Winter 2015 Project

MUILTI-NAVIGATION SATELLITE SYSTEM CONSTELLATION SIMULATOR

Discussion
Calculations
Summary
Codes

Ebrahim Ghaderpour*
James Dorland
Franck Kapoko

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ebig2@yorku.ca
www.ghader.org

EXECUTIVE SUMMARY

In this work, we introduce the various types of files containing satellite information (Ephemeris and Almanac for GPS and GLONASS constellations), and we study GNSSs and RNSSs. We show how to compute satellite orbits and how to display them in a global map. We also show how to calculate satellite-receiver line-of-sight and Dilution of Precisions for a single position (receiver) and a period of time. We demonstrate the sky plots, world views for GPS and GLONASS orbits. We also produce a static global map that shows the number of satellites in view for a given set of constellations and a given time and date. Also, we illustrate the results of DOP values on a static global map for a given set of constellations and a given period of time and date. We show that the DOP values are less than 3 (excellent) when we include all current operating GPS and GLONASS satellites. A robust GUI is generated to show all of the above tasks, and we describe its functionality in details.

LIST OF ABBREVIATIONS

Dilution of Precision = DOP
Earth Centered Earth Fixed = ECEF
Geometric Dilution of Precision = GDOP
Global Navigation Satellite System = GNSS
Global Navigation Satellite System (Russia Satellites) = GLONASS
Global Positioning System = GPS
Graphical User Interface = GUI
Horizontal Dilution of Precision = HDOP
International Terrestrial Reference Frame = ITRF
Position Dilution of Precision = PDOP
Precise Point Positioning = PPP
Pseudorandom Noise = PRN
Receiver Independent Exchange = RINEX
Regional Navigation Satellite Systems = RNSS
Space Vehicle = SV
Standard Positioning Service = SPS
Time Dilution of Precision = TDOP
Vertical Dilution of Precision = VDOP

Table of Contents

List of Figures

List of Tables

INTRODUCTION AND OBJECTIVE

GPS was originally designed for the purpose of guiding aircrafts, soldiers and sailors in 1960s. Today GPS is used for civilian more than military purposes. In order to find positions, we need to know where the satellites are, so we have to first find the satellite positions.

There are a number of files that contain the satellite information that either include the actual observations or they contain parameters that can be used to find the satellite orbits (ellipses with small eccentricities). The broadcast ephemerides are generated by the GPS Control Segment that uses the Kalman filter to estimate satellite position, velocity, solar radiation, pressure coefficient, clock bias, clock drift and clock drift rate (e.g., Warren and Raquet, 2003). The estimated parameters can be used to predict the satellite orbits. The broadcast ephemerides for GLONASS, on the other hand, contain the satellite positions, velocity and accelerations and the GLONASS orbit may be determined by either extrapolating or using GLONASS Almanac that contain the Keplerian parameters (e.g., Diggelen, 2009). In this work, we only use the ephemeris and almanac files to calculate the satellite orbits for any given time. Other files like SP3 are used for PPP that we do not need in this work.

GPS nominal constellation includes 6 orbital planes that approximately have $55°$ inclination (with respect to the equatorial plane). Each plane previously contained 4 satellites with altitude of 20,200 km (e.g., page 178 Parkinson and Spilker, 1996). The current Almanac and Ephemeris (precise) files contain 32 GPS satellites (PRNs), and the GUI in this work includes all of them. Current GLONASS constellation includes 3 orbital planes with approximately $64.8°$ inclination. Each orbit contains 8 satellites (slot number) with altitude of 19,100 km. GLONASS' orbit is suitable for usage in high latitudes (north or south), where getting a GPS signal is problematic.

The question is now is how accurate is GPS. The specified SPS accuracy is given in terms of "minimum performance levels;" that is, accuracy will be no worse than a certain level for a certain percentage of time. For any point on the Earth, the horizontal accuracy is equal to or better than 100 meters. This means that over a 24-hour period, the horizontal coordinates of a position determined by GPS will be within 100 meters of the true position about 95 percent of the time. The corresponding specified accuracy for heights is 156 meters and 340 nanoseconds for time transfer (cf., Langley, 1999). We refer to Parkinson and Spilker (1996) for more details on GPS accuracy. An improved representation of satellite visibility is given by the sky plots (e.g., Leick, 2004 and Hofmann 2001). DOP values provide a certain confidence for the accuracy of user positions (receivers) which will be described briefly in this report.

The tasks of this work are summarized below:

Objective:

- Read existing ephemeris and almanac files for GPS and GLONASS satellites.

- Compute and verify satellite orbits.

- Compute line-of-sight from any point on or above the Earth's surface to any existing satellite (create a world view showing kinematically the satellites orbits)

- Calculation of satellite visibility, elevation angle, azimuth and range from a selected location and a given elevation mask. (create a kinematic sky plot)

- Compute GDOP, HDOP, PDOP, TDOP and VDOP from a selected location with a set elevation mask and selected SVs.

- Produce a static global map with a number of satellites in view for a given set of constellations and a given time and date.

- Produce a static global map with any DOP values for a given set of constellations and a given time and date (Time period is also included in the last four figures in this report).

- Create a GUI that does all the above tasks.

READING OF EPHEMERIS FILES

The RINEX files are updated every 5 minutes and can be downloaded in Unix compressed format at ftp://cddis.gsfc.nasa.gov/gps/data/daily. The names of the files are given by brdcddd0.yyg for GLONASS broadcast ephemeris and brdcddd0.yyn for GPS broadcast ephemeris, where brdc stands for broadcast, ddd time of the year and yy is year.
GPS ephemeris parameters are given by Table 1.

Code	Parameter	Definition
PRN	PRN	Pseudo Random Noise
M0	M_0	Mean anomaly at a reference time
e	e	Eccentricity of the satellite orbit
Delta_n	Δn	Mean Motion Difference from Computed Value
sqrt_A	$\sqrt{a}$	Square Root of the Semi-Major Axis of the satellite orbit
OMEGA	Ω_0	Longitude of Ascending Node of Orbit Plane at Weekly Epoch
io	i_0	Inclination Angle at Reference Time
omega	ω	Argument of Perigee
omega_dot	$\dot{\Omega}$	Rate of Right Ascension
i_dot	i	Rate of Inclination Angle
Cuc	C_{us}	Amplitude of the Sine Harmonic Correction Term to the Argument of Latitude
Cus	C_{uc}	Amplitude of the Cosine Harmonic Correction Term to the Argument of Latitude
Crs	C_{rs}	Amplitude of the Sine Harmonic Correction Term to the Orbit Radius
Crc	C_{rc}	Amplitude of the Cosine Harmonic Correction Term to the Orbit Radius
Cis	C_{is}	Amplitude of the Sine Harmonic Correction Term to the Angle of Inclination
Cic	C_{ic}	Amplitude of the Cosine Harmonic Correction Term to the Angle of Inclination
toe	t_{oe}	Reference Time Ephemeris
TOW	TOW	Time of the week
GPS_seconds	calculated	We calculate GPS_second based on UTC given in broadcast ephemeris

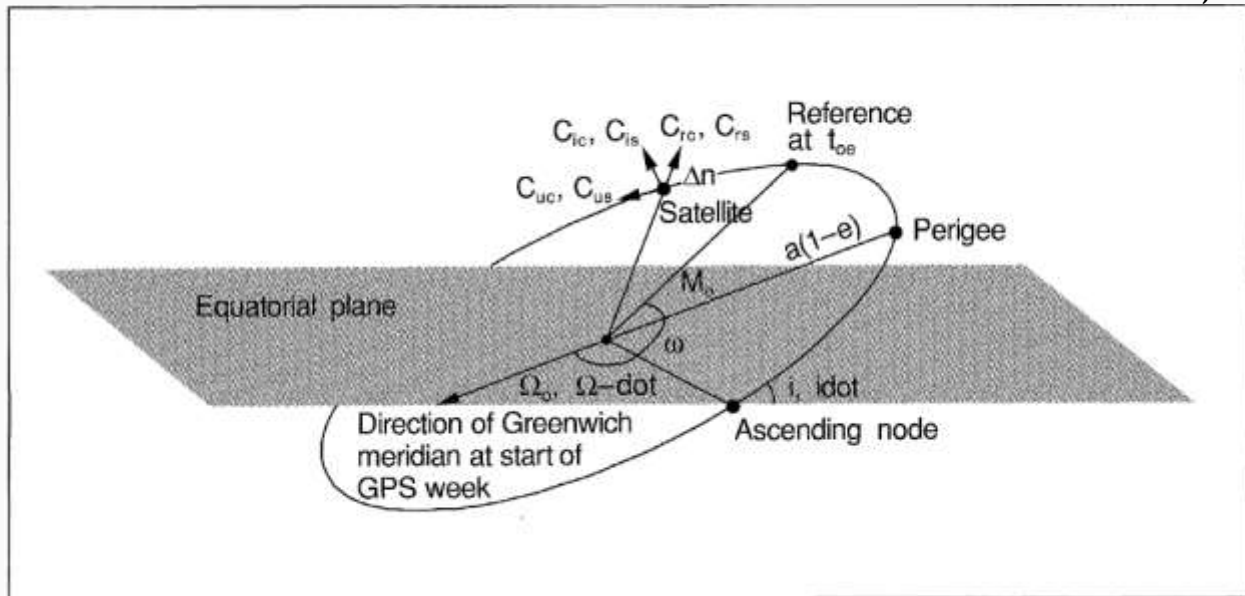Table 1. Definition of Keplerian Elements for Calculating Satellite Orbit

Figure 1. Parameters of the GPS Broadcast Ephemeris. (Langley, 1991)

Figure 1 graphically describes the essential parameters needed to calculate a satellite orbit.

