Equalizers for PCC-OFDM with Overlapping Symbol Periods*

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Abstract - **Polynomial cancellation coded orthogonal frequency division multiplexing (PCC-OFDM) with overlapping symbol periods is a modulation technique which overcomes many of the disadvantages of OFDM. Overlap PCC-OFDM is much less sensitive to frequency offset and Doppler spread and has better performance in channels with long delay spread than OFDM. This paper analyzes the equalizers that are required. Results are presented for linear and decision feedback equalizers of varying length in the presence of additive white Gaussian noise. The performance is degraded by two effects: the non orthogonality of the basis functions, and the limited span of the equalizers. Despite these effects, for the same bandwidth efficiency overlap PCC-OFDM systems can be designed to have similar performance to OFDM with a cyclic prefix even when the only channel impairment is noise.**

I. BACKGROUND ON POLYNOMIAL CANCELLATION CODING

Polynomial cancellation coding (PCC) is a coding technique for orthogonal frequency division multiplexing (OFDM) in which the data to be transmitted is mapped onto weighted groups of subcarriers rather than individual subcarriers. PCC-OFDM has been shown to be much less sensitive than OFDM to frequency offset and Doppler spread [1, 2, 3]. In its simplest form the spectral efficiency of PCC-OFDM is approximately half that of OFDM. It has previously been shown [4-6] that PCC-OFDM can be used without any loss in spectral efficiency if intersymbol interference is deliberately introduced at the transmitter by overlapping the symbol periods and the data is recovered at the receiver using an equalizer. Such systems have been shown to have better performance than OFDM in channels subject to long delay spread [6]. This paper analyzes the equalizers that are required. Expressions are derived for linear and decision feedback equalizers. Simulation results are presented for equalizers of varying complexity and for PCC-OFDM with varying symbol overlap.

Fig. 1 shows the transmitter for a PCC-OFDM system with overlapping symbol periods. In the rest of the paper this will be referred to as 'overlap PCC-OFDM'. Compared with a standard OFDM system the transmitter has two extra functional blocks. The block preceding the inverse discrete Fourier transform (DFT) maps the data onto weighted groups of subcarriers rather than individual subcarriers. In this paper mapping of data onto pairs of subcarriers will be considered. There is also an extra block to overlap and add adjacent symbols.

Fig. 2 shows the form of the overlapped transmitted symbols. Mapping the data onto adjacent subcarriers results in a windowing effect [1, 5] with the energy of each symbol concentrated at the centre of the symbol period. The amount of overlap can be varied. Fig 2 shows an overlap of $T/2$, where T is the symbol period. However smaller overlaps can also be used.

Fig. 3 shows the receiver for the system. The input signal is converted to serial and then passed through a delay line so that T length sections of the input signal, centred on each transmitted symbol, are input to the DFT. For an overlap of $T/2$ one DFT operation is performed every $T/2$ so that the output vector $Z(i)=z_0(i)\cdots z_{N-1}(i)$ depends mainly on the corresponding input vector $A(i)=a_0(i)\cdots a_{N-1}(i)$ but also partly on the adjacent input vectors $A(i-1)$ and $A(i+1)$. N is the number of subcarriers. Reducing the overlap reduces the dependence on the adjacent input vectors. At the receiver the transmitted data sequence is estimated from the sequence of vectors $Z(i)$. In the systems considered in this paper $Z(i)$ is then input to a weighting and adding block, which combines adjacent subcarrier pairs to give vector $V(i)$.

