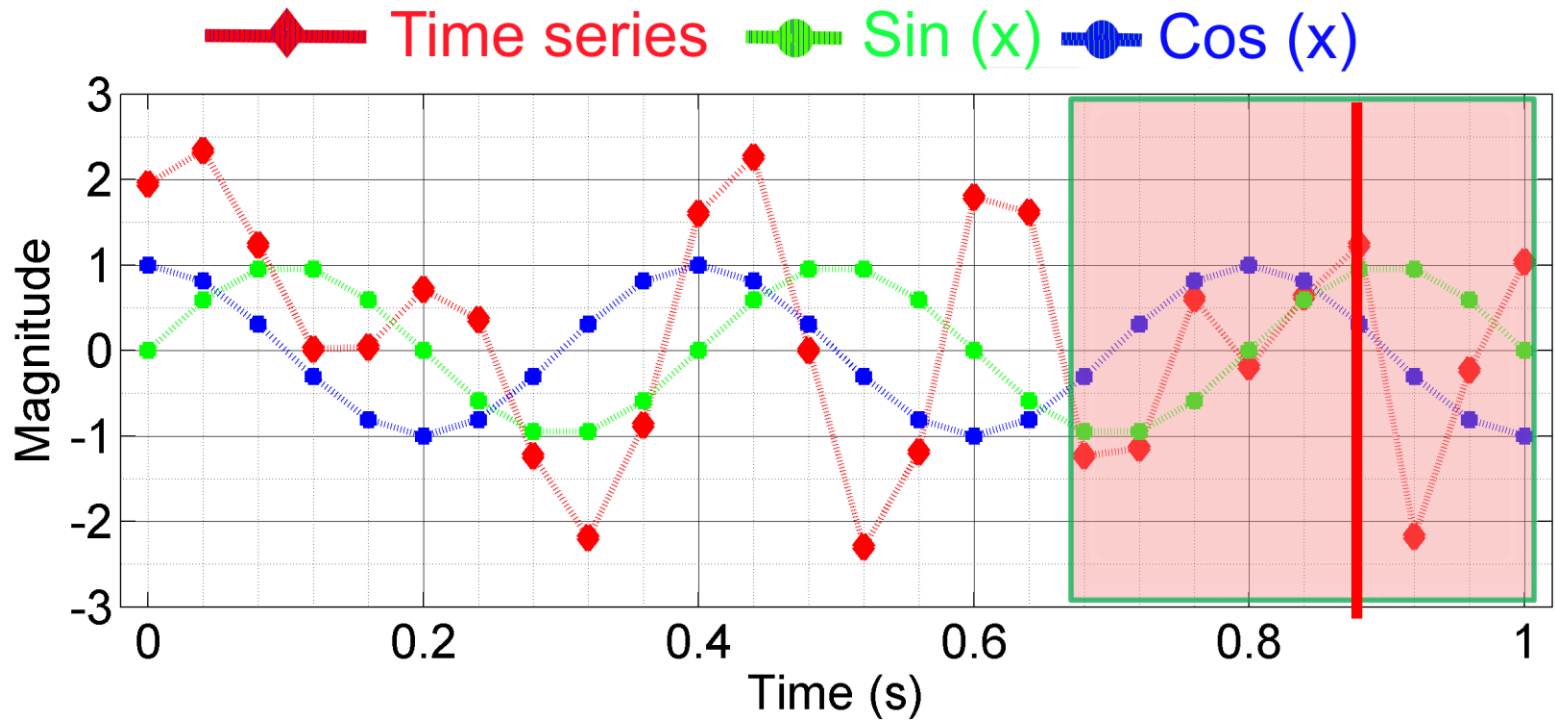 - **Dilates and shrinks**
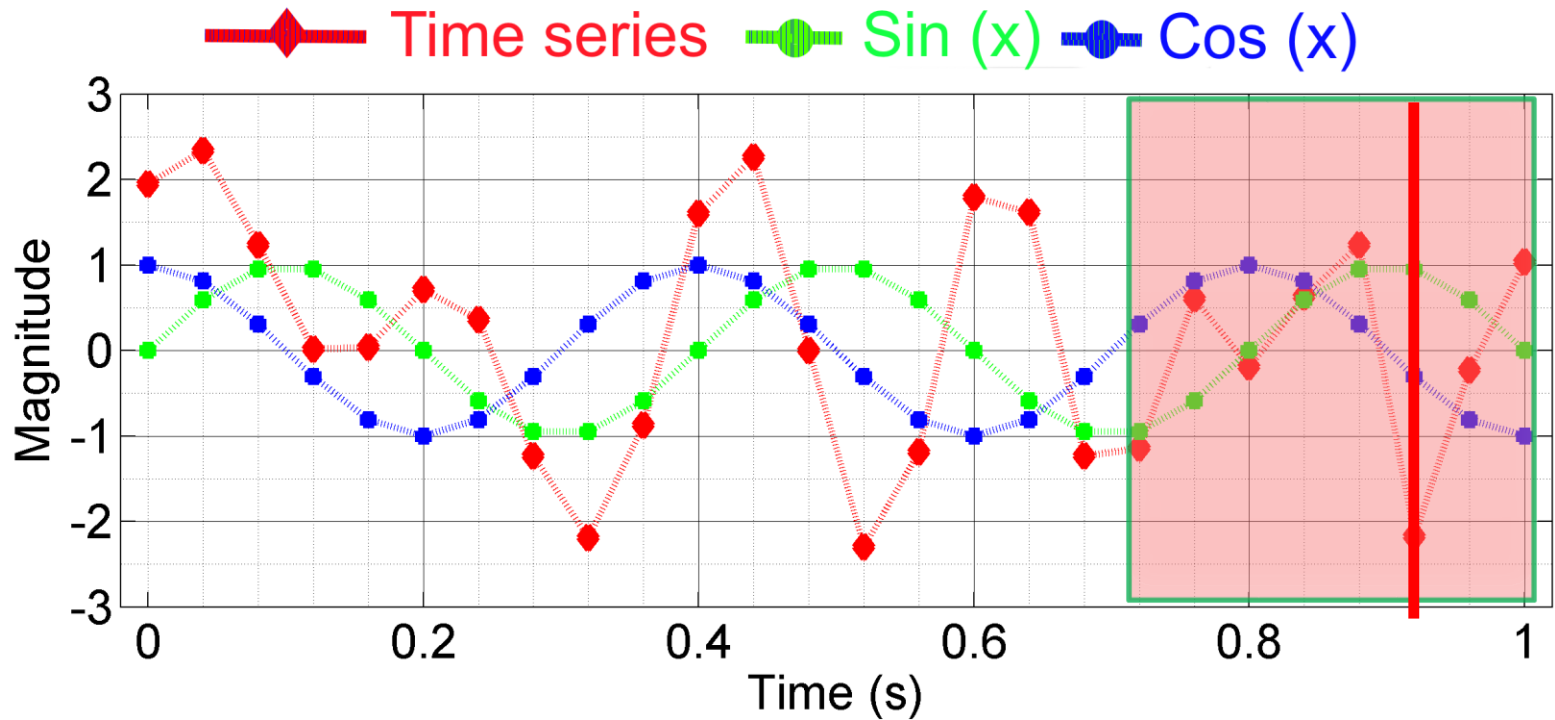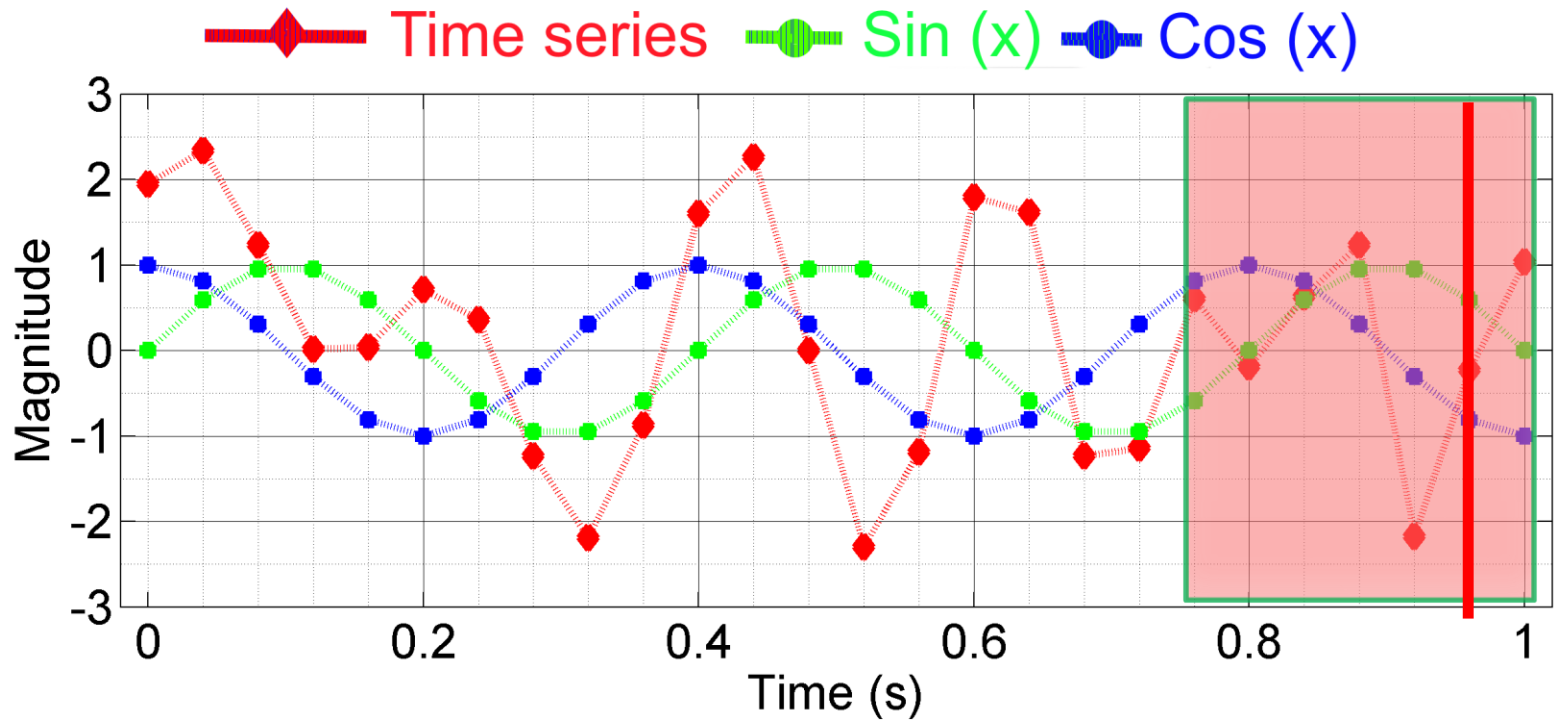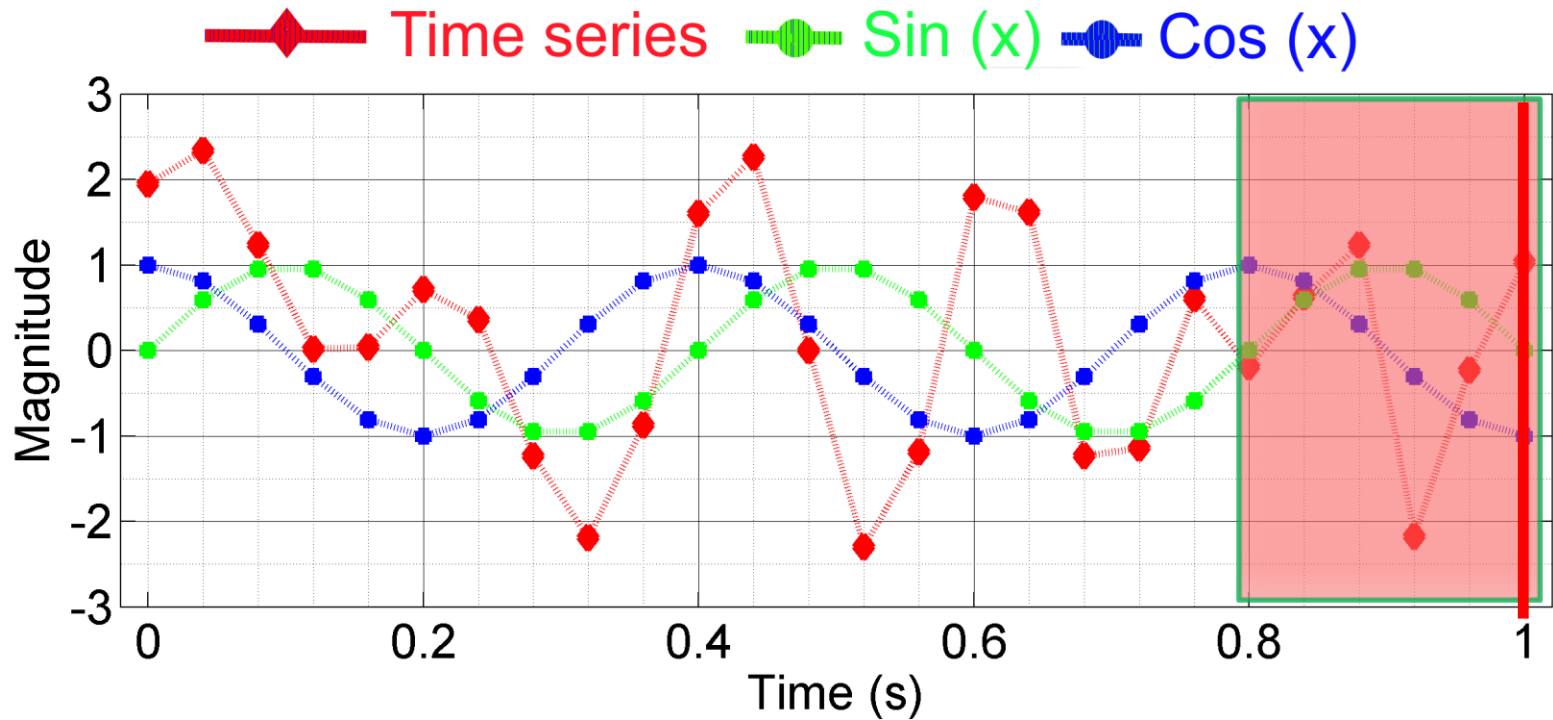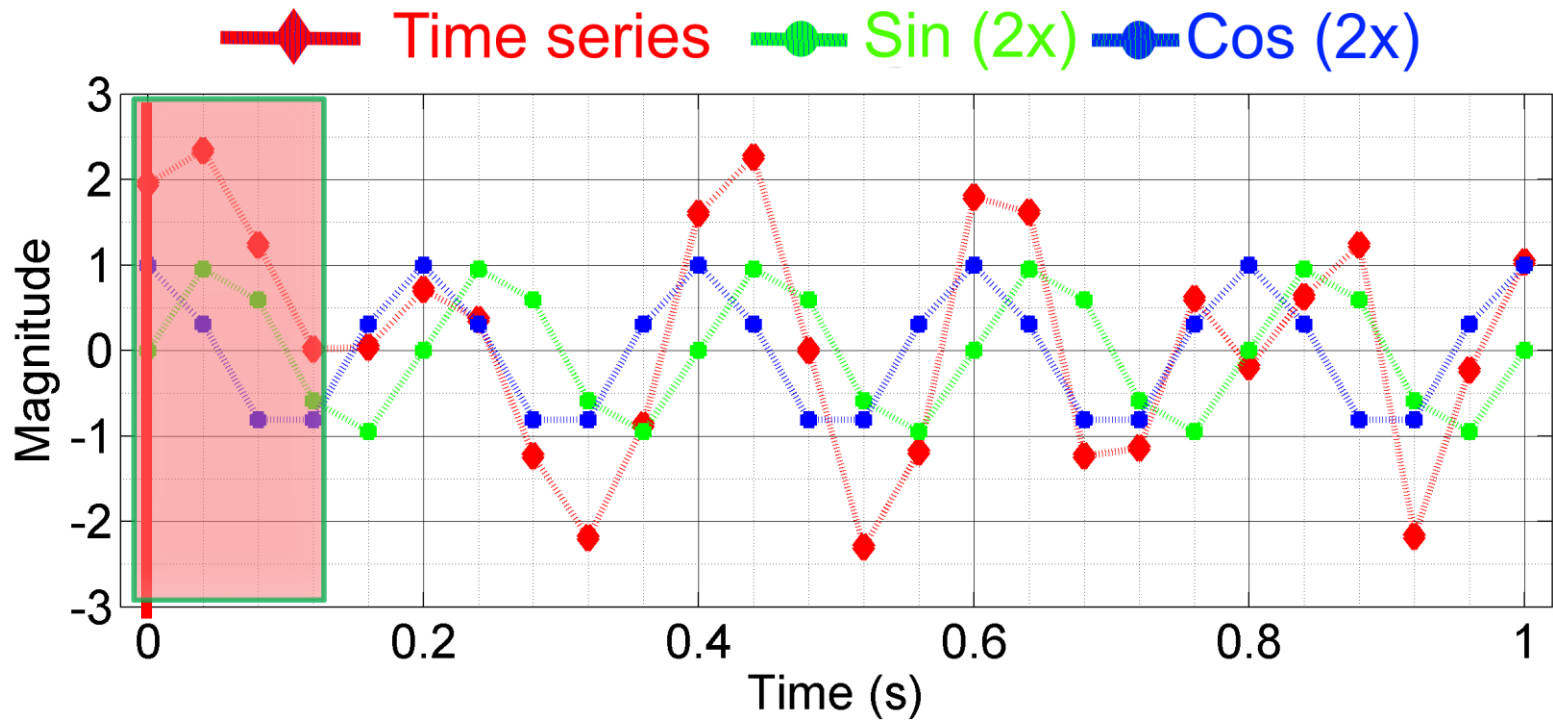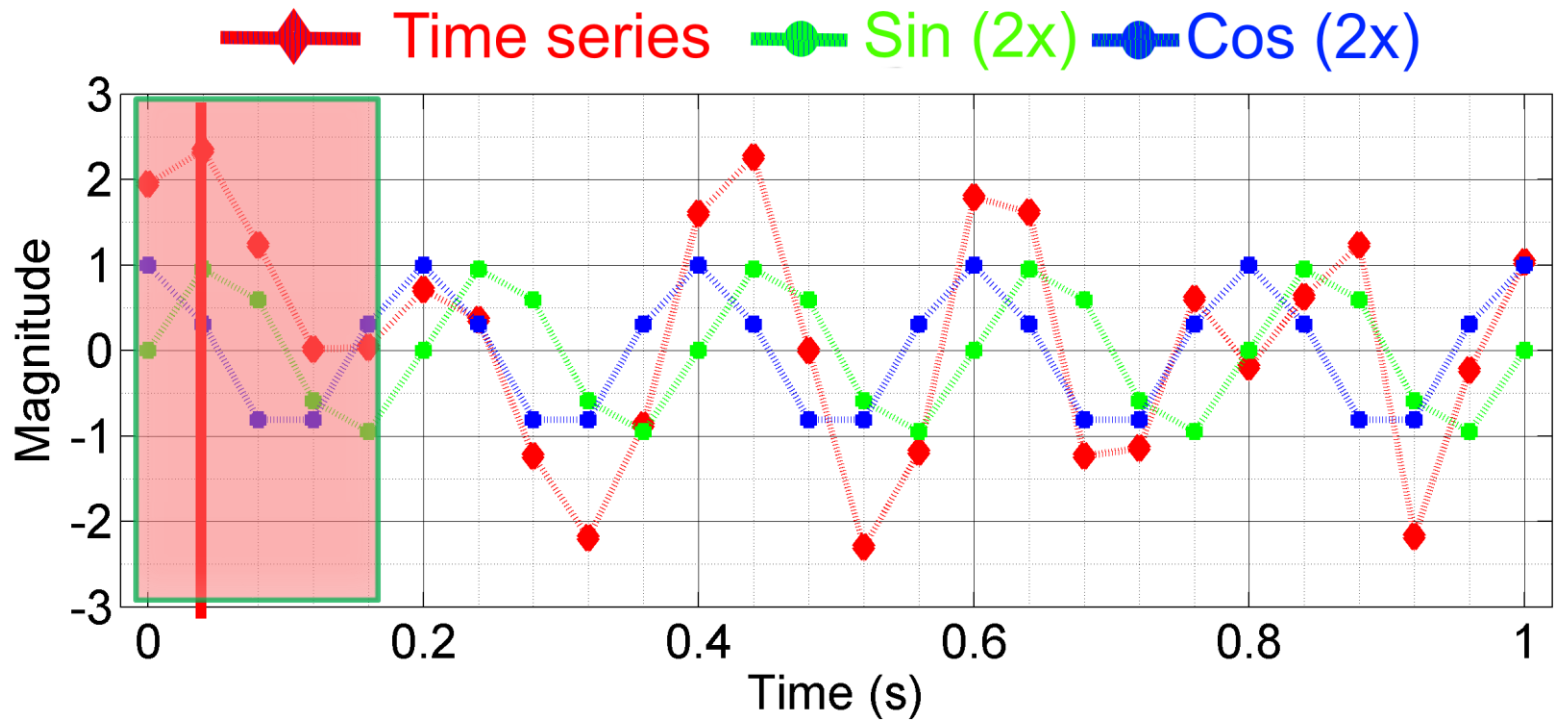 - **Translates** (slides)

Translating window for an equally spaced time series

Translating window for an equally spaced time series

Translating window for an equally spaced time series
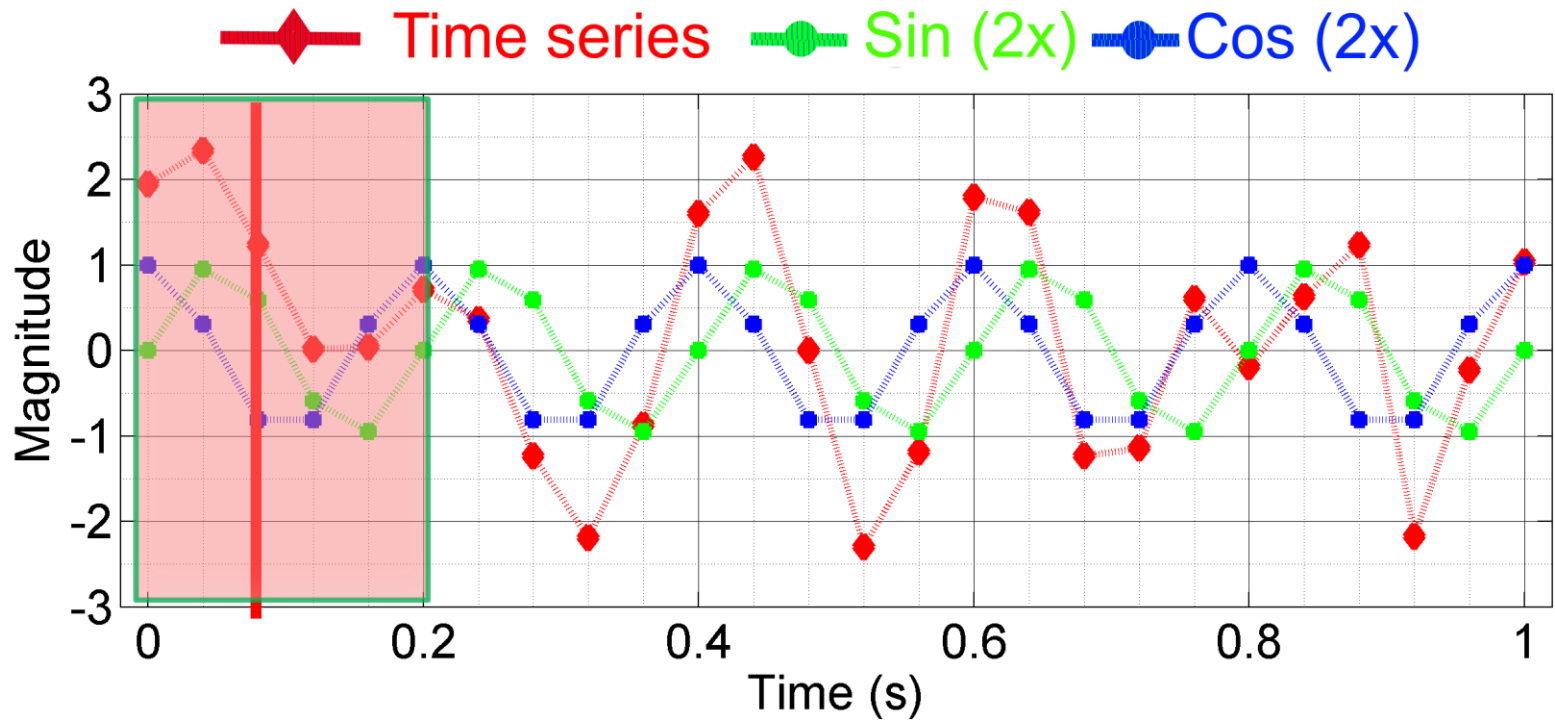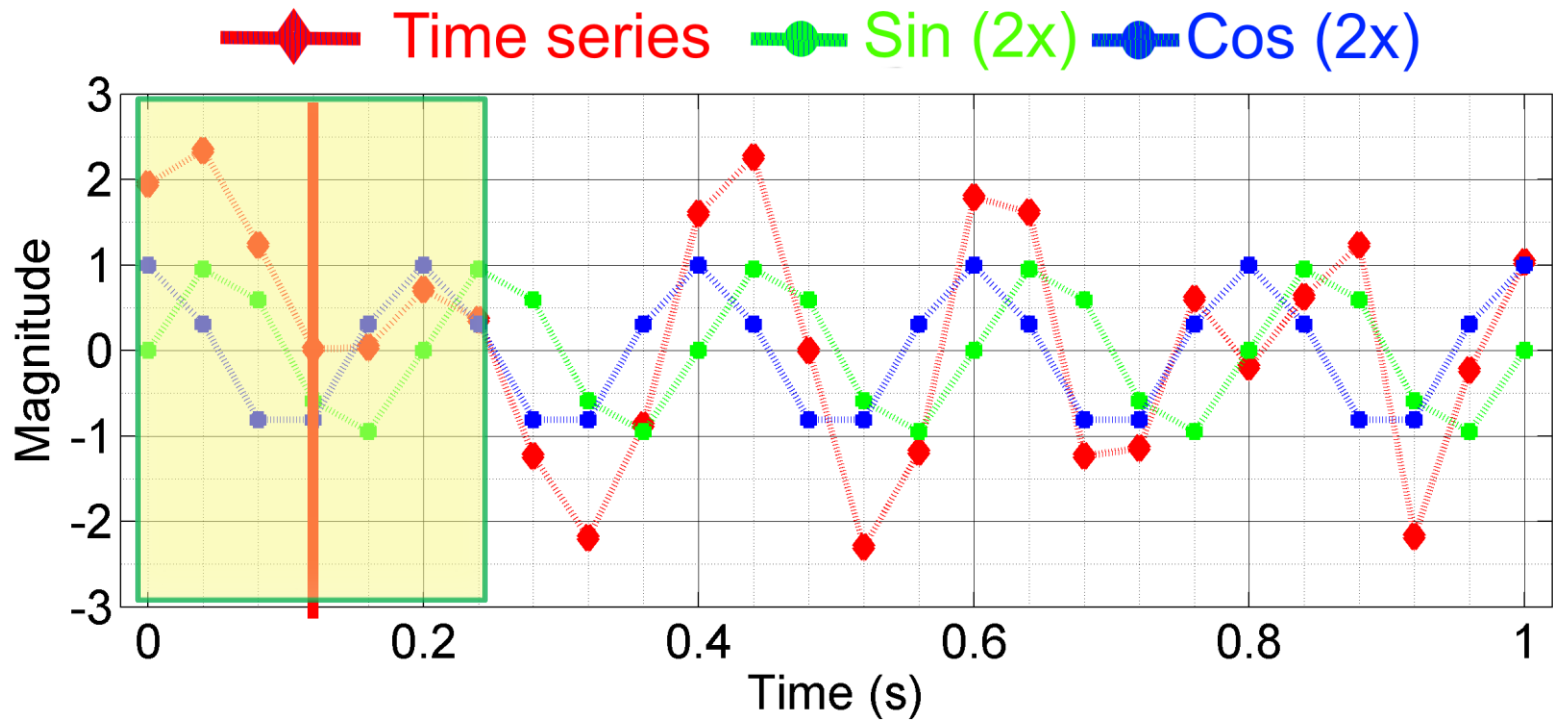
Translating window for an equally spaced time series