Note that GPS time was zero at 0h 6-Jan-1980 and since it is not affected or altered by leap seconds, GPS is now ahead of UTC by 15 seconds. We convert UTC time given in broadcast ephemeris to GPS seconds. The reference time ephemeris is fraction of GPS week.

Almanac data is course orbital parameters for all SVs. Each SV broadcasts Almanac data for ALL SVs. This Almanac data is not very precise, and it is valid for up to several months useful for prediction of satellite orbits for planning tasks. On the other hand Ephemeris data is very precise orbital and clock correction for each SV, and it is necessary for precise positioning. It should be noted that each SV broadcasts only its own Ephemeris data. This data is only considered valid for about 30 minutes.

There are a number of formats currently available for GPS Almanac such as YUMA (used in this project), CEM and AGP.

GPS Almanac in YUMA format contains less parameters compare to GPS Ephemeris. For instance for PRN=1, GPS Almanac broadcasts:

```
ID:                      01
Health:                  000
Eccentricity:            0.4049301147E-002
Time of Applicability(s): 233472.0000
Orbital Inclination (rad):  0.9617783903
Rate of Right Ascen(r/s):  -0.8080336578E-008
SQRT(A)   (m 1/2):        5153.605469
Right Ascen at Week(rad): -0.1049829711E+001
```

```
Argument of Perigee(rad):    0.378966283
Mean Anom(rad):              0.2282708721E+001
Af0(s):                     -0.6675720215E-005
Af1(s/s):                    0.0000000000E+000
week:                        815
```

The current GPS Almanac in YUMA format can be downloaded at
http://www.navcen.uscg.gov/?pageName=gpsYuma.
We also bring the descriptions of the parameters in here (e.g., Montenbruck and Gill, 2012):

ID: PRN of the SV
Health: 000=usable
Eccentricity: This shows the amount of the orbit deviation from circular (orbit). It is the distance between the foci divided by the length of the semi-major axis (our orbits are very circular).
Time of Applicability: The number of seconds in the orbit when the almanac was generated. Kind of a time tag.
Orbital Inclination: The angle to which the SV orbit meets the equator (GPS is at approx. 55 degrees). Roughly, the SV's orbit will not rise above approx. 55 degrees latitude. The number is part of an equation: #= pi/180 = the true inclination.
Rate of Right Ascension: Rate of change in the measurement of the angle of right ascension as defined in the Right Ascension mnemonic.
SQRT(A) Square Root of Semi-Major Axis: This is defined as the measurement from the center of the orbit to either the point of apogee or the point of perigee.
Argument of Perigee: An angular measurement along the orbital path measured from the ascending node to the point of perigee, measured in the direction of the SV's motion.
Mean Anomaly: Angle (arc) traveled past the longitude of ascending node (value= 0-180 degrees or 0-negative 180 degrees). If the value exceeds 180 degrees, subtract 360 degrees to find the mean anomaly. When the SV has passed perigee and heading towards apogee, the mean anomaly is positive. After the point of apogee, the mean anomaly value will be negative to the point of perigee.
Af(0): SV clock bias in seconds
Af(1): SV clock Drift in seconds per seconds
week: GPS week (0000-1024), every 7 days since 22 Aug 1999

We bring the GLONASS Ephemeris parameters in Table 2. As it can be seen in Table 2, the positions (directly), velocity and accelerations of GLONASS satellites are provided rather than Keplerian parameters.

Code	Parameter	Definition
R	R	GLONASS slot number
SP_X	X	Satellite Position X (km) ITRF (ECEF)
SP_Y	Y	Satellite Position Y (km) ITRF
SP_Z	Z	Satellite Position Z (km) ITRF
X_v	$\dot{X}$	Satellite X Velocity (km/sec) ITRF

Y_v	$\dot{Y}$	Satellite Y Velocity (km/sec) ITRF
Z_v	$\dot{Z}$	Satellite Z Velocity (km/sec) ITRF
X_a	$\ddot{X}$	Satellite X Acceleration (km/sec^2) ITRF
Y_a	$\ddot{Y}$	Satellite Y Acceleration (km/sec^2) ITRF
Z_a	$\ddot{Z}$	Satellite Z Acceleration (km/sec^2) ITRF
GPS_seconds	Calculated	We calculate GPS seconds based on the given date and time

Table 2. Parameters given in GLONASS ephemeris to calculate satellite orbits

GLONASS Almanac in YUMA format is not currently available, however, other formats like AGL is available that its parameters are slightly different from GPS Almanac. The code to read GLONASS Almanac is brought in Appendix, but its use for the satellite orbit prediction is subject to future work. The YUMA format for GLONASS Almanac is described in page 345 in Diggelen (2009).

CALCULATION OF SATELLITE ORBITS

In this part, we briefly explain how to find satellite orbits.

GPS Satellite Orbit based on the broadcast Ephemeris : Once we obtained the Keplerian parameters from RINEX (Navigation) files, we use Table 3 (e.g., Hofmann, 2001) to calculate the GPS satellite orbits. Note that in order to find the eccentric anomaly in Table 3, we set $E_k = M_k$, and we iterate. Also, if $t_k < -302400$ (half week second), then we add 604800 to t_k, and if $t_k > 302400$, we subtract 604800 from t_k.

Constants and Formulas	Name
$\mu = 3.986005 \times 10^{14} meters^3/sec^2$	WGS-84 value of the earth's universal gravitational parameter for GPS user
$\dot{\Omega}_e = 7.2921151467 \times 10^{-6} rad/sec$	WGS-84 value of the earth's rotation rate
$A = (\sqrt{A})^2$	Semi-major axis
$n_0 = \sqrt{\dfrac{\mu}{A^3}}$	Computed mean motion (rad/sec)
$t_k = t - t_{oe}$	Time of ephemeris reference epoch
$n = n_0 + \Delta n$	Corrected mean motion
$M_k = M_0 + nt_k$	Mean Anomaly
$E_k = M_k + e\sin E_k$	Eccentric Anomaly
$v_k = tan^{-1} \left\{ \dfrac{(\sqrt{1-e^2}\sin E_k)}{(\cos E_k - e)} \right\}$	True Anomaly
$\Phi_k = v_k + \omega$	Argument of Latitude
$\delta u_k = c_{us}\sin 2\Phi_k + c_{uc}\cos 2\Phi_k$	Argument of Latitude correction
$\delta r_k = c_{rs}\sin 2\Phi_k + c_{rc}\cos 2\Phi_k$	Radius correction
$\delta i_k = c_{is}\sin 2\Phi_k + c_{ic}\cos 2\Phi_k$	Inclination correction
$u_k = \Phi_k + \delta u_k$	Corrected Argument of Latitude
$r_k = A(1 - e\cos E_k) + \delta r_k$	Corrected radius
$i_k = i_0 + \delta i_k + (IDOT)t_k$	Corrected Inclination
$x'_k = r_k\cos u_k$	x position of orbital plane
$y'_k = r_k\sin u_k$	y position of orbital plane
$\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e)t_k - \dot{\Omega}_e t_{oe}$	Corrected longitude of ascending node

$x_k = x'_k cos\Omega_k - y'_k cosi_k sin\Omega_k$	x-coordinate ECEF of Satellite
$y_k = x'_k sin\Omega_k + y'_k cosi_k cos\Omega_k$	y-coordinate ECEF of Satellite
$z_k = y'_k sini_k$	z-coordinate ECEF of Satellite

Table 3. Formulas for Calculating the Satellite Orbit

GPS Satellite Orbit based on Almanac (Yuma): After extracting the Keplerian parameters form Almanac files, we again use Table 3 to calculate the satellite orbits, and we set the ungiven parameters to zero.

$$\Delta n = IDOT = c_{us} = c_{uc} = c_{rs} = c_{rc} = c_{is} = c_{ic} = 0.$$

For converting UTC to GPS seconds we use the following:

$JD = \lfloor 365.25 \times year \rfloor + \lfloor 30.6001 \times (month + 1) \rfloor + day + hour + \frac{min}{60} + \frac{sec}{60} + 1720981.5,$

$GPS_{week} = \lfloor (JD - 2444244.5)/7 \rfloor,$

$GPS_{sec} = round(((\frac{JD-2444244.5}{7} - GPS_{week}) \times 604800)/.5) \times .5),$

where JD stands for Julian Day, and 2444.244.5 is JD in 00:00:00 on January 6th 1980. We use the above equations to calculate GPS seconds for both Ephemeris and Almanac.