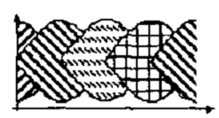

Fig. 2. PCC-OFDM with overlapping in time domain

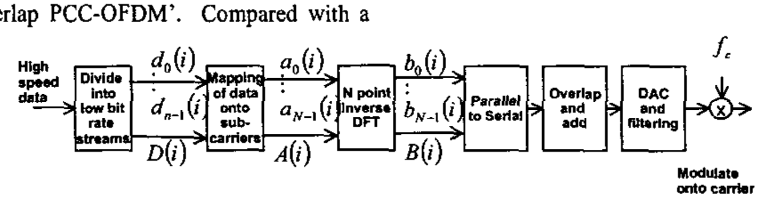

Fig. 1. Block diagram of transmitter for PCC-OFDM with overlapping symbol periods

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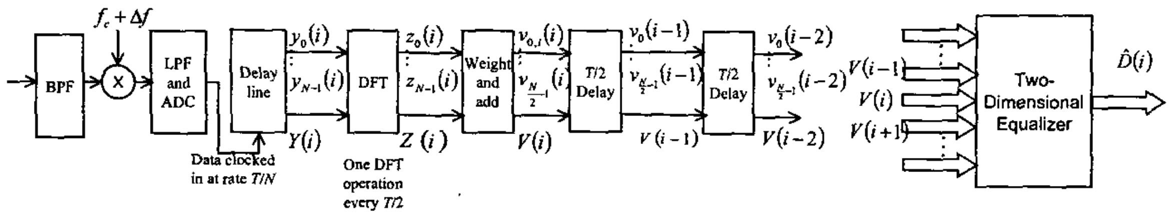

Fig. 3 PCC-OFDM receiver

This results in the receiver being matched to the PCC waveforms and also contributes to the ICI cancellation properties of the technique [3]. However it also results in the noise components in V_i being correlated with those in V_{i-1} and V_{i+1}. The sequence of V vectors are then input to an equalizer to recover the transmitted data sequence.

II. ANALYSIS OF OVERLAP PCC-OFDM SYSTEM

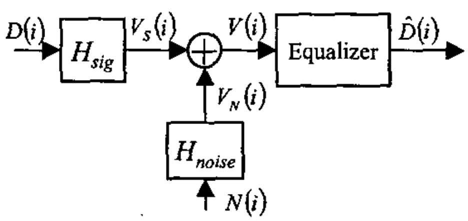

Fig. 4 Block diagram of PCC-OFDM system

Fig. 4 shows a simplified block diagram of an overlap PCC-OFDM system. The H_{sig} block represents the signal path between the data input in the transmitter and the equalizer input in the receiver. This includes the mapping, inverse DFT, overlapping-and-adding, the channel, the DFT and the weighting-and-adding block. The inputs are the sequence of column vectors $\cdots D(i-1), D(i), D(i+1)\cdots$ each of length $N/2$. The outputs are the vectors $\cdots V_S(i-1), V_S(i), V_S(i+1)\cdots$ each also column vectors of length $N/2$. These represent the wanted signal component at the input to the equalizer. The overall vector impulse response for the signal path is given by

$$H_{sig} = [H_S(-K_1) \;\cdots\; H_S(0) \;\cdots\; H_S(+K_2)]. \quad (1)$$

K_1 is the number of non-zero vector precursor terms and K_2 is the number of non-zero postcursors. For an overlap of $T/2$ and a distortionless channel, $K_1 = K_2 = 1$. Multipath spread of up to $T/2$ will increase K_2 to 2. The H_S matrices are of size $N/2$ by $N/2$. The elements of $H_s(0)$ give the contribution of each element of $D(i)$, to each element in $V_S(i)$. Similarly the $H_s(-1)$ matrix gives the contribution of each element of $D(i+1)$, to each element in $V_S(i)$. The values of the matrix elements depend on both the channel and the degree of overlap of symbols. For a distortionless channel they have a high degree of symmetry, particularly for the case of an overlap of $T/2$. For a distortionless channel the $H_s(0)$ matrix is an identity matrix.

$$V_S(i) = [H_S(K_2) \;\cdots\; H_S(0) \;\cdots\; H_S(-K_1)]\begin{bmatrix} D(i-K_2) \\ \vdots \\ D(i) \\ \vdots \\ D(i+K_1) \end{bmatrix} \quad (2)$$