Translating window for an equally spaced time series

Translating window for an equally spaced time series

Translating window for an equally spaced time series
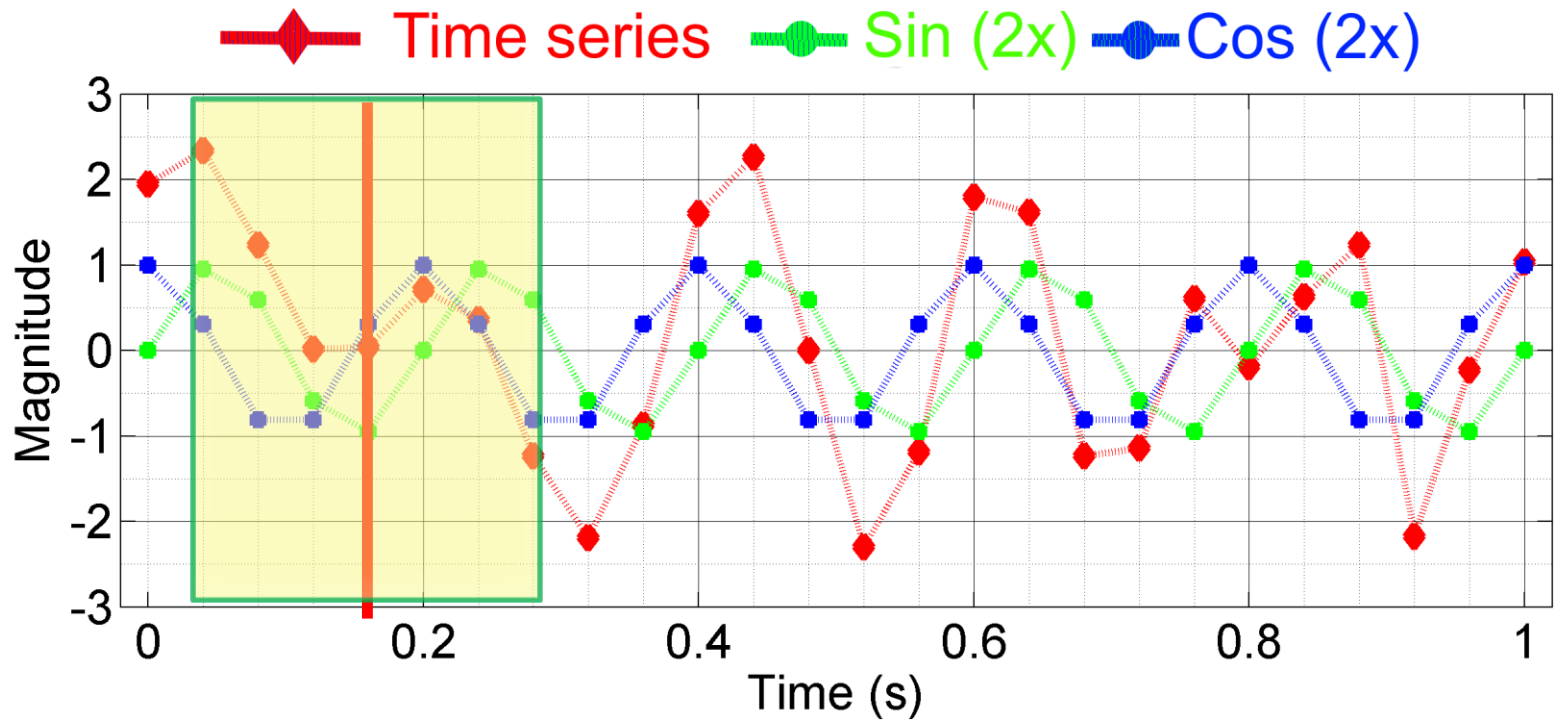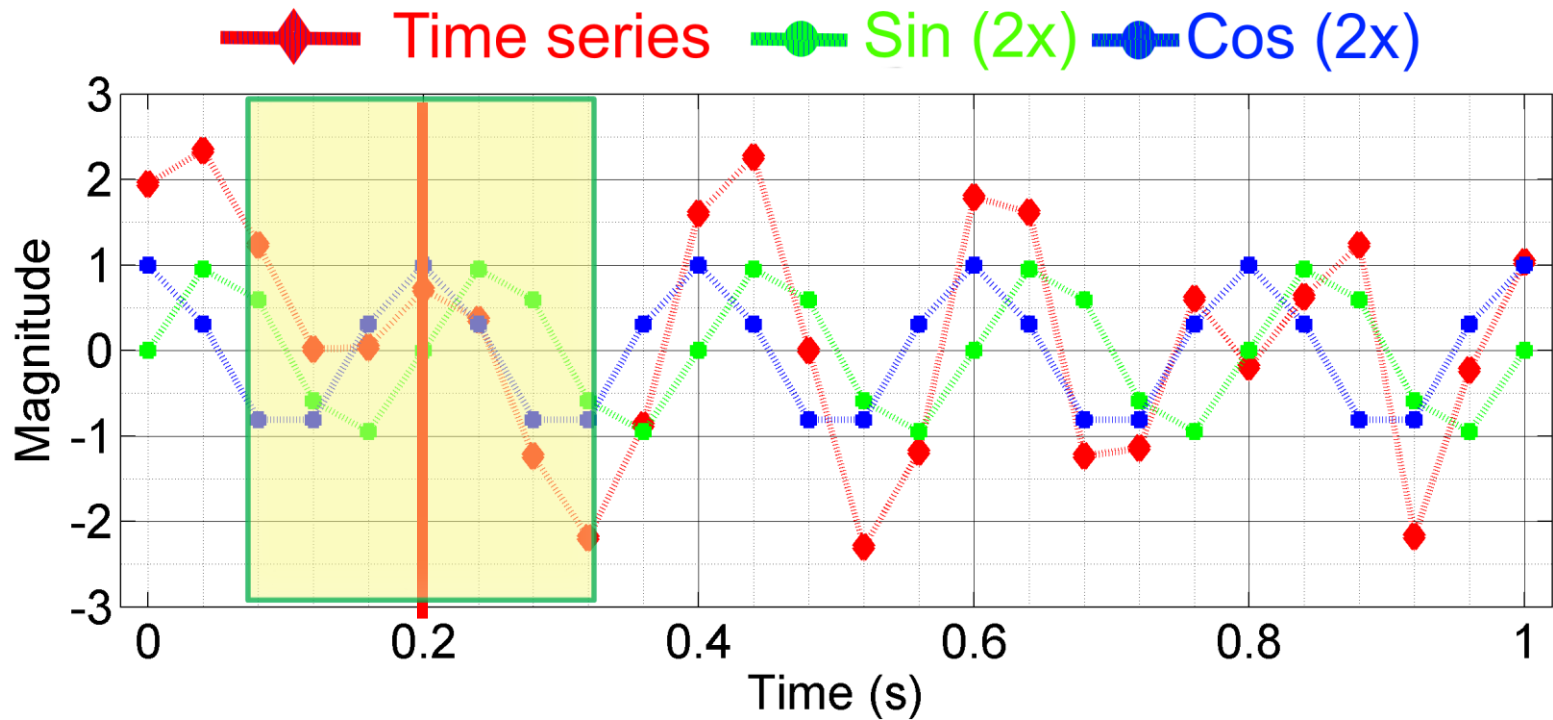
Translating window for an equally spaced time series

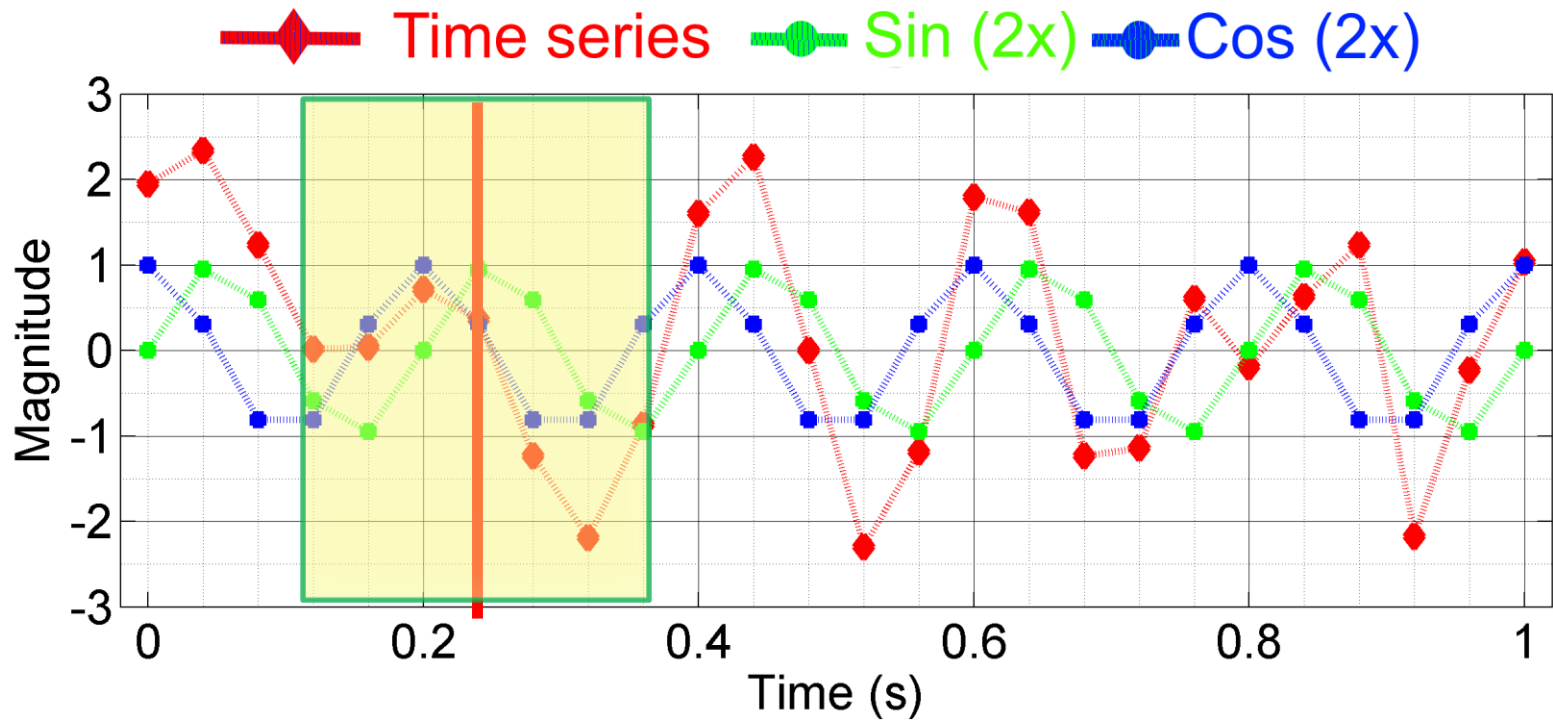# Translating window for an equally spaced time series

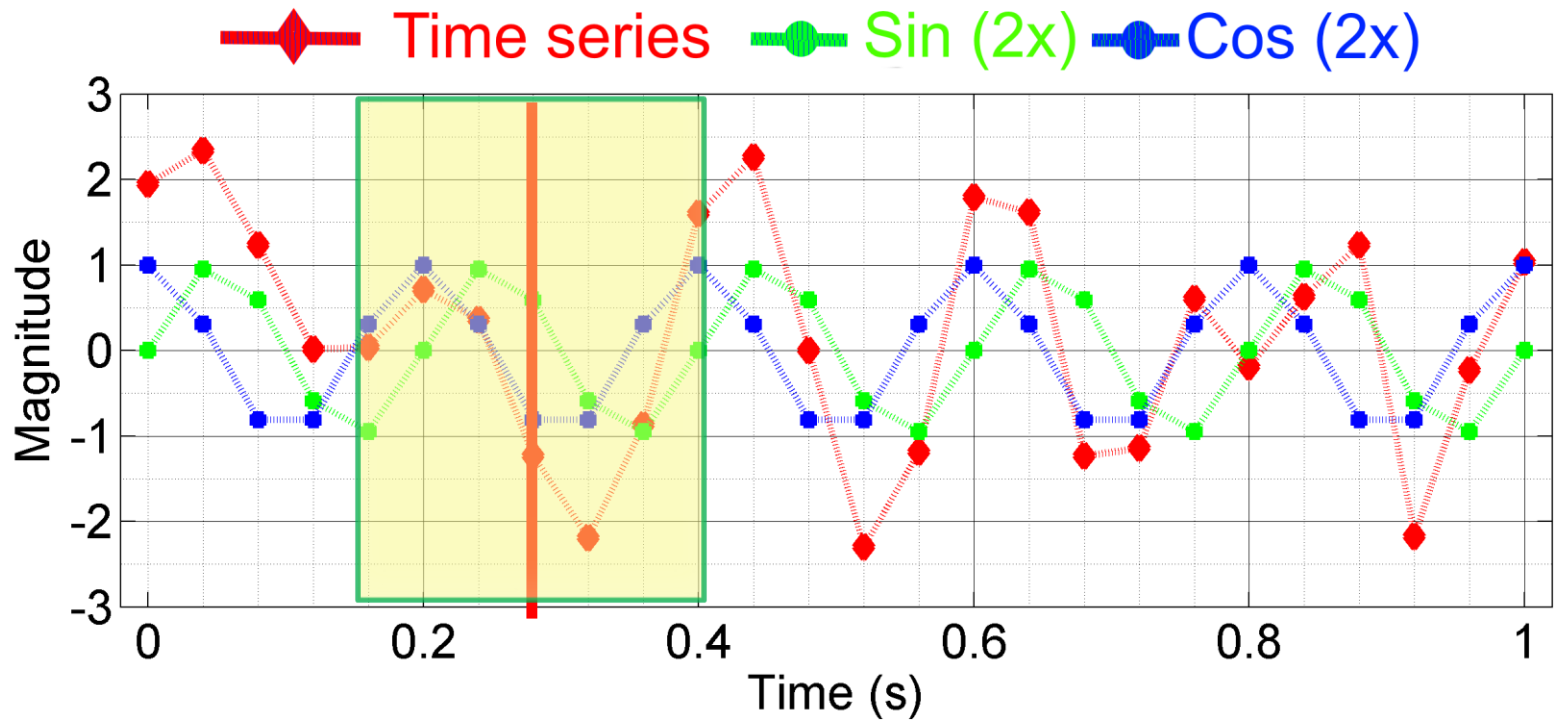# Translating window for an equally spaced time series

Translating window for an equally spaced time series

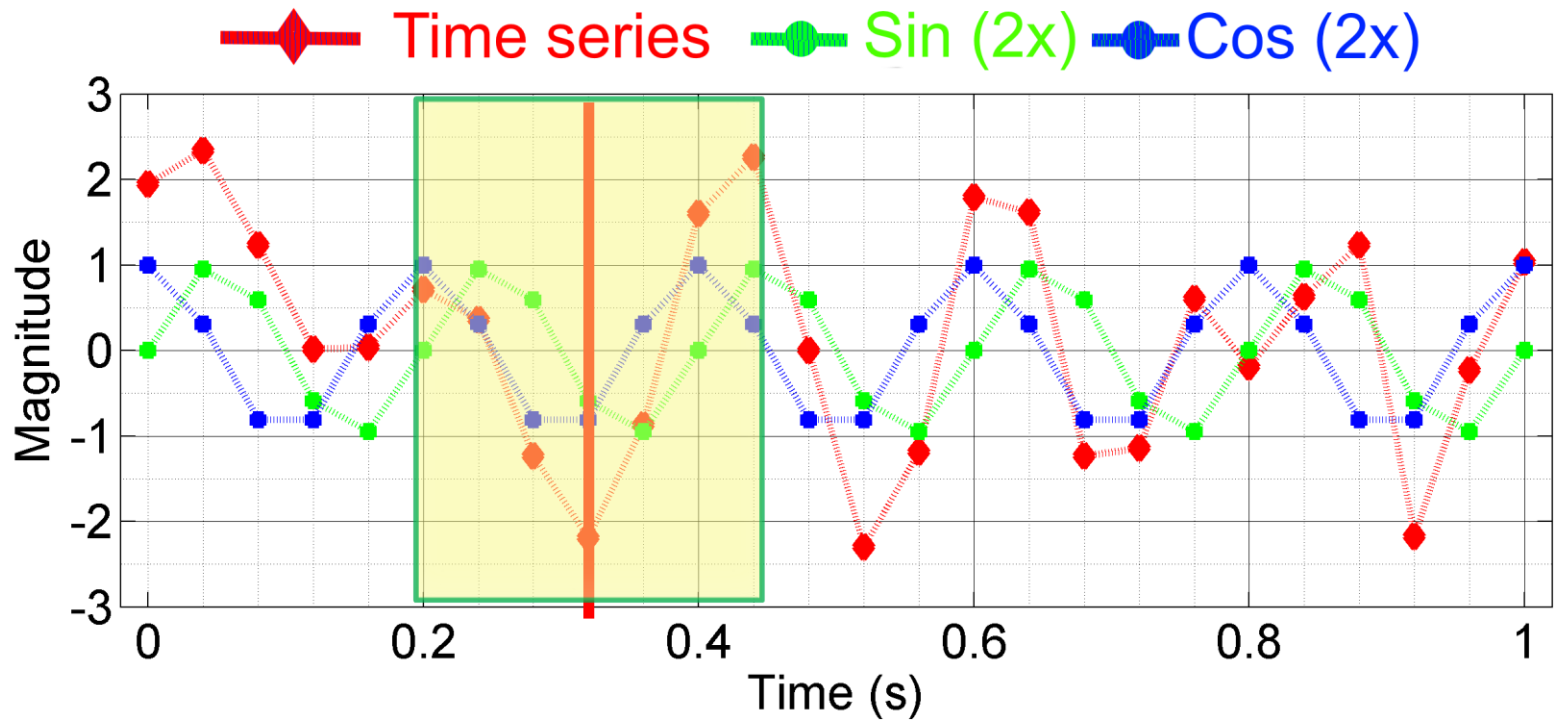# Translating window for an equally spaced time series

Translating window for an equally spaced time series

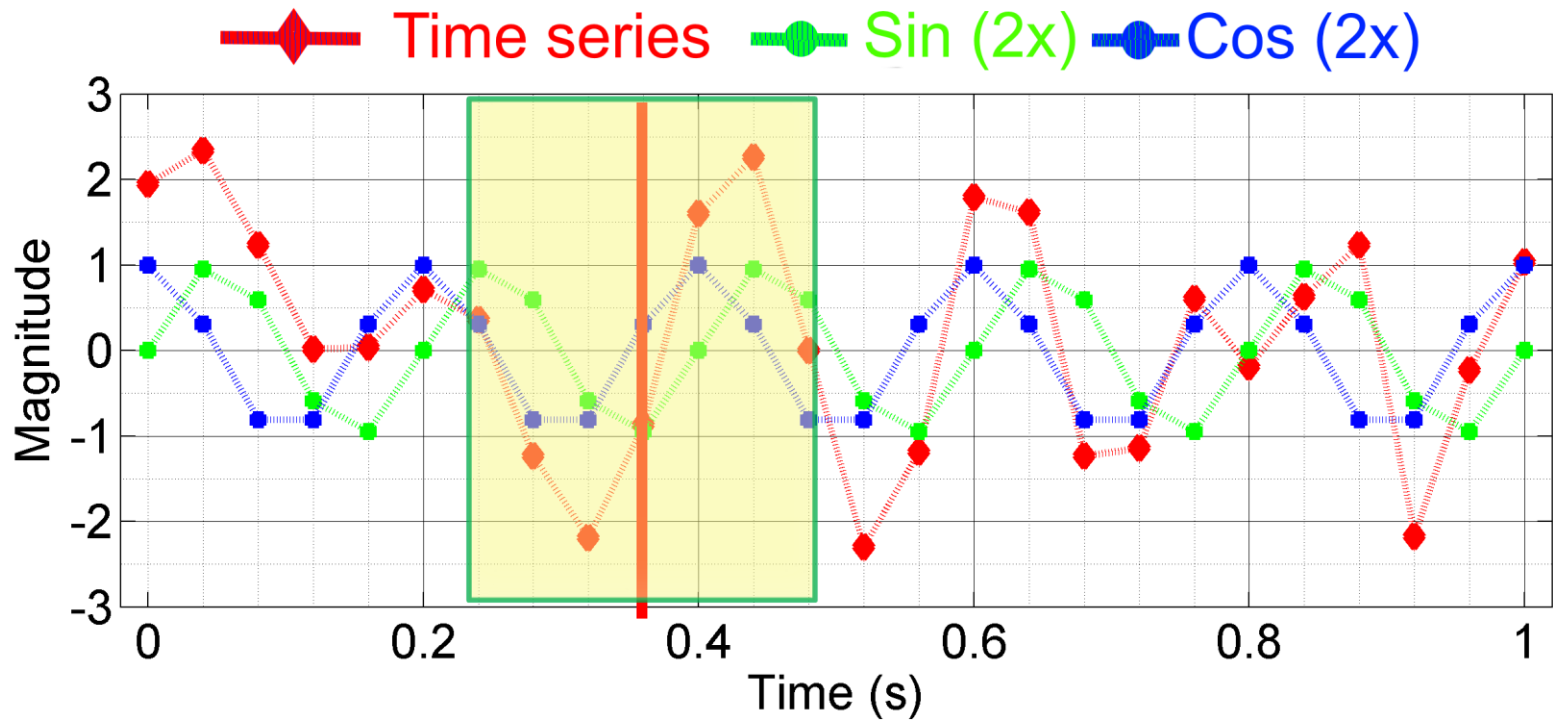# Translating window for an equally spaced time series

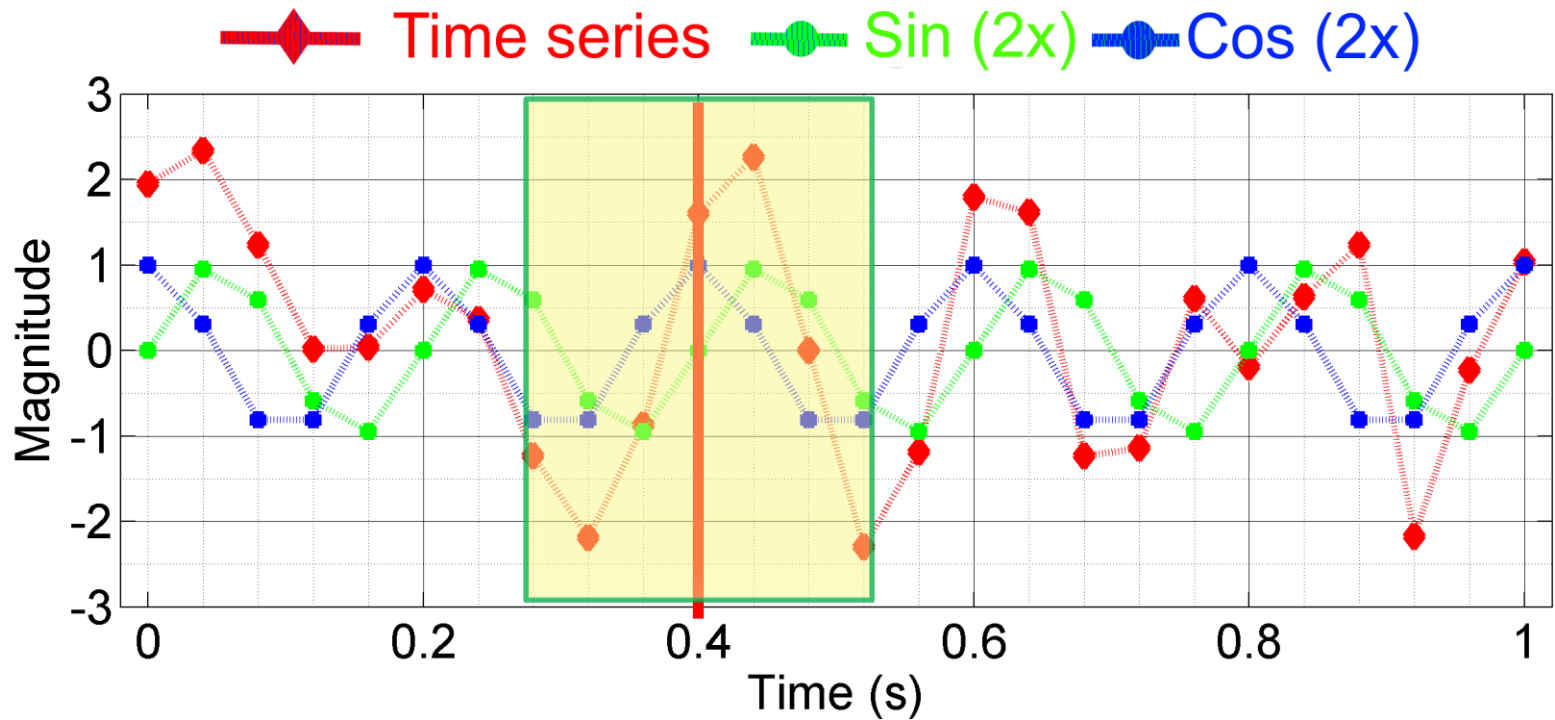# Translating window for an equally spaced time series

Translating window for an equally spaced time series

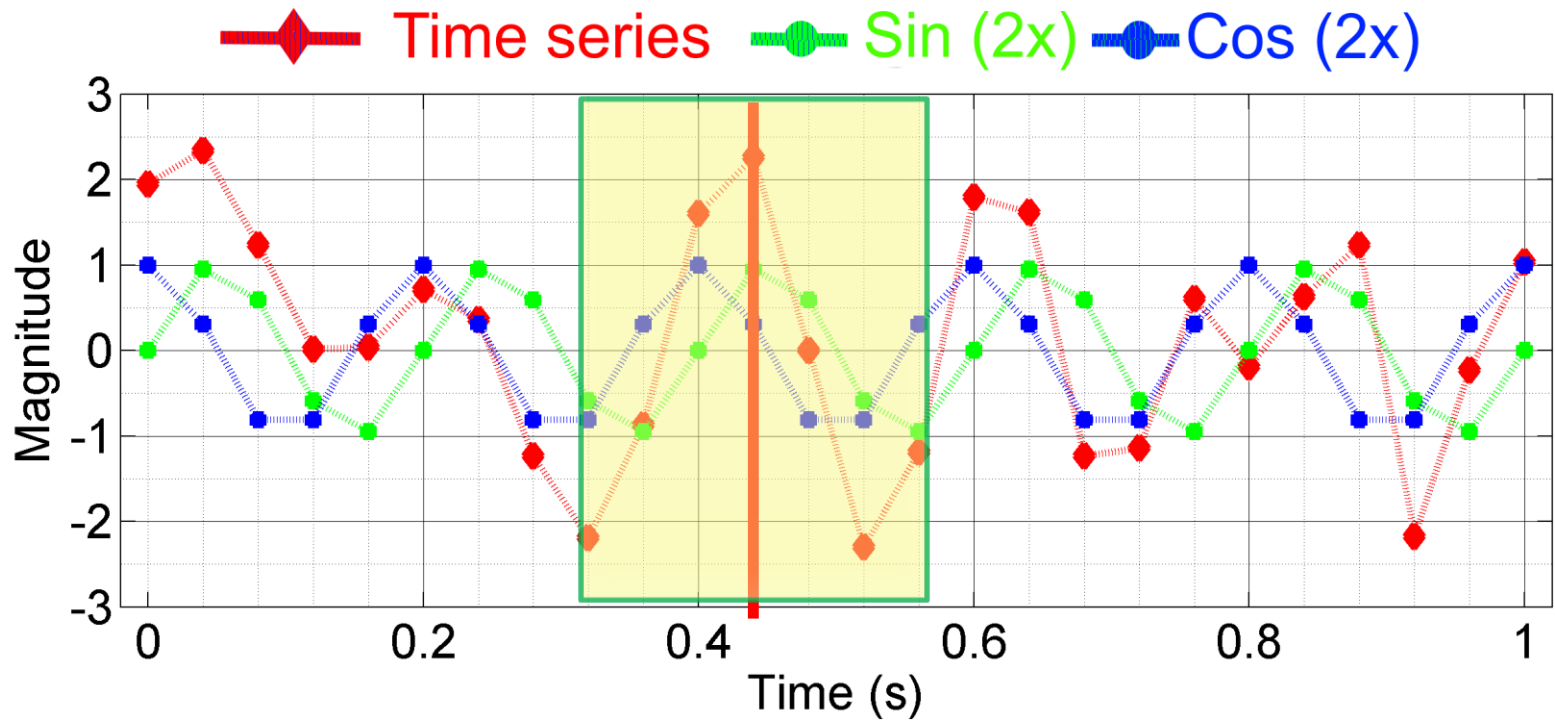# Translating window for an equally spaced time series