GPS Satellite Orbit based on the broadcast Ephemeris: Since GLONASS ephemeris contains position, velocity and acceleration of GLONASS satellites in some given epochs, we use the following equations (from Newton's law) for other times within the start time and end time in the ephemeris file:

$X_t = 0.5 \times \ddot{X} \times (t - t_0)^2 + \dot{X} \times (t - t_0) + X_{t_0} ,$

$Y_t = 0.5 \times \ddot{Y} \times (t - t_0)^2 + \dot{Y} \times (t - t_0) + Y_{t_0} ,$

$Z_t = 0.5 \times \ddot{Z} \times (t - t_0)^2 + \dot{Z} \times (t - t_0) + Z_{t_0} ,$

where t_0 is a given epoch in the ephemeris file. Unlike GPS ephemeris, the GLONASS satellite positions will not be accurate for times exceeding the time range in GLONASS ephemeris because we do not have access to Keplerian parameters. Note that the above equations give reasonable positions only for times close to the given epochs in GLONASS ephemeris.

Figure 2 shows the GPS satellite positions for PRN 17 from a broadcast ephemeris on Jan 1th 2015. These positions were compared with the results obtained by Garrett and John, and the difference in the positions is about ± 1 (cf., Figure 3) which is due to clock corrections that is not applied in this project.
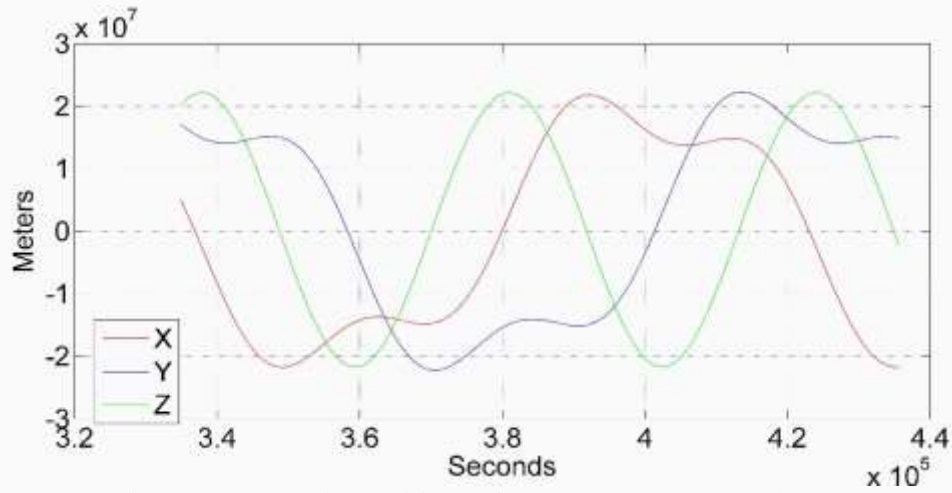

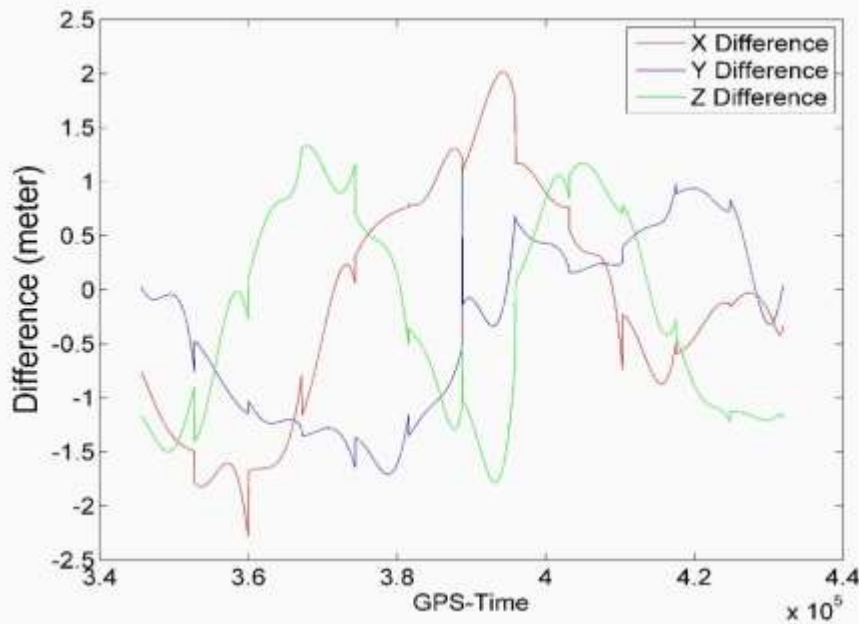Figure 2. X, Y and Z positions of GPS PRN 17 on Jan 1th 2015

Figure 3. Comparison of satellite positions in Figure 2 with results obtained by Garrett and John

In order to compare the areas that GPS and GLONASS satellites cover on a global map, we illustrate their orbits in Figure 4 using broadcast ephemeris files on April 3th 2015 from 12pm to 6 pm Eastern Time (UTC-4). The inclination of GPS and GLONASS orbits are approximately $55°$ and $64.8°$ respectively. This means that GLONASS satellites can cover areas toward north and south better than GPS satellites. There are currently 32 GPS satellites and 24 GLONASS satellites in the broadcast ephemeris. The satellite orbits were compared with the orbits obtained by the Trimble software http://www.trimble.com/gnssplanningonline. To generate Figure 4, we first normalize the satellite positions that are calculated in ECEF. Suppose that for a SV and an epoch, we have its position: $r = (X, Y, Z)$. We find the right ascension α and declination δ as follows:

$$\sin \delta = \frac{Z}{\|r\|} \rightarrow \delta = \sin^{-1}(\frac{Z}{\|r\|}) \quad \text{and} \quad \cos \alpha = \frac{X}{\|r\| \cos \delta} \rightarrow \alpha = d\cos^{-1}(X/(\|r\| \cos \delta)),$$

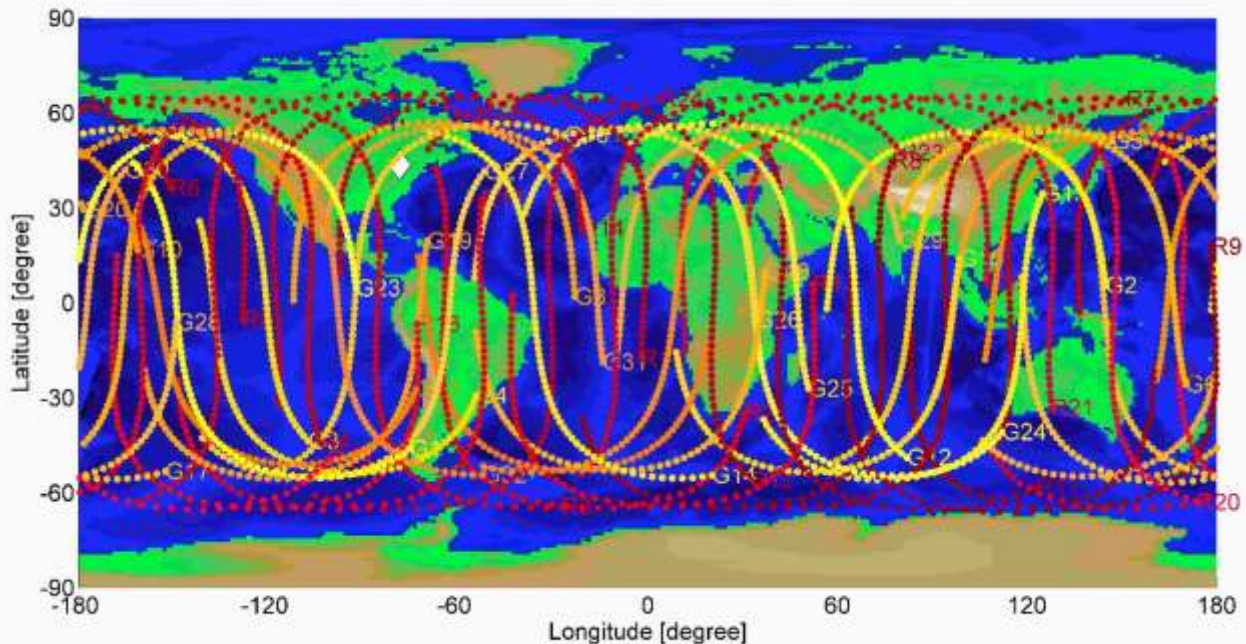where $d = 1$ if $Y > 0$ and $d = -1$ if $Y \leq 0$.

Figure 4. World view of GPS (yellowish G) and GLONASS (reddish R) satellites from 12 pm to 6 pm Eastern Time on Apr 3th 2015. Note that all satellites travel from the west to the east of the map.