The weighting and adding block in the receiver results in the noise at the input to the equalizer being correlated. This effect is represented by the block,

$$H_{noise} = [H_N(-1) \;\; H_N(0)]. \quad (3)$$

In the analysis, the noise at the input to the receiver is considered as consisting of vectors of white Gaussian noise samples $\cdots N(i-1), N(i), N(i+1)\cdots$. Each vector is of length $N-l$ where lT/N is the time duration of the overlap. The noise component at the input to the equalizer is given by $\cdots V_N(i-1), V_N(i), V_N(i+1)\cdots$ H_{noise} does not depend on the channel, it depends only on the degree of overlap of symbols.

$$V_N(i) = [H_N(0) \;\; H_N(-1)]\begin{bmatrix} N(i) \\ N(i+1) \end{bmatrix} \quad (4)$$

Column vectors $\cdots V(i-1), V(i), V(i+1)\cdots$ are input to the equalizer where $V(i) = V_S(i) + V_N(i)$. The output from the equalizer is a sequence of vectors $\cdots \hat{D}(i-1), \hat{D}(i), \hat{D}(i+1)\cdots$ which are the estimates of the input data $\cdots D(i-1), D(i), D(i+1)\cdots$.

III. LINEAR EQUALIZERS

A number of forms of equalizer are possible for example linear equalizers or decision feedback equalizers. We will first consider a linear equalizer. In this case, a linear combination of a sequence of vectors $V(i-L_2)\cdots V(i)\cdots V(i+L_1)$ is used to calculate the estimate $\hat{D}(i)$. The number of vectors used will be described as the number of 'stages' in the equalizer. So the number of stages in this case is L_1+L_2+1. For the moment we will consider the case where every element in each vector is used. In this case the equalizer consists of L_1+L_2+1 stages with $(N/2)\times(N/2)$ taps per stage. The data estimate is given by

$$\hat{D}(i)=[C(L_2)\ \cdots\ C(0)\ \cdots\ C(-L_1)]\begin{bmatrix} V(i-L_2) \\ \vdots \\ V(i) \\ \vdots \\ V(i+L_1) \end{bmatrix} \tag{5}$$

Combining (2), (4) and (5) gives (6).

$$\hat{D}(i)=C_T\left(FD_T(i)+GN_T(i)\right) \tag{6}$$

where

$C_T=\left[C(L_2)\ \cdots\ C(0)\ \cdots\ C(-L_1)\right]$,

$$F=\begin{bmatrix} H_s(K_2) & \cdots & H_s(-K_1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & H_s(K_2) & \cdots & H_s(-K_1) & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & H_s(K_2) & \cdots & H_s(-K_1) \end{bmatrix}$$

$$G=\begin{bmatrix} H_N(0) & H_N(-1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & H_N(0) & H_N(-1) & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & H_N(0) & H_N(-1) \end{bmatrix},$$

$$D_T(i)=\begin{bmatrix} D(i-K_2-L_2) \\ \vdots \\ D(i) \\ \vdots \\ D(i+K_1+L_1) \end{bmatrix} \text{ and } N_T(i)=\begin{bmatrix} N(i-L_2) \\ \vdots \\ N(i+L_1+1) \end{bmatrix}.$$

The vector impulse response of the equalizer is $\left[C(-L_1)\ \cdots\ C(0)\ \cdots\ C(L_2)\right]$.

The estimation error vector is given by

$$\varepsilon(i)=\hat{D}(i)-D(i)=C_T(FD_T(i)+GN_T(i))-AD_T(i) \tag{7}$$

where A is defined so that $D(i)=AD_T(i)$.

For a minimum mean square equalizer we are seeking to find the value of C_T which minimizes the mean square error. Each element of the error vector depends on the corresponding row of the C_T matrix, so the problem can be simplified to $N/2$ separate optimizations. The error in estimating the k-th element of $D(i)$ is given by

$$\varepsilon_k(i)=\hat{d}_k(i)-d_k(i)=C_{Tk}\left(FD_T(i)+GN_T(i)\right)-A_kD_T(i) \tag{8}$$

where C_{Tk} is the k-th row of C_T and $d_k(i)=A_kD_T(i)$.

