

# Solutionbank C2

## Edexcel Modular Mathematics for AS and A-Level

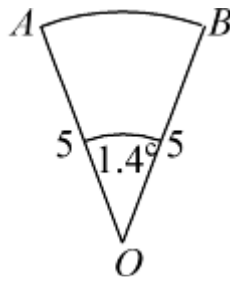
### Practice paper Exercise 1, Question 1

#### Question:

The sector  $AOB$  is removed from a circle of radius 5 cm.  
The  $\angle AOB$  is 1.4 radians and  $OA = OB$ .

- (a) Find the perimeter of the sector  $AOB$ . (3)  
(b) Find the area of sector  $AOB$ . (2)

#### Solution:



(a)

$$\begin{aligned}\text{Arc length} &= r\theta = 5 \times 1.4 = 7 \text{ cm} \\ \text{Perimeter} &= 10 + \text{Arc} = 17 \text{ cm}\end{aligned}$$

(b) Sector area =  $\frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times 1.4 = 17.5 \text{ cm}^2$

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## Edexcel Modular Mathematics for AS and A-Level

### Practice paper Exercise 1, Question 2

#### Question:

Given that  $\log_2 x = p$ :

- (a) Find  $\log_2 (8x^2)$  in terms of  $p$ . (4)
- (b) Given also that  $p = 5$ , find the value of  $x$ . (2)

#### Solution:

(a)  $\log_2 x = p$

$$\log_2 (8x^2) = \log_2 8 + \log_2 x^2 = 3 + 2 \log_2 x = 3 + 2p$$

(b)  $\log_2 x = 5 \Rightarrow x = 2^5 \Rightarrow x = 32$

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### Practice paper Exercise 1, Question 3

#### Question:

- (a) Find the value of the constant  $a$  so that  $(x - 3)$  is a factor of  $x^3 - ax - 6$ . (3)
- (b) Using this value of  $a$ , factorise  $x^3 - ax - 6$  completely. (4)

#### Solution:

(a) Let  $f(x) = x^3 - ax - 6$   
If  $(x - 3)$  is a factor then  $f(3) = 0$   
i.e.  $0 = 27 - 3a - 6$   
So  $3a = 21 \Rightarrow a = 7$

(b)  $x^3 - 7x - 6$  has  $(x - 3)$  as a factor, so  
 $x^3 - 7x - 6 = (x - 3)(x^2 + 3x + 2) = (x - 3)(x + 2)(x + 1)$

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### Practice paper Exercise 1, Question 4

#### Question:

(a) Find the coefficient of  $x^{11}$  and the coefficient of  $x^{12}$  in the binomial expansion of  $(2 + x)^{15}$ . (4)

The coefficient of  $x^{11}$  and the coefficient of  $x^{12}$  in the binomial expansion of  $(2 + kx)^{15}$  are equal.

(b) Find the value of the constant  $k$ . (3)

#### Solution:

$$(a) (2 + x)^{15} = \dots \binom{15}{11} 2^4 x^{11} + \binom{15}{12} 2^3 x^{12} + \dots$$

$$\text{Coefficient of } x^{11} = \binom{15}{11} \times 16 = 1365 \times 16 = 21\,840$$

$$\text{Coefficient of } x^{12} = \binom{15}{12} \times 8 = 455 \times 8 = 3640$$

$$(b) 21\,840k^{11} = 3640k^{12}$$

$$\text{So } k = \frac{21\,840}{3640}$$

$$\text{i.e. } k = 6$$

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## Edexcel Modular Mathematics for AS and A-Level

### Practice paper Exercise 1, Question 5

#### Question:

(a) Prove that:

$$\frac{\cos^2 \theta}{\sin \theta + \sin^2 \theta} \equiv \frac{1 - \sin \theta}{\sin \theta}, 0 < \theta < 180^\circ. \quad (4)$$

(b) Hence, or otherwise, solve the following equation for  $0 < \theta < 180^\circ$ :

$$\frac{\cos^2 \theta}{\sin \theta + \sin^2 \theta} = 2$$

Give your answers to the nearest degree. (4)

