Linear Symbol Precoding for Low Complexity V-BLAST OFDM Systems

Himal A. Suraweera, Jaunty T. Y. Ho and Jean Armstrong

***Abstract*— In this paper we propose a low complexity linear precoded Vertical Bell Labs Layered Space-Time (V-BLAST) orthogonal frequency division multiplexing (OFDM) system based on QR decomposition. A conventional uncoded V-BLAST OFDM system exhibits significant system degradation in a frequency selective fading channel as it does not fully exploit the available frequency diversity. The new precoded scheme exhibits considerable error performance improvement over a conventional V-BLAST OFDM QR system. Simulation results for both linear and maximum likelihood estimators indicate a trade off between the allowable complexity against the achievable system performance. An extended analysis for error propagation across the different layers in the case of V-BLAST OFDM is also presented.**

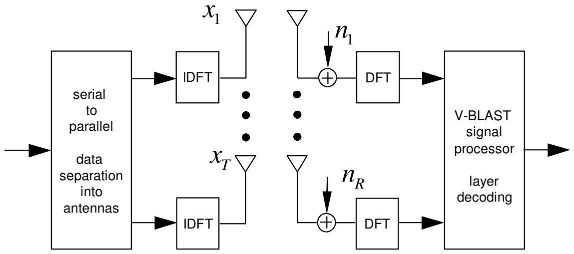
I. INTRODUCTION

Communication systems employing multiple-input multiple-output (MIMO) techniques are capable of increasing the information capacity in wireless fading channels. One such method is the *Vertical Bell Labs Layered Space-Time* (V-BLAST) architecture, where possibly uncoded data streams are transmitted from different antennas. At the receiver signals are decoded using a decision feedback algorithm. Orthogonal frequency division multiplexing (OFDM) is robust against multipath fading and offer low complexity equalization. OFDM in combination with V-BLAST has been proposed as a method to combat the performance degradation due to MIMO frequency selective fading channels [1]–[3]. As OFDM converts a broadband fading channel into a set of narrow band parallel MIMO channels, the V-BLAST algorithm can be applied per subcarrier basis [4].

The performance of an uncoded V-BLAST OFDM system is limited as it does not fully exploit the available transmit and/or frequency diversity in a frequency selective fading channel [5]. Another drawback of the conventional BLAST algorithm is the optimization required at the interference cancellation stages in order to minimize the risk of error propagation (a large number of pseudo matrix inversion computations are needed with proper ordering). Some of the solutions presented in earlier literature for the first problem include applying forward error correction codes across different OFDM *layers* or the use of precoding in combination with advanced signal processing at the receiver [6]. OFDM data symbols sent through *each* individual transmit antenna is referred to as a *layer* in the V-BLAST system. Most of these techniques are computationally intense and require intensive hardware processing. For the diagonal BLAST a precoding technique applied across the OFDM space-frequency grid combined with sphere decoding was presented in [5]. Achieving both the spatial and frequency diversity amidst complexity involved was the motivation in [5].

In this paper we consider a V-BLAST OFDM system based on QR decomposition [7], [8] and propose a low complexity precoding scheme to improve the system performance. The QR based technique is different from the conventional V-BLAST detection as it only requires one pseudo inverse calculation [4]. The OFDM symbols are grouped and mapped through a matrix transformation before passing through the inverse discrete Fourier transform (DFT) at the transmitter. We use the QR decomposition of the channel matrix to reduce the complexity substantially at the receiver processing [7]. For decoding the nth transmitted layer precoded subcarriers the performance of both computationally complex maximum likelihood (ML) and a linear estimator is compared. As observed from the simulation results our method shows a trade off between the complexity and the system performance.

The rest of this paper is organized as follows. In Section II we present the V-BLAST OFDM system model based on QR decomposition and an error propagation analysis. Section III describes the proposed precoding technique. Simulation results are given in Section IV and finally Section V concludes the paper with some remarks.



1: Block diagram of a V-BLAST OFDM transmission system.

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II. V-BLAST OFDM MODEL WITH QR DECOMPOSITION

We assume a multiple antenna system with T transmit and R receive antennas. At the transmitter incoming bits are multiplexed to T transmit antennas, serial/parallel converted, mapped onto a modulation constellation. Following passing through an inverse discrete Fourier transform (IDFT) a cyclic prefix is inserted before upconverting to a radio frequency and transmitted to the channel.