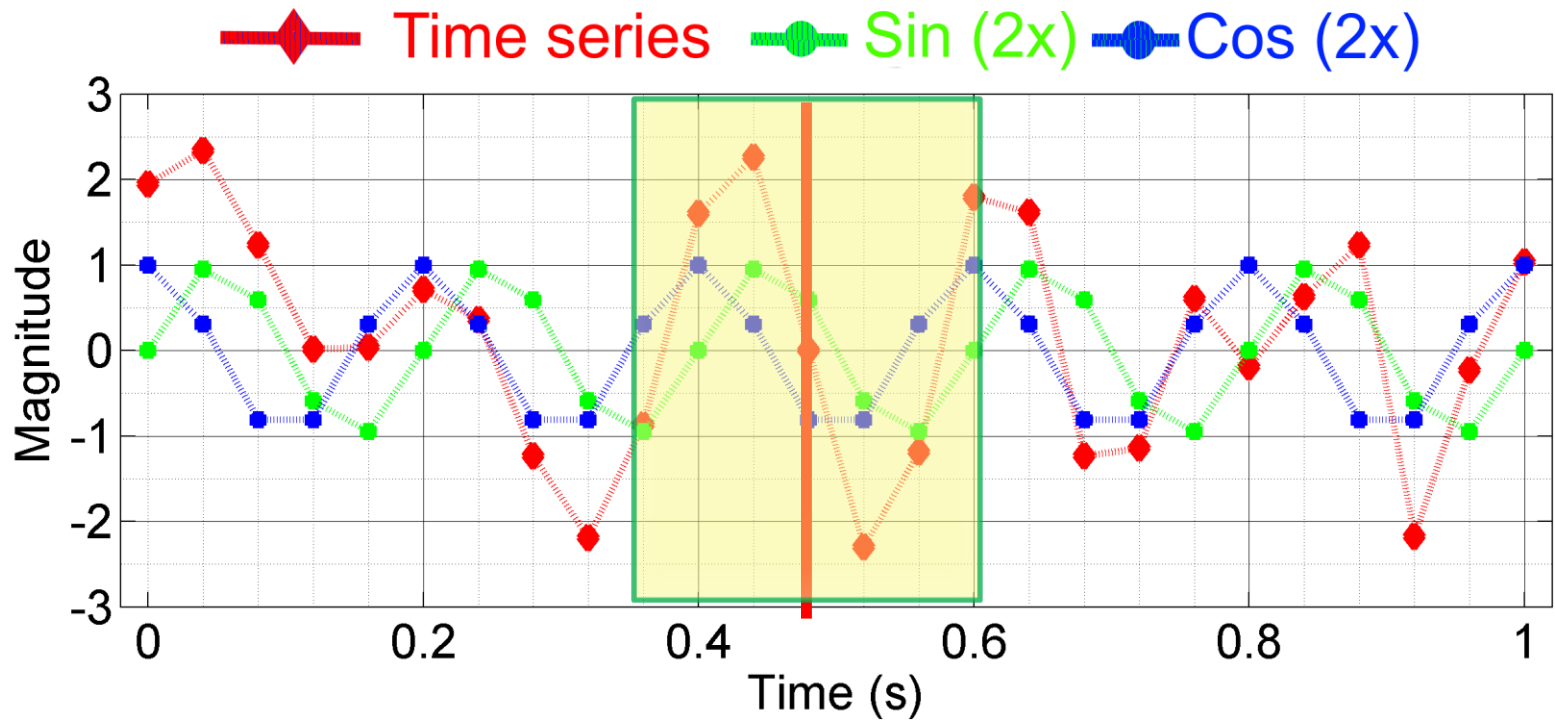# Translating window for an equally spaced time series

Translating window for an equally spaced time series

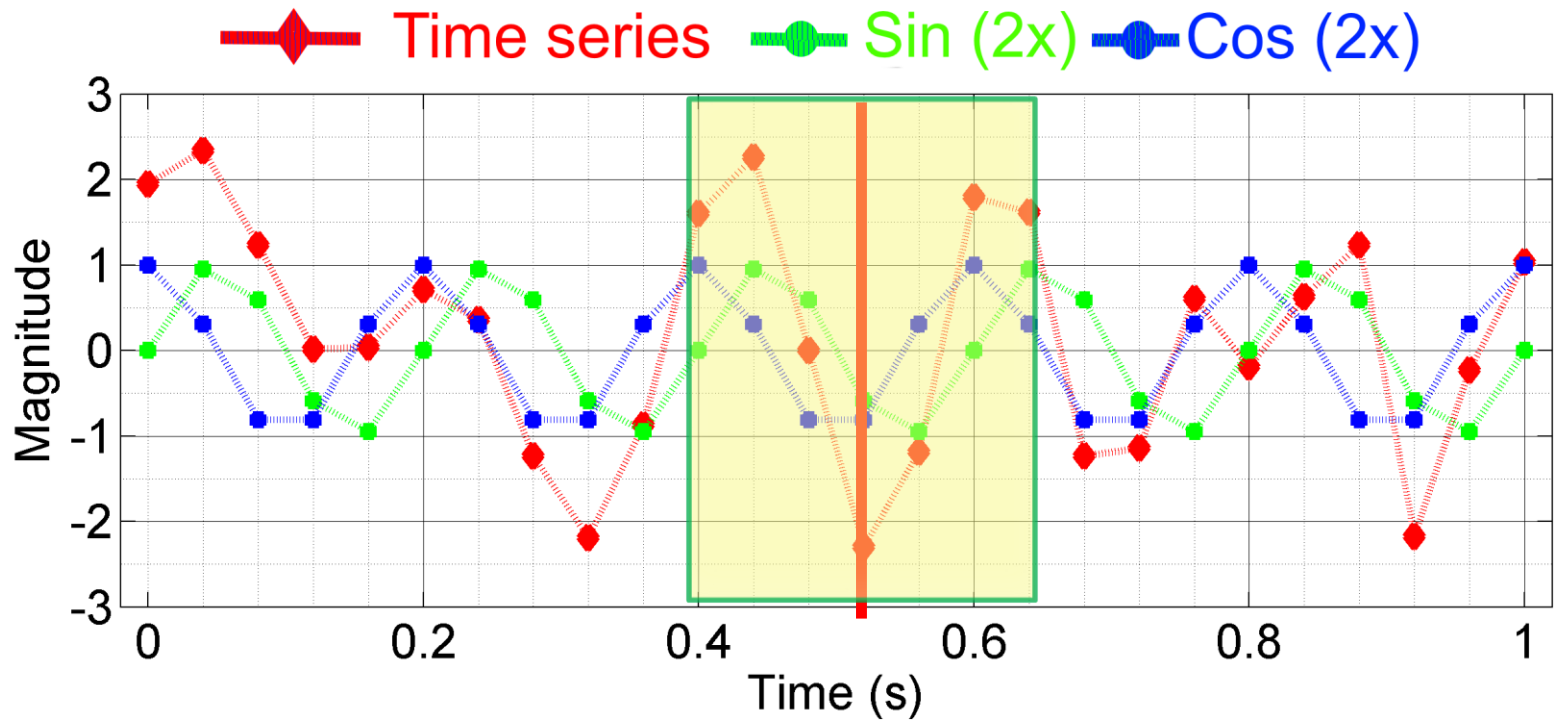# Translating window for an equally spaced time series

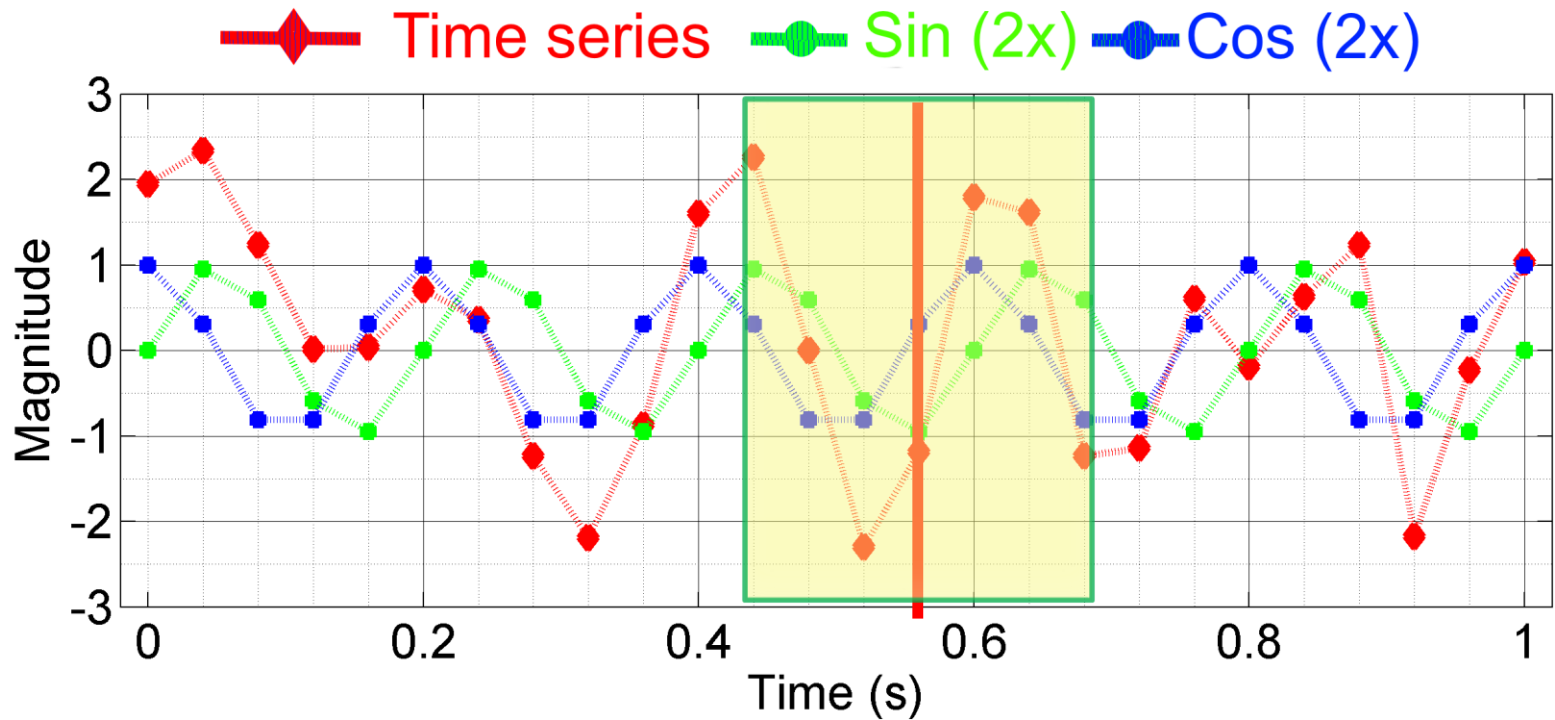# Translating window for an equally spaced time series

Translating window for an equally spaced time series

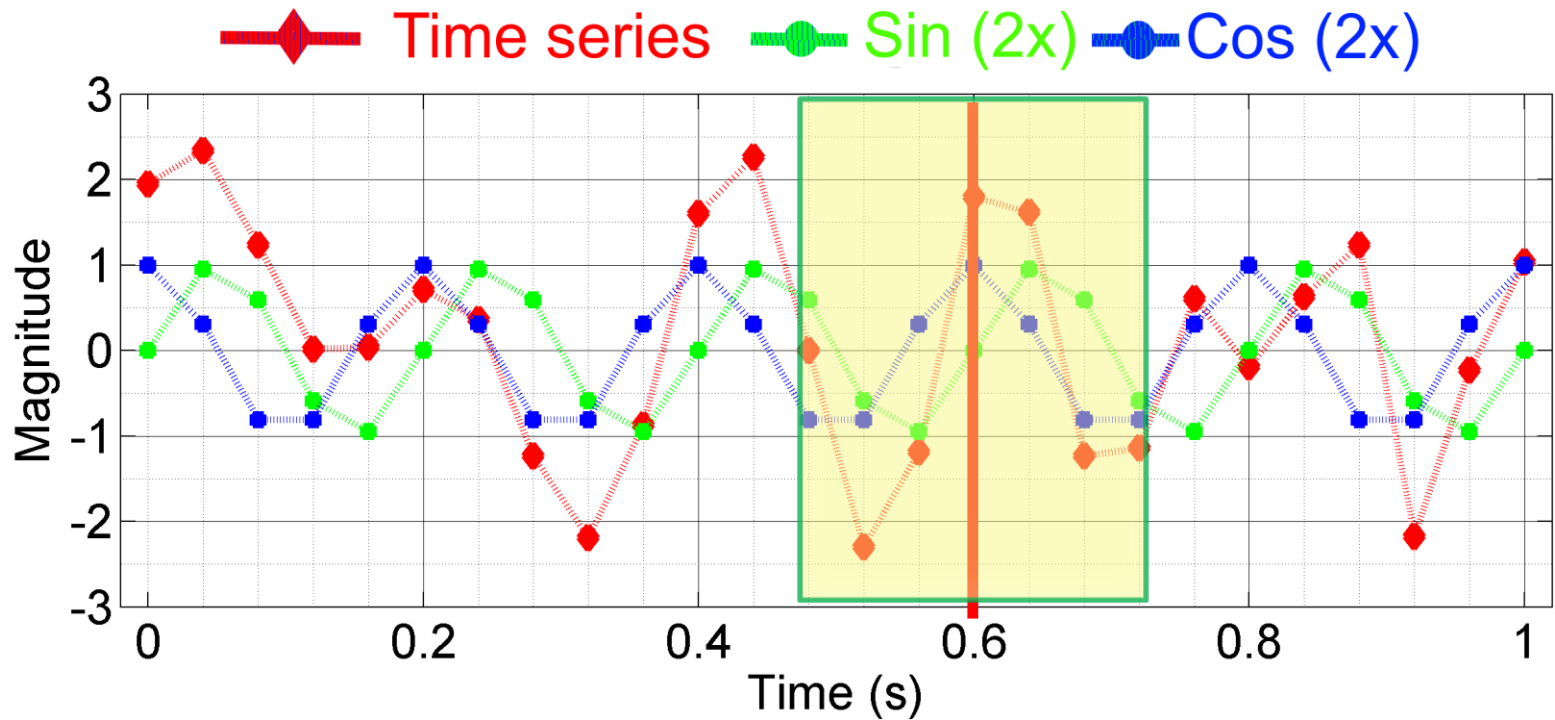# Translating window for an equally spaced time series

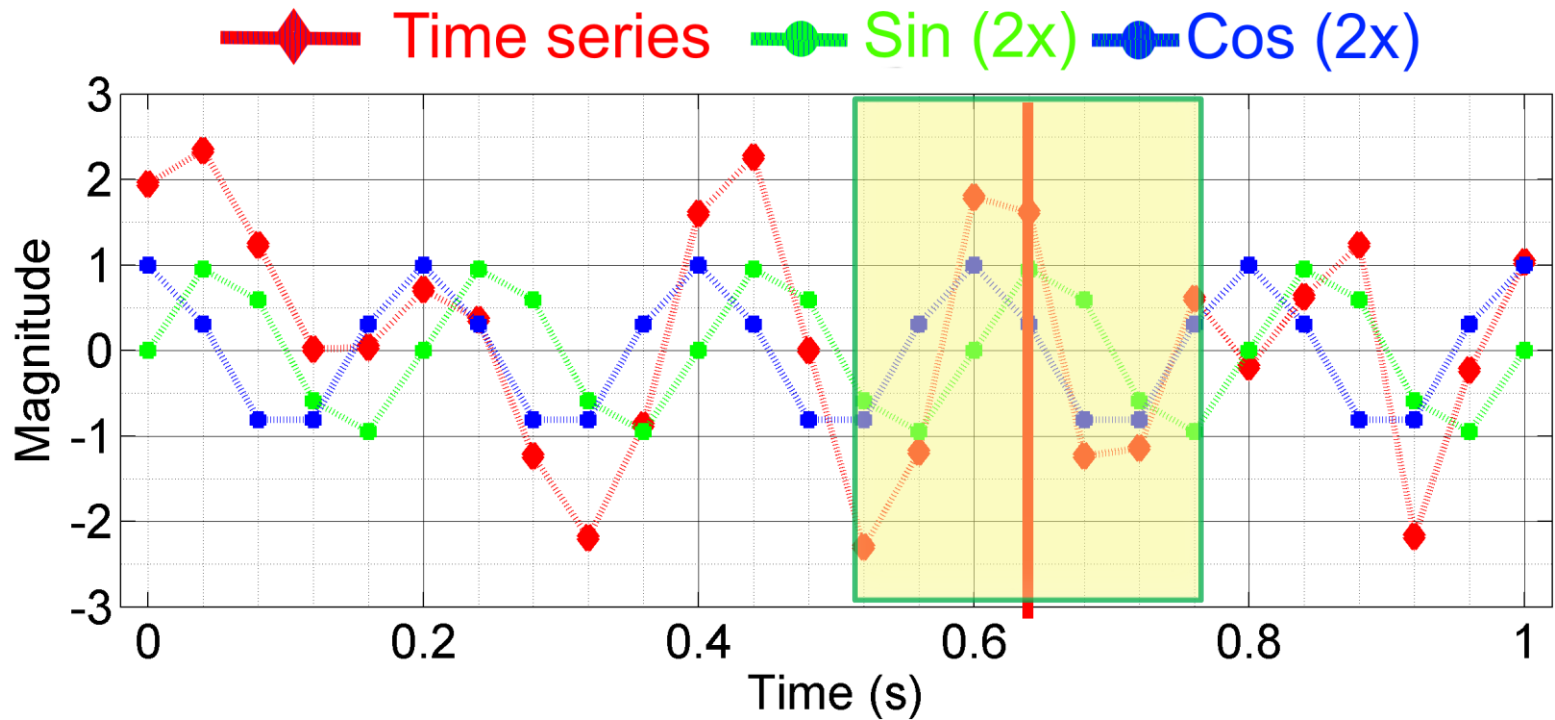# Translating window for an equally spaced time series

Translating window for an equally spaced time series

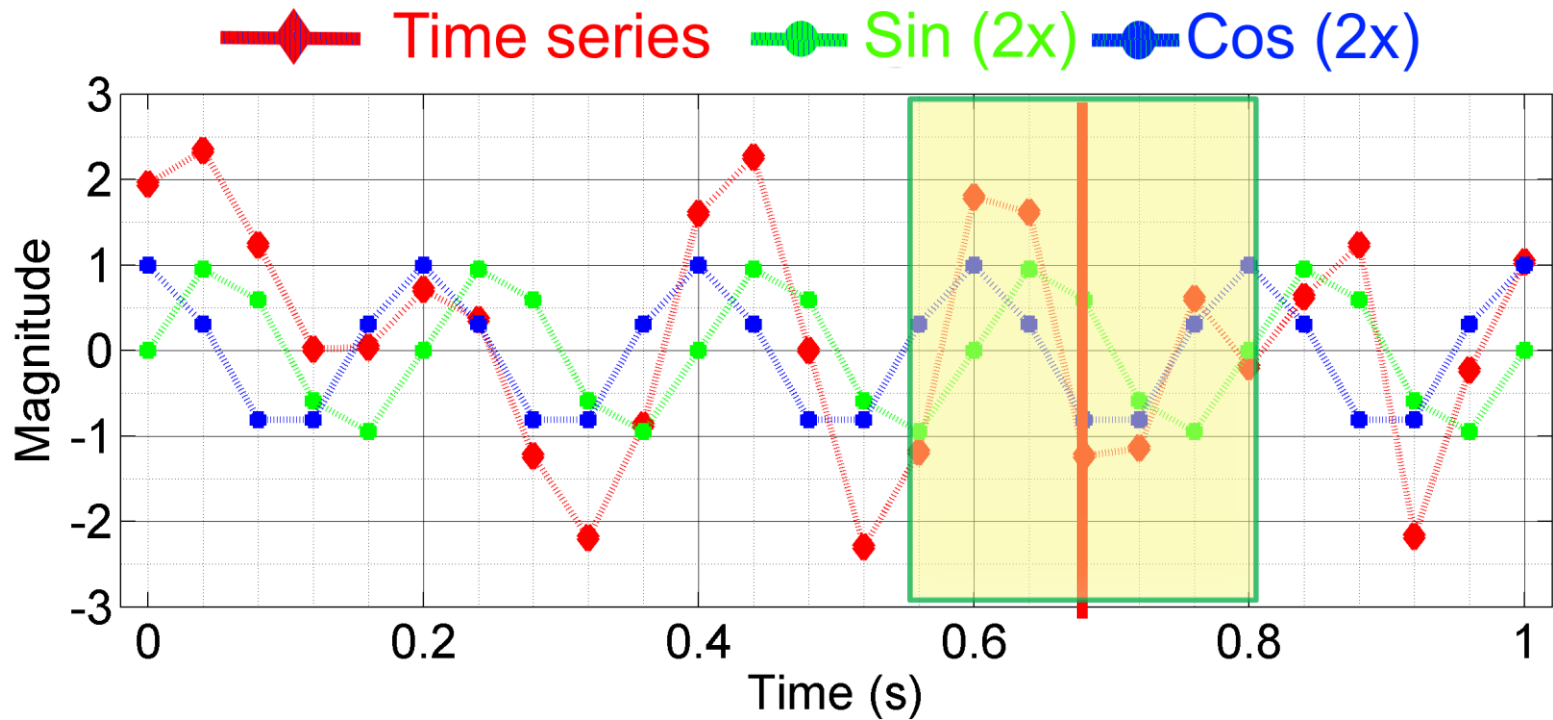# Translating window for an equally spaced time series

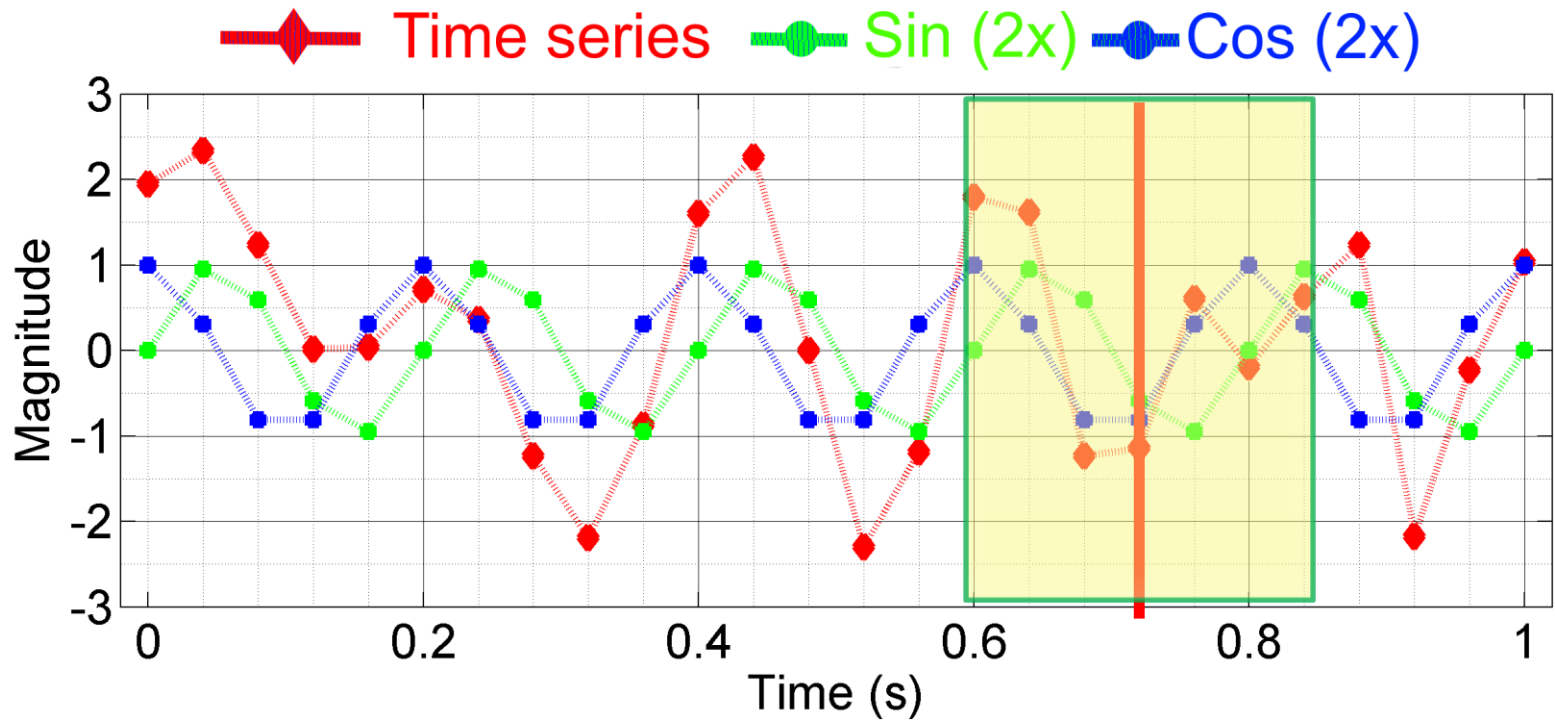# Translating window for an equally spaced time series