CALCULATION OF LINE-OF-SIGHT

In order to determine a "line of sight" or vector in 3D space, it is required to define the start and end points. In this case the start point is the hypothetical location of a receiver on the surface of the Earth and the end point is the location of an individual GNSS satellite. Given that the broadcast locations of the GNSS satellites are provided in an ECEF three dimensional orthogonal coordinate system, this problem is reduced to a simple vector calculation once the approximate location of the receiver is known in this system. This relies on the assumption that both GLONASS and GPS satellite positions are broadcast in the same reference frame. It has been determined that the PZ90 and WGS84 mapping ellipsoids are equivalent to within a few meters of position and under 500 milliseconds of rotation (Boucher et al., 2001) making this a reasonable assumption given the scope of the project.

The first step is to transform Curvilinear to Cartesian Geodetic Coordinates. More precisely, let ϕ, λ and h be the latitude, longitude (in radian) and height (in meter from ellipsoid WGS84) of the user position. For ellipsoid WGS84, we have $f = \frac{a-b}{a} = 1/298.257223563$, and $a = 6378137$ m, where f is flattering of the ellipsoid, and a and b are the semi-major and semi-minor axes of the ellipsoid, respectively. The user position in Cartesian Geodetic Coordinates can be calculated from the following equations:

$$x = (N + h) \cos \phi \cos \lambda,$$
$$y = (N + h) \cos \phi \sin \lambda,$$
$$z = \left((1 - f^2)N + h \right) \sin \phi,$$

where $N = a/\sqrt{1 - f(2 - f)\sin^2\phi}$ is the radius of curvature in the prime vertical (e.g., Ghaderpour, 2014). The next step is to find the coordinate differences from the satellite position (X, Y, Z) to the user position or receiver (x, y, z). Since both positions are now in ITRF (ECEF), we have

$$U = \left(\frac{X-x}{R}, \frac{Y-y}{R}, \frac{Z-z}{R} \right),$$

where $R = \sqrt{(X - x)^2 + (Y - y)^2 + (Z - z)^2}$. Then, we write the axes of local coordinate frame:

$$n = (-\sin \phi \cos \lambda, -\sin \phi \sin \lambda, \cos \phi),$$
$$e = (-\sin \lambda, \cos \lambda, 0),$$
$$v = (\cos \phi \cos \lambda, \cos \phi \sin \lambda, \sin \phi),$$

where n, e and v stand for the north, east and vertical. Denote by E and Az, the elevation angle and the azimuth angle. We have (e.g., page 149 Hofmann, 1992 for the geometry):

$$U \cdot n = \cos E \cos Az,$$

$$U \cdot e = \cos E \sin Az \,,$$
$$U \cdot v = \sin E \,,$$

where "·" denotes the dot product. From the above equations, we can obtain E and Az (see below and the code in Appendix). The simplified and expanded equations can be found in page 46 Leick (2001) and are presented here for convenience:

$$Az = \tan^{-1}\left(\frac{-\sin \lambda \Delta x + \cos \lambda \Delta y}{-\sin \varphi \cos \lambda \Delta x - \sin \varphi \sin \lambda \Delta y + \cos \varphi \, \Delta z}\right),$$

$$E = \sin^{-1}\left(\frac{\cos \varphi \sin \lambda \, \Delta x + \cos \varphi \sin \lambda \, \Delta y + \sin \varphi \, \Delta z}{R}\right),$$

where $\Delta x = X - x, \ \Delta y = Y - y, \ \Delta z = Z - z$.

If we want to determine if a satellite is visible from a certain location, we need to determine if the elevation angle to that satellite is above some threshold angle which is typically 10 degrees. A threshold or mask once set is used to filter a group of line of sight calculations such that any with a vertical angle less than the provided threshold are deemed not visible. If the satellite is deemed not visible is not included in any of the various plots or in the calculation of DOP values.

We compare the elevation and azimuth angles calculated via our software with the Trimble software. To do so, we select the GPS satellite with PRN number 16 and the GLONASS satellite with slot number 11. We choose the time duration to be from 1pm to 5pm Eastern Time on April 16th 2015 in Toronto (Latitude: 43.7°, Longitude: -79.4°, Height: zero), and we choose a cut-off-angle 10°. We illustrate the elevation angles of these two satellites using Trimble and our software in Figure 5. We also show the elevation differences in Figure 6. The differences are within ±1°. There are two reasons for the elevation differences:

First, we use ephemeris files, whereas the Trimble software uses Almanac file (using Almanac file gives less accurate satellite positions than using broadcast ephemeris files). Second, the elevation (and the azimuth) angles are integer numbers in the Trimble software, whereas the elevation and azimuth angles calculated via our software are real numbers.
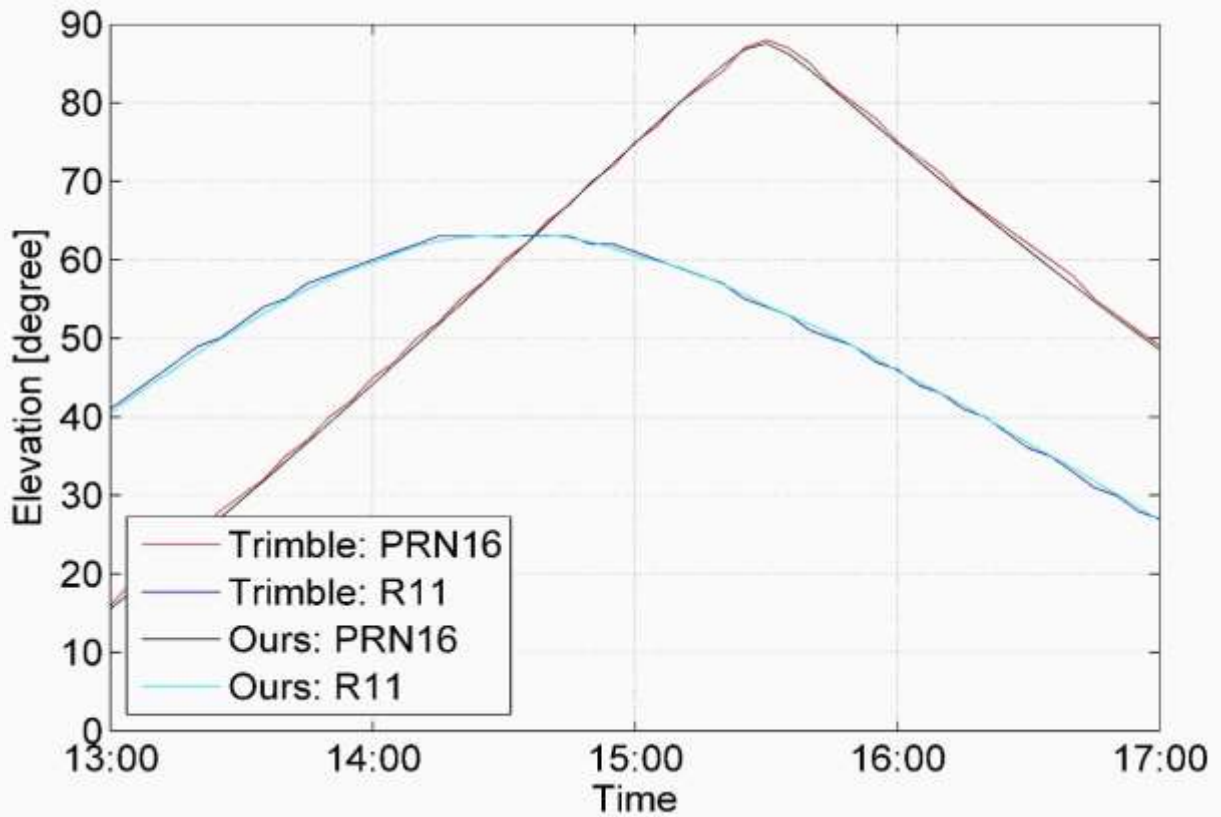
Figure 5. The elevation angles of GPS and GLONASS satellites in Toronto on Apr 16th 2015.
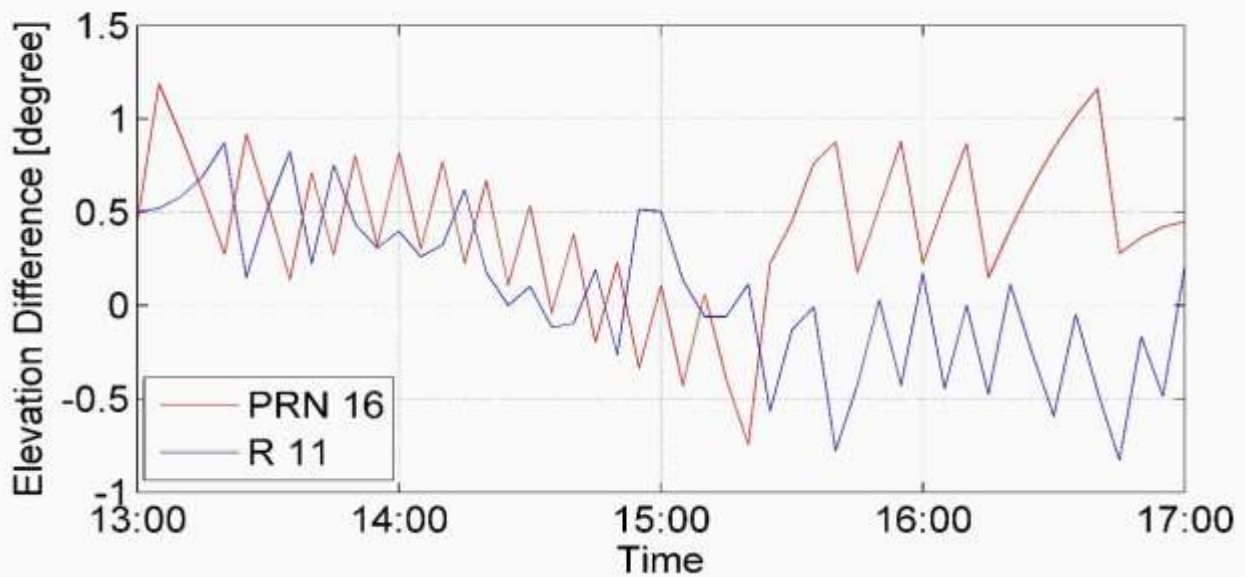

Figure 6. The elevation angle differences of GPS and GLONASS satellites in Toronto on Apr 16th 2015.