Let $\mathbf{x}[n]$ denote the discrete time $T \times 1$ transmitted OFDM signal vector and $\mathbf{y}[n]$ be the received $R \times 1$ signal vector. We assume a wideband frequency selective channel. Hence the received signal $\mathbf{y}[n]$ can be expressed as

$$\mathbf{y}[n] = \sum_{l=0}^{L-1} \tilde{\mathbf{H}}_l \mathbf{x}[n-l] \tag{1}$$

where the complex $R \times T$ matrix $\tilde{\mathbf{H}}_l$ represents entries from the lth tap. $\tilde{\mathbf{H}}_l$ contains circularly symmetric Gaussian variables with zero mean and variance σ_l^2, where σ_l^2 is derived from the channel power delay profile. The entries within the matrix $\tilde{\mathbf{H}}_l$ can be correlated, however elements of $\tilde{\mathbf{H}}_l$ due to different propagation paths are assumed to be uncorrelated. The cyclic prefix length is assumed to be greater than the channel memory.

The received signal is converted to the discrete frequency domain by using a DFT. Hence the decision symbol for the kth OFDM subcarrier at the output of the DFT is given by

$$\mathbf{Y}(k) = \mathbf{H}(k)\mathbf{X}(k) + \mathbf{N}(k) \tag{2}$$

where $\mathbf{X}(k) = [X_k^{(0)}, X_k^{(1)}, \cdots, X_k^{(M_T-1)}]$, $X_k^{(i)}$ denotes the kth OFDM subcarrier transmitted from ith antenna and $k = 0, 1, \cdots, N-1$. N is the total number of OFDM subcarriers and $N(k)$ is the additive white Gaussian noise (AWGN) for the kth subcarrier with variance $E\left\{\mathbf{N}(k)\mathbf{N}^{\dagger}(k)\right\} = \sigma_n^2 \mathbf{I}_R$. $E(\cdot)$ is the statistical expectation, $\mathbf{I}_R$ is the $R \times R$ identity matrix and $(\cdot)^{\dagger}$ is the conjugate transpose of a matrix. $\mathbf{H}(k)$ is the channel transfer function for the kth subcarrier.

$$\mathbf{H}(k) = \sum_{l=0}^{L-1} \tilde{\mathbf{H}}_l \mathrm{exp}\left(\frac{-j2\pi lk}{N}\right) \tag{3}$$

Eq.(3) shows how OFDM converts the wideband MIMO facing channel into a set of parallel narrowband frequency flat channels. Hence we can apply the original V-BLAST algorithm per subcarrier basis for the OFDM system.

We consider the method based on QR decomposition of the channel to realize the V-BLAST algorithm. The V-BLAST algorithm calculates a series of computationally intense pseudo inverses of the initial channel and the modified $\mathbf{H}(k)$ matrices. A reduced complexity method as opposed to the conventional method using the QR decomposition of the channel matrix was proposed in [7], [8]. In this method due to the upper triangular structure of the resulting matrix only one inverse calculation is performed. This saves the required computational effort. However the optimum ordering is still crucial in minimizing the error propagation. QR decomposition applied to the kth subcarrier channel matrix $\mathbf{H}$ can be expressed by $\mathbf{H} = \mathbf{QR}$.

$$\mathbf{H} = \underbrace{\begin{pmatrix} q_{1,1} & \cdots & q_{1,T} \\ \vdots & \ddots & \vdots \\ q_{R,1} & \cdots & q_{R,R} \end{pmatrix}}_{\mathbf{Q}} \underbrace{\begin{pmatrix} r_{1,1} & \cdots & r_{1,T} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_{T,T} \end{pmatrix}}_{\mathbf{R}} \tag{4}$$

where $\mathbf{Q}$ matrix has unit norm orthogonal columns and $\mathbf{R}$ is upper triangular. The QR factorization can be calculated using many methods. e.g. using the Householder transformation or Givens rotation [9]. By left multiplying $\mathbf{r}$ with $\mathbf{Q}^{\dagger}$ we obtain

$$\bar{\mathbf{Y}} = \mathbf{R}\mathbf{X}(k) + \tilde{\mathbf{N}}(\mathbf{k}) \tag{5}$$

