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Efficient solver for stabilized formulations of the Maxwell's equations

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SUMMARY

A problem of spurious modes in numerical solutions of Maxwell's equations is well known (Smith, 1996). The problem originates because of the rich null-space of the curl operator and violation of the conservation of current due to limited finite precision of the hardware. This deteriorates accuracy of the solutions and the convergence of the generall purpose solvers. The latter requires additional cleaning of the solution (known as the divergence correction introduced by Smith (1996)) or development of adhoc preconditioners (Arnold et al., 2000; Grayver and Kolev, 2015). Although these preconditioners help solve numerical problems efficiently, they do not guarantee that the final solution itself has no spurous modes and thus the problem of inaccurate solution remains. While the issue of spurious modes is typically attributed to the low-frequency regime, this is not the only factor causing instability. Other factors include problem formulation (E or H), conductivity value and spatial scale. Therefore, applications that need EM fields calculated accurately in the air (airborne EM, global induction modeling at satellites). Explicit augmentation of the Maxwell's equations with convervation of current condition helps solve the problem (Schwarzbach, 2009), but the efficient solver for this problem has not been presented. Using direct solvers (Streich et al., 2010) hinders scalability of the algorithm. This work presents stabilized formulation and efficient solver based on the multigrid preconditioner that can be applied to both E and H field formulations. High-order finite element method on arbitrary hexahedral meshes is used to derive discrete formulation and ensure high accuracy.

Keywords: forward modeling, solver, stabilization, multigrid, finite elements

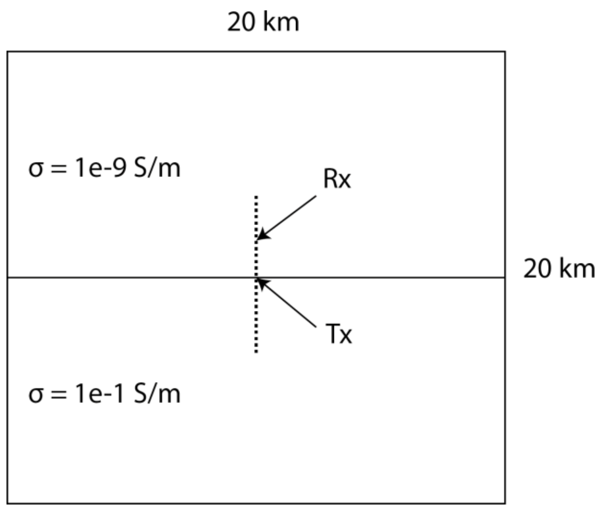

Figure 1. Schematic illustration of the model used for numerical experiment shown in Figure 2.

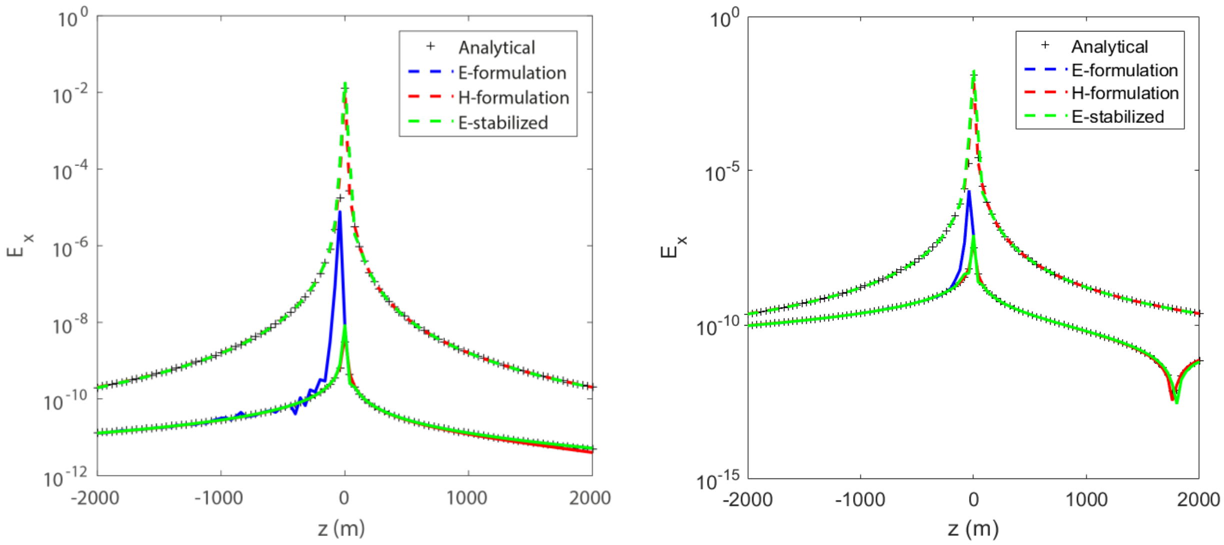

Figure 2. Comparison of analytical and numerical electric fields for model shown in Figure 1 using conventional E, H and E-stabilized formulations. Real and Imaginary parts of the Ex components for x-unit dipole at frequencies of 0.05 (left) and 0.5 Hz (right) are shown respectively with dashed and solid lines. Note instability in imaginary part for the E-field formulation.

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