We also plot the azimuth differences obtained from the two software in Figure 7. The differences are within ±1° for the same reasons mentioned above. Note that the peak that appears for GPS satellite with PRN number 16 at time 15:25 in Figure 7 is that the elevation of the satellite at that time is close to 90° (cf., Figure 5), and so any small difference in satellite position from the Trimble software and our software results in a large azimuth angle difference.
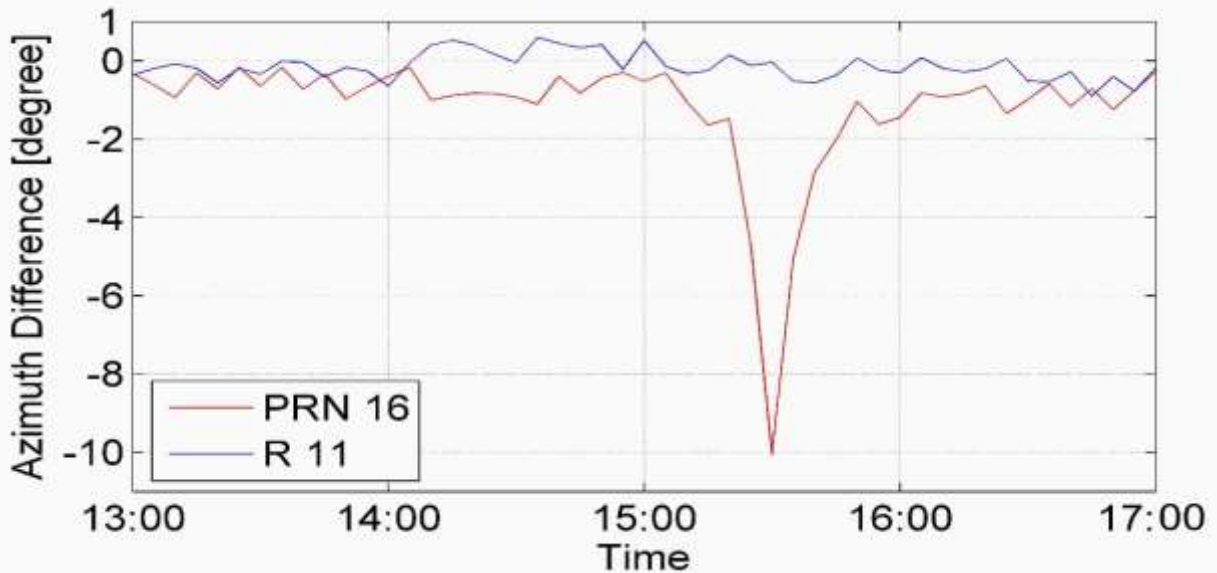

Figure 7. The azimuth angle differences of GPS and GLONASS satellites in Toronto on Apr 16th 2015. The peak for PRN 16 at 15:25 is for the sensitivity of the azimuth angles to a small change in satellite position when the elevation angle is close to 90° (cf., Figure 5).

DOP CALCULATIONS

For the purpose of positional measurement precisions, we need to calculate DOPs. When we found satellite positions, we need to find the receiver position (point positioning). Now the question that arises here is that how much we are confident for the position that we calculate.
The observation equation is given by (neglecting ionosphere, troposphere and other minor effects):

$$\rho = \sqrt{(X_i - \bar{x})^2 + (Y_i - \bar{y})^2 + (Z_i - \bar{z})^2} + c(dt_r - dt_s),$$

where (X_i, Y_i, Z_i) is the satellite position in ECEF, $(\bar{x}, \bar{y}, \bar{z})$ the user position being estimated (unknown), c speed of light, dt_r reciver clock bias (unknown) and dt_s satellite clock bias.
Solving the above non-parametric model by the least-squares method (after linearization), we obtain:

$$\delta = (A^T P A)^{-1} A^T P w, \qquad (1)$$

where $P = I$, w is the misclosure vector, and $\hat{x} = (x, y, z, dt_{r0}) + \delta$ will be next candidate for iteration. A is the deign matrix:

$$A = \begin{bmatrix} \dfrac{X_1 - x}{R_1}, \dfrac{Y_1 - y}{R_1}, \dfrac{Z_1 - z}{R_1}, c \\ \dfrac{X_2 - x}{R_2}, \dfrac{Y_2 - y}{R_2}, \dfrac{Z_2 - z}{R_2}, c \\ \dfrac{X_3 - x}{R_3}, \dfrac{Y_3 - y}{R_3}, \dfrac{Z_3 - z}{R_3}, c \\ \dfrac{X_4 - x}{R_4}, \dfrac{Y_4 - y}{R_4}, \dfrac{Z_4 - z}{R_4}, c \\ \cdots \\ \cdots \end{bmatrix}$$

with $R_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2}$. Note that (x, y, z) in this project is the converted user position (See GUI description) to Cartesian Geodetic Coordinates which can be seen as initial position for point positioning in which we verify DOPs (see below).

The first three columns of matrix A are the components of the unit vector

$$U_i = (\frac{X_i - x}{R_i}, \frac{Y_i - y}{R_i}, \frac{Z_i - z}{R_i}),$$

If the elements of the forth column are all c, then TDOP (see below) will always be one, but if the forth column be -1, then TDOP will be calculated properly.

Using the Covariance Law for Equation (1), one obtains

$$C_\delta = N^{-1} = (A^T A)^{-1}$$

The combinations of the diagonal elements of C_δ gives us DOPs. However, calculating DOPs in this way is in ECEF. When the topocentric local coordinate system with its axes along the local north, east and vertical is used (left handed), we must first convert the position correction δ to the new coordinate system $Q = R\delta$, where

$$R = \begin{bmatrix} -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi & 0 \\ -\sin\lambda & \cos\lambda & 0 & 0 \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Now using the Covariance Law, we obtain $C = C_Q = RC_\delta R^T$. Denote the diagonal elements of C by $C_{11}, C_{22}, C_{33}, C_{44}$. The DOPs are defined as follows (e.g., Hofmann et. al., 1992):

$$\begin{aligned} \text{PDOP} &= \sqrt{C_{11} + C_{22} + C_{33}}\,, \\ \text{GDOP} &= \sqrt{C_{11} + C_{22} + C_{33} + C_{44}}\,, \\ \text{HDOP} &= \sqrt{C_{11} + C_{22}}\,, \\ \text{VDOP} &= \sqrt{C_{33}}\,, \\ \text{TDOP} &= \sqrt{C_{44}}\,. \end{aligned}$$

To understand the concept of DOPs, for instance GDOP, consider a square pyramid is formed by lines joining four satellites with the receiver at the tip of the pyramid. When the volume of this pyramid gets larger, GDOP will decrease (better), and vice versa. If the values of DOPs are less than 1, then we have the highest confidence for precision (Ideal). The values between 1 and 2 are excellent. The values from 2 to 5 are good, 5 to 10 are moderate, 10 to 15 are fair, and >15 are poor meaning measurements are inaccurate and should be discarded.