$\tilde{\mathbf{N}}(k) = \mathbf{Q}^H \mathbf{N}(k)$. Detection of the symbols in vector $\mathbf{X}$ can be described by the following steps.

$$\bar{Y}_T = Y_{T,T} X_T + \tilde{N}_T \tag{6a}$$

$$\hat{X}_T = Q^{'}\{\bar{Y}_T\} \tag{6b}$$

$$\bar{Y}_{T-1} = r_{T-1,T-1} X_{T-1} + r_{X_{T-1},T} \hat{X}_T + \tilde{N}_T \tag{6c}$$

$$\hat{X}_{T-1} = Q^{'}\{\bar{Y}_{T-1}\} \tag{6d}$$

Generally the nth element of $\bar{\mathbf{Y}}$ is given by

$$\bar{Y}_n = r_{n,n} X_n + \sum_{i=n+1}^{T} r_{n,i} \hat{X}_i + \tilde{N}_n \tag{7}$$

$Q^{'}\{\cdot\}$ denotes the detection. Symbol detection starts from (6a). As seen from (6b) $\hat{X}_T$ which is totally free from interference can be initially estimated. The *assumed* correct decisions are nulled from the upper layers to detect symbols $\hat{X}_{T-1}, \hat{X}_{T-2}, ..., \hat{X}_1$ respectively. If interference can be perfectly nulled in the upper layers (assuming symbols are detected correctly in the lower layers), then the signal-to-noise ratio for the nth layer is governed by the diagonal element $|r_{n,n}|^2$ in $\mathbf{R}$ [7]. Similar to the original V-BLAST algorithm the detection order is critical in minimizing the error propagation. That is proper ordering of the elements $r_{n,n}$ is essential.

A. Error Propagation Analysis

In this section we analyze the error propagation due to QR factorization applied to a V-BLAST OFDM system. For narrowband conventional V-BLAST systems recent literature have studied the effects of layer decoding and error propagation results. Assuming no error propagation, the average symbol error rate (SER) for the nth layer S_n of a given OFDM subcarrier k is written as [10],

$$\bar{S}_n = \int_0^{\infty} S_n(\mu)\gamma_n(\mu) d\mu \tag{8}$$

Instantaneous SER, $S_n(\mu)$ depends on a particular modulation scheme (for a given instantaneous SNR μ) and the frequency selectivity of the MIMO channel. $\gamma_n(\mu)$ is the probability density function of nth layer SNR for kth subcarrier. Note that we have dropped the dependency of k from the notation of μ for simplicity.

$$\mu = |r_{n,n}|^2 \frac{E_s}{\sigma_n^2} \tag{9}$$

In the case of M-QAM modulation $S_n(\mu)$ is given by [11]

$$S_n(\mu) = 1 - \left\{ 1 - \left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\mu}\right)\right\}^2 \tag{10}$$

where $Q(x) = \text{erfc}(x/\sqrt{2})$. Note however that we are interested in evaluating the total SER. Now considering the error propagation effects throughout the layers, the actual total SER for nth layer can be expressed by considering the conditional error probability events. Hence for the $(T-1)$th layer total SER is given by

$$P_{s,T-1} = \hat{P}(T-1|T)P_{s,T} + (1 - P_{s,T})S_{T-1} \tag{11}$$

where $\hat{P}(T-1|T)$ is the conditional SER for $(T-1)$th layer assuming that Tth layer was decoded erroneously. By following the same steps, for the $(T-2)$th layer, the total SER can be expressed as,

$$\begin{aligned} P_{s,T-2} = {} & (1 - P_{s,T})(1 - P_{s,T-1})S_{T-2} \\ & + \hat{P}(T-2|T, T-1)P_{s,T-1}P_{s,T} \\ & + \hat{P}(T-2|T)\{P_{s,T-1}(1 - P_{s,T-2})\} \\ & + \hat{P}(T-2|T-1)\{P_{s,T-1}(1 - P_{s,T})\} \end{aligned} \tag{12}$$

Expressions for the remaining layers can be derived following similar steps. Error propagation is mainly due to single layer error decisions under high SNR conditions. (mathematically this can be justified by neglecting all the cross error probability terms). Our simulation results indicate that this is indeed true in most cases of interest. Hence we can express (12) in a more simpler format.