Now, finding $E\{\varepsilon_k(i)\varepsilon_k^*(i)\}$ and differentiating with respect to C_{Tk} it can be shown that the taps for the MMSE equalizer are given by

$$C_{Tk_{mmse}}=\frac{E\{d^2\}}{E\{n^2\}}A_kF^*\left(\frac{E\{d^2\}}{E\{n^2\}}FF^*+GG^*\right)^{-1} \tag{9}$$

where $E\{d^2\}$ is the expected power of the data and $E\{n^2\}$ is the expected power of the noise.

IV. Decision Feedback Equalizers

The performance of an overlap PCC-OFDM system can be considerably improved by using a decision feedback equalizer (DFE) rather than a linear equalizer. In this case the components from previously decoded symbols can be subtracted before the vectors are input to the linear feedforward section of the equalizer. In single carrier systems, DFEs may suffer from error propagation when incorrect decisions are fed back. In this application the decisions are vectors rather than scalars and error-correcting codes can be used across each vector to ensure that the majority of the errors are corrected before the decision feedback occurs [7]. For the DFE case the operation of the linear section of the equalizer is described by:

$$\hat{D}(i)=\left[C(0)\ \cdots\ C(-L_1)\right]\begin{bmatrix} V_{FB}(i) \\ \\ V_{FB}(i+L_1) \end{bmatrix} \tag{10}$$

where $V_{FB}(i)$ is the i-th vector output from the decision feedback section of the equalizer and

$$V_{FB}(i)=V(i)-\hat{D}_{FEC}(i-1)C(1)-\cdots-\hat{D}_{FEC}(i-L_2)C(L_2) \tag{11}$$

where $\hat{D}_{FEC}(i)$ is the estimate of $D(i)$ after forward error correction. For the feedback stages, $C(-k)=H_S(k)$ for equalizers using all of the taps in each stage. The derivation for the MMSE linear feedforward stage is exactly the same as for the linear case except that the F matrix becomes.

$$F=\begin{bmatrix} H_s(0) & \cdots & H_s(-K_1) & \mathbf{0} & \cdots & \mathbf{0} \\ H_s(1) & H_s(0) & \cdots & H_s(-K_1) & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ H_s(K_2) & \cdots & \cdots & \cdots & \cdots & H_s(-K_1) \end{bmatrix} \tag{12}$$

V. Equalizers with Fewer Taps per Stage

Because of the properties of PCC-OFDM the spectrum of each subchannel has a very rapid roll-off. So interchannel interference resulting from intersymbol interference (ISI) is limited to the immediately adjacent subchannels. The equalizers can be simplified by using fewer taps per stage to generate each data estimate. For example if three taps are used, not all of the elements of $V(i)$ are used in calculating, $\hat{d}_k(i)$, only the three elements $v_{k-1}(i)$, $v_k(i)$, $v_{k+1}(i)$ would be used from this vector. To obtain optimum performance, the equalizer tap values must be recalculated taking into account that different inputs are used to estimate each output. Simply using some of the taps calculated in (9) does not give optimum performance. The calculation for the MMSE linear sections with reduced taps proceeds exactly as above. For the decision feedback section with reduced taps, the feedback taps are simply derived using the signal impulse response for the significant terms.

VI. PERFORMANCE OF EQUALIZERS

The benefits of overlap PCC-OFDM in channels subject to frequency offset, Doppler spread and delay spread has been demonstrated in earlier papers [2,4,6]. In this paper only the performance of the equalizers in additive white Gaussian noise (AWGN) is considered. This is to show the degradation caused by the use of non-orthogonal basis functions and equalizers with limited span. For comparison the performance of an orthogonal modulation scheme (for example OFDM without a cyclic prefix) and OFDM with a cyclic prefix (CP) of $T/8$ for the same signal constellation are also shown. The systems compared do not necessarily have the same bandwidth efficiency. For an overlap of $T/2$, overlap PCC-OFDM has a bandwidth efficiency equal to the orthogonal modulation scheme. OFDM with a cyclic prefix of $T/8$, and PCC-OFDM with smaller overlaps have lower bandwidth efficiency.