Translating window for an equally spaced time series

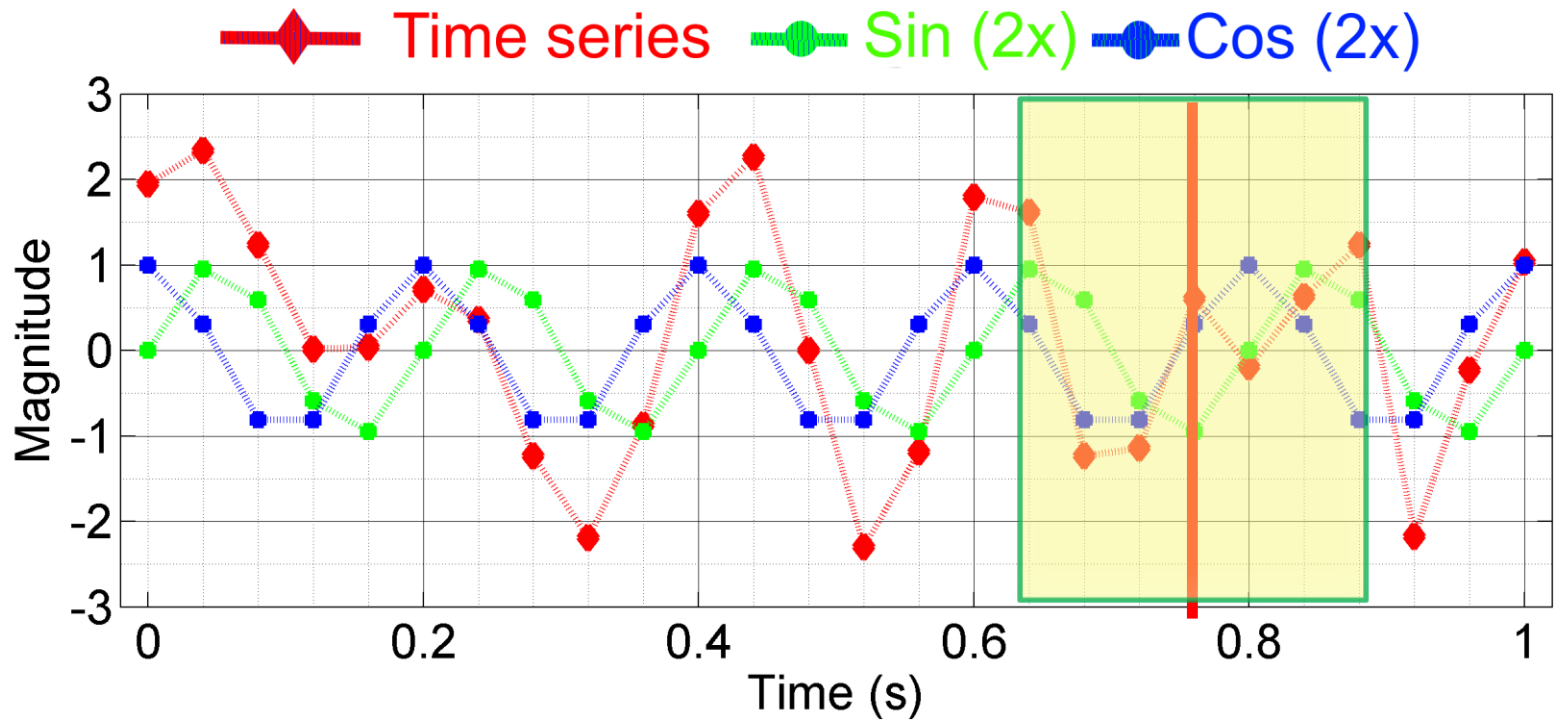# Translating window for an equally spaced time series

Translating window for an equally spaced time series

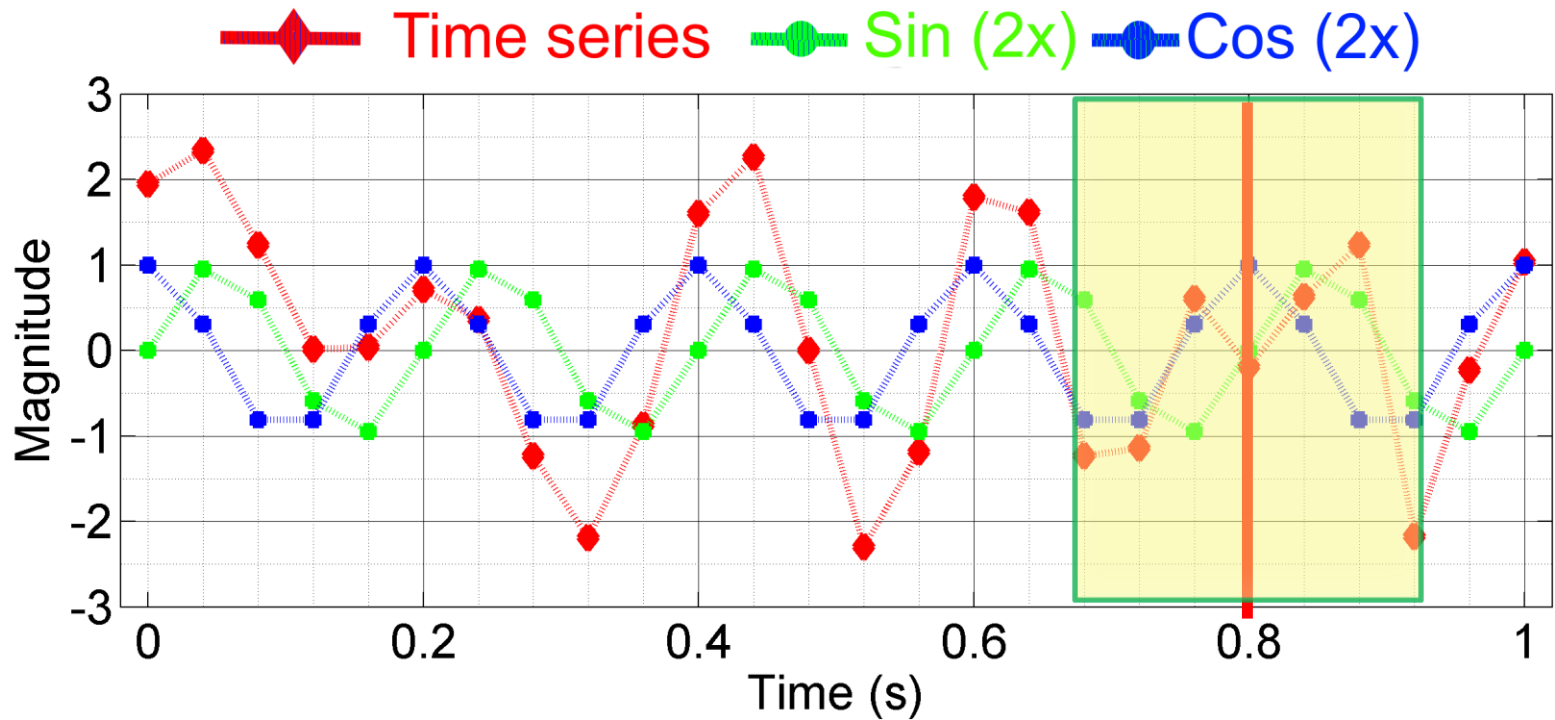# Translating window for an equally spaced time series

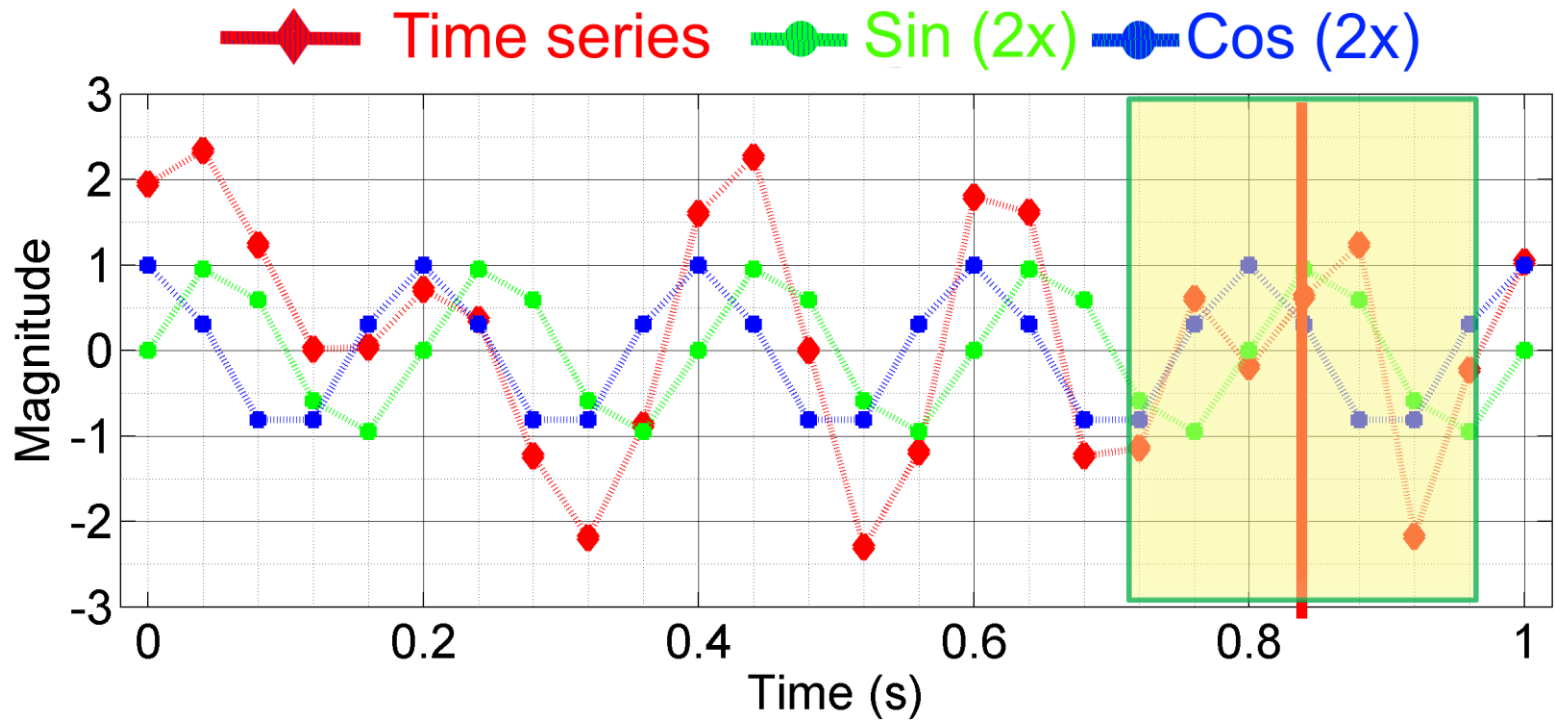# Translating window for an equally spaced time series

Translating window for an equally spaced time series

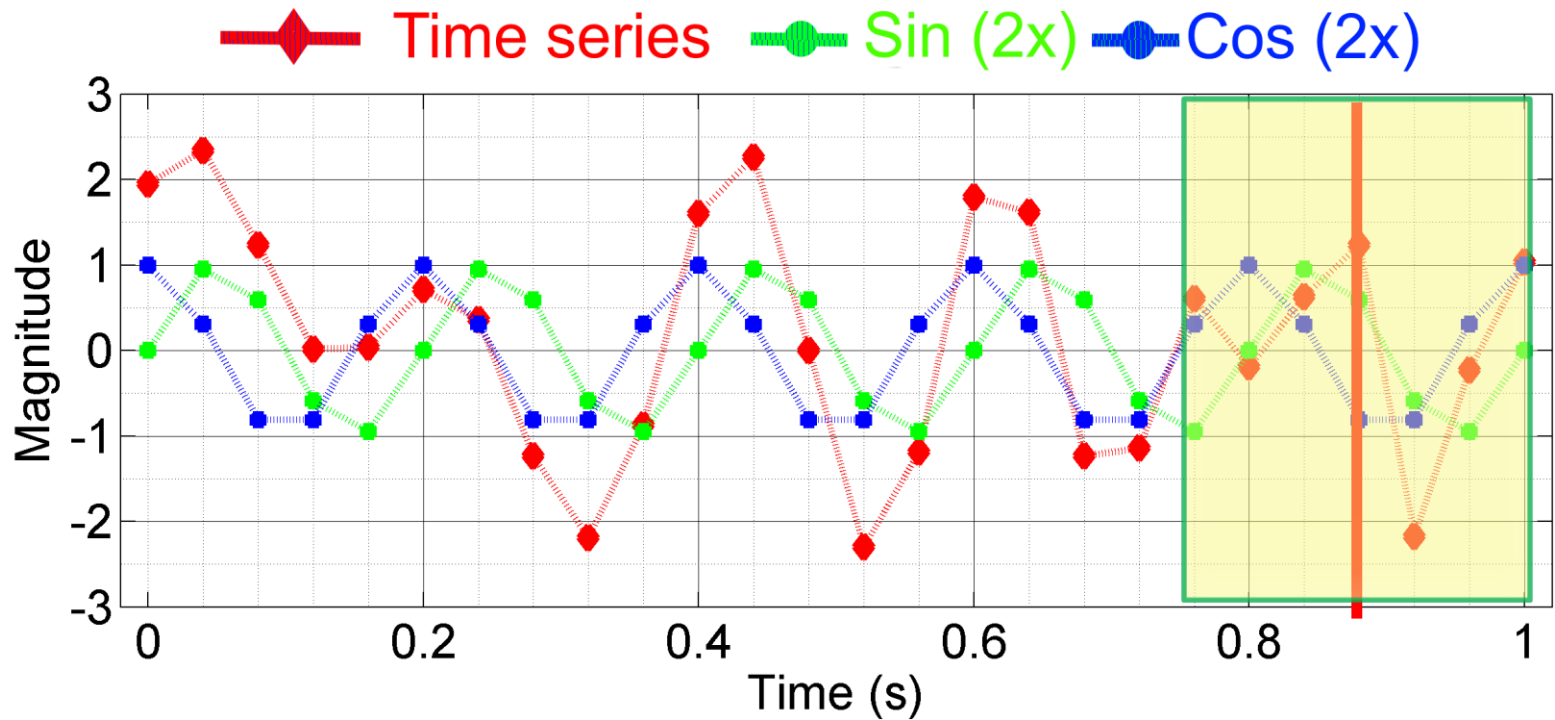# Translating window for an equally spaced time series

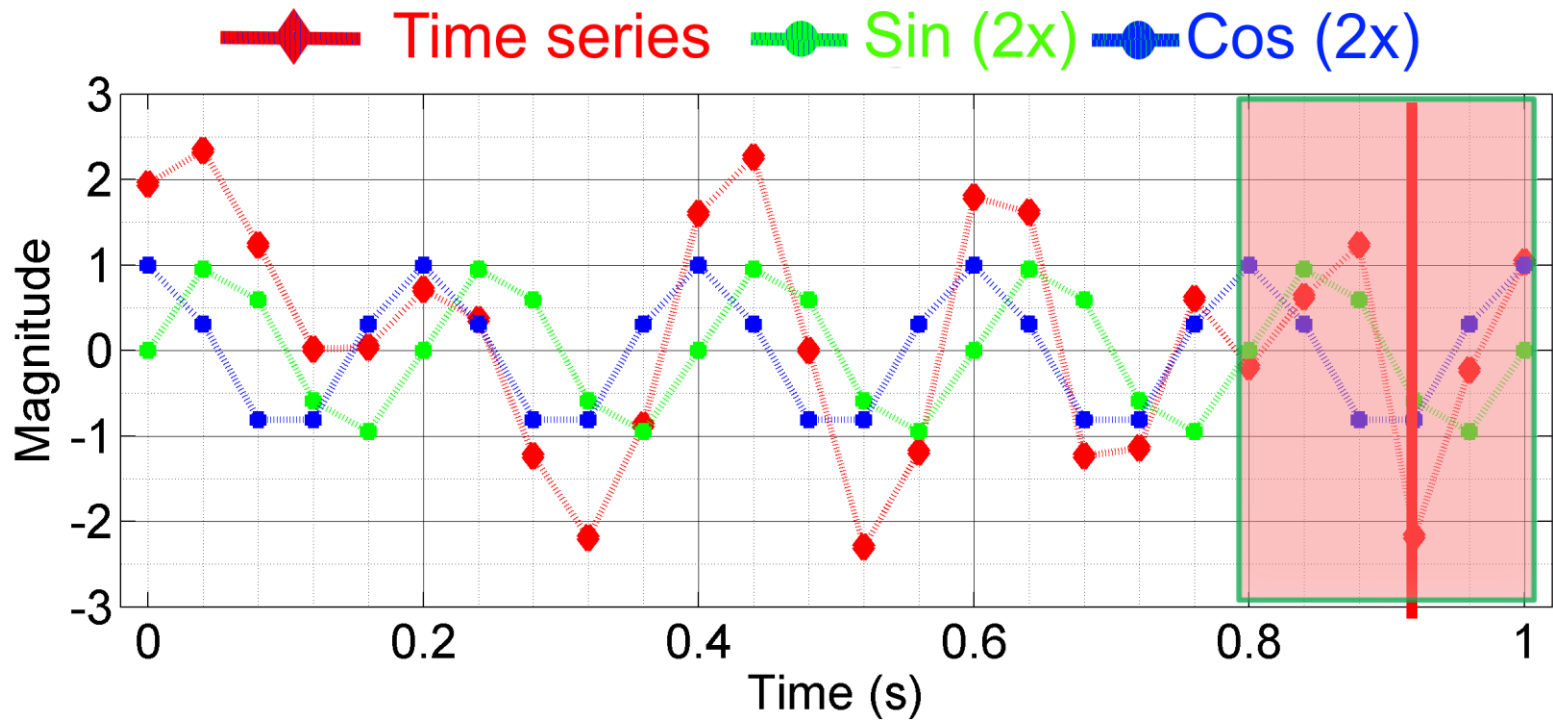# Translating window for an equally spaced time series

Translating window for an equally spaced time series

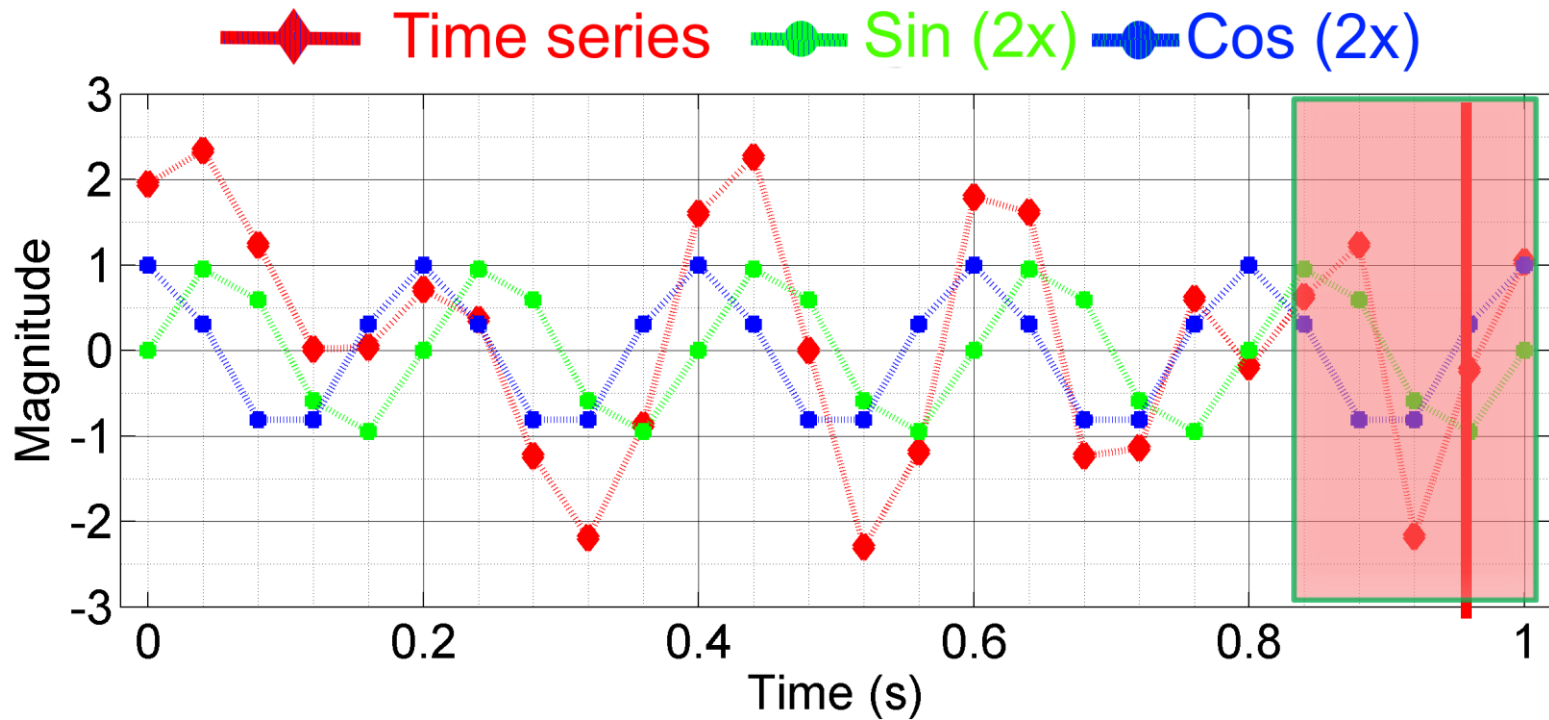# Translating window for an equally spaced time series

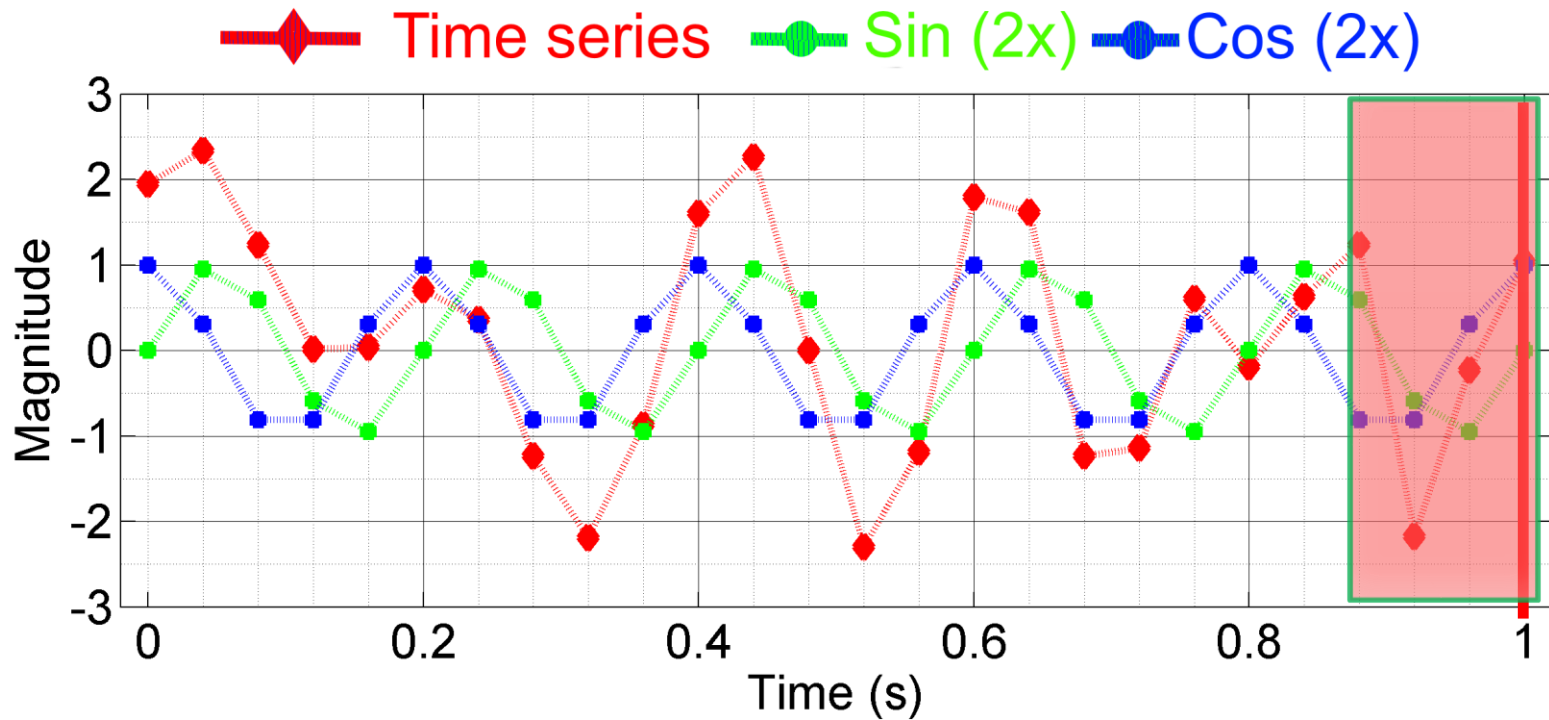# Translating window for an equally spaced time series

Translating window for an equally spaced time series

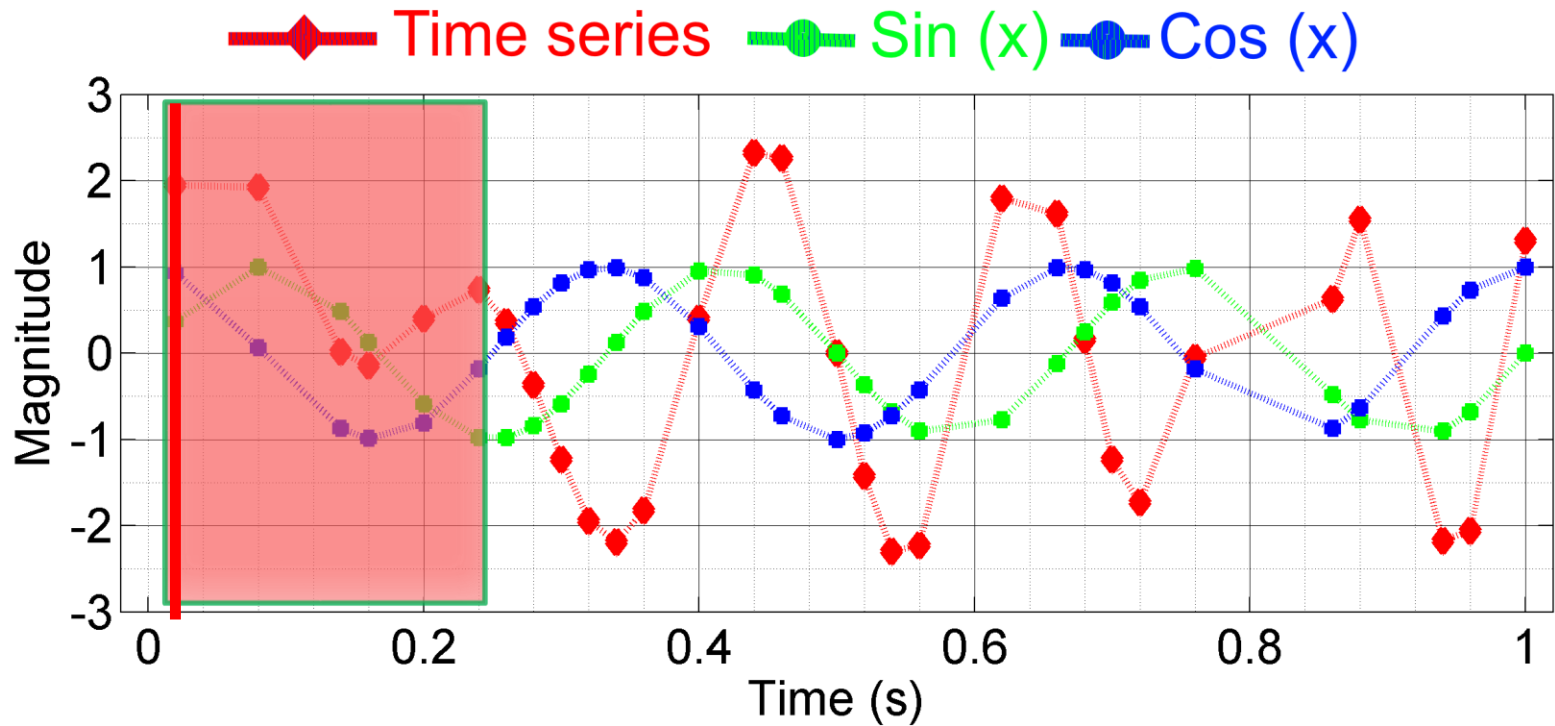# Translating window for an equally spaced time series