We compare the DOP values with Trimble software for all GPS satellites except PRN 8 and 26. DOP values were calculated on April 16th, 2015 from time 1pm to 5pm Eastern Time in Toronto with cut-off-angle 10° and zero elevation. We show the DOP values calculated with our software using RINEX Navigation file and calculated with Trimble software using Almanac in Figure 8. The difference in DOP values between our software and Trimble software is shown in Figure 9. The reason for the peaks (large DOP difference) appearing in this figure is that some satellites set or rise a little earlier (from the cut-off-angle) in our software compare to Trimble software because we use the RINEX ephemeris file, and we do not round the DOP values to two decimal numbers as Trimble does (our DOP values are more accurate than the ones calculated via Trimble software). For instance the GPS satellite with PRN number 29 at time 14:55 has 10º elevation, and it is considered in the calculation of DOPs in our software (resulting in better DOP

values), whereas this satellite is not considered in the calculation of DOPs in Trimble software. For other peaks in the figure, we have similar reasons. Note that the time increment for both software is 5 minutes. The differences in DOP values between Trimble and our software (by the assumption of the same number of satellites availability) is ±0.05.
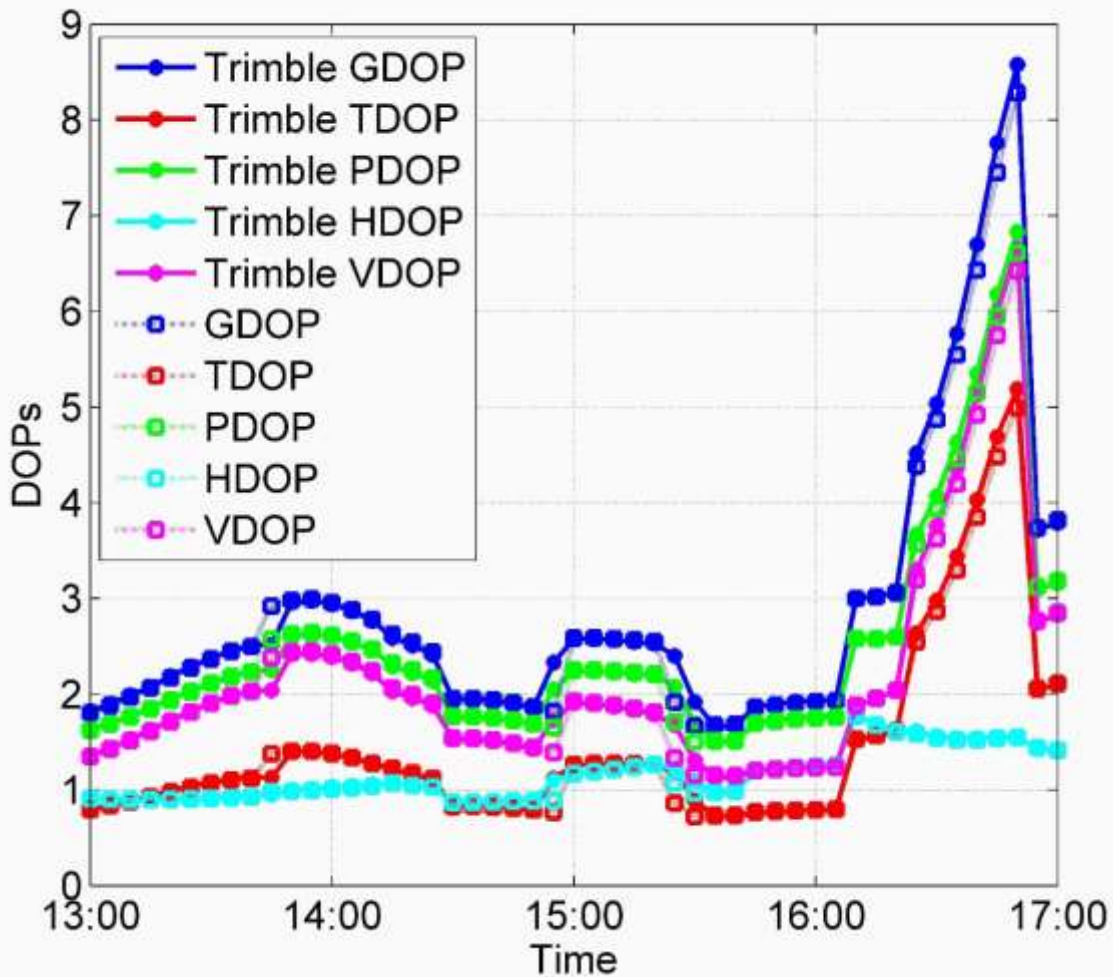

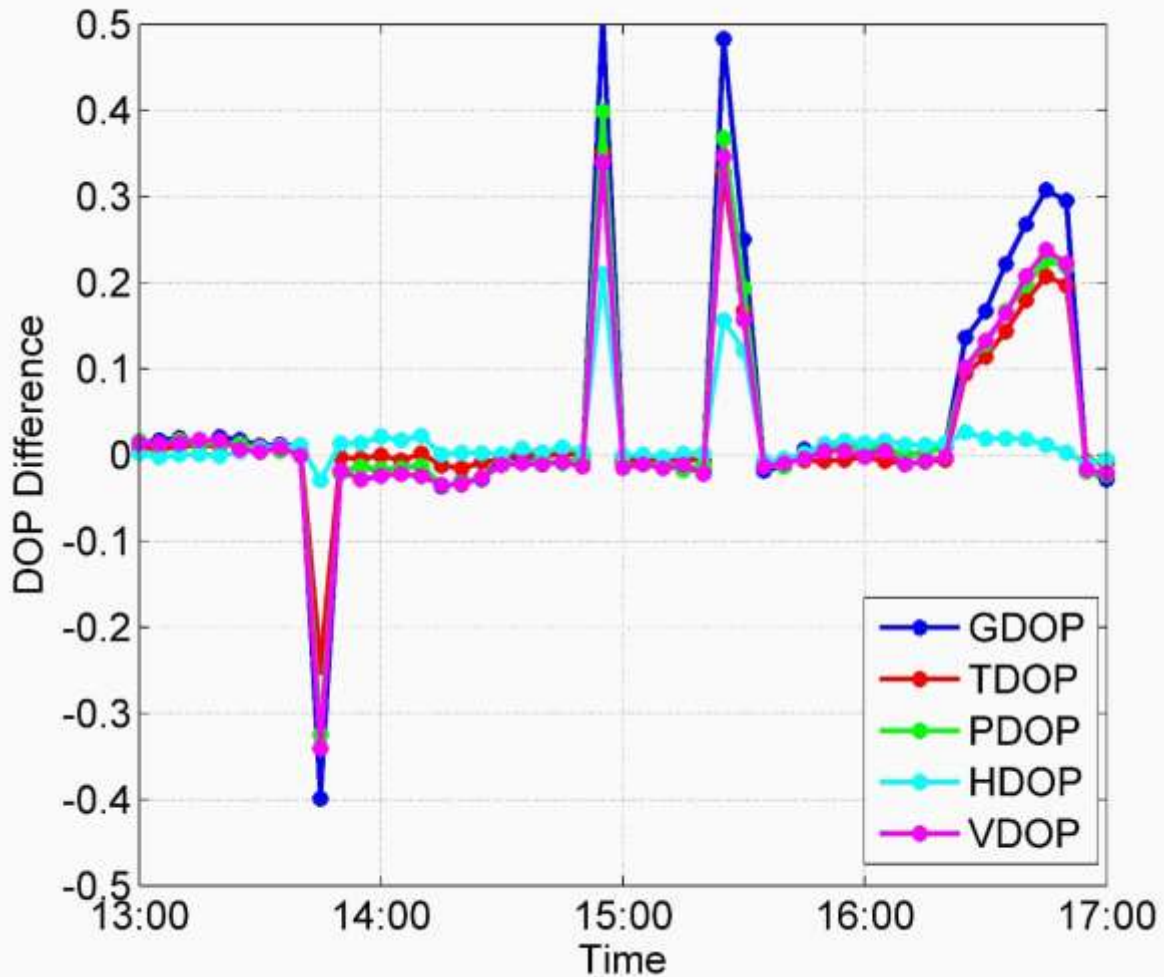Figure 8. The DOP values calculated from the Trimble software and ours in Toronto on Apr 16th 2015.

Figure 9. DOP value differences for GPS satellites (All PRNs except PRN 8 and PRN 26) calculated from our software and the Trimble software in Toronto on Apr 16th 2015. The peaks in this plot is due to visibility of satellites above the cut off angle 10°.

For creating DOP values for a static global map in this project, DOPs are calculated by considering a cut-off-angle and a given epoch for positions on the Earth with 1-10 degrees (selected by the user) incrementing in latitude and longitude.