Fig 5 shows the bit error rate (BER) versus E_b/N_0 for 4QAM. Where E_b is the energy per bit and N_0 is the single sided noise spectral density. The results are for linear equalizers of varying numbers of stages with all the taps used in each stage. In each case $L_1 = L_2$. The overlap between symbols is $l = N/2$. The BER for the PCC-OFDM systems is significantly worse for this case than for OFDM with a cyclic prefix. There are two effects limiting the performance of the PCC-OFDM systems: the non-orthogonality of the basis functions causing noise enhancement in the equalizer and the residual ISI due to the limited span of the equalizers. The first is more important at low E_b/N_0 so increasing the length of the equalizer has little effect, whereas the second becomes more important at higher E_b/N_0. The graphs plateau as E_b/N_0 increases.

Fig. 6. shows the results for three different overlaps when the constellation is increased to 64QAM. Comparing Figs. 5. and 6. shows that the degradation has a much greater effect on BER for a larger constellation. Comparing the results for $l = 24N/64$, $l = 28N/64$ and $l = N/2$, shows that the performance is very dependent on overlap. So there is a simple trade-off between bandwidth efficiency and performance in AWGN. An overlap of $l = 28N/64$ has the same bandwidth efficiency as the 'OFDM with CP'.

Fig. 7 shows the performance of a DFE for 4QAM and $l = N/2$ and varying number of feedforward stages, L_1. For the cases considered one feedback stage is enough to remove all postcursor interference. The simulation is for the case where correct decisions are fed back and so there is no error propagation. Comparing Figs. 5 and 7 shows that the DFE has much better performance because only the feedforward stages cause any degradation.

Fig. 8 shows the results for DFEs and 64QAM and $l = N/2$ For the DFEs, increasing the equalizer length gives more improvement in performance than for the linear case, because the limiting factor is equalizer span rather than noise enhancement. In Fig. 9, the overlap has been decreased to $l = 28N/64$. For equalizers with four or more feedforward stages the performance is approximately the same as for an OFDM based system with the same spectral efficiency.

Fig. 10 shows the results when only three taps are used in each of the stages to estimate each data value. Reducing the number of taps drastically reduces the complexity with little degradation in performance.

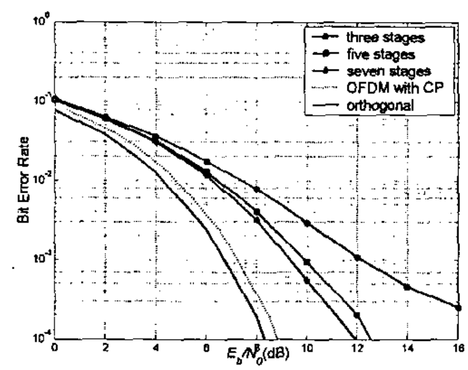

Fig. 5. BER versus E_b/N_0 for linear equalizers of varying length. $N = 64$, overlap $l = 32$, modulation 4QAM

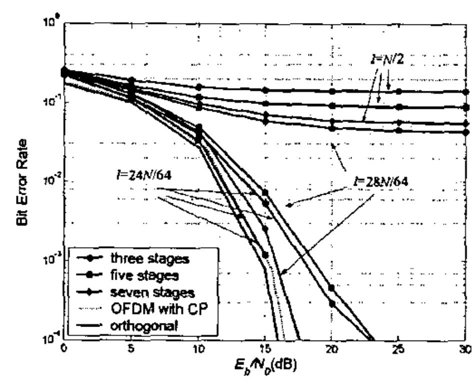

Fig. 6. BER versus E_b/N_0 for linear equalizers of varying length. $N = 64$, varying symbol overlap and modulation 64QAM

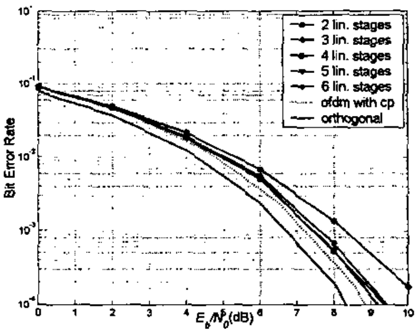

Fig. 7. BER versus E_b/N_0 for DFEs of varying length. $N=64$, overlap $l=32$, modulation 4QAM.