Translating window for an equally spaced time series

Translating window for an equally spaced time series

Translating window for an equally spaced time series

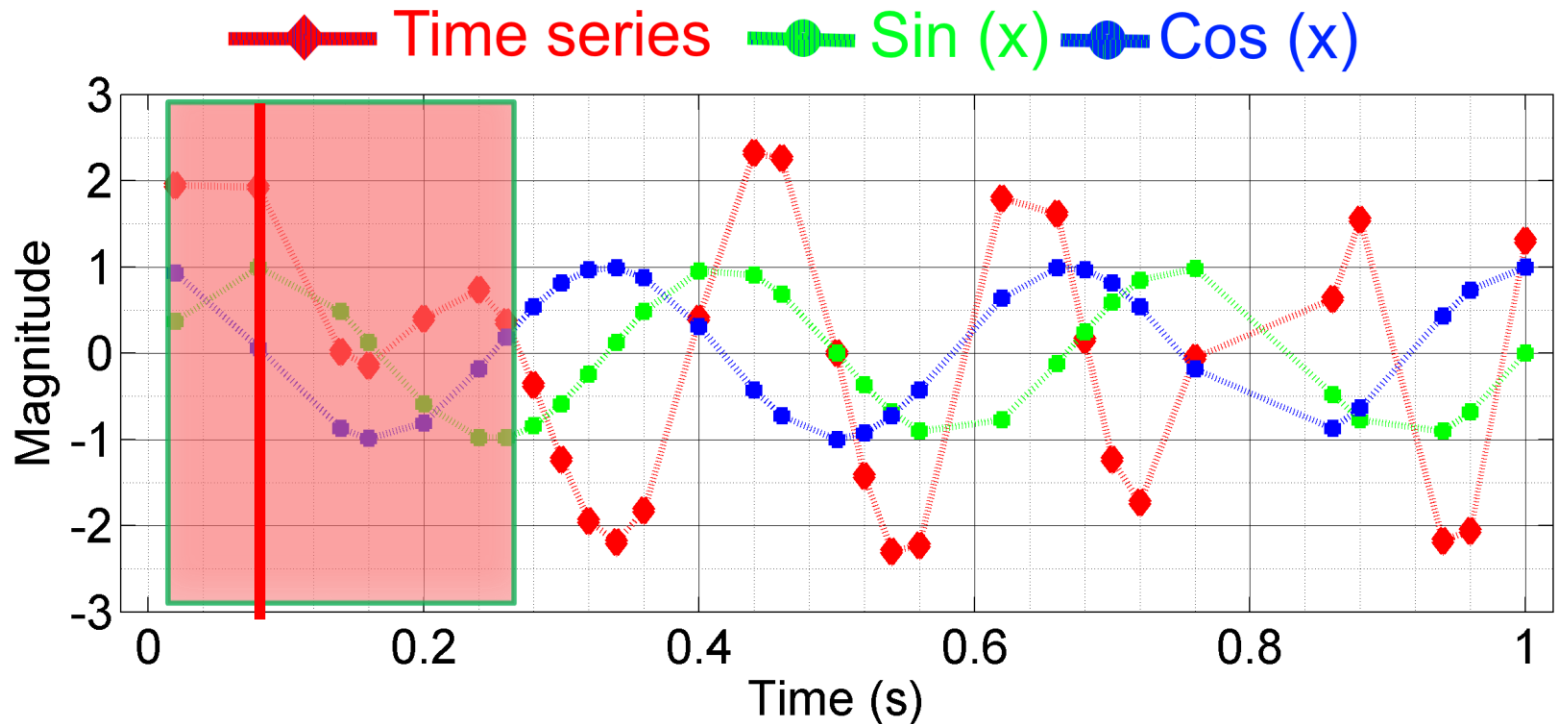# Translating window for an equally spaced time series

Translating window for an equally spaced time series

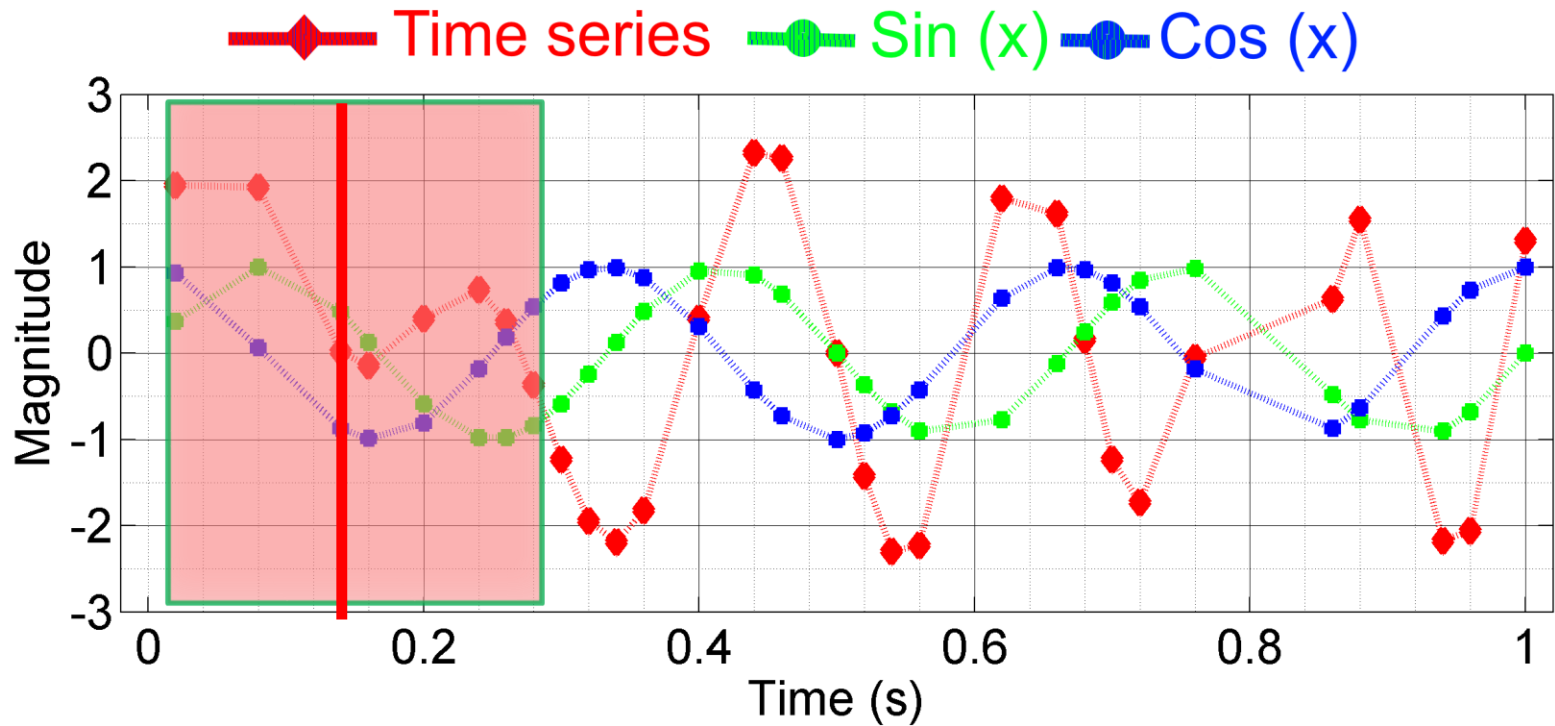# Translating window for an equally spaced time series

Translating window for an equally spaced time series

Translating window for an equally spaced time series
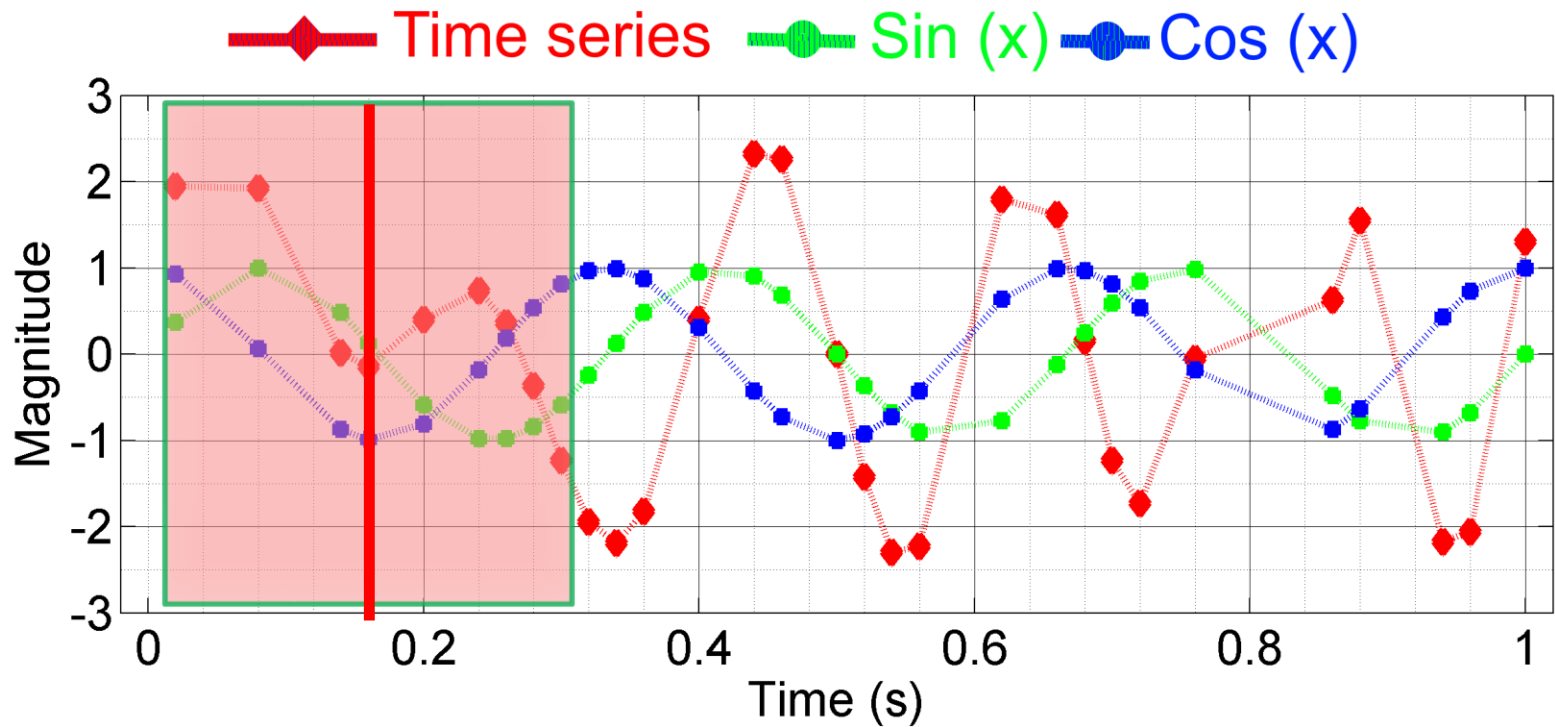
Translating window for an equally spaced time series

Translating window for an equally spaced time series
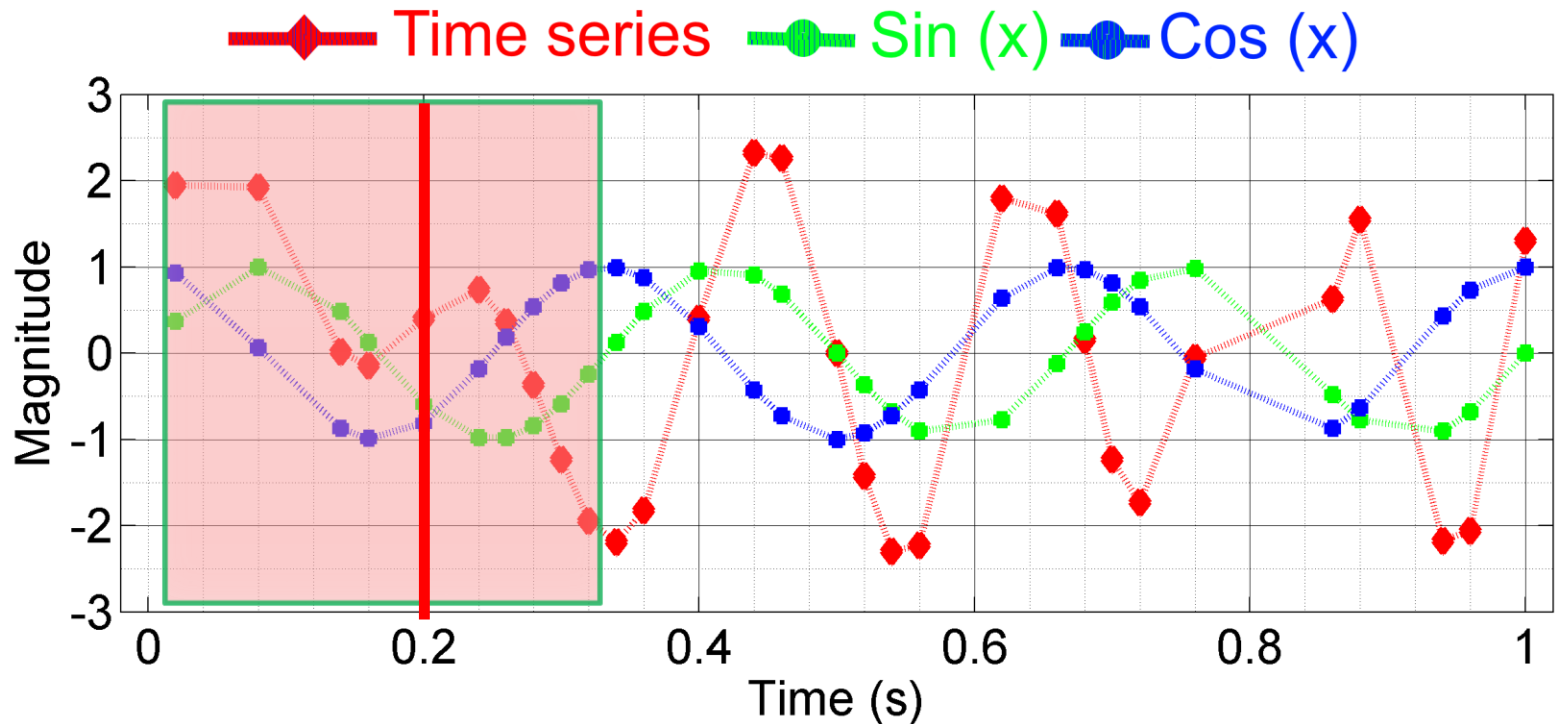
Translating window for an equally spaced time series

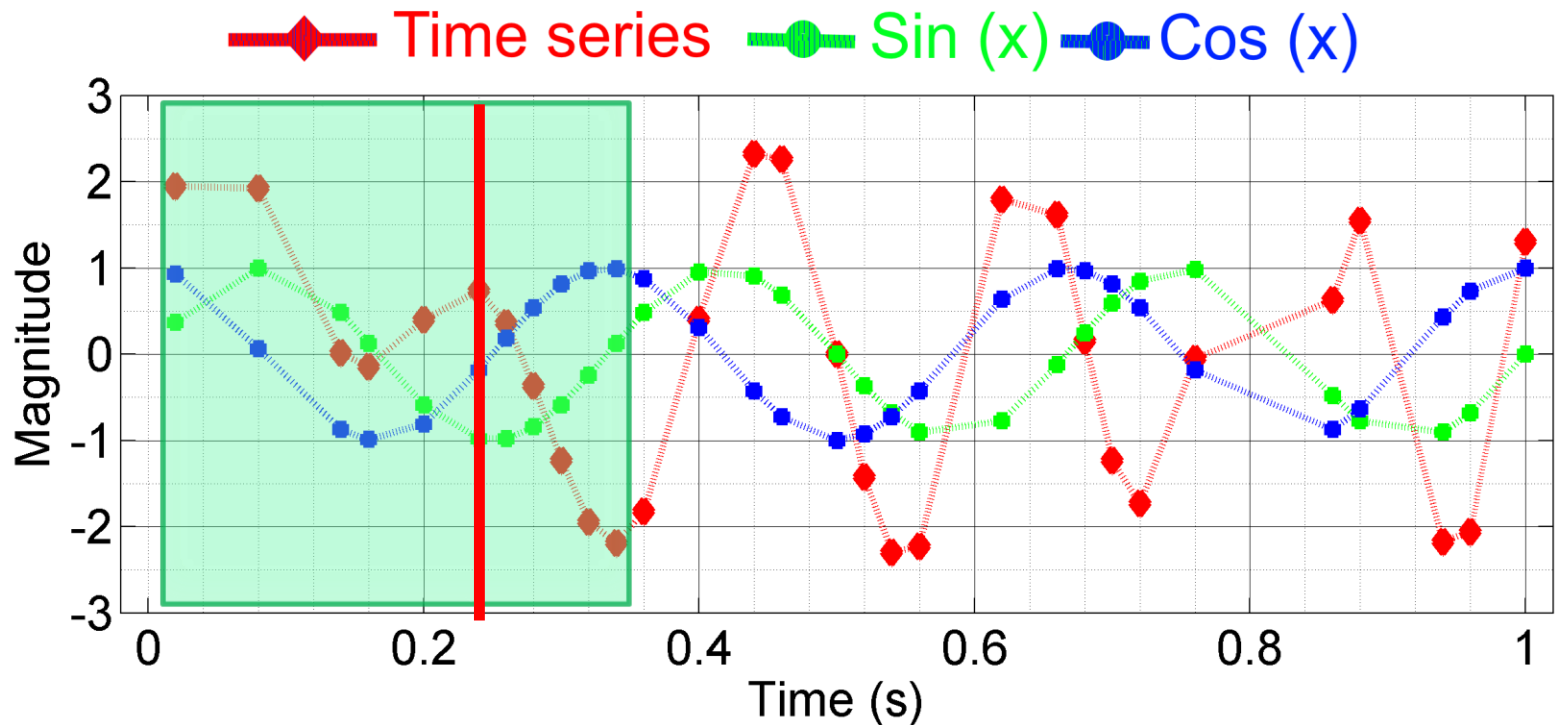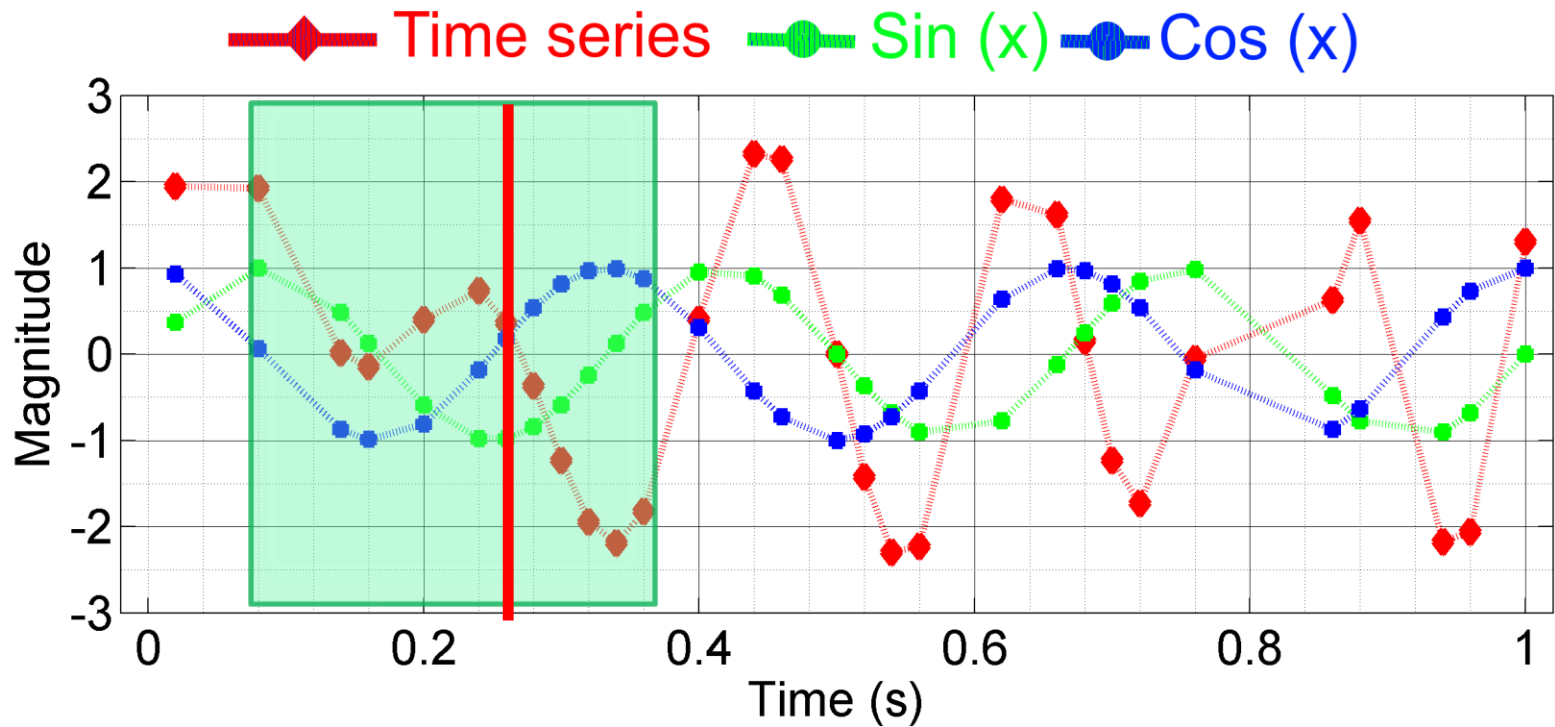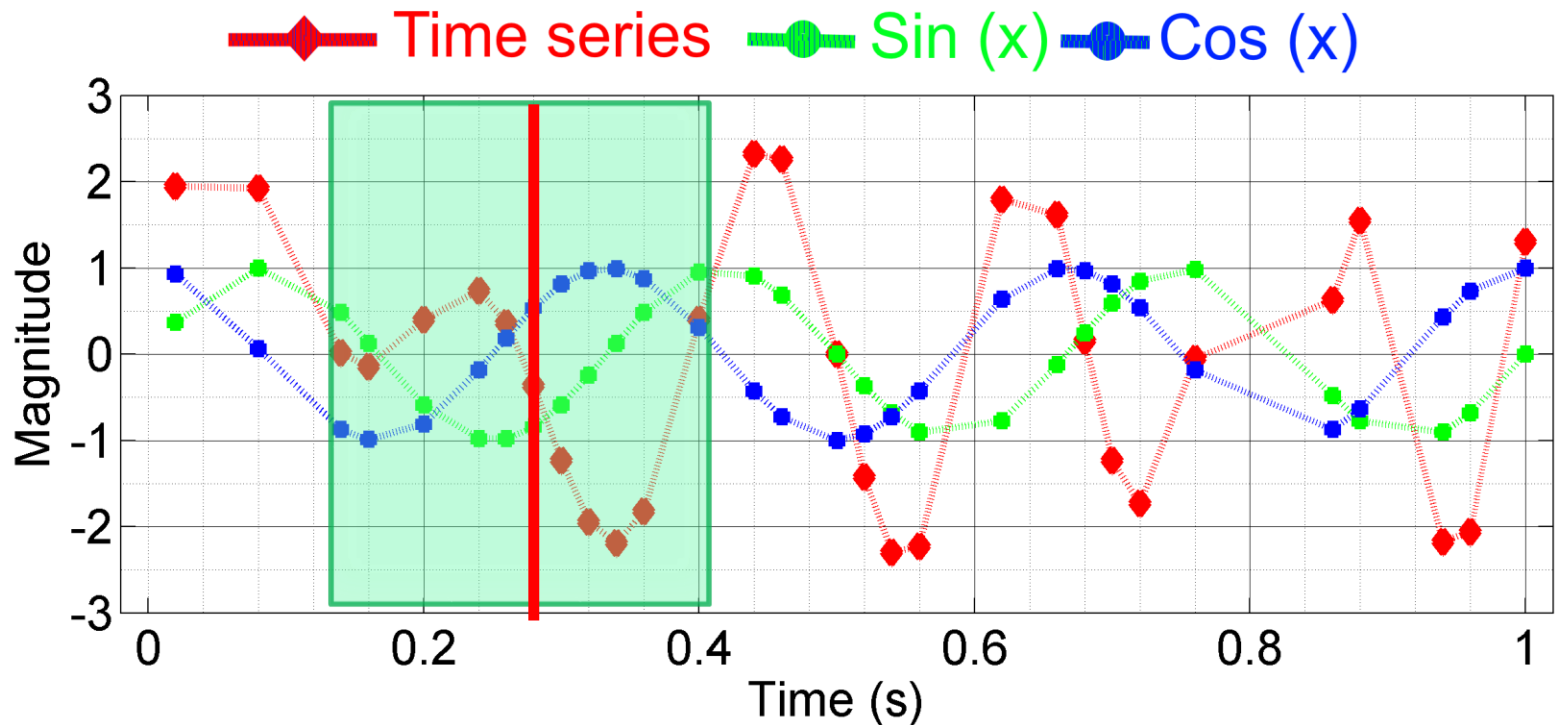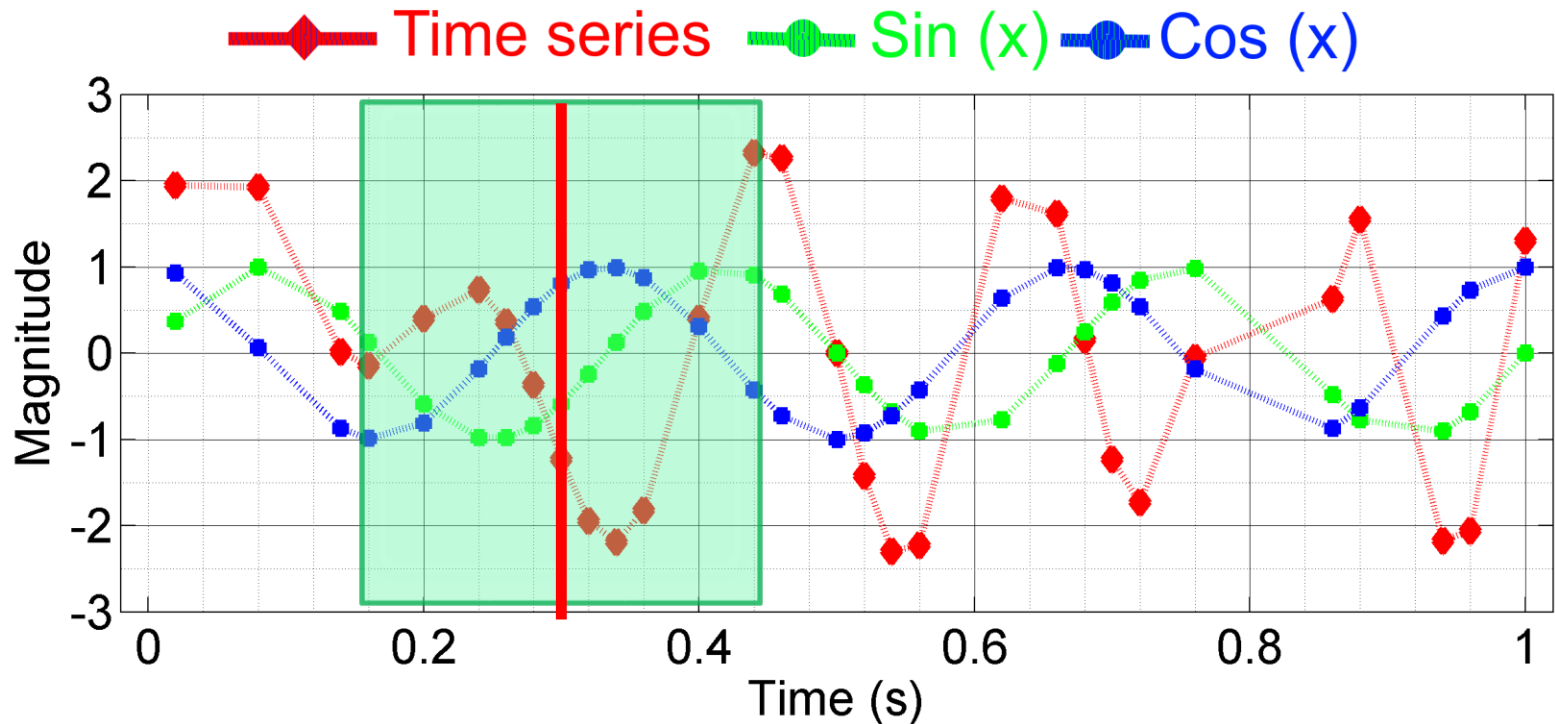# Translating window for an equally spaced time series

Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series
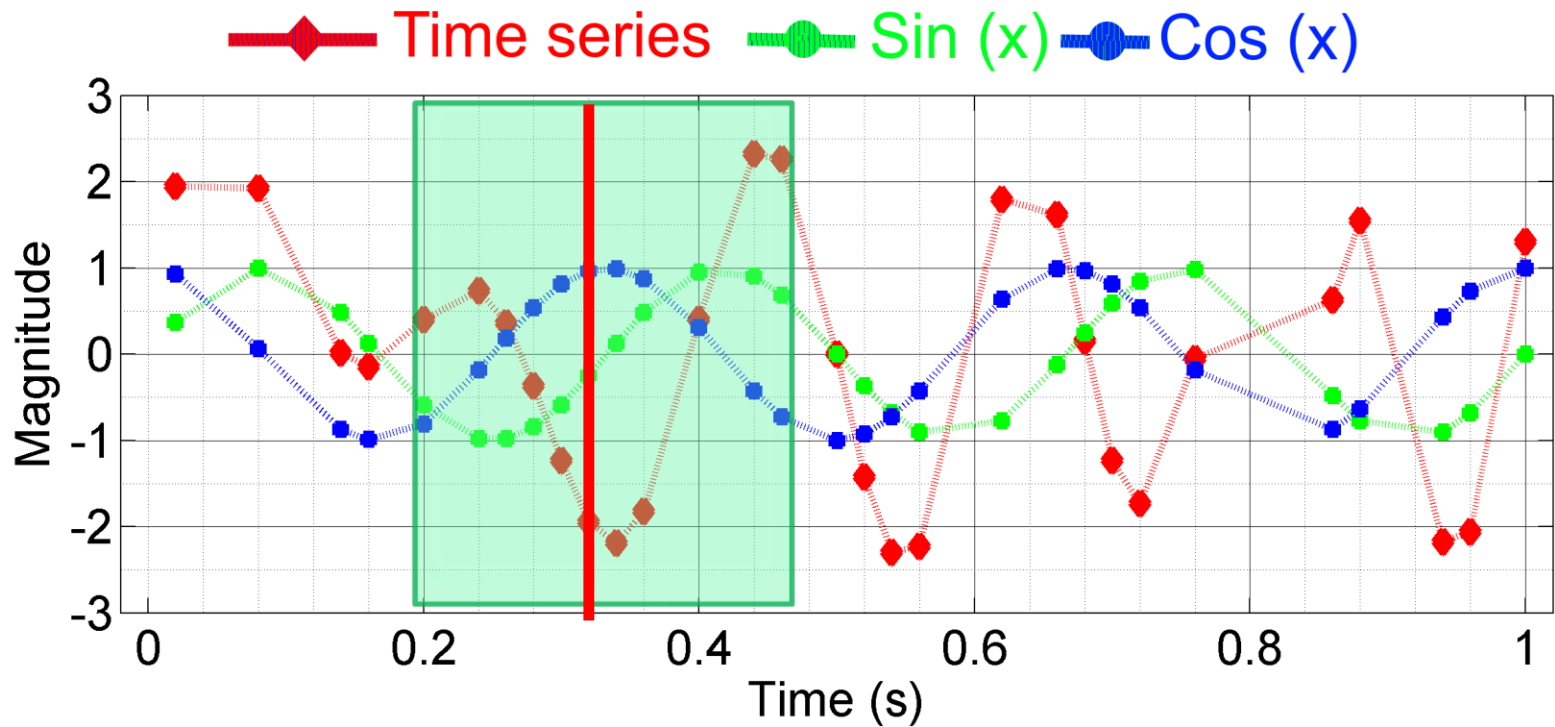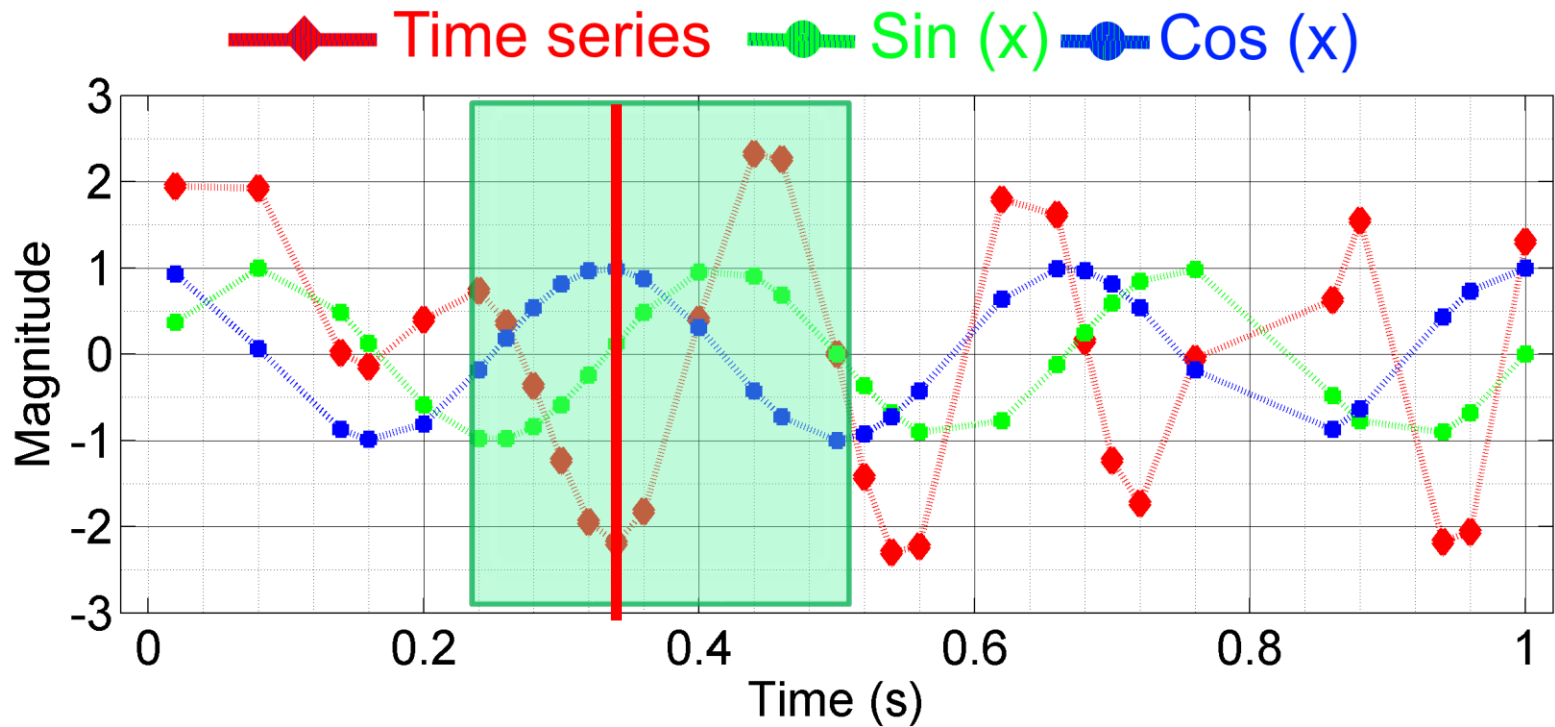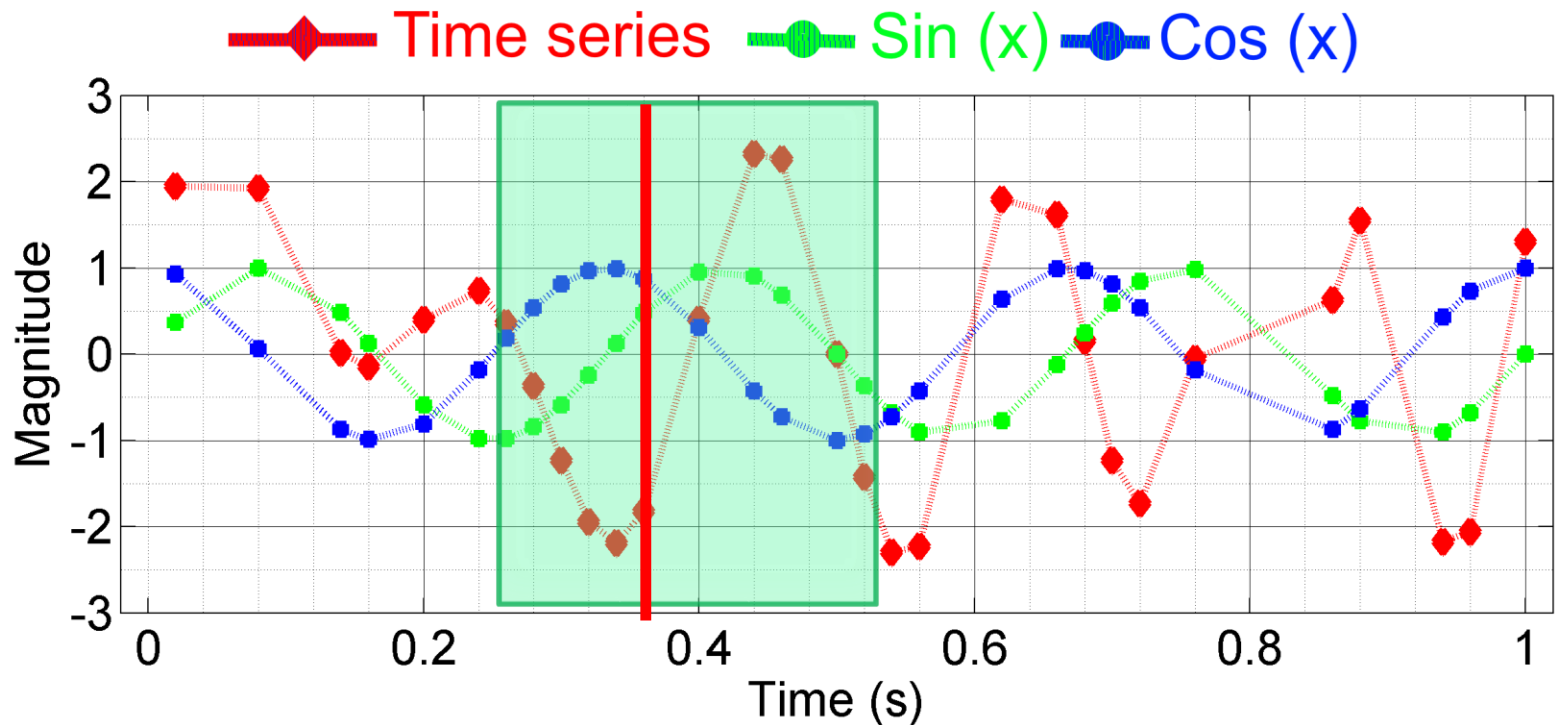
Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

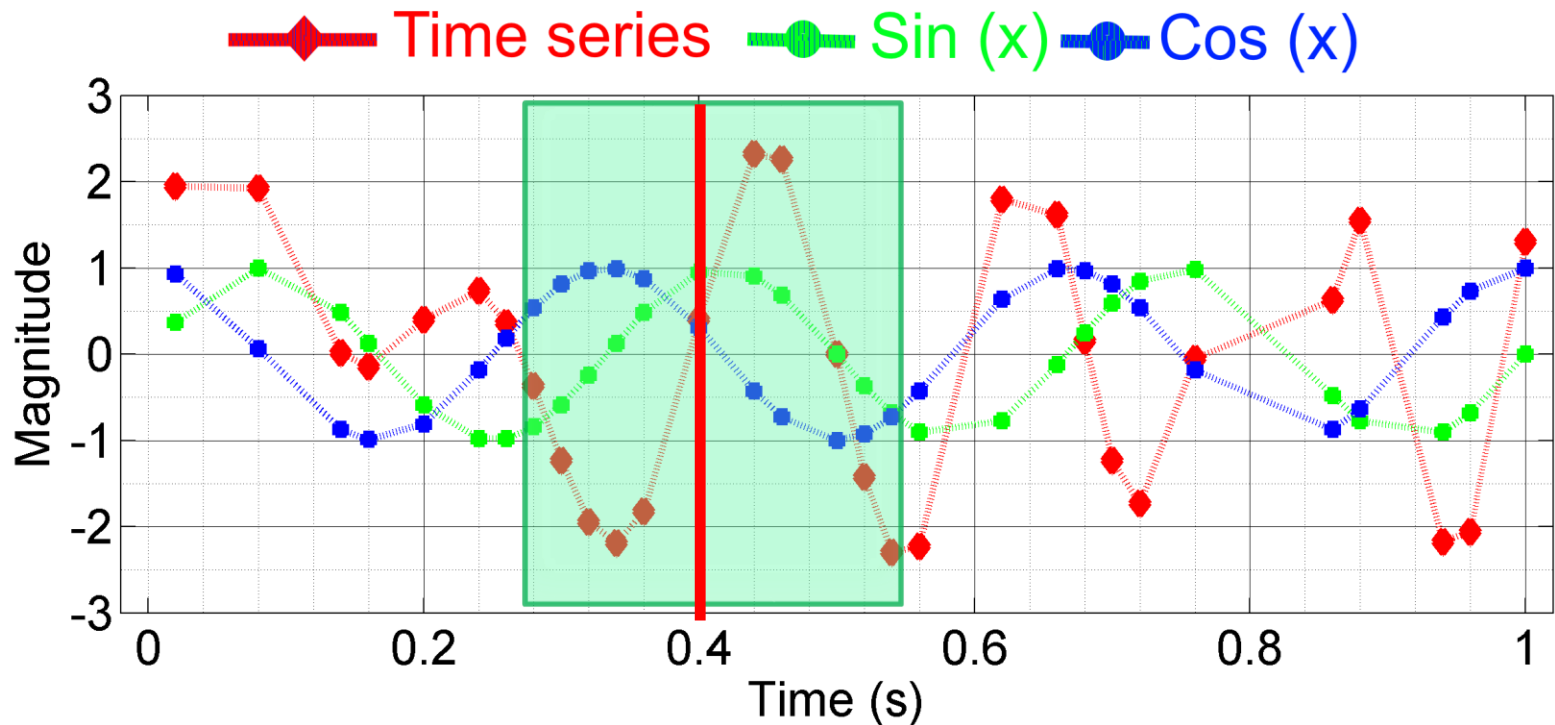# Translating window for an **unequally spaced** time series

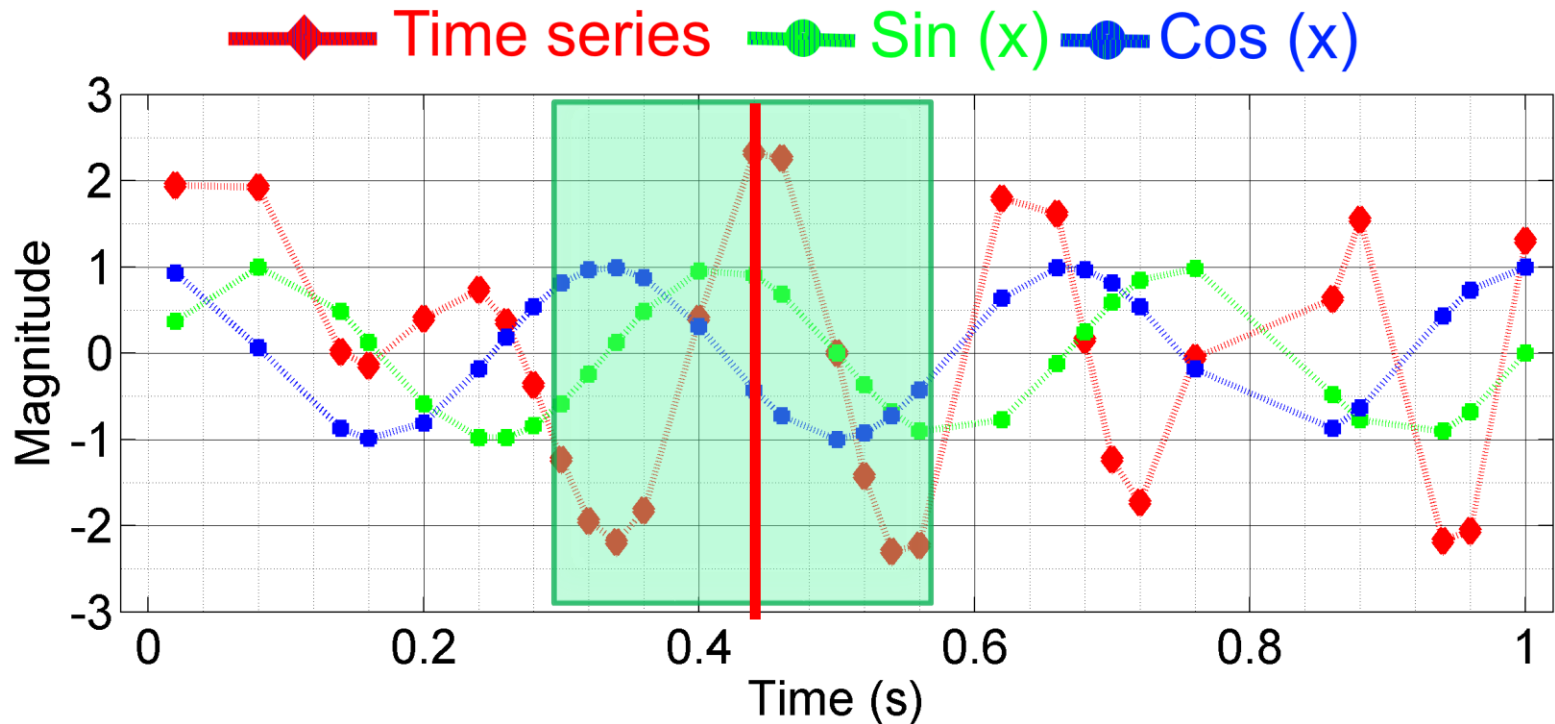# Translating window for an **unequally spaced** time series

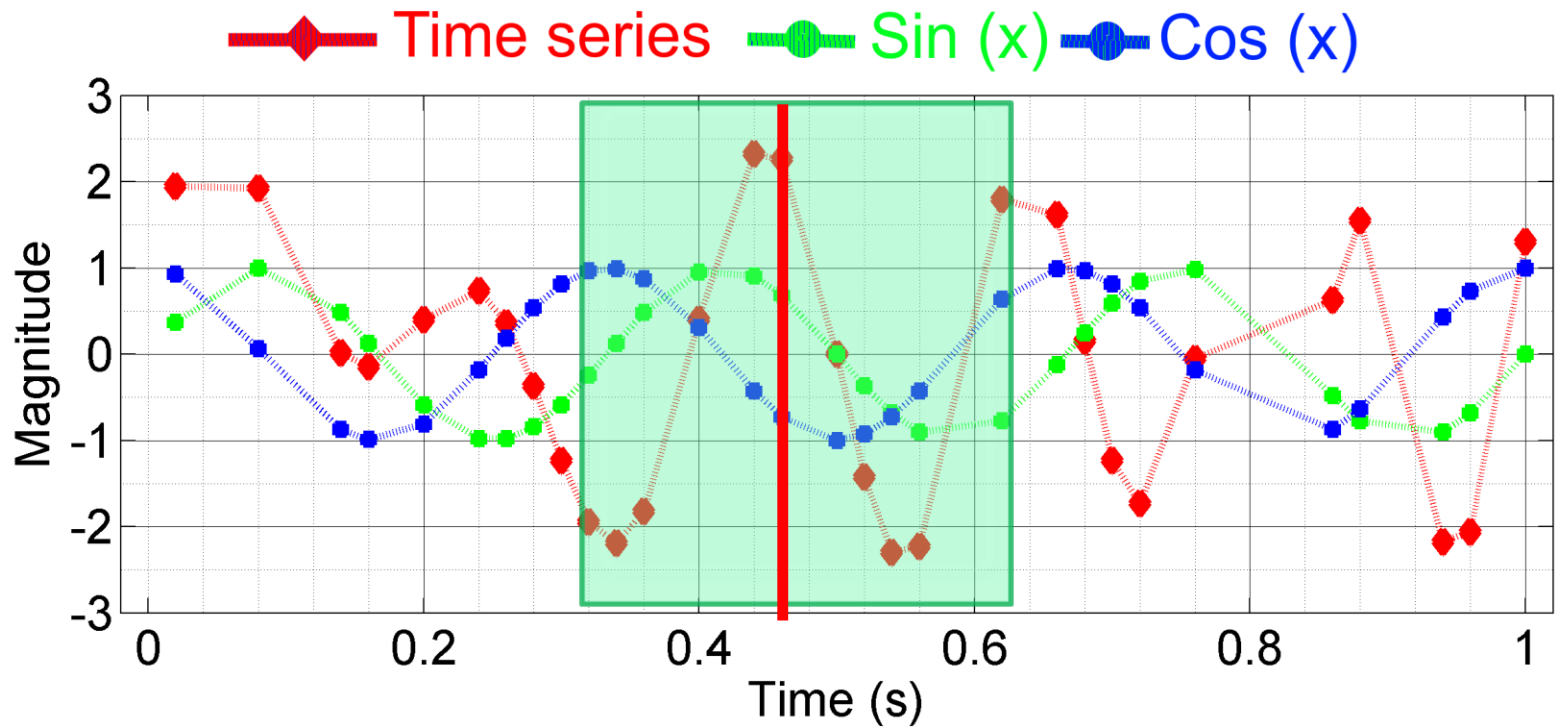# Translating window for an **unequally spaced** time series

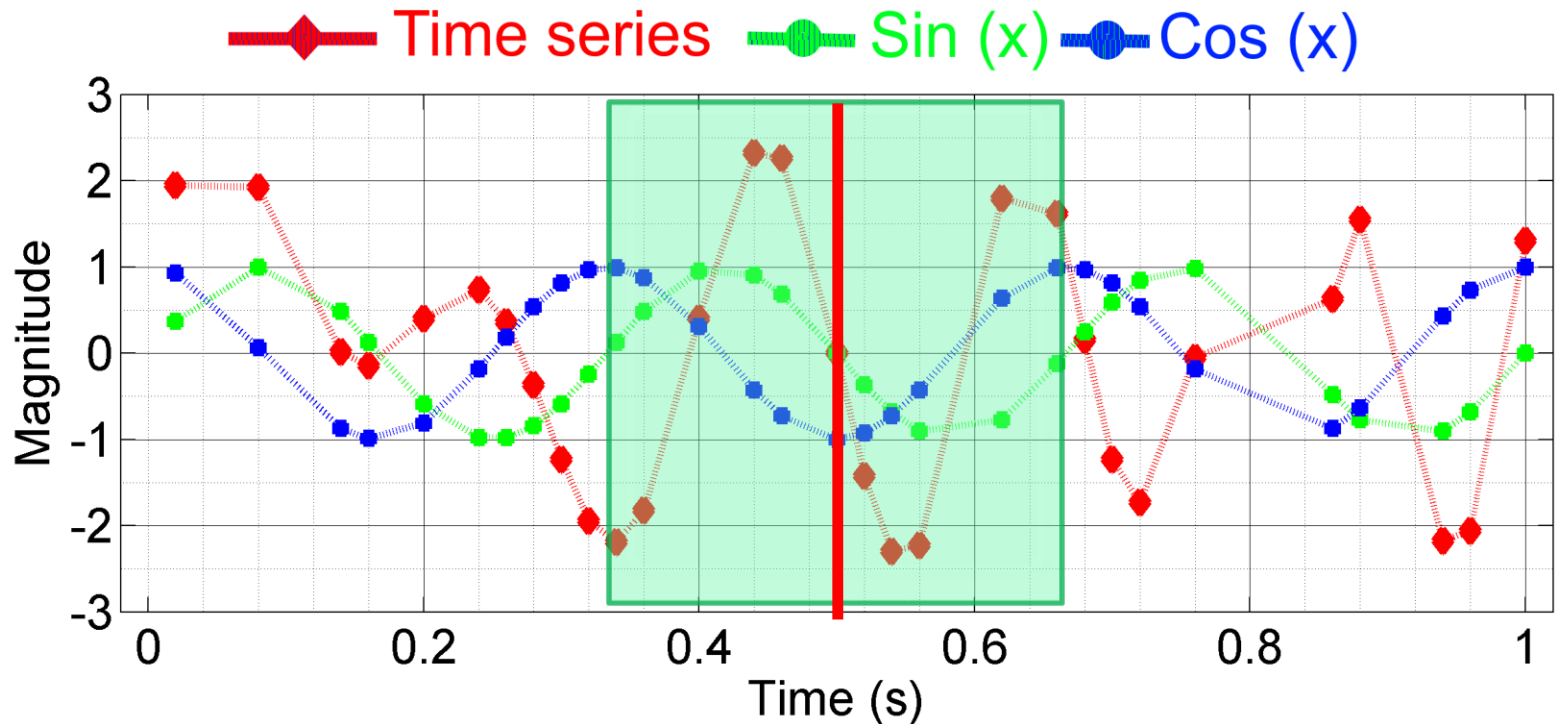# Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

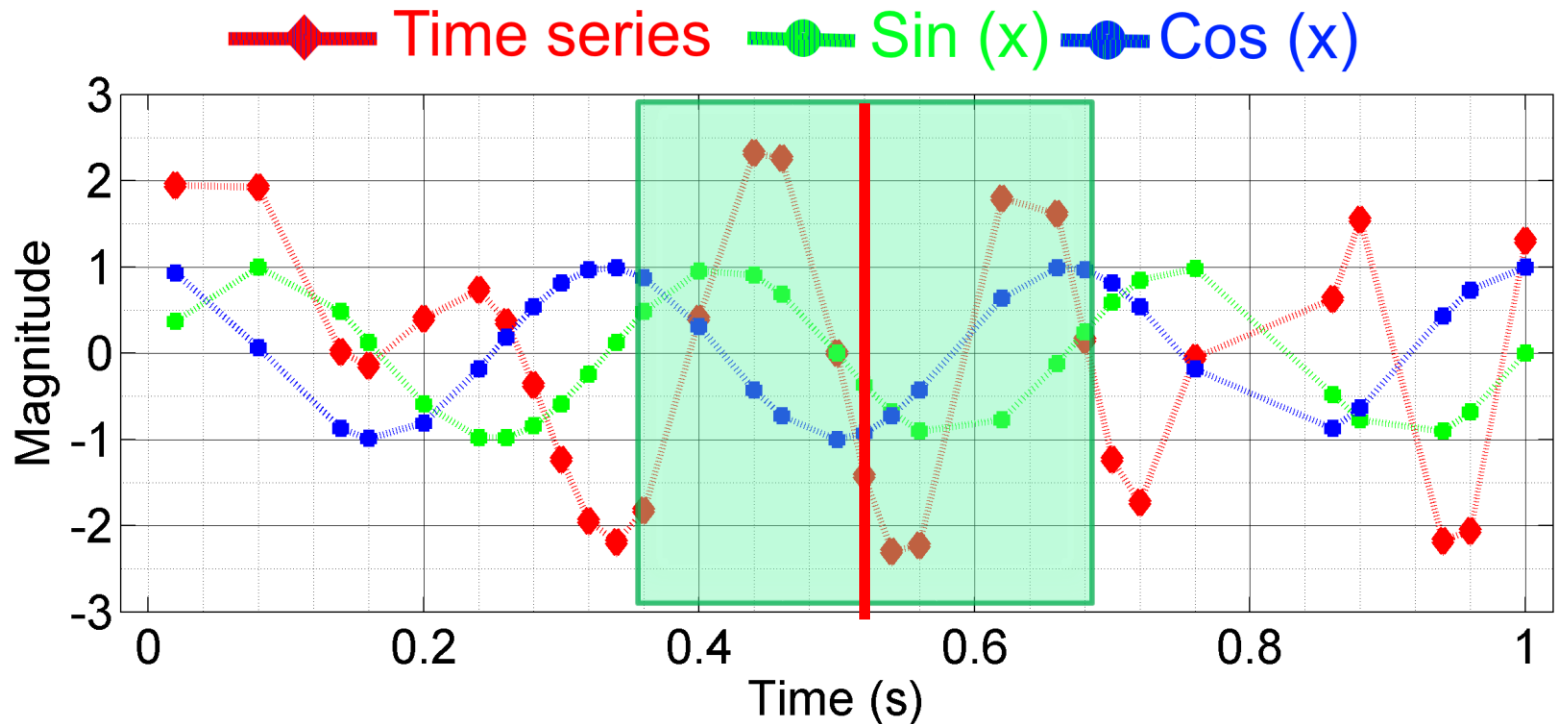# Translating window for an **unequally spaced** time series

