Elmwood Press

Core Mathematics C3

Paper J

(Question Paper)

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Core Mathematics C3 For Edexcel
Advanced Level

Paper J
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. Simplify

$$\frac{4x^2 - 25}{x^2 + x} \div \frac{2x^2 - x - 10}{x^2 + 3x + 2}.$$ **(6)**

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2. (*a*) Given $x = \tan y$, find $\frac{dx}{dy}$ and hence find $\frac{dy}{dx}$ in terms of x. **(4)**

(*b*) Show that $(1 + x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$. **(3)**

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3. The n^{th} term of an arithmetic progression is $\ln(pq^{n-1})$ where p and q are positive integers.

(*a*) What is the first term of the sequence? **(1)**

(*b*) Show that the common difference is $\ln q$. **(2)**

(*c*) Find, in terms of $\ln p, \ln q$ and n, the sum of the first n terms of the series formed by the terms of the sequence. **(3)**

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4. The root of the equation $f(x) = 0$, where

$$f(x) = 2x + \ln 3x - 5$$

is to be estimated by using an iterative formula.

(*a*) Show that the root α, such that $f(\alpha) = 0$, lies in the interval [1, 2]. **(2)**

(*b*) Show that $f(x) = 0$ can be rewritten as

$$x = \frac{1}{2}(5 - \ln 3x).$$ **(2)**

(*c*) Use the iteration

$$x_{n+1} = \frac{1}{2}(5 - \ln 3x_n) \quad \text{with} \quad x_0 = 1.5,$$

to obtain the values of x_1, x_2, x_3 and x_4. **(2)**

(*d*) Give the value of α correct to 3 decimal places. **(1)**

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5. (*a*) Given that

$$\cos(2x - 60) = 2\sin(2x + 30),$$

prove that $\tan 2x = -\frac{1}{\sqrt{3}}$. **(5)**

(*b*) Using the result from part (*a*), find two values of x, $0 < x < 180°$, which satisfy the equation

$$2\sin(2x + 30) - \cos(2x - 60) = 0.$$ **(3)**

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6. (*a*) On the same axes sketch the graphs of C_1, $y = e^{\frac{1}{2}x}$, and C_2, $y = e^{-2x}$.

The graphs intersect at the point A. **(4)**

(*b*) State the coordinates of A. **(1)**

(*c*) Prove that the tangent at point A to the curve C_1 is the normal to the curve C_2 at the same point. **(5)**

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7. Differentiate with respect to x,

(*a*) $(3x + 1)^7$, **(3)**

(*b*) $\ln\sqrt{(4x + 1)}$, **(3)**

(*c*) $\cos 7x$. **(3)**

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8. The function f is defined by

$$\mathrm{f}: x \mapsto |2x - 1| - 4, \quad x \in \mathbb{R}.$$

(*a*) Sketch the graph of $y = \mathrm{f}(x)$. **(2)**

(*b*) Solve the equation $\mathrm{f}(x) = 3$. **(3)**

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 - 8x + 17, \quad x \geq 0.$$

(*c*) Find the range of g. **(3)**

(*d*) Find $\mathrm{gf}(3)$. **(2)**

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9. (*a*) (i) Express

$$9\cos\theta - 40\sin\theta \quad \text{in the form}$$

$R\cos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(ii) Hence solve the equation

$$9\cos\theta - 40\sin\theta = 6,$$

for $0 < \theta < \frac{\pi}{2}$, giving your answer to 2 decimal places. **(3)**

(*b*) Solve the equation

$$13 + 10\cot\theta = 3\tan\theta,$$

for $0 < \theta < \frac{\pi}{2}$, giving your answer to 2 decimal places. **(5)**

END **TOTAL 75 MARKS**