#### Solution:

$$\begin{aligned} \text{(a) LHS} &= \frac{\cos^2 \theta}{\sin \theta + \sin^2 \theta} \\ &= \frac{1 - \sin^2 \theta}{\sin \theta + \sin^2 \theta} \text{ (using } \sin^2 \theta + \cos^2 \theta \equiv 1) \\ &= \frac{(1 - \sin \theta)(1 + \sin \theta)}{\sin \theta(1 + \sin \theta)} \text{ (factorising)} \\ &= \frac{1 - \sin \theta}{\sin \theta} \text{ (cancelling } [1 + \sin \theta] \text{)} \end{aligned}$$

= RHS

$$\begin{aligned} \text{(b) } 2 &= \frac{\cos^2 \theta}{\sin \theta + \sin^2 \theta} \\ \Rightarrow 2 &= \frac{1 - \sin \theta}{\sin \theta} \\ \Rightarrow 2 \sin \theta &= 1 - \sin \theta \text{ (can multiply by } \sin \theta \because 0 < \theta < 180) \\ \Rightarrow 3 \sin \theta &= 1 \\ \Rightarrow \sin \theta &= \frac{1}{3} \end{aligned}$$

So  $\theta = 19.47 \dots^\circ, 160.5 \dots^\circ = 19^\circ, 161^\circ$  (to nearest degree)

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### Practice paper

#### Exercise 1, Question 6

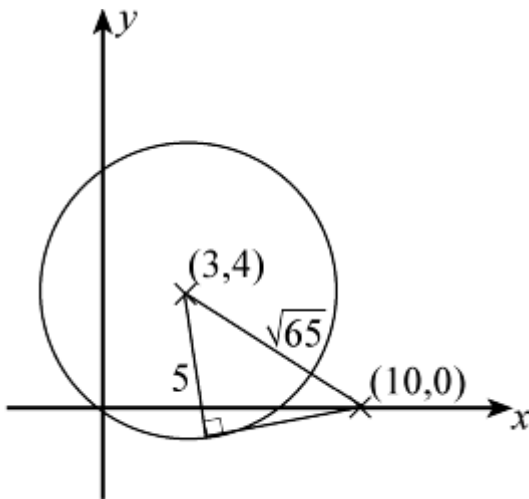
#### Question:

- (a) Show that the centre of the circle with equation  $x^2 + y^2 = 6x + 8y$  is  $(3, 4)$  and find the radius of the circle. (5)
- (b) Find the exact length of the tangents from the point  $(10, 0)$  to the circle. (4)

#### Solution:

$$\begin{aligned} \text{(a) } x^2 - 6x + y^2 - 8y &= 0 \\ \Rightarrow (x - 3)^2 + (y - 4)^2 &= 9 + 16 \\ \text{i.e. } (x - 3)^2 + (y - 4)^2 &= 5^2 \\ \text{Centre } (3, 4), \text{ radius } &5 \end{aligned}$$

$$\text{(b) Distance from } (3, 4) \text{ to } (10, 0) = \sqrt{7^2 + 4^2} = \sqrt{65}$$



$$\text{Length of tangent} = \sqrt{\sqrt{65}^2 - 5^2} = \sqrt{40} = 2\sqrt{10}$$

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## Edexcel Modular Mathematics for AS and A-Level

### Practice paper

#### Exercise 1, Question 7

#### Question:

A father promises his daughter an eternal gift on her birthday. On day 1 she receives £75 and each following day she receives  $\frac{2}{3}$  of the amount given to her the day before. The father promises that this will go on for ever.