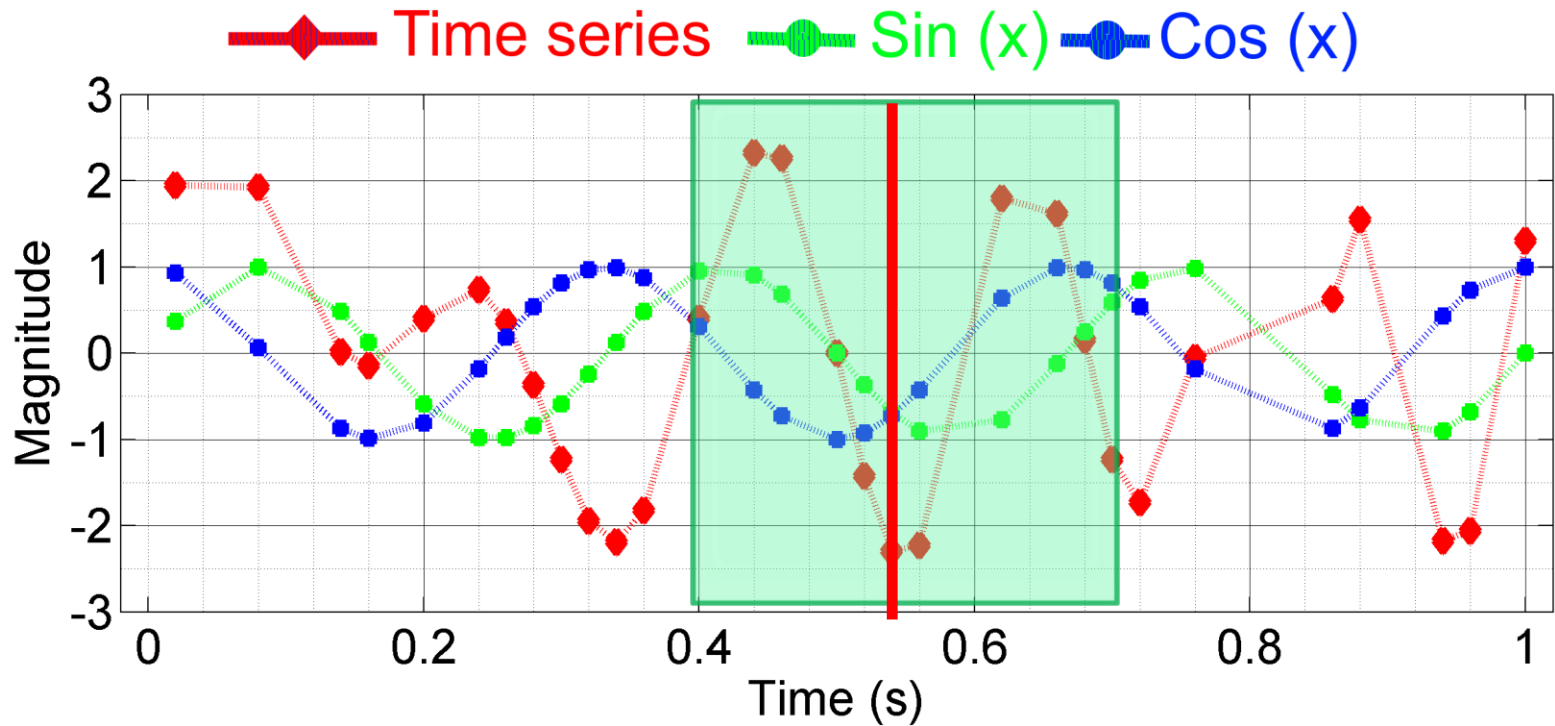# Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

Translating window for an
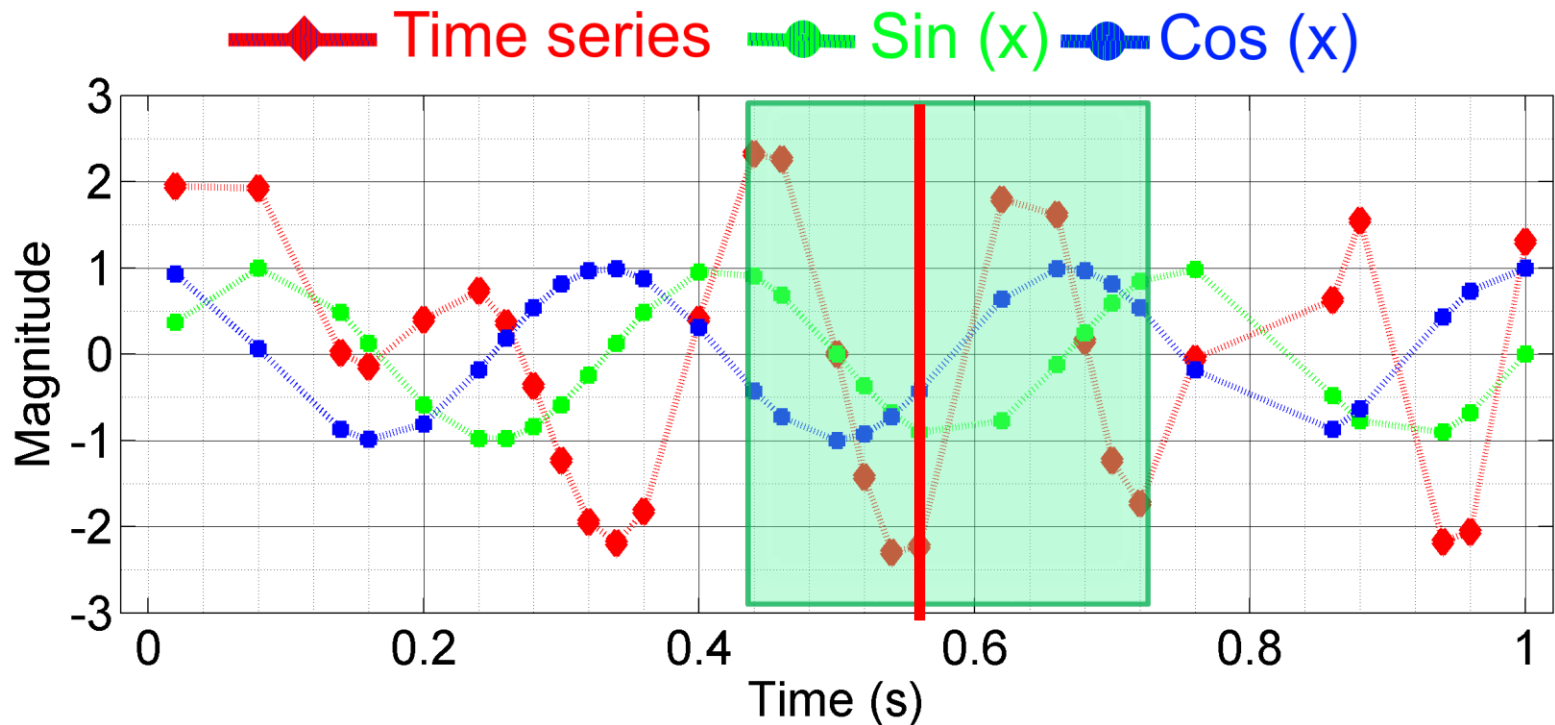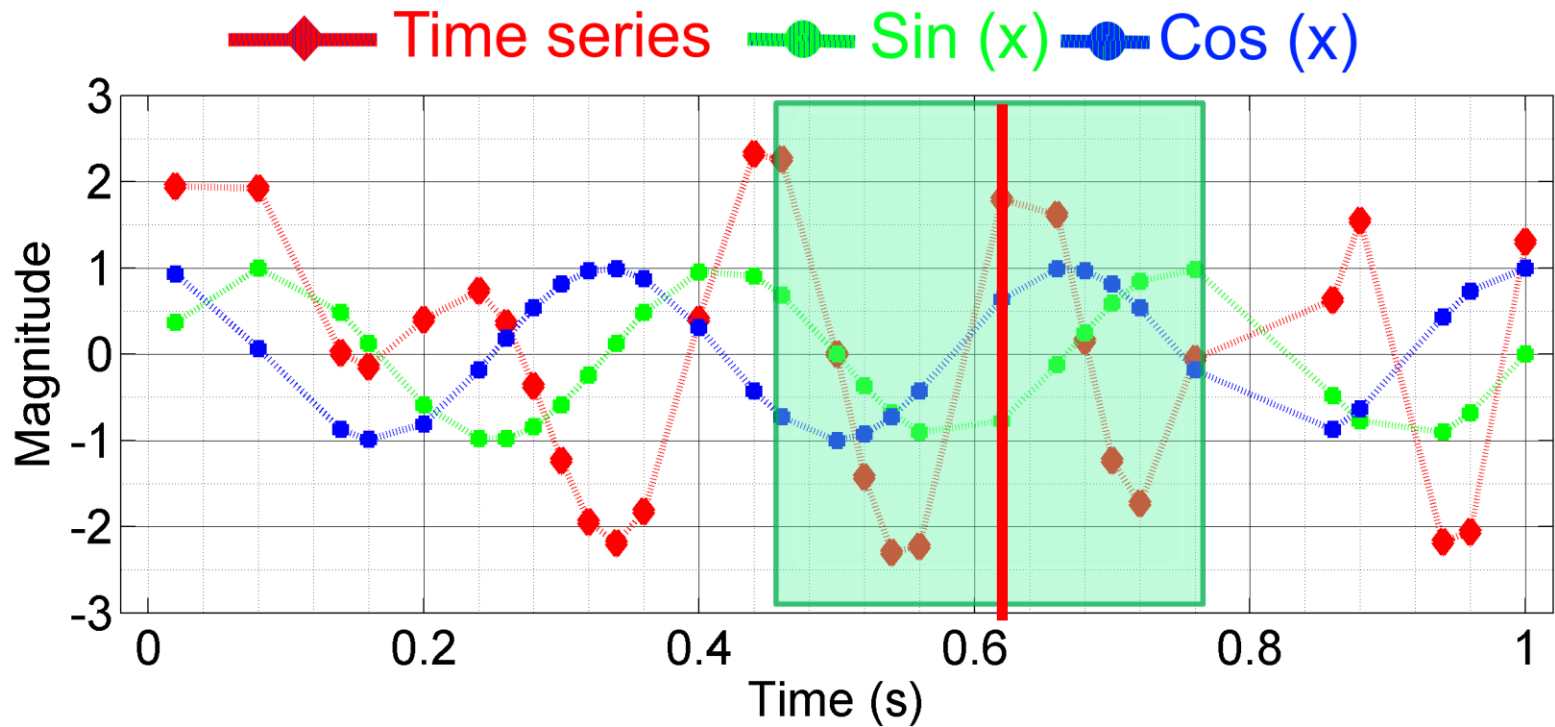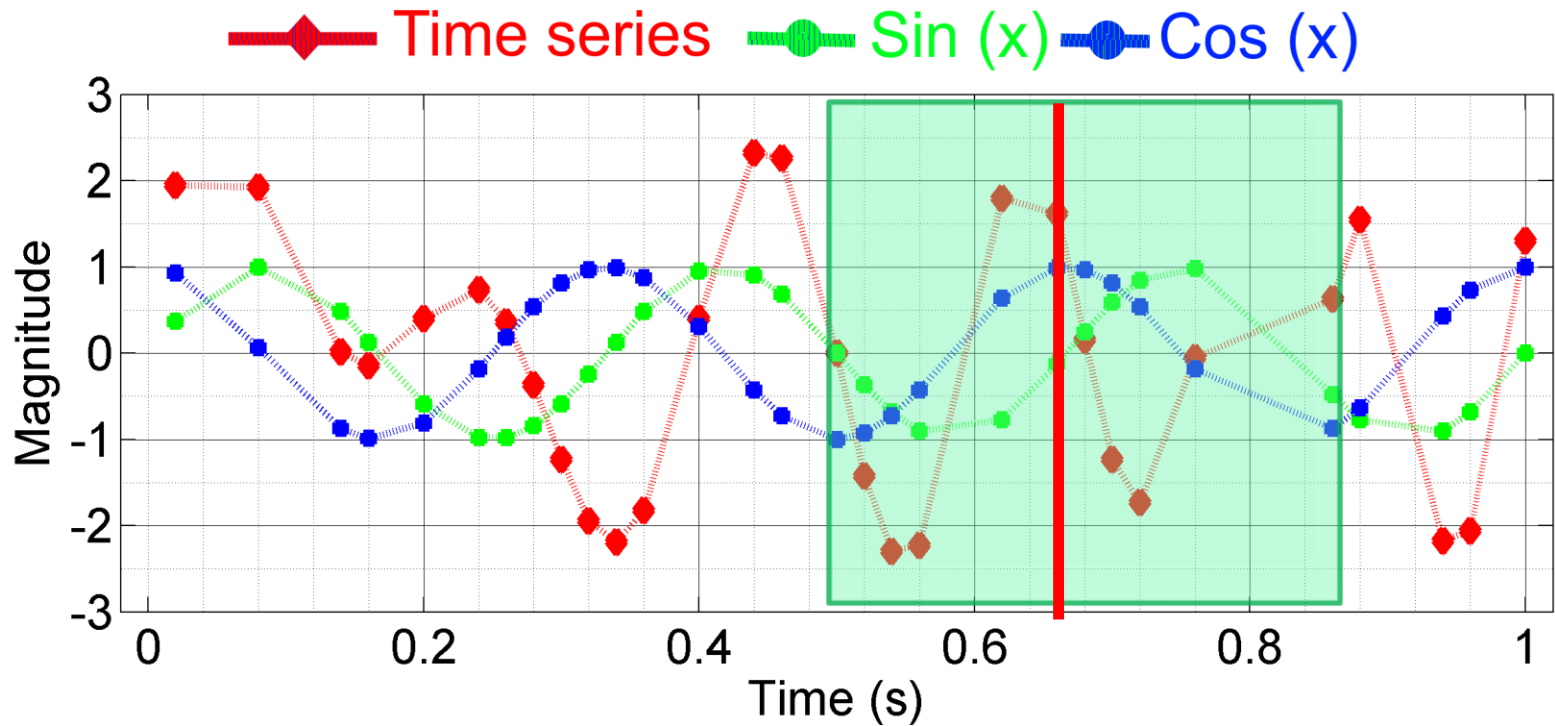## **unequally spaced** time series

Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series
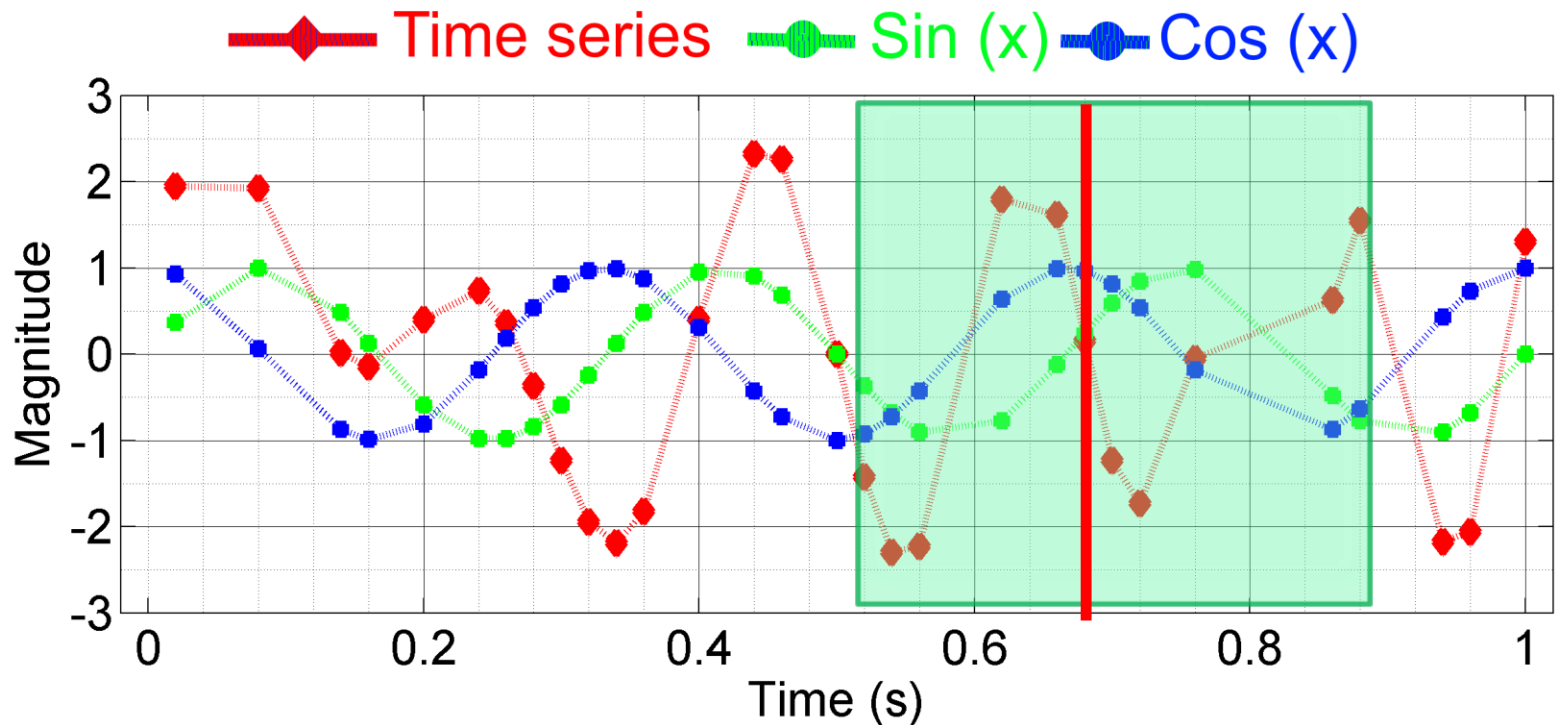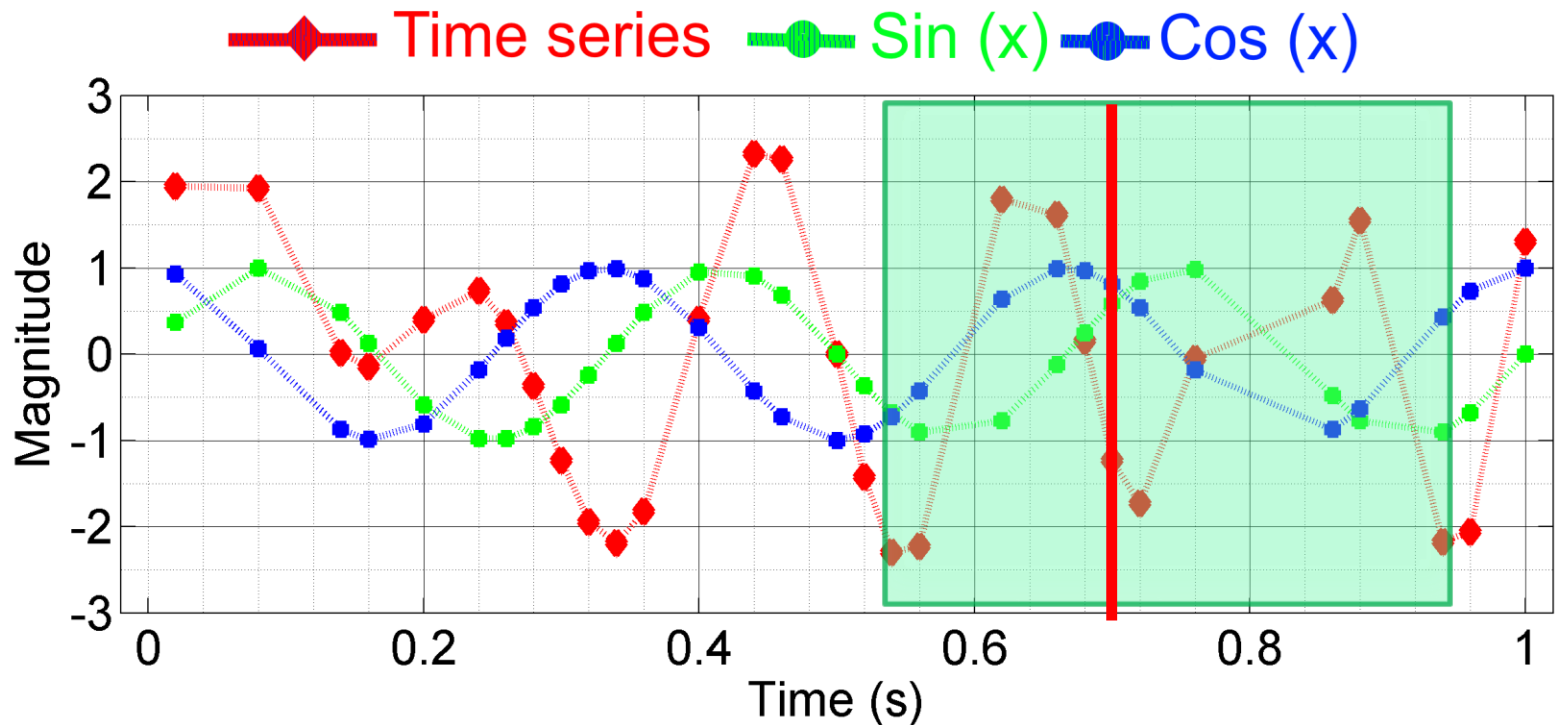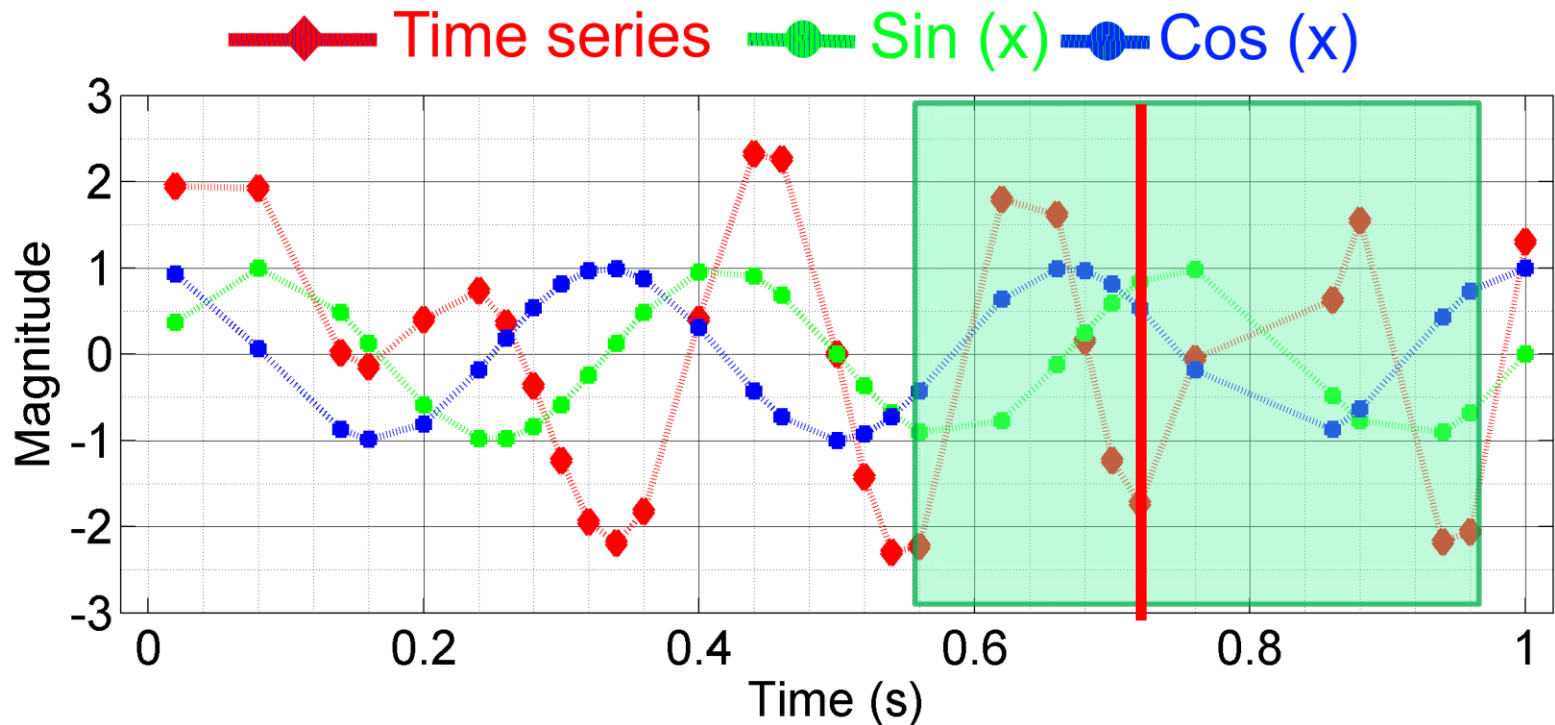
Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