GENERATION OF SATELLITE SKY PLOT

After the calculation of Azimuth (*Az*) and Elevation (*E*) for a SV and a given epoch (in Line-of-Sight) and using a cut-off-angle (*Mask*), we use the following polar equations to generate a sky plot:

$$\alpha = r \cos\theta, \qquad \beta = r \sin\theta,$$

where $r = 90 - E \times 180/\pi$, $\theta = Az$ with the condition $0 \leq r \leq 90 - Mask$. Figure 10 shows a sky plot for both all GPS and GLONASS using the broadcast ephemeris on April 3th 2015. It can be seen that GLONASS satellites cover north part of Toronto better than GPS satellites, however, the number of GPS satellites are more than the number of GLONASS satellites (cf., Figure 11). We illustrate the elevation of satellites above 10° horizon in Toronto in Figure 12.
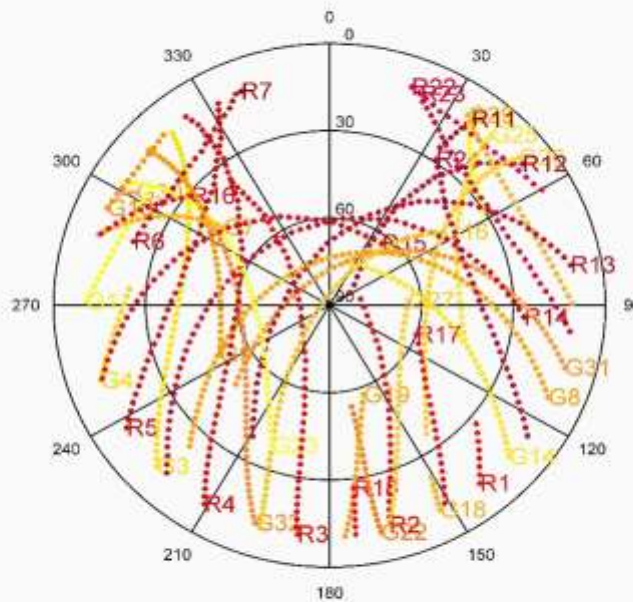

Figure 10. Sky Plot for GPS (G) and GLONASS (R) satellites in Toronto from 12 to 6 pm on Apr 3th 2015
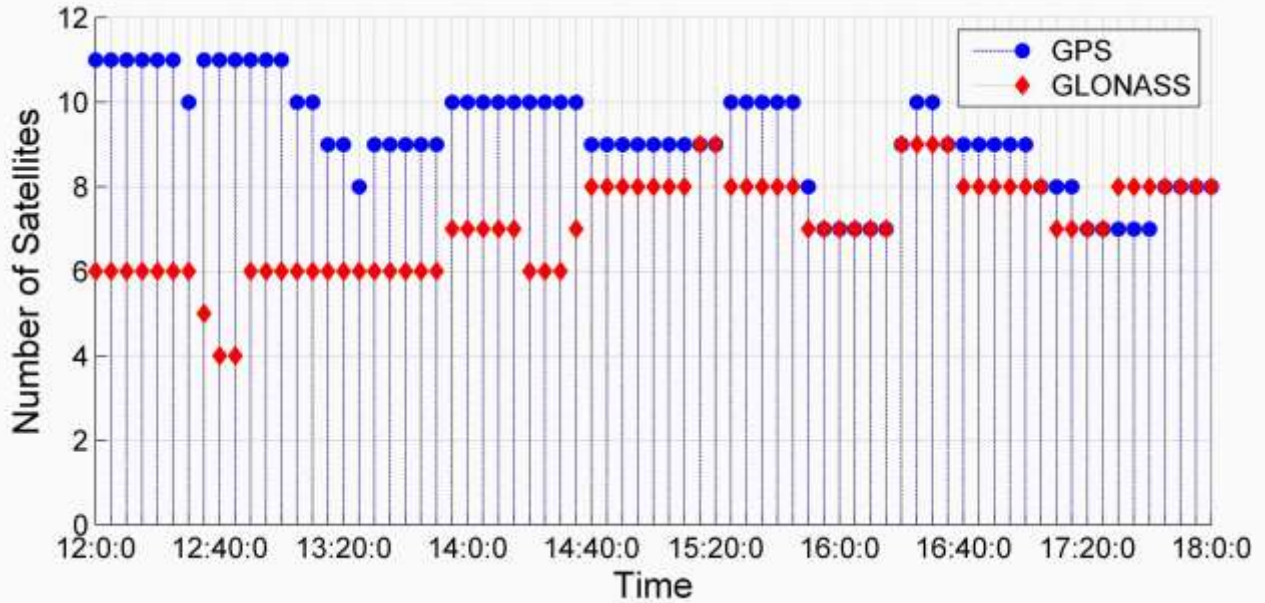
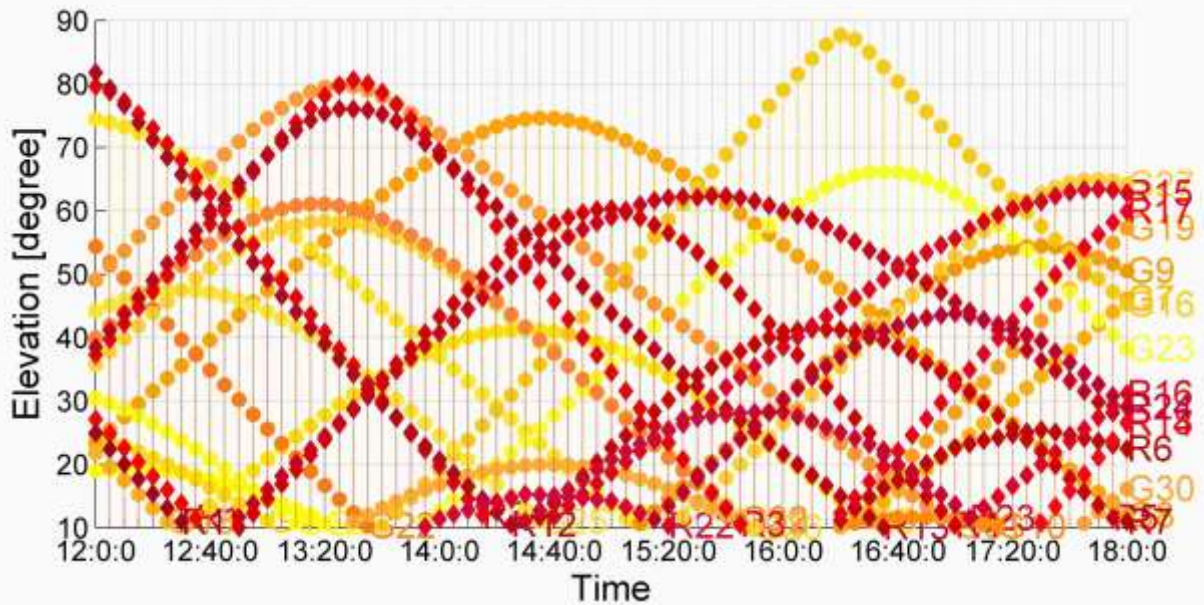Figure 11. Number of visible satellites on Apr 3th 2015 in Toronto (Eastern Time)

Figure 12. Elevation angles of GPS (G) and GLONASS (R) satellites from 12 to 6 pm Eastern Time on Apr 3th 2015 in Toronto.

In order to see the number of visible satellite above some elevation mask on the entire Earth, we use a Mercator map and for a given epoch, we count the number of satellites, so we need the elevation angles of GPS and GLONASS satellites calculated in Section (Calculation of Line-of – Sight). Then we count those satellites that are above a given elevation mask (cut-off-angle). We show the number of visible satellites in Toronto at noon (Eastern Time) with $10^{°}$ elevation mask in Figure 13. It can be seen that the minimum number of visible satellites in that time is 11. However, having a large number of satellites does not necessary mean having good DOPs. In order to see DOPs, we illustrate VDOP in Figure 14. Considering both GPS and GLONASS satellites decreases DOPs, and for $10^{°}$ eleveation mask their values are less than 2 which is excellent for positioning. We also show PDOPs for four different times 12 pm, 12:30 pm, 1pm and 1:30 pm in Figures 15 and 16. The values of PDOPs (and other DOPs) change from time to time, but they are less than 2.5. DOP values for other times were checked (not shown here), and they were all less than 3 for all positions on the globe. We emphasize that we put into consideration the elevation mask for all positions on the global map, and position increment is $1^{°}$ for these figures along latitude and longitude.
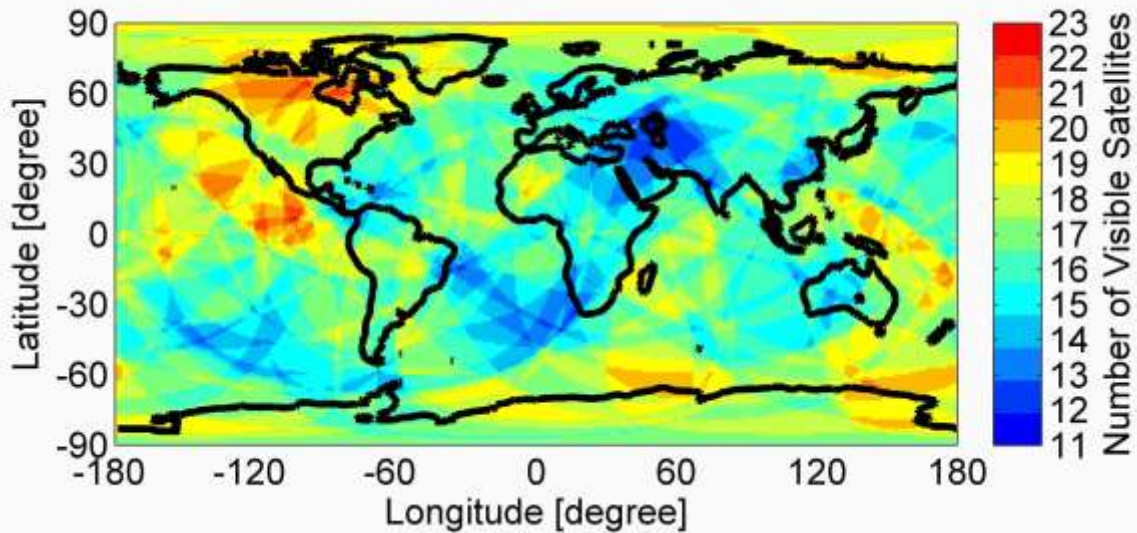