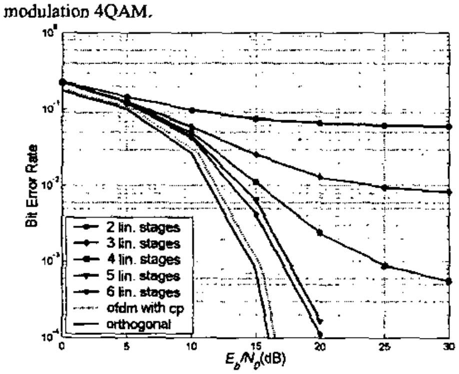

Fig. 8. Bit Error rate versus E_b/N_0 for DFEs of varying length. $N=64$, overlap $l=32$, modulation 64QAM.

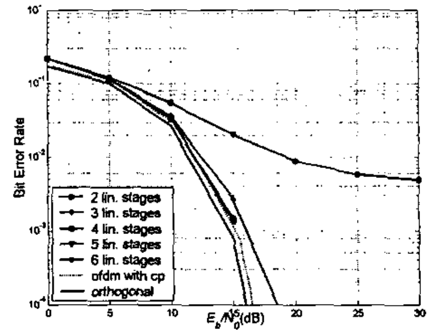

Fig. 9. BER versus E_b/N_0 for DFEs of varying length. $N=64$, overlap $l=28T/64$, modulation 64QAM, 2000 symbols simulated.

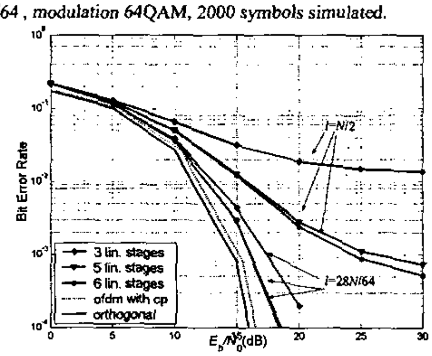

Fig. 10. BER versus E_b/N_0 for DFEs of varying length, with each data value estimated using only three non-zero taps per stage. $N=64$, modulation 64QAM

VII. Conclusions

Overlap PCC-OFDM is a modulation scheme that has previously been shown to outperform OFDM in channels subject to multipath, frequency offset and Doppler spread. In this paper the equations describing the equalizers that are required in a PCC-OFDM system are derived. To give a better understanding of the effect of various parameters in a PCC-OFDM system, simulation results are presented for linear and decision feedback equalizers of varying complexity in AWGN. Results are presented for 4QAM and 64QAM.

Two effects limit the performance of overlap PCC-OFDM in AWGN, the non-orthogonality of the basis functions and the limited span of the equalizers. The non-orthogonality is the more important effect for linear equalizers whereas the span is more important for DFEs. Despite these effects, when a DFE is used with overlap PCC-OFDM, systems can be designed, which for the same bandwidth efficiency, give similar performance in AWGN to OFDM with a cyclic prefix. For a given equalizer complexity the BER decreases as the symbol overlap decreases. This allows a simple trade-off between bandwidth efficiency and BER.

An important property of PCC-OFDM is that the subchannels have very sharp spectral roll-off. This means that interchannel interference caused by intersymbol interference is significant only in the immediately adjacent subchannels. As a result equalizers with fewer taps per stage can be used. This very much reduces the complexity of the equalizers with comparatively little degradation in performance.

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