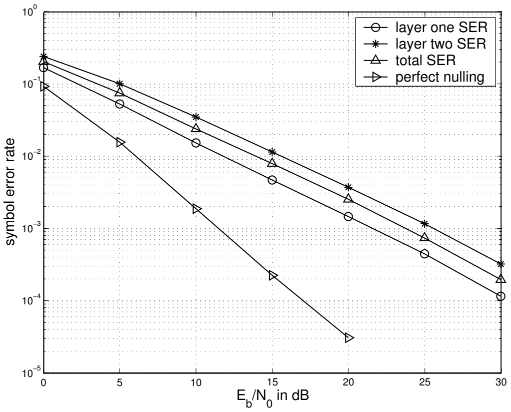
$$\begin{aligned} P_{s,T-2} \le {} & S_{T-2} \\ & + \hat{P}(T-2|T)P_{s,T} + \hat{P}(T-2|T-1)P_{s,T-1} \end{aligned} \tag{13}$$

In general SER for any nth layer is approximately given by

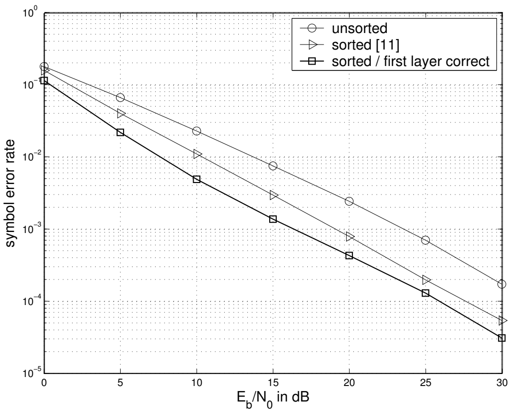
$$P_{s,n} \le S_n + \sum_{m=T}^{n-1} \hat{P}(n|m)P_{s,m} \tag{14}$$

Averaging the mean SER for all individual N OFDM subcarriers we obtain the total SER.

$$\bar{P}_s = \frac{1}{NT}\sum_{m=0}^{N-1}\sum_{n=1}^{T} P_{s,n}(m) \tag{15}$$



2: SER against E_b/N_0 for an uncoded 4-QAM modulated 2×2 system with and without error propagation.



3: SER against E_b/N_0 for an uncoded 4-QAM modulated 4×4 V-BLAST OFDM system.

III. PRECODED V-BLAST OFDM SYSTEM

The previous analysis shows that the degradation in QR decomposition technique is mainly due to the error performance of the first few decoded layers. Hence if one can improve the detection reliability in those layers it is expected that overall performance of the system will improve. One way of achieving this idea is to employ a coding technique across the OFDM subcarriers in a layer. However such methods can be complex and the achievable performance depends on the specific code structure and decoding technique used. Hence we do not consider the coding approach in this paper. Instead the proposed technique uses a combination of subcarrier grouping (symbol precoding) and layer ordering to exploit the available frequency diversity in an OFDM system to improve the performance.

Symbol grouping or precoding techniques have been addressed previously to solve the high peak-to-average power ratio problem in OFDM signals and to reduce decoding complexity in BLAST OFDM systems [5], [6], [12]. Symbol grouping when properly designed will improve the SER performance compared to the conventional per sub carrier decoding method. Precoding to a certain extent has the potential to average out the effects of *good* and the *bad* subcarriers at the V-BLAST OFDM decoder. In this way channel fading effects can be mitigated because the OFDM symbols are spread across several subcarriers and recovered jointly.

At the V-BLAST OFDM transmitter, the proposed method using linear precoding first divides the N subcarrier data in the nth layer into N_g number of sub groups. Elements in each of these sub groups are denoted by the $1 \times u$ row vector $\mathbf{X}_n$, where $u = N/N_g$. Note that we only apply precoding across the individual layers. vth group vector $\mathbf{X}_{v,n}$ is expressed by

$$\mathbf{X}_{v,n} = [X_{v,n}(0), X_{v,n}(1), \cdots, X_{v,n}(u)] \tag{16}$$

After grouping the individual groups are multiplied by a $u \times u$ precoding matrix $\mathbf{\Theta}$. No redundancy is introduced as the precoder outputs a vector of $1 \times u$. All the N_g modified vectors $\tilde{\mathbf{X}}_{v,n}$, are assembled and then possibly interleaved before processing with the IDFT.