Figure 13. Number of visible GPS and GLONASS satellites above 10° local horizon at noon (Eastern Time) on Apr 3th 2015
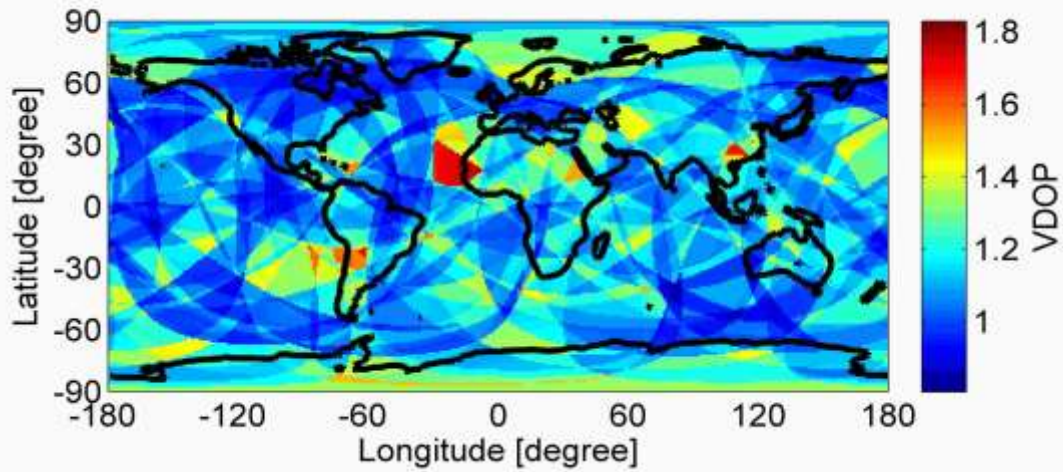
Figure 14. VDOP considering both GPS and GLONASS satellites at noon in Toronto on Apr 3th 2015
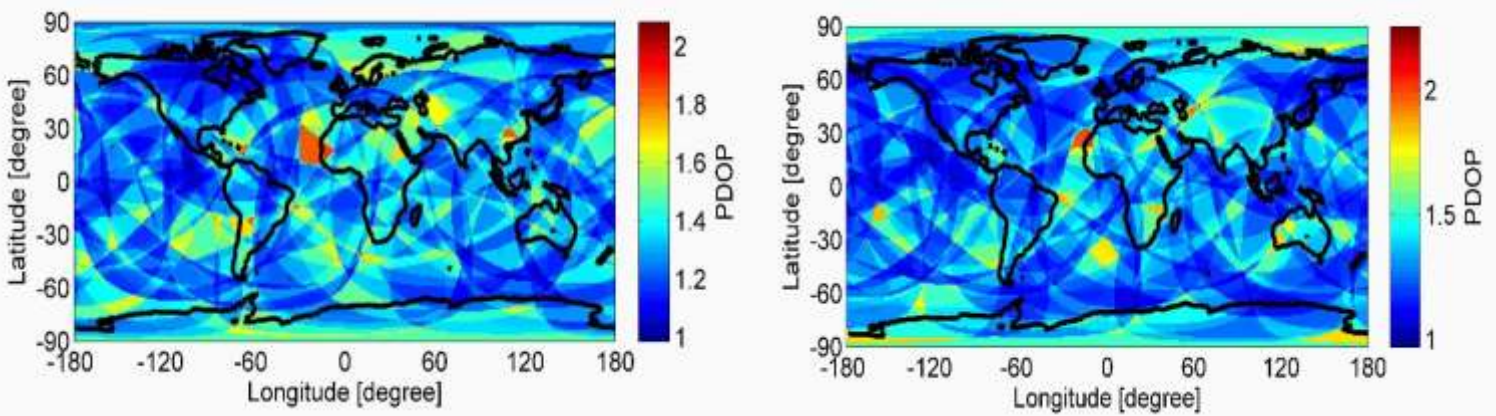

Figure 15. PDOP for all GPS and GLONASS satellites at 12 pm (left panel) and 12:30 pm (right panel) on Apr 3th 2015
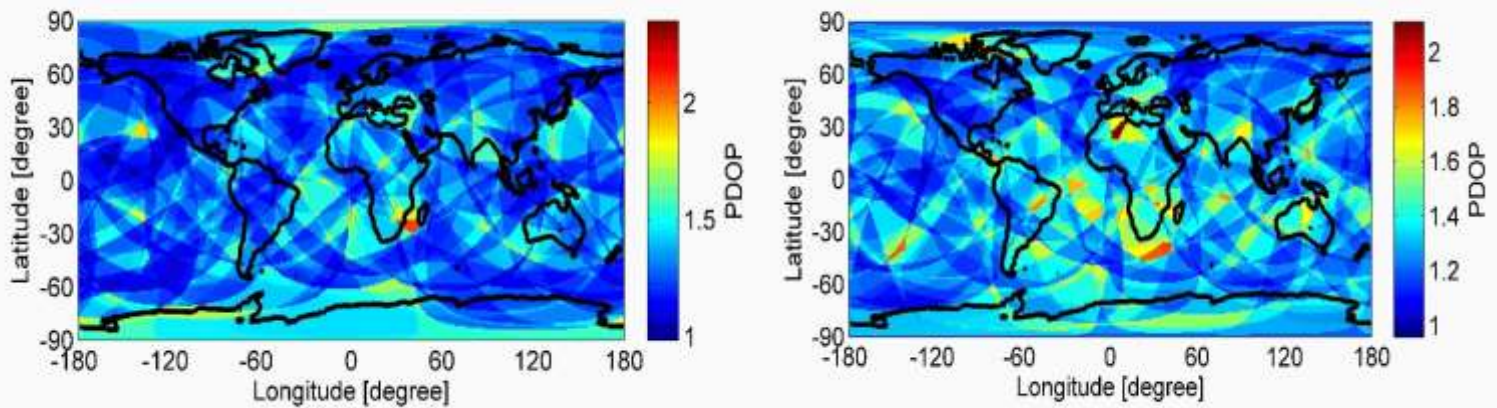

Figure 16. PDOPs for all GPS and GLONASS satellites at 1 pm (left panel) and 1:30 pm (right panel) on Apr 3th 2015

The GUI contains following buttons which also include the instructions:

Menu

1) File : GPS, GLONASS (use Ephemeris file option or Almanac file)
2) Satellites: GPS 32 PRNs and GLONASS 24 SVs and Cut off Angle.
3) User Position: Latitude, Longitude, Height (above the ellipsoid WGS84)
4) Time Period: Date, Start and End Time, Time increment for satellite positions, Standard Time Zone (Note that Day lights are not considered) and Time Label for Plots (GPS or Local time)
5) DOPs: PDOP, VDOP, HDOP, GDOP, TDOP for a static global map that uses the Start Time in Time Period. It also has Visibility (Map) button which generates the number of selected visible satellites that can be observed above a given Cut off Angle for positions on the Earth that are incremented x degrees, $1 \le x \le 10$, for both latitude and longitude. Note that DOPs create a static global map.

Toolbar

1) **Apply:** After entering the data in menu press this button. The GUI is created smart that guide beginners even if they start with entering wrong data or wrong button though.
2) Stop plotting: this button is used when the user wants to pause plotting at a particular time for investigation. Note that Sky Plot, World View and Elevations will be generated kinematically.
3) Elevation: for a given user position and a given time period. It uses all the information in Menu.
4) Visibility: for a given user position and a given time period. It uses all the information in Menu.
5) Sky Plot: for a given user position and a given time period. It uses all the information in Menu. It also shows the running time for the positions being displayed.
6) World View: for a given user position and a given time period. It uses all the information in Menu. It also shows the running time for the positions being displayed
7) DOP: This is different from DOPs in Menu. It plots DOPs for a given user position and a given time period. It uses all the information in Menu. If DOPs are greater 15 or the matrix of normal equation **N** is singular, then the value of DOPs will be set to 15. The latter is also considered in DOPs in Menu (number 5).

CONCLUSIONS AND FUTURE WORK

In this project, the various formats of Broadcast Ephemeris (RINEX Navigation) files was tested successfully as well as Almanac (YUMA, CEM, AGL). The GPS and GLONASS satellites positions are obtained and checked by the existing software from Trimble. The line-of-sight and DOPs are successfully implemented. A static global map with a number of satellites in view is created as well as a static global map for all DOPs values for a given time and elevation mask. A GUI containing all the above tasks is generated, and it operates intelligently in such a way that it guides the beginners to get the solution.

In future work, the DOPs values will be calculated based on given obstructions (such as buildings). Relative DOPs will be also calculated (with respect to the user position, input). Almanac for GLONASS will be added to the GUI. A static global map for the ionosphere and for a given time (period) will be also added to the GUI. The current GUI contains only GPS and GLONASS satellites. It is planned to add Galileo and BeiDou to the GUI. Including these to the GUI, makes the GUI a great tool for GNSS planning purposes.

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APPENDIX:
FLOWCHART OF COMPUTER CODE