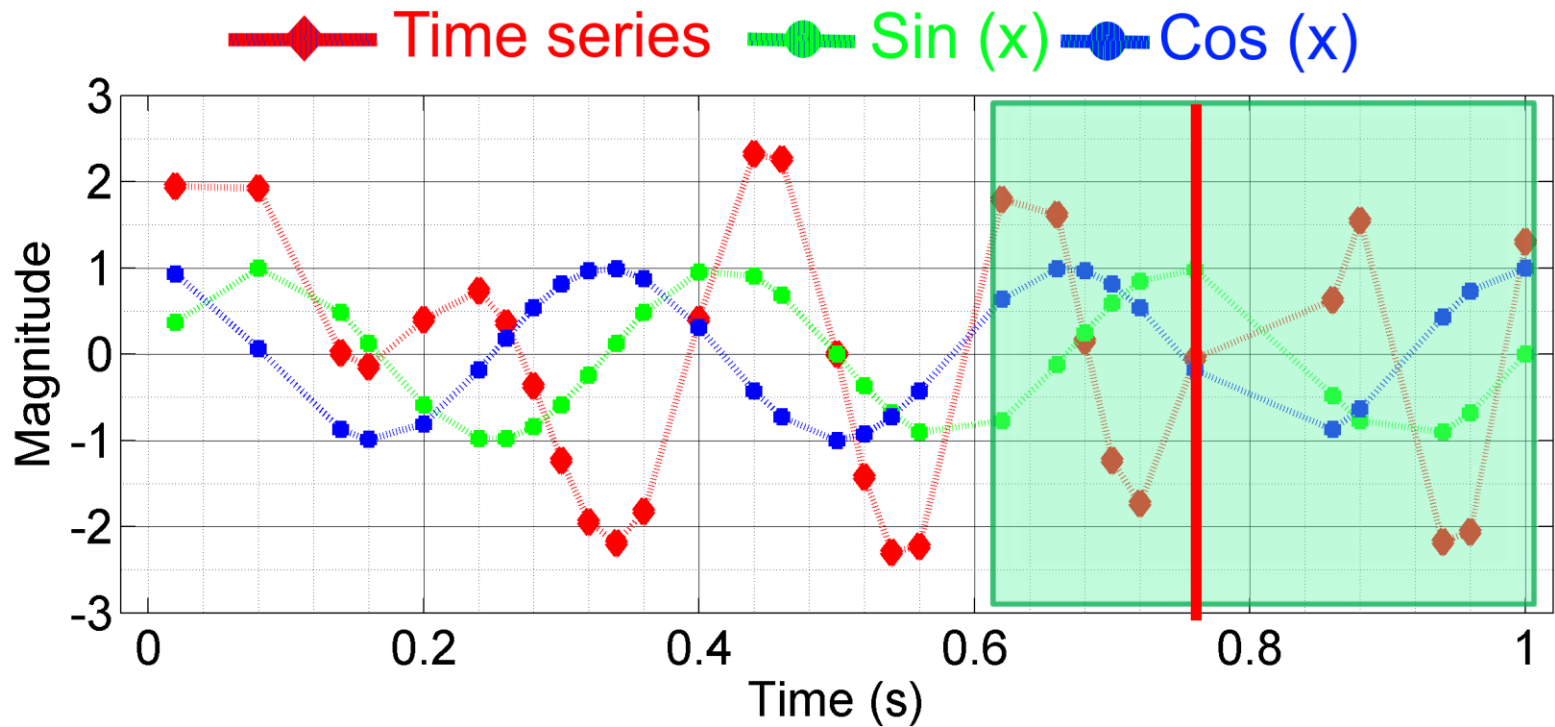# Translating window for an **unequally spaced** time series

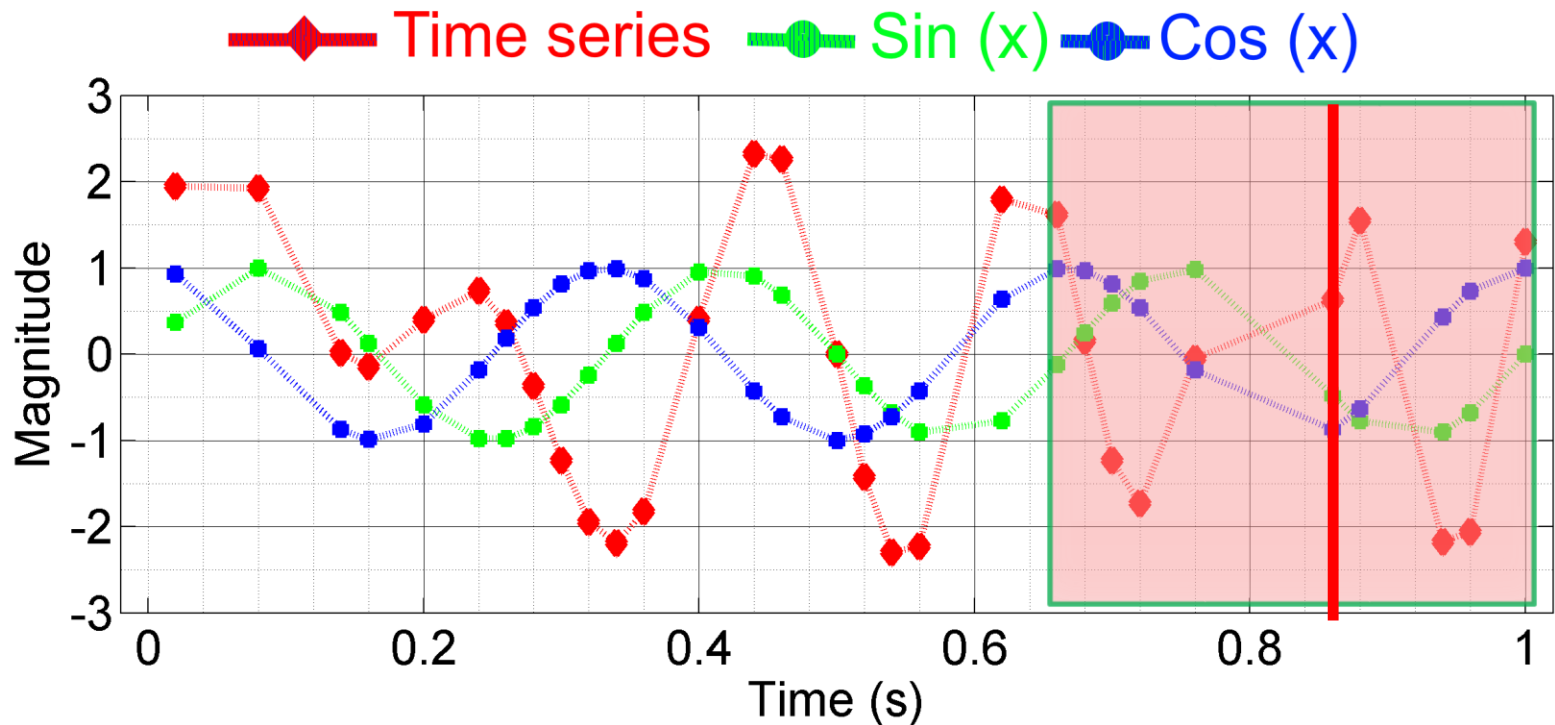# Translating window for an **unequally spaced** time series

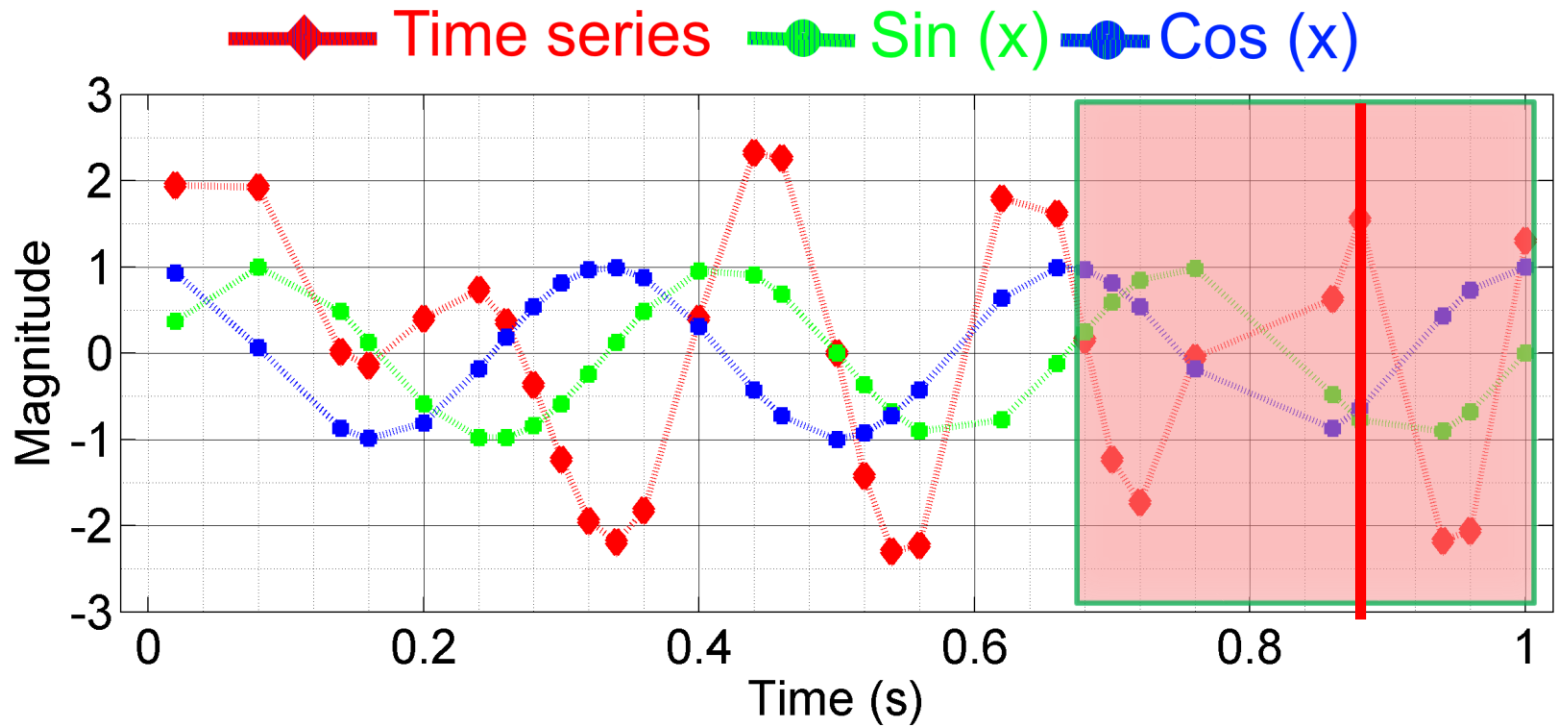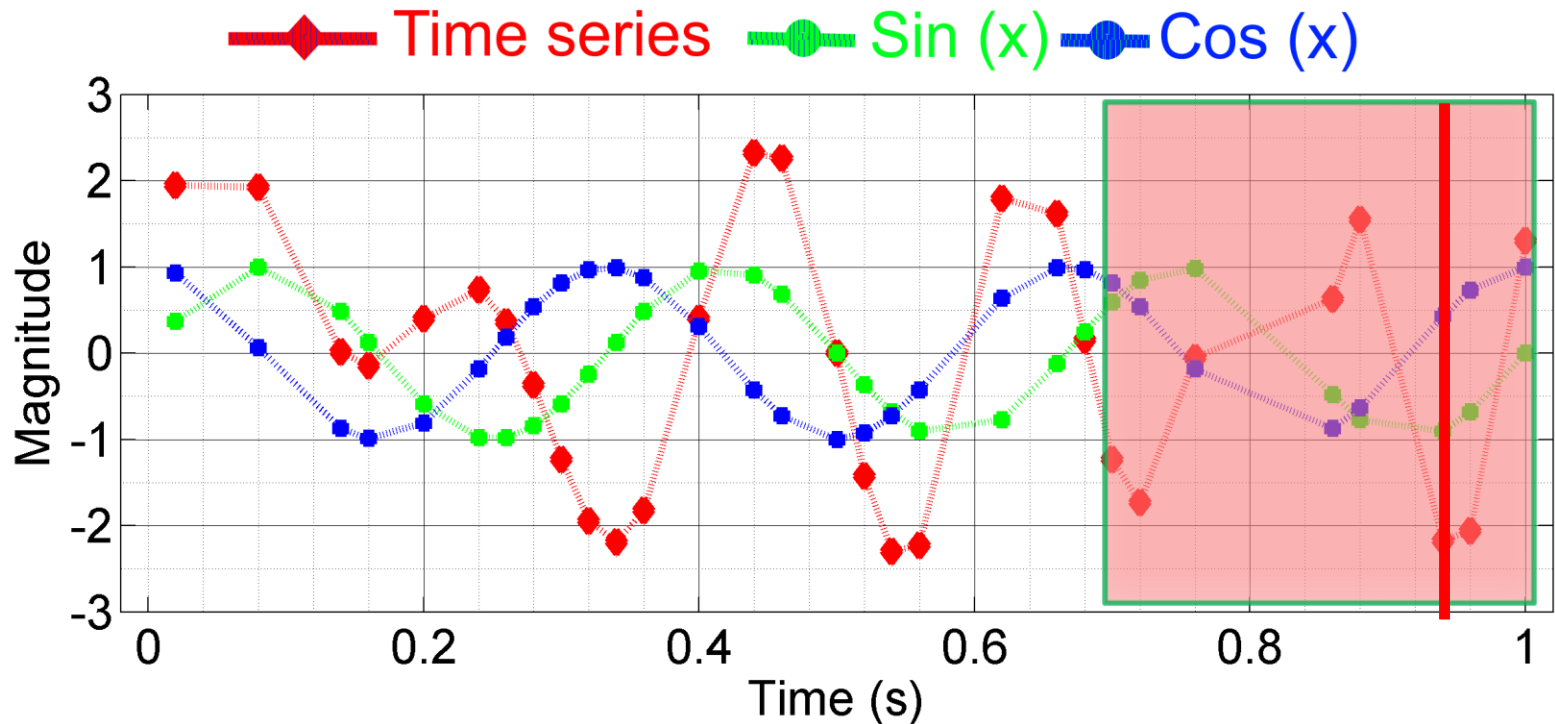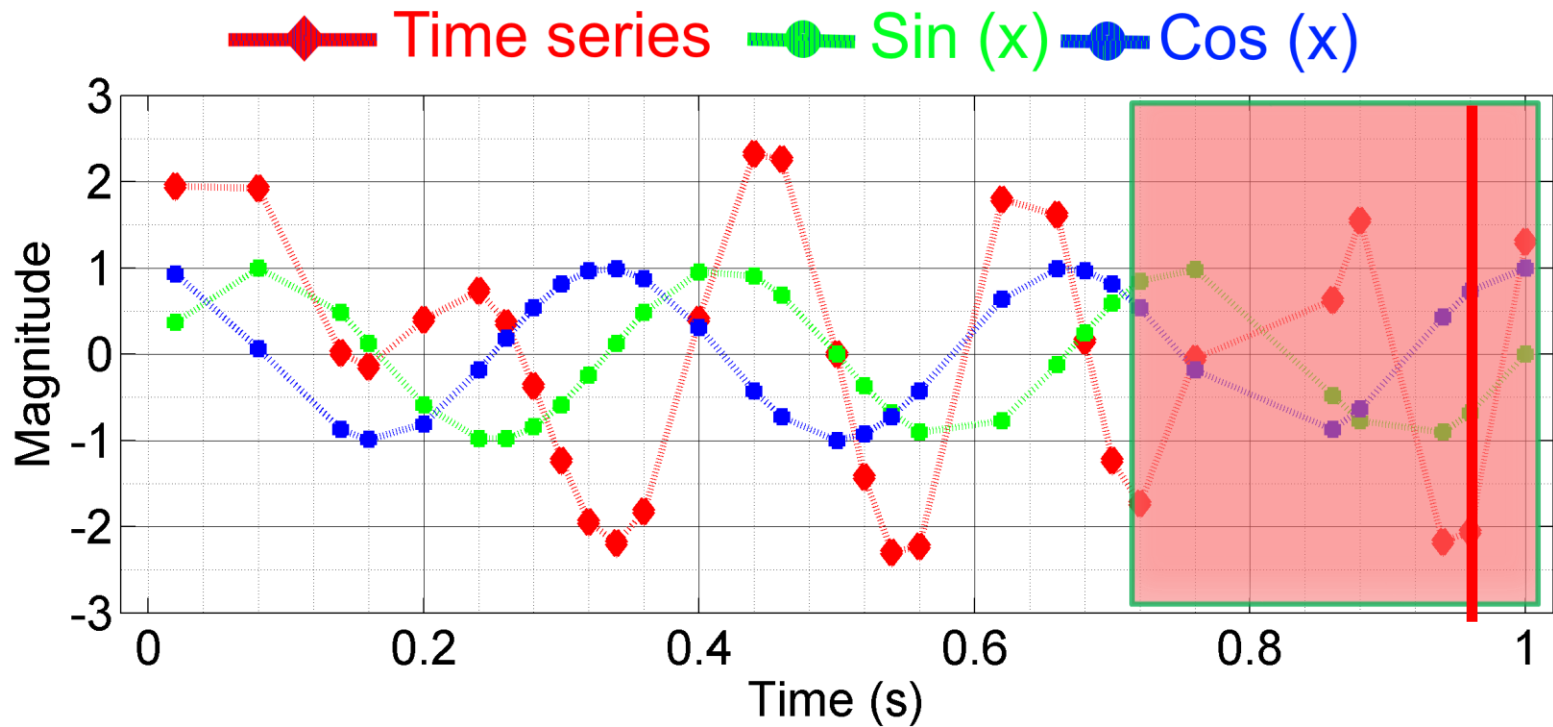# Translating window for an **unequally spaced** time series
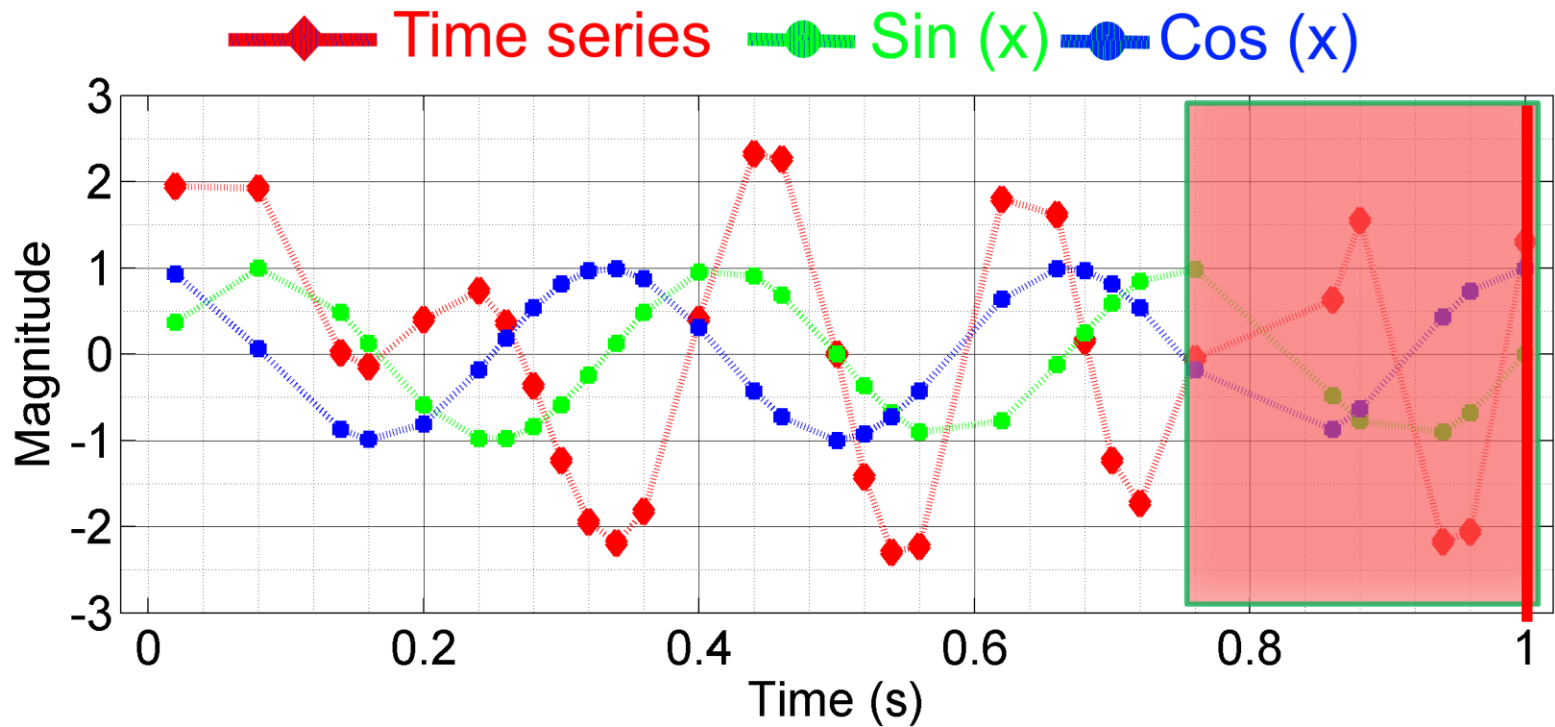
Translating window for an **unequally spaced** time series

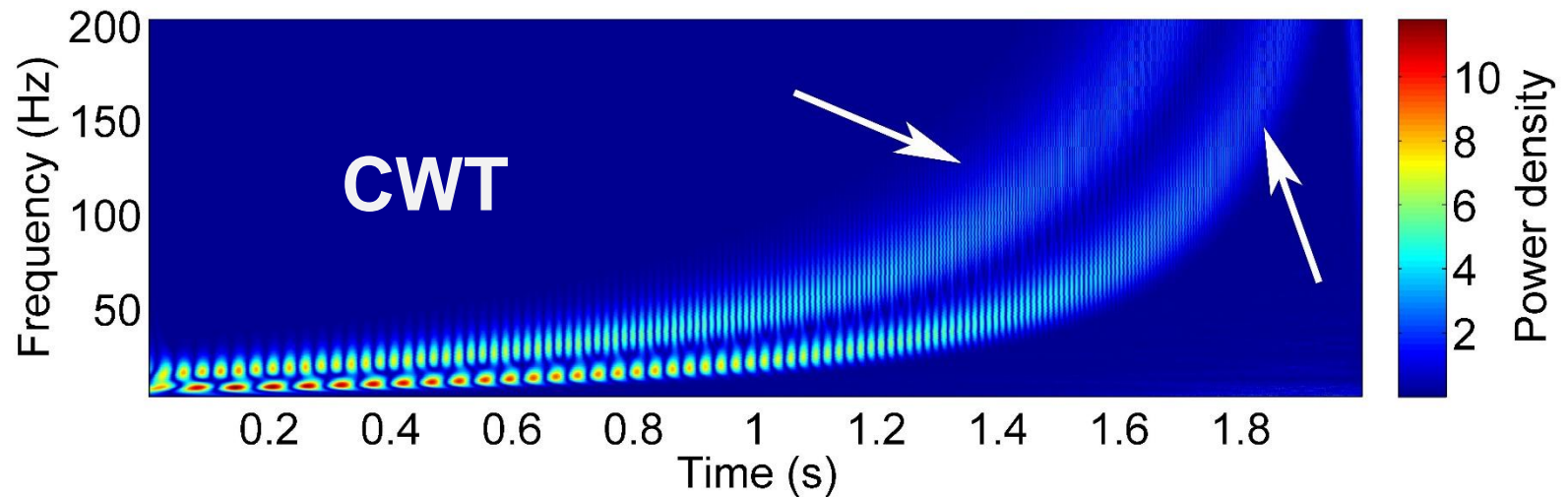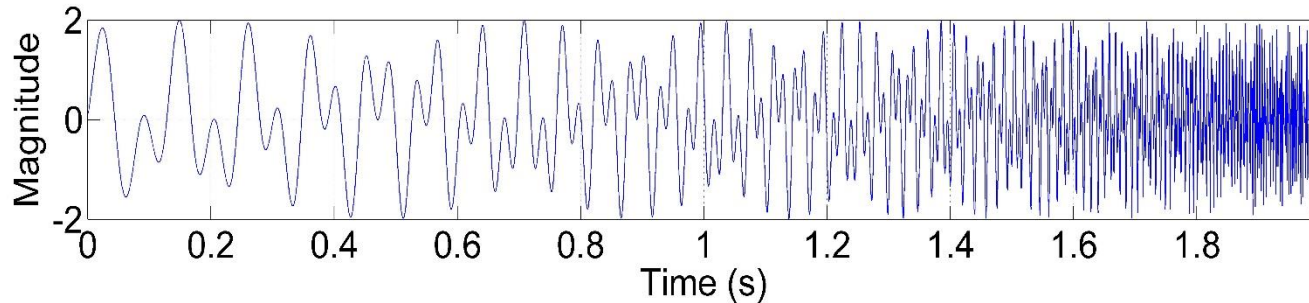# Translating window for an **unequally spaced** time series
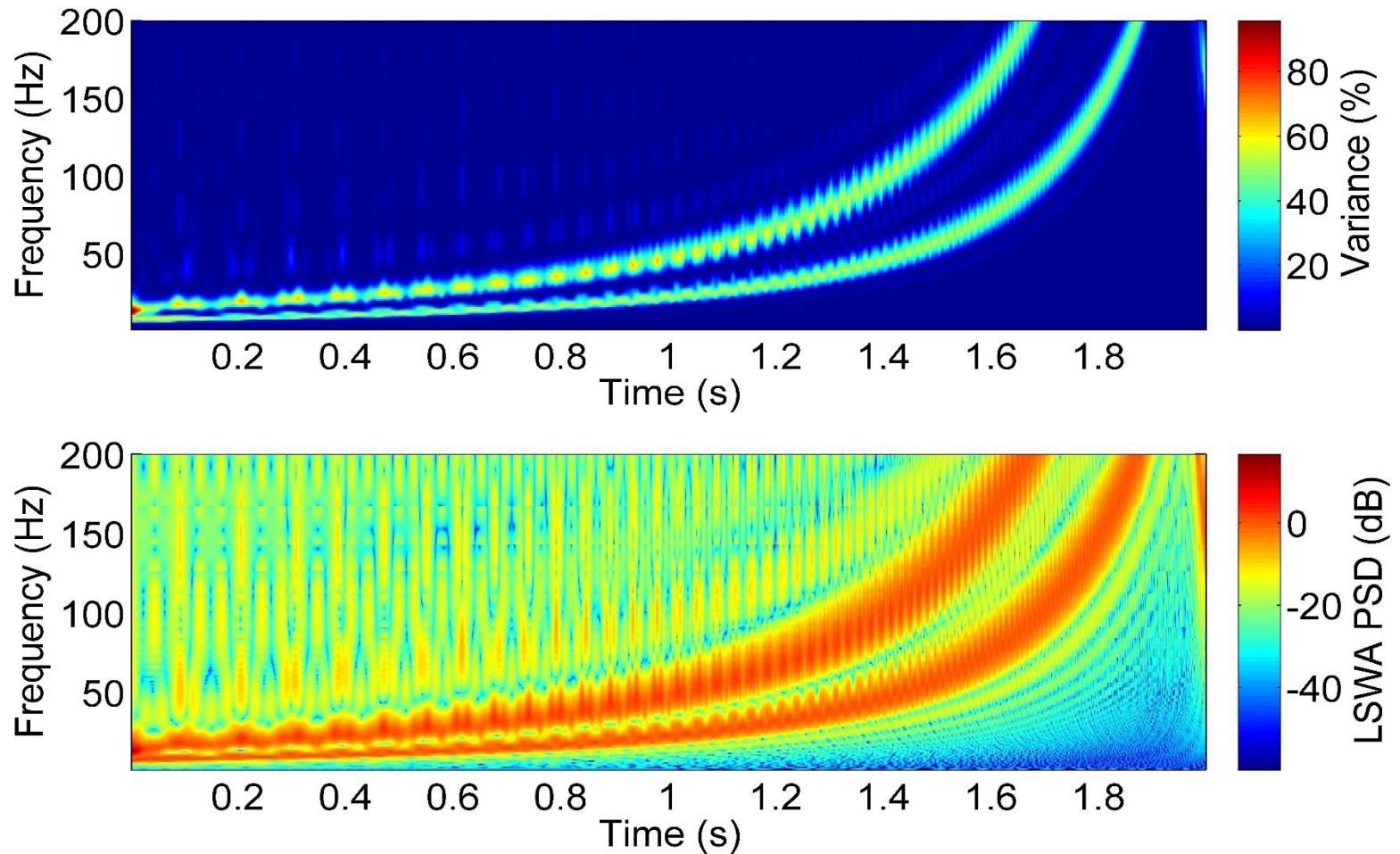
Translating window for an **unequally spaced** time series

Translating window for an **unequally spaced** time series

Example: $f(t) = \cos(\frac{230}{2.3-t}) + \cos(\frac{500}{2.3-t})$

LSWA Spectrograms: Variance (%) & PSD(dB)

Conclusions

- **In the LSWA, time series do NOT have to be equally spaced.**

- **The LSWA considers trends, datum shifts and any systematic noise in time series.**

- **The LSWA shows perfectly the variability in amplitude and frequency of the components of the time series.**

- **The LSWA rigorously analyzes any time series.**

Thank you for your attention!

Any questions or comments?