- (a) Show that after 2 days the daughter will have received £125. (2)
- (b) Find how much money the father should set aside to ensure that he can cover the cost of the gift. (3)  
After  $k$  days the total amount of money that the daughter will have received exceeds £200.
- (c) Find the smallest value of  $k$ . (5)

#### Solution:

(a) Day 1 = £ 75, day 2 = £ 50, total = £ 125

(b)  $a = 75$ ,  $r = \frac{2}{3}$ —geometric series

$$S_{\infty} = \frac{a}{1-r} = \frac{75}{1-\frac{2}{3}}$$

Amount required = £ 225

$$(c) S_k = \frac{a(1-r^k)}{1-r}$$

$$\text{Require } \frac{75 \left[ 1 - \left( \frac{2}{3} \right)^k \right]}{1 - \frac{2}{3}} > 200$$

$$\text{i.e. } 225 \left[ 1 - \left( \frac{2}{3} \right)^k \right] > 200$$

$$\Rightarrow 1 - \left( \frac{2}{3} \right)^k > \frac{8}{9}$$

$$\Rightarrow \frac{1}{9} > \left( \frac{2}{3} \right)^k$$

$$\text{Take logs: } \log \left( \frac{1}{9} \right) > k \log \left( \frac{2}{3} \right)$$

Since  $\log \left( \frac{2}{3} \right)$  is negative, when we divide by this the inequality will change around.

$$\text{So } k > \frac{\log \left( \frac{1}{9} \right)}{\log \left( \frac{2}{3} \right)}$$

i.e.  $k > 5.419 \dots$

So need  $k = 6$

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### Practice paper Exercise 1, Question 8

#### Question:

Given  $I = \int_1^3 \left( \frac{1}{x^2} + 3\sqrt{x} \right) dx$ :

(a) Use the trapezium rule with the table below to estimate  $I$  to 3 significant figures. (4)

$x$	1	1.5	2	2.5	3
$y$	4	4.119	4.493	4.903	5.307

(b) Find the exact value of  $I$ . (4)

(c) Calculate, to 1 significant figure, the percentage error incurred by using the trapezium rule as in part (a) to estimate  $I$ . (2)

#### Solution:

(a)  $h = 0.5$

$$\begin{aligned}
 I &\approx \frac{0.5}{2} \left[ 4 + 2 \left( 4.119 + 4.493 + 4.903 \right) + 5.307 \right] \\
 &= \frac{1}{4} \left[ 36.337 \right] \\
 &= 9.08425
 \end{aligned}$$

(b)  $I = \int_1^3 \left( x^{-2} + 3x^{\frac{1}{2}} \right) dx$

$$\begin{aligned}
 &= \left[ \frac{x^{-1}}{-1} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^3 \\
 &= \left[ -\frac{1}{x} + 2x^{\frac{3}{2}} \right]_1^3 \\
 &= \left( -\frac{1}{3} + 2 \times 3\sqrt{3} \right) - \left( -1 + 2 \right) \\
 &= 6\sqrt{3} - \frac{4}{3}
 \end{aligned}$$

(c) Percentage error =  $\frac{|6\sqrt{3} - \frac{4}{3} - 9.08425|}{6\sqrt{3} - \frac{4}{3}} \times 100 = 0.279 \dots \% = 0.3 \%$

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### Practice paper

#### Exercise 1, Question 9

#### Question:

The curve  $C$  has equation  $y = 6x^{\frac{7}{3}} - 7x^2 + 4$ .

(a) Find  $\frac{dy}{dx}$ . (2)

(b) Find  $\frac{d^2y}{dx^2}$ . (2)

(c) Use your answers to parts (a) and (b) to find the coordinates of the stationary points on  $C$  and determine their nature. (9)

#### Solution:

(a)  $y = 6x^{\frac{7}{3}} - 7x^2 + 4$

$$\frac{dy}{dx} = 6 \times \frac{7}{3} x^{\frac{4}{3}} - 14x$$

$$\frac{dy}{dx} = 14x^{\frac{4}{3}} - 14x$$

(b)  $\frac{d^2y}{dx^2} = \frac{56}{3} x^{\frac{1}{3}} - 14$

(c)  $\frac{dy}{dx} = 0 \Rightarrow x^{\frac{4}{3}} - x = 0 \Rightarrow x \left( x^{\frac{1}{3}} - 1 \right) = 0$

So  $x = 0$  or  $1$

$x = 0 \Rightarrow \frac{d^2y}{dx^2} = -14 < 0 \therefore (0, 4)$  is a maximum

$x = 1 \Rightarrow \frac{d^2y}{dx^2} = \frac{56}{3} - 14 > 0 \therefore (1, 3)$  is a minimum