$$\tilde{\mathbf{X}}_{v,n} = \mathbf{\Theta}\mathbf{X}_{v,n}^t \tag{17}$$

where $(\cdot)^t$ describes the matrix transpose. Designing the proper precoding matrices in order to achieve high system performance is not a trivial problem and is complex. In fact it involves the *pairwise error probability* analysis and then the precoding matrix can be designed using different criterion. The effect of the precoder is to spread the transmitted energy among the individual subcarriers in the group and its effectiveness to improve the SER performance strictly depends on the state of the channel condition. Interleaving applied across the OFDM subcarriers will further enhance the ability of the precoder to improve the system performance. In our simulations we simply use the results reported in [12].

$$\mathbf{\Theta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{j(\pi/4)} \\ 1 & e^{j(5\pi/4)} \end{pmatrix} \tag{18}$$

$$\mathbf{\Theta} = \frac{1}{2} \begin{pmatrix} 1 & e^{j(\pi/8)} & e^{j(2\pi/8)} & e^{j(3\pi/8)} \\ 1 & e^{j(5\pi/8)} & e^{j(10\pi/8)} & e^{j(15\pi/8)} \\ 1 & e^{j(9\pi/8)} & e^{j(18\pi/8)} & e^{j(27\pi/8)} \\ 1 & e^{j(13\pi/8)} & e^{j(26\pi/8)} & e^{j(39\pi/8)} \end{pmatrix}$$

A. Modified Decoding Approach

A modified approach to the existing sorted QR decomposition (SQRD) algorithm [13] is presented. It jointly uses the SQRD for layer ordering and linear symbol decoding. SQRD calculates the elements of the $\mathbf{R}$ matrix from top to bottom ($r_{1,1}$ to $r_{T,T}$) iteratively to achieve small SNR for the top layers thereby reordering the layer detection sequence. In doing so at ith step SQRD calculates minimum norm of the column vectors of $\mathbf{Q}$ from i to T and exchange with the current ith column vector (see Fig. 1 (3-4) in [13]). In our proposed method we modify this criterion by calculating a groupwise norm for the precoded group. Also the precoded symbols are decoded using ML and a linear equalizer. The performance improvement in our method is due to precoding (achieves frequency diversity) and the modified ordering employed for the layers in the precoded group. The proposed extended algorithm is described by the following steps for the Gth group in layer n.

Step 1: A modified ordering criterion for SQRD is determined. $\alpha_{i,l}$ at lth step per subcarrier basis in SQRD is defined by

$$\alpha_{i,l} = \|(\mathbf{q}_l)_i\|^2 \tag{19}$$

where $i = 1, 2, ..., u$ and $(\mathbf{q}_l)_i$ is the lth column vector for the i subcarrier $\mathbf{Q}$ matrix. Groupwise norms for different subcarrier $\mathbf{Q}$ matrices are defined and the layer having the maximum groupwise norm is chosen. This will determine the swapping column index $k^{'}$ at the lth step of the modified SQRD.

$$k^{'} = \text{arg max}_{\,l=i,...,T} \sum_{i=0}^{u-1} \alpha_{i,l} \tag{20}$$

The remaining $(T-1)$ ordering steps in the SQRD algorithm are determined by repeating step 1. This will ensure that groupwise the low SNR layers are positioned in the upper left hand corner of the $\mathbf{R}$ matrices.

Step 2: After ordering the layers are detected from bottom to top using interference suppression. Let the bottom layer to be detected is given by the $u \times 1$ vector $\bar{\mathbf{y}}_T = [\bar{y}_{0,T}, \bar{y}_{1,T}, ..., \bar{y}_{u-1,T}]^T$.

$$\bar{\mathbf{y}}_T = \text{diag}\left(r_{T,T}^0, r_{T,T}^1, ..., r_{T,T}^u\right)\mathbf{\Theta}\mathbf{X}_v^t \tag{21}$$

$r_{T,T}^i$ describes $r_{T,T}$ for the ith subcarrier in the group and ω is the Tth layer ordering index. Transmitted data can be recovered by using ML detection and minimizing the following cost function $f(\mathbf{X}_v^t)$ [6].

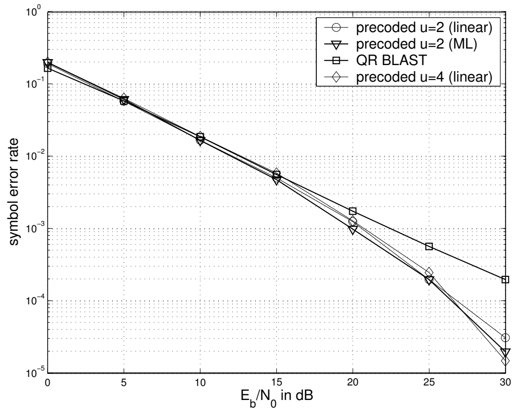
$$f(\mathbf{X}_v^t) = \left\|\bar{\mathbf{y}}_T - \text{diag}\left(r_{T,T}^0, ..., r_{T,T}^{u-1}\right)\mathbf{\Theta}\mathbf{X}_v^t\right\|^2 \tag{22}$$

A linear equalizer can also be used for data detection. In [14] it was shown that a linear equalizer followed by hard decision can also exploit the maximum multipath diversity in a linearly precoded OFDM system. In this case the equalizer is given by the pseudoinverse of diag$(r_{n,n})\mathbf{\Theta}$. Although this produces sub-optimum results compared to the ML approach, it is less computationally complex. In our simulations we have presented results using both of these approaches.

Step 3: Estimate precoded subcarrier symbols $\hat{\tilde{\mathbf{X}}}$ for the ωth layer are subtracted from the top layers.

$$\hat{\tilde{\mathbf{X}}}_v = \mathbf{\Theta}\hat{\mathbf{X}}_v^t \tag{23}$$

Signals in the top layers are detected following cancellation of the estimated signals at the bottom layers and using steps. 2-3 recursively.



4: SER versus E_b/N_0 of the precoded 2×2 system in a 2 path Rayleigh fading channel using 4-QAM modulation.

IV. SIMULATION RESULTS

We have considered a V-BLAST OFDM system using 4-QAM modulation and $N = 64$. $E_b/N_0 = T/\log_2(M)\sigma_n^2$. Channel state information is assumed at the receiver with perfect synchronization. Power delay profile of the channel is exponential with uniform tap spacing in OFDM sample interval. Cyclic prefix length was set to 6 (in OFDM samples). The impulse response of the channel was assumed to be constant during one OFDM frame, and variant for different frames.

Figs. 2-3 show the error propagation results for 2×2 and 4×4 transmit/receive antenna systems using the QR BLAST algorithm. Using (11) and (15), the layer one SER, the layer two SER and the total SER are plotted in Fig. 2. If a correct decision is made in layer one, the total SER reduces significantly. Fig. 4 shows the performance gain achieved by optimally ordering the layers using SQRD. Note that the gain from perfectly nulling the first detected layer is not as great as that in a 2×2 system for the simulated channel, due to the possible error propagation in the remaining three layers.

Fig. 4 shows the SER results against E_b/N_0 for the linearly symbol precoded 2×2 QR V-BLAST system. The precoded systems exhibit low SER for $E_b/N_0 >$ 15 dB and at high E_b/N_0 the difference between the conventional QR and the proposed techniques is comparable. The ML estimator outperforms the linear estimator however the difference is only visible for high E_b/N_0 values. The complexity of the ML estimator increases exponentially as the number of subcarriers per precoded group and for higher order modulation schemes. In that case the linear estimator is attractive if the performance degradation can be tolerated.

V. CONCLUSIONS

In this paper we have analyzed the performance of a QR decomposition based V-BLAST OFDM system. Error propagation results show that to maintain a low system SER correct decisions at the first few decoded layers are crucial. An approximate expression for the resulting V-BLAST OFDM SER performance was derived

A modified approach using symbol precoding and the SQRD algorithm was also presented. The method exhibits better error performance than a conventional QR V-BLAST OFDM system. Precoding was employed to spread the good and the bad channel effects evenly across the OFDM subcarriers. A modified SQRD algorithm was used to find the optimum ordering for the precoded groups. SER results for the ML and the linear equalizer showed a tradeoff between the complexity and